Lecture 3:
Access Methods

Logistics

- Turing Point quiz details

Recap

- Storage Management
- Smart Pointers and Pages
- Slotted Page
- Buffer Management
- 2Q Policy

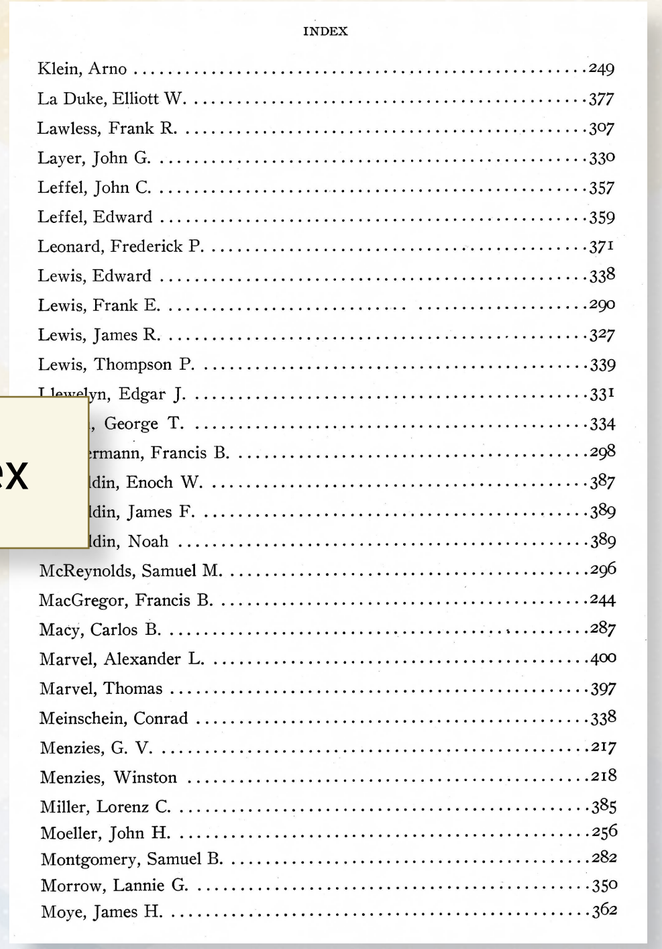
Lecture Overview

- Indexing
- Hash Table
- Double Hashing
- Range Query
- B+Tree
- Trie
- RTree

Index

Indexing in Database Systems

Book Index



INDEX

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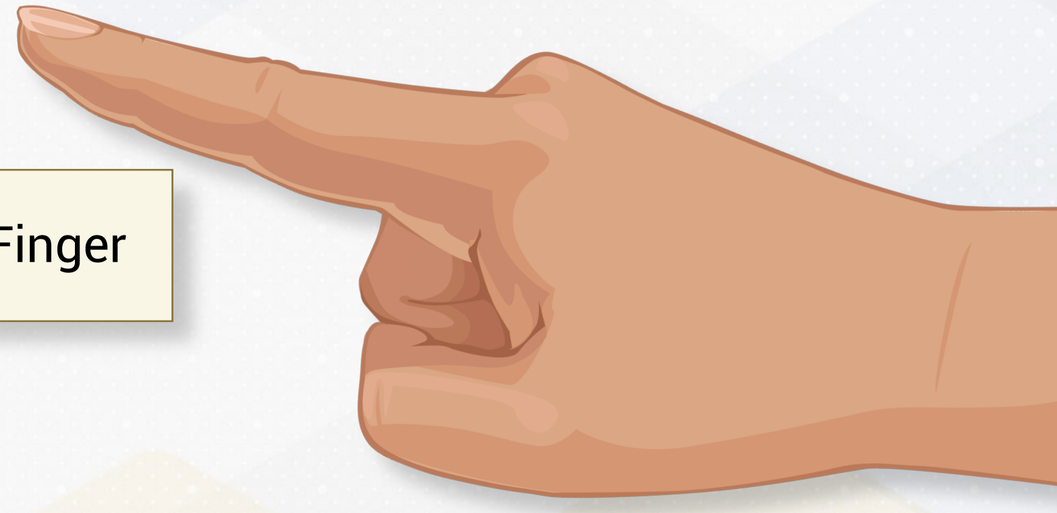
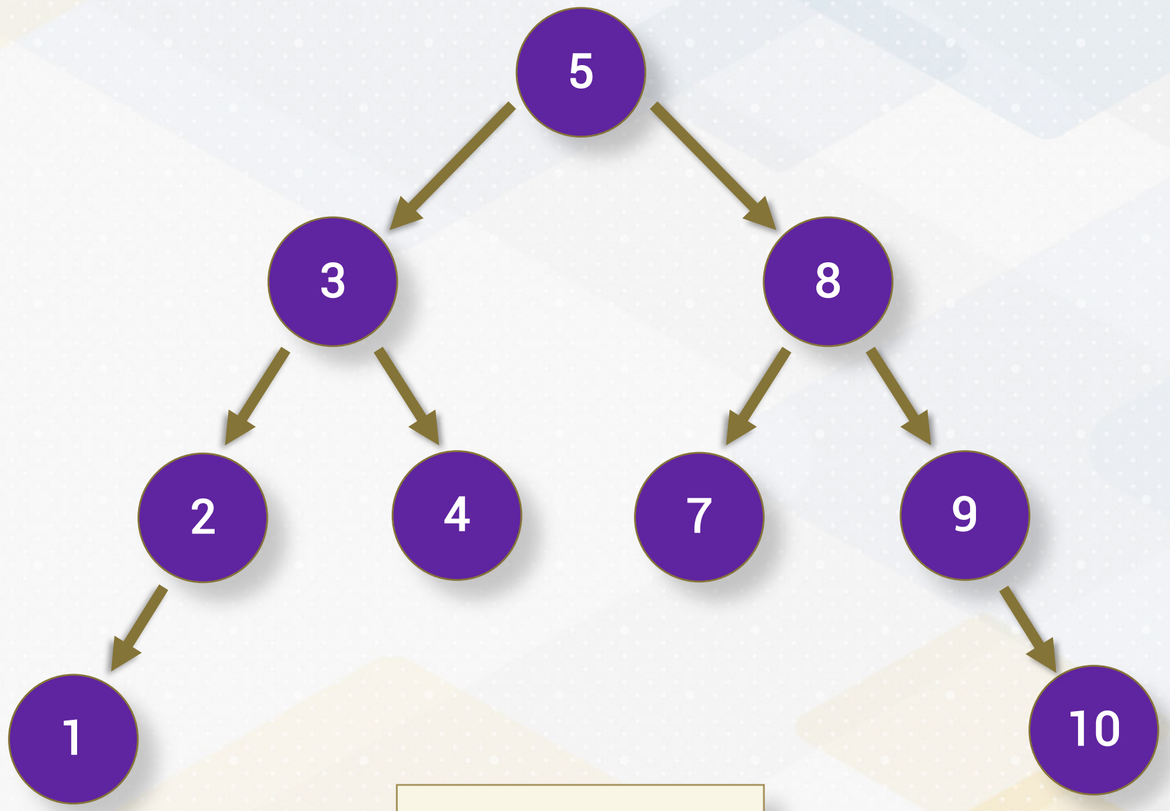
Index Finger



Indexing in C++: ordered_map

Key	Value
1	Apple
2	Banana
3	Cherry
4	Date
5	Elderberry
6	Fig
7	Grape
8	Honeydew
9	Ilama
10	Jackfruit



O (log N)

Indexing in C++: unordered_map

Key	Value
2	Banana
6	Fig
1	Apple
3	Cherry
4	Date
5	Elderberry
8	Honeydew
7	Grape
10	Jackfruit
9	llama

Avg Case	Worst Case
O (1)	O (N)

map vs unordered_map

	MAP	UNORDERED MAP
Ordering	Sorted based on key	Not sorted based on key
Implementation	Binary Search Tree	Hash Table
Average Case Time	O(log N)	O(1)
Worst Case Time	O(log N)	O(N)

HashIndex in BuzzDB

insertOrUpdate

getValue

```
class HashIndex {
private:
  std::unordered_map<int, int> hash_index;
public:
  void insertOrUpdate(int key, int value) {
        hash_index[key] = value;
  }
  int getValue(int key) const {
    auto it = hash_index.find(key);
    return it != hash_index.end() ? it->second : -1;
  }
};
```

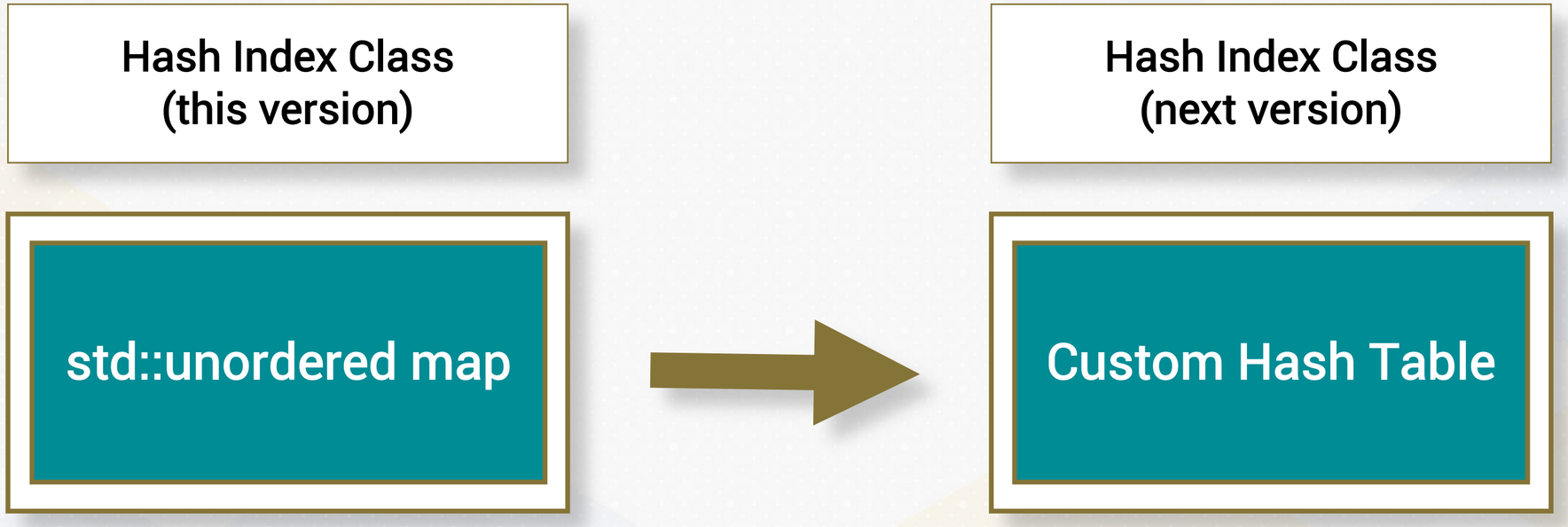
Integration of HashIndex in BuzzDB

During the scanning phase, each tuple's key-value pair is inserted into the HashIndex.

```
void BuzzDB::scanTableToBuildIndex() {
  // Iterating over pages and tuples
  for (size_t page_itr = 0; page_itr < num_pages; page_itr++) {
    // Extract key-value pairs and insert them into the index
    int key = loadedTuple->fields[0]->asInt();
    int value = loadedTuple->fields[1]->asInt();

    index.insertOrUpdate(key, value);
  }
}
```

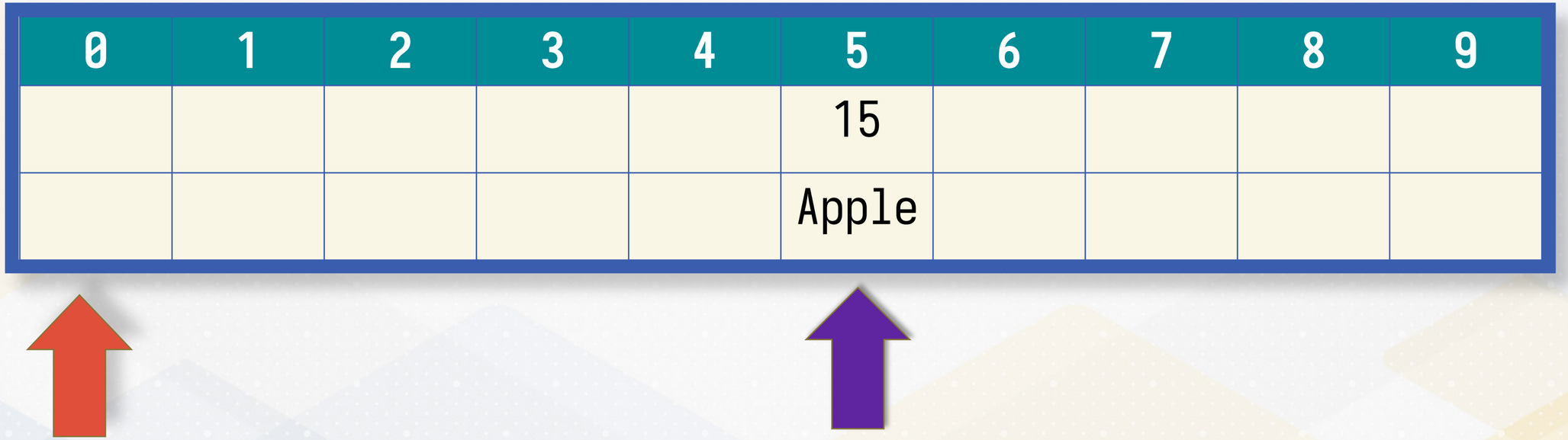
Custom HashTable



Hash Table

Hash Table

Think of a **hash table** like a **cabinet of drawers**.
Each drawer can hold a piece of paper with the **key** and **associated value**.



DIRECT ACCESS USING HASH FUNCTION

Inserting Key-Value Pair

Insert (15, Apple)

0	1	2	3	4	5	6	7	8	9
					15				
					Apple				

HASH(KEY) = KEY % 10 = 15 % 10 = 5 (since 15 = 10 * 1 + 5)

Inserting Key-Value Pair

Insert (26, Grape)

0	1	2	3	4	5	6	7	8	9
					15	26			
					Apple	Grape			

HASH(KEY) = KEY % 10 = 26 % 10 = 6 (since 26 = 10 * 2 + 6)

Inserting Key-Value Pair with Collision

Insert (56, Kiwi)

0	1	2	3	4	5	6	7	8	9
					15	26	56		
					Apple	Grape	Kiwi		

HASH(KEY) = KEY % 10 = 56 % 10 = 6

Finding Key-Value Pair

Find value associated with key 15 = Apple

0	1	2	3	4	5	6	7	8	9
					15	26	56		
					Apple	Grape	Kiwi		

HASH(KEY) = KEY % 10 = 15 % 10 = 5

Finding Key-Value Pair

Find value associated with key 56 = Kiwi

0	1	2	3	4	5	6	7	8	9
					15	26	56		
					Apple	Grape	Kiwi		

HASH(KEY) = KEY % 10 = 56 % 10 = 6

Finding Key-Value Pair

Find value associated with key 86 = KEY NOT FOUND

0	1	2	3	4	5	6	7	8	9
					15	26	56		
					Apple	Grape	Kiwi		

HASH(KEY) = KEY % 10 = 86 % 10 = 6

Hash Function

Hash Function

Hash function maps keys to slots in the hash table

```
size_t hashFunction(int key) {
  return key % totalSlots; // Our simple formula
}
```

```
HASH(KEY) = KEY % TABLE_SIZE
```

Collision Handling

Linear Probing — Move to the next available slot until we find one empty.

0	1	2	3	4	5	6	7	8	9
80	11				15	26	56	18	79
Lime	Date				Apple	Grape	Kiwi	Fig	Pear

If slot 8 is taken, check slot 9, then 0, 1, and so on.

"Linear" Probing

Formula to find the index of the next slot I after K “probes" is “linear" with respect to K.

```
INDEX I = (HASH(KEY) + K) % TABLE_SIZE

HASH(KEY) = KEY % TABLE_SIZE
```

HashEntry Struct

HashEntry Struct	Represents a piece of paper containing **key** and **value**.

```
struct HashEntry {
  int key, value;
  HashEntry(int k, int v) : key(k), value(v) {}
};
```

HashIndex

HashTable is simply an array of drawers. Each drawer can either hold a **HashEntry** or be **empty** (indicating no value).

```
class HashIndex {
  std::vector<std::optional<HashEntry>> hashTable;

  static const size_t capacity = 100; // Hard-coded capacity
  HashIndex() { hashTable.resize(capacity); }
};
```

Inserting Key-Value Pair

insertOrUpdate	Uses linear probing to find the next available slot for insertion or locate an existing key for updating.

```
void insertOrUpdate(int key, int value) {
  size_t index = hashFunction(key);
  do {
    if (!hashTable[index]) {
      hashTable[index] = HashEntry{key, value, true}; // Insert new entry
      break;
    } else if (hashTable[index]->key == key) {
      hashTable[index]->value += value; // Update existing entry
      break;
    }
    index = (index + 1) % capacity; // Linear probing to next slot
  } while (index != originalIndex); // Returned to starting point
}
```

Finding Key-Value Pair

getValue	The retrieval process employs linear probing as well to navigate through potential collision sequences.

```
int getValue(int key) const {
  size_t index = hashFunction(key);
  do {
    if (hashTable[index] && hashTable[index]->key == key) {
      return hashTable[index]->value; // Key found
    }
    index = (index + 1) % capacity; // Continue probing
  } while (index != originalIndex);
  return -1; // Key not found
}
```

Operation Complexity

N = Number of key-value pairs in the hash table.

	NO/FEW COLLISIONS	LOTS OF COLLISIONS
INSERT	O(1)	O(N)
FIND	O(1)	O(N)

Clustering Problem with High Collision Rates

Bad Hash Function: HASH(KEY) = 5

0	1	2	3	4	5	6	7	8	9
					15	26	56		
					Apple	Grape	Kiwi		

HASH(KEY) = 5

Quadratic Probing

Quadratic Probing

Formula to find the index of the next slot I after K “probes” is “quadratic" with respect to K.

$$\text{INDEX I} = (\text{HASH(KEY)} + \text{K}^2) \% \text{ TABLE\_SIZE}$$

$$\text{HASH(KEY)} = \text{KEY} \% \text{ TABLE\_SIZE}$$

Inserting 2

Insert (2, Apple)

0	1	2	3	4	5	6	7	8	9
		2							
		Apple							

HASH(2) = 2; K = 0; INDEX I = (2 + 0^2) % 10 = 2

Inserting 12

Insert (12, Grape)

0	1	2	3	4	5	6	7	8	9
		2	12						
		Apple	Grape						

HASH(12) = 2; K = 1; INDEX I = (2 + 1^2) % 10 = 3

Inserting 22

Insert (22, Kiwi)

0	1	2	3	4	5	6	7	8	9
		2	12			22			
		Apple	Grape			Kiwi			

HASH(22) = 2; K = 2; INDEX I = (2 + 2^2) % 10 = 16 % 10 = 6

Inserting 32

Insert (32, Fig)

0	1	2	3	4	5	6	7	8	9
	32	2	12			22			
	Fig	Apple	Grape			Kiwi			

HASH(32) = 2; K = 3; INDEX I = (2 + 3^2) % 10 = 11 % 10 = 1

Quadratic Probing

Probe sequence follows a quadratic formula.

```
void insertOrUpdate(int key, int value) {
  size_t originalIndex = hashFunction(key);
  bool inserted = false;
  int i = 0; // Attempt counter
  do {
    if (!hashTable[index].exists) {
      ...
    } else if (hashTable[index].key == key) {
      ...
    }
    i++;
    index = (originalIndex + i * i) % capacity; // Quadratic probing
  } while (index != originalIndex && !inserted);
}
```

Benefits of Quadratic Probing

Better spread of keys; higher search efficiency

0	1	2	3	4	5	6	7	8	9
	32	2	12			22			
	Fig	Apple	Grape			Kiwi			

Limitation of Quadratic Probing

Secondary clustering of all keys hashed to 2.

0	1	2	3	4	5	6	7	8	9
	32	2	12			22			
	Fig	Apple	Grape			Kiwi			

Double Hashing

Double Hashing

Double hashing	Uses a secondary hash function to calculate the probe step, offering a unique probe sequence for each key.

```
INDEX I = (HASH1(KEY) + K * HASH2(KEY)) % TABLE_SIZE

HASH1(KEY) = KEY % TABLE_SIZE

HASH2(KEY) = 1 + KEY % (TABLE_SIZE - 1)
```

Inserting 2

Insert (2, Apple)

0	1	2	3	4	5	6	7	8	9
		2							
		Apple							

HASH1(2) = 2; HASH2(2) = 1 + (2 % 9) = 1 + 2 = 3
K = 0; INDEX I = (2 + 0 * 3) % 10 = 2

Inserting 12

Insert (12, Grape)

0	1	2	3	4	5	6	7	8	9
		2				12			
		Apple				Grape			

HASH1(12) = 2; HASH2(12) = 1 + (12 % 9) = 1 + 3 = 4
K = 1; INDEX I = (2 + 1 * 4) % 10 = 6 % 10 = 6

Inserting 22

Insert (22, Kiwi)

0	1	2	3	4	5	6	7	8	9
		2				12	22		
		Apple				Grape	Kiwi		

HASH1(22) = 2; HASH2(22) = 1 + (22 % 9) = 1 + 4 = 5
K = 1; INDEX I = (2 + 1 * 5) % 10 = 7 % 10 = 7

Inserting 32

Insert (32, Fig)

0	1	2	3	4	5	6	7	8	9
		2				12	22	32	
		Apple				Grape	Kiwi	Fig	

HASH1(32) = 2; HASH2(32) = 1 + (32 % 9) = 1 + 5 = 6
K = 1; INDEX I = (2 + 1 * 6) % 10 = 8 % 10 = 8

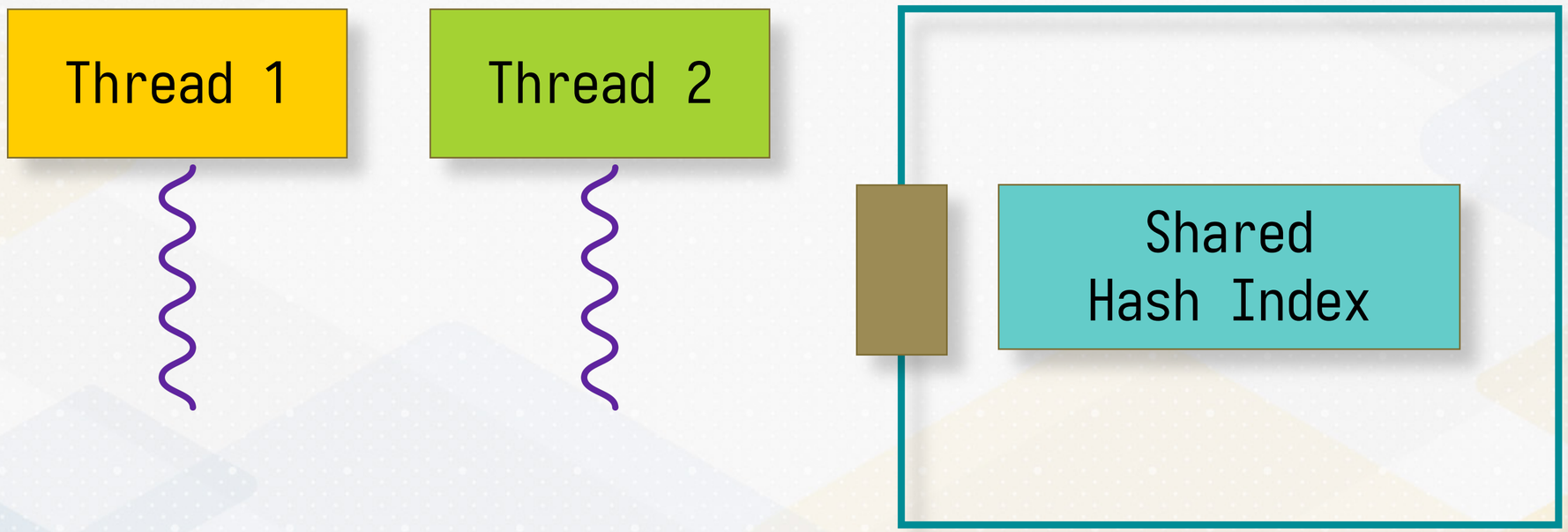
Benefit of Double Hashing

0	1	2	3	4	5	6	7	8	9
		2				12	22	32	
		Apple				Grape	Kiwi	Fig	

Parallel Index Construction

Parallel Index Construction

With multi-core CPUs, **parallelizing index construction** offers a significant performance boost by **distributing** the workload across **multiple threads**.

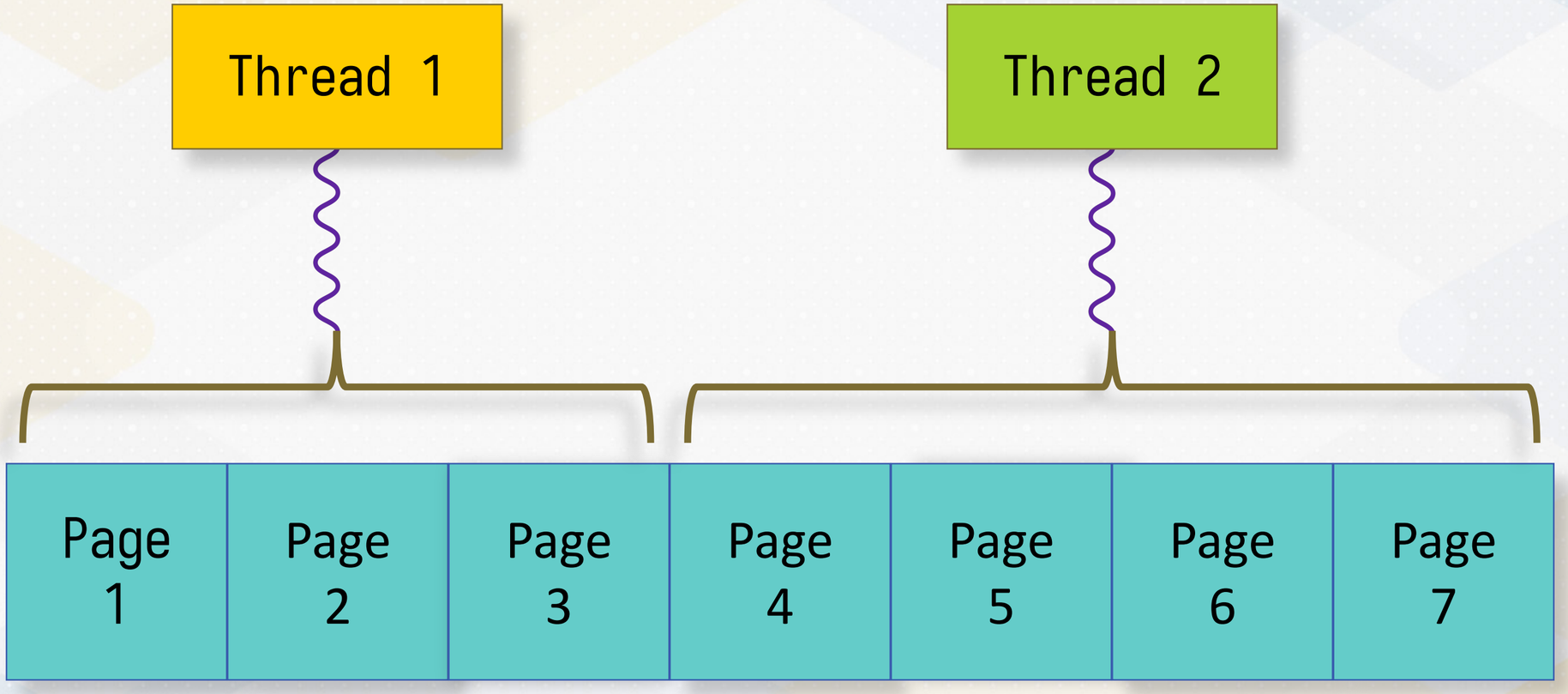


Page Assignment

Divide the total number of pages (num_pages) by the number of available threads (num_threads), assign each thread a specific range of pages to process.

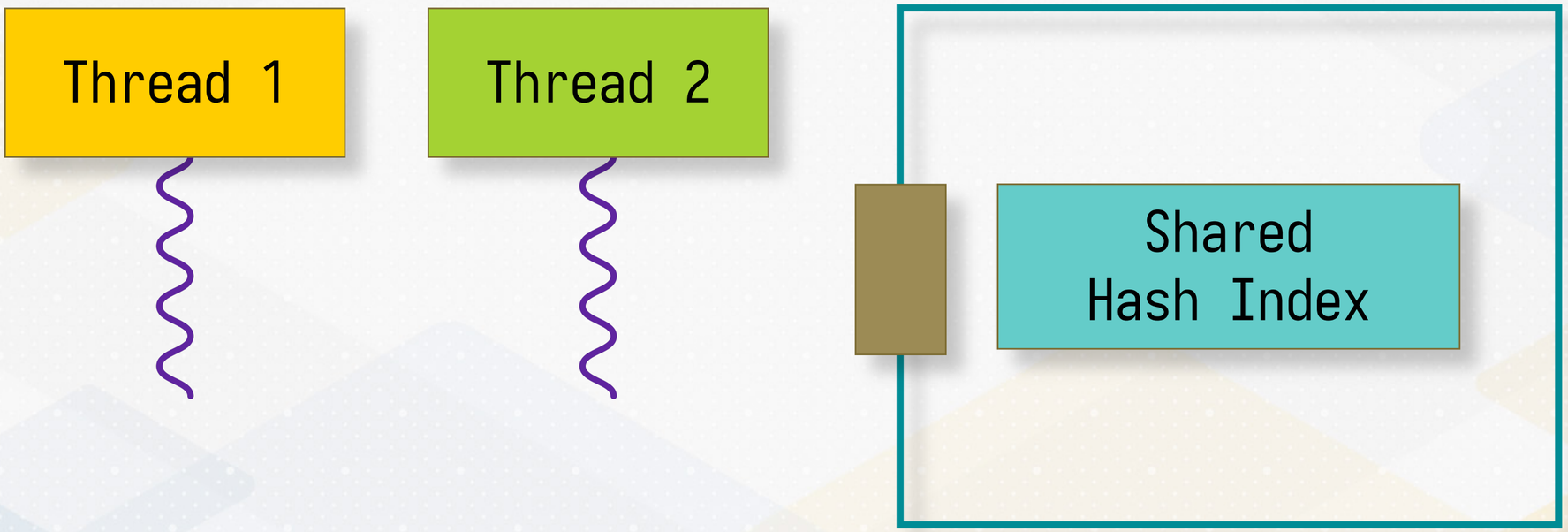
```
void parallelProcessPages(size_t num_threads = 5) {
  auto num_pages = buffer_manager.getNumPages();
  size_t pages_per_thread = num_pages / num_threads;
  std::vector<std::thread> threads;
  for (size_t i = 0; i < num_threads; i++) {
    size_t start_page = i * pages_per_thread;
    size_t end_page =...; // Last thread gets any remaining pages
    threads.emplace_back(&BuzzDB::processPageRange, this, start_page,
end_page);
  }
  ...
}
```

Page Assignment



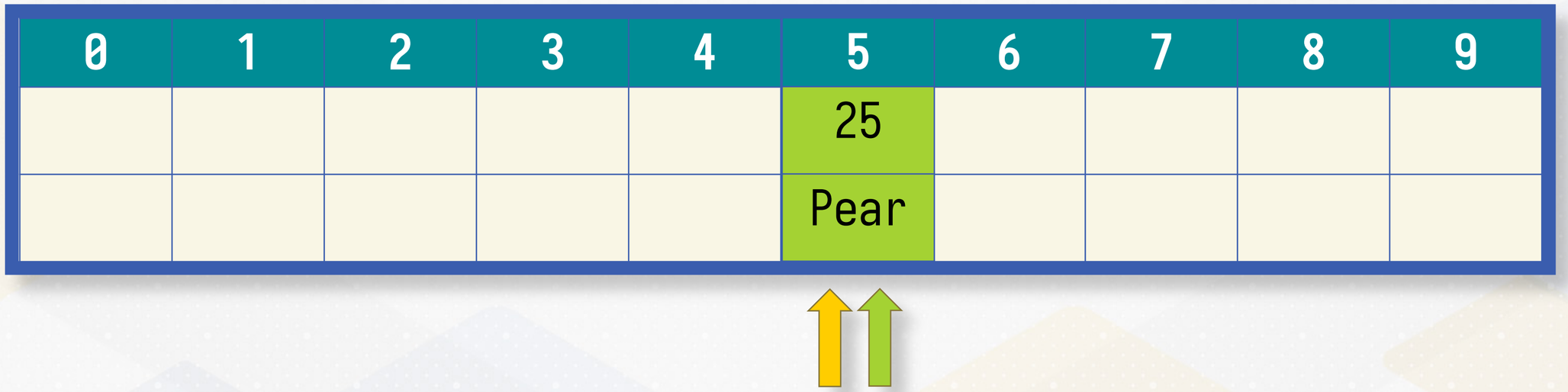
Thread Safety

With **parallel index construction**, the challenge is that **concurrent operations** on the index by multiple threads may lead to **inconsistent state**.



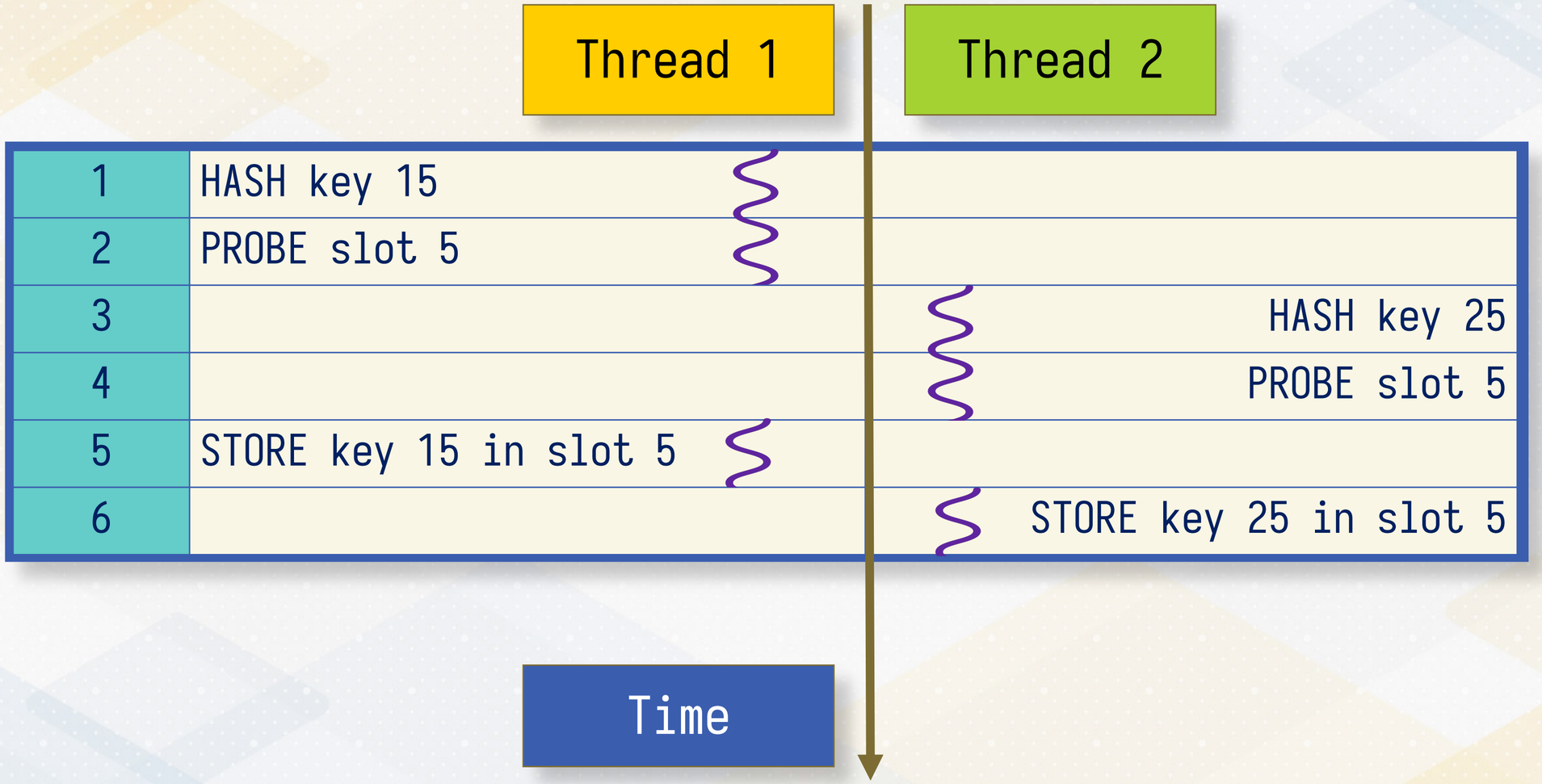
Thread Safety

Thread 1: Insert (15, Fig) **Thread 2**: Insert (25, Pear)



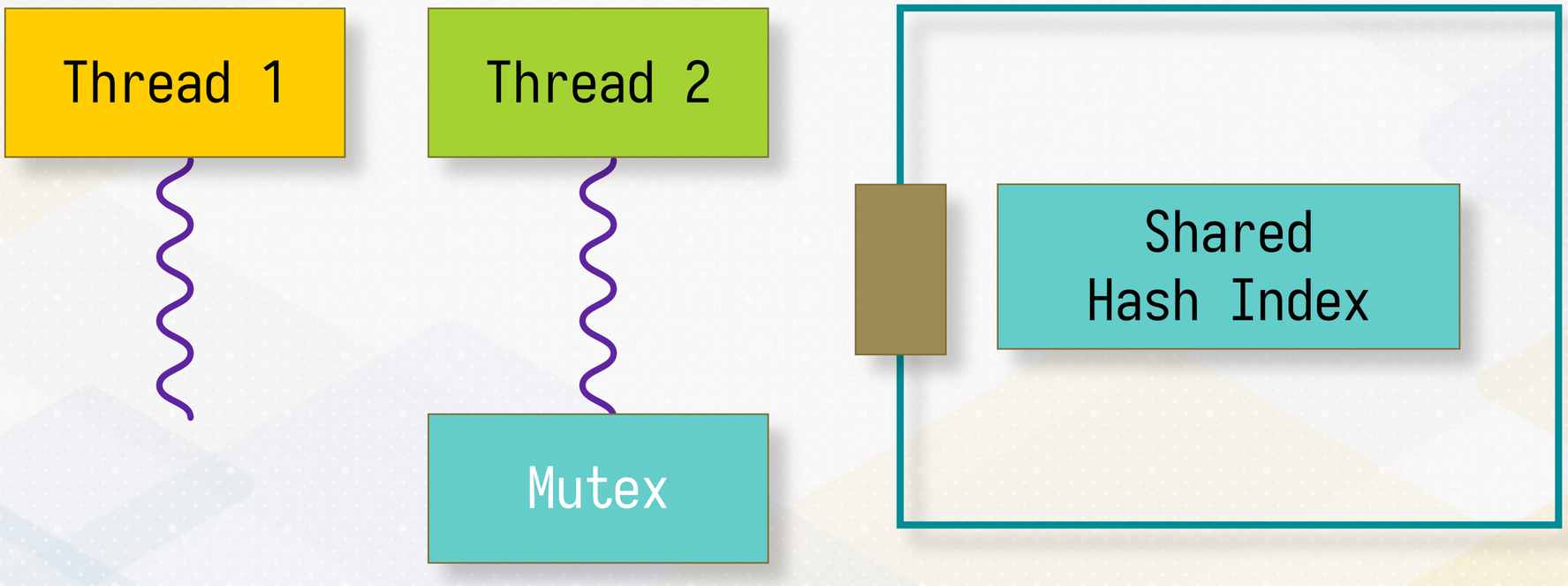
HASH(KEY) = KEY % 10

Race Condition



std::mutex

Mutex "serializes" access to the shared index structure.



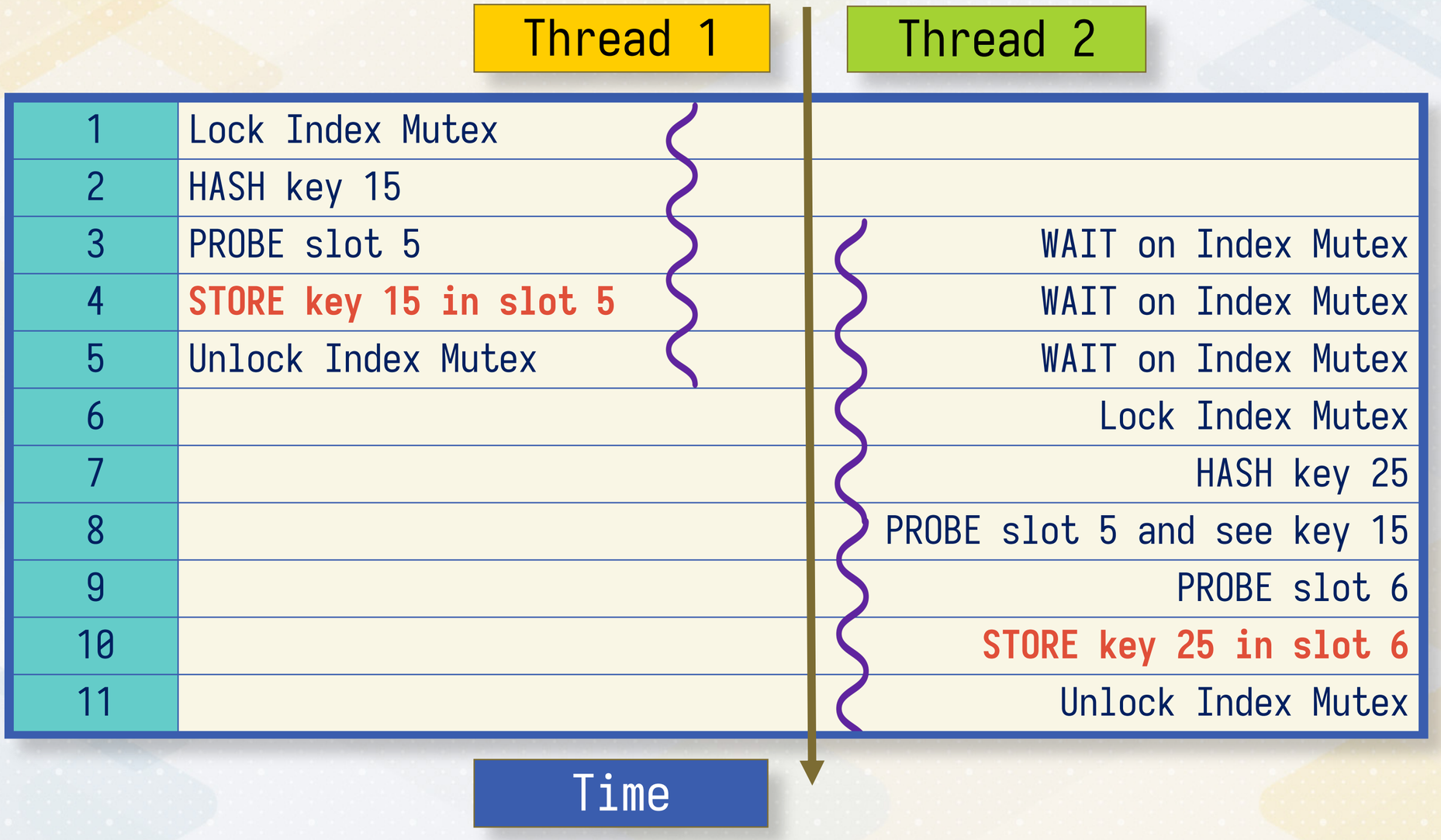
std::lock_guard

lock_guard automatically acquires a lock on creation and releases it on destruction.

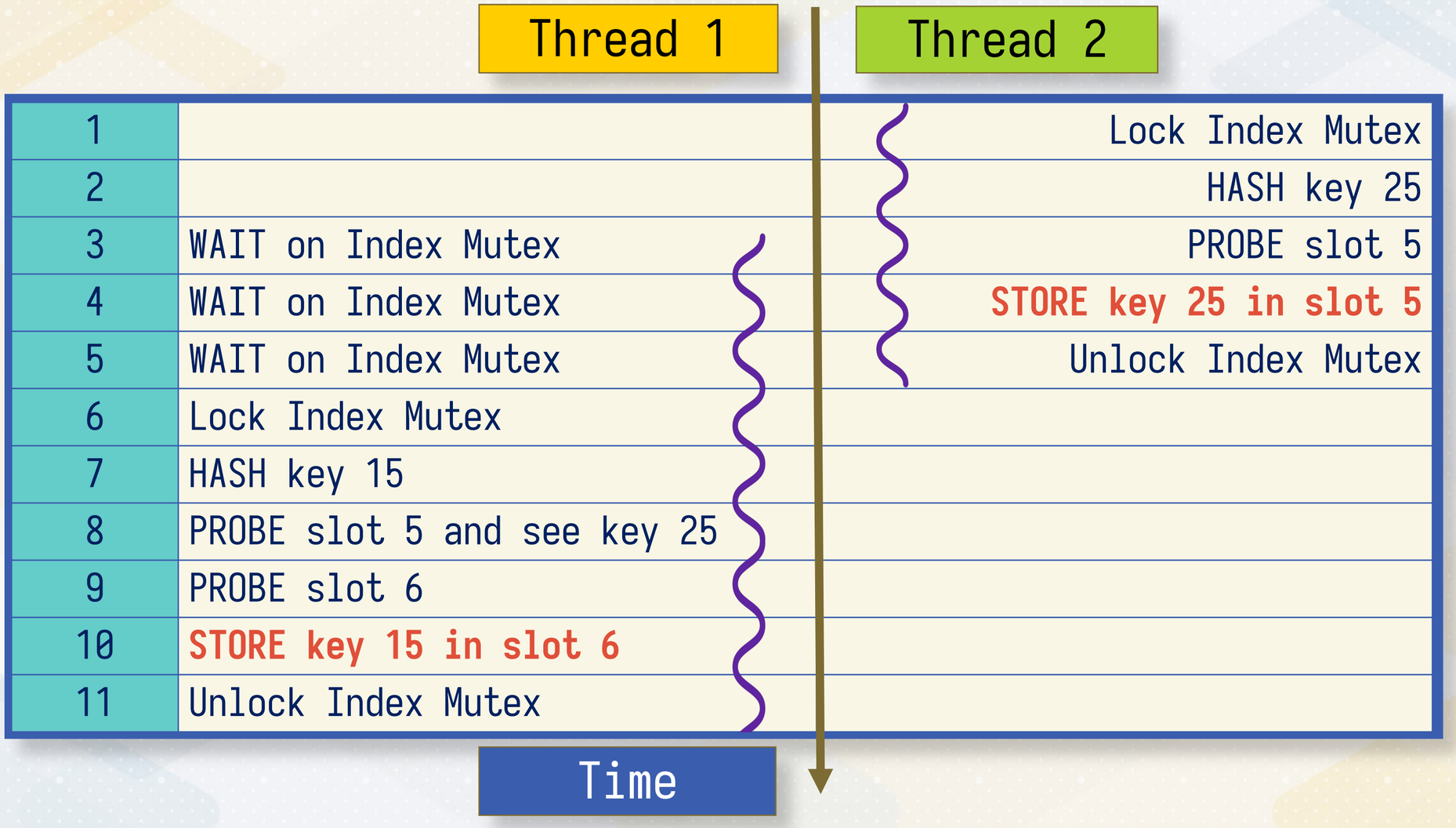
```
class HashIndex {
private:
  mutable std::mutex indexMutex; // Mutex for thread-safe access

  void insertOrUpdate(int key, int value) {
    // RAII-style mutex management
    std::lock_guard<std::mutex> guard(indexMutex);
    // Perform thread-safe update on the index
  }
};
```

Thread Safety with Mutex



Thread Safety with Mutex



Order of Thread Execution

0	1	2	3	4	5	6	7	8	9
					15	25			
					Fig	Pear			

0	1	2	3	4	5	6	7	8	9
					25	15			
					Pear	Fig			

Range Query

Range Query vs Point Query

Range Query

Tuples where a column is in a Range of Values

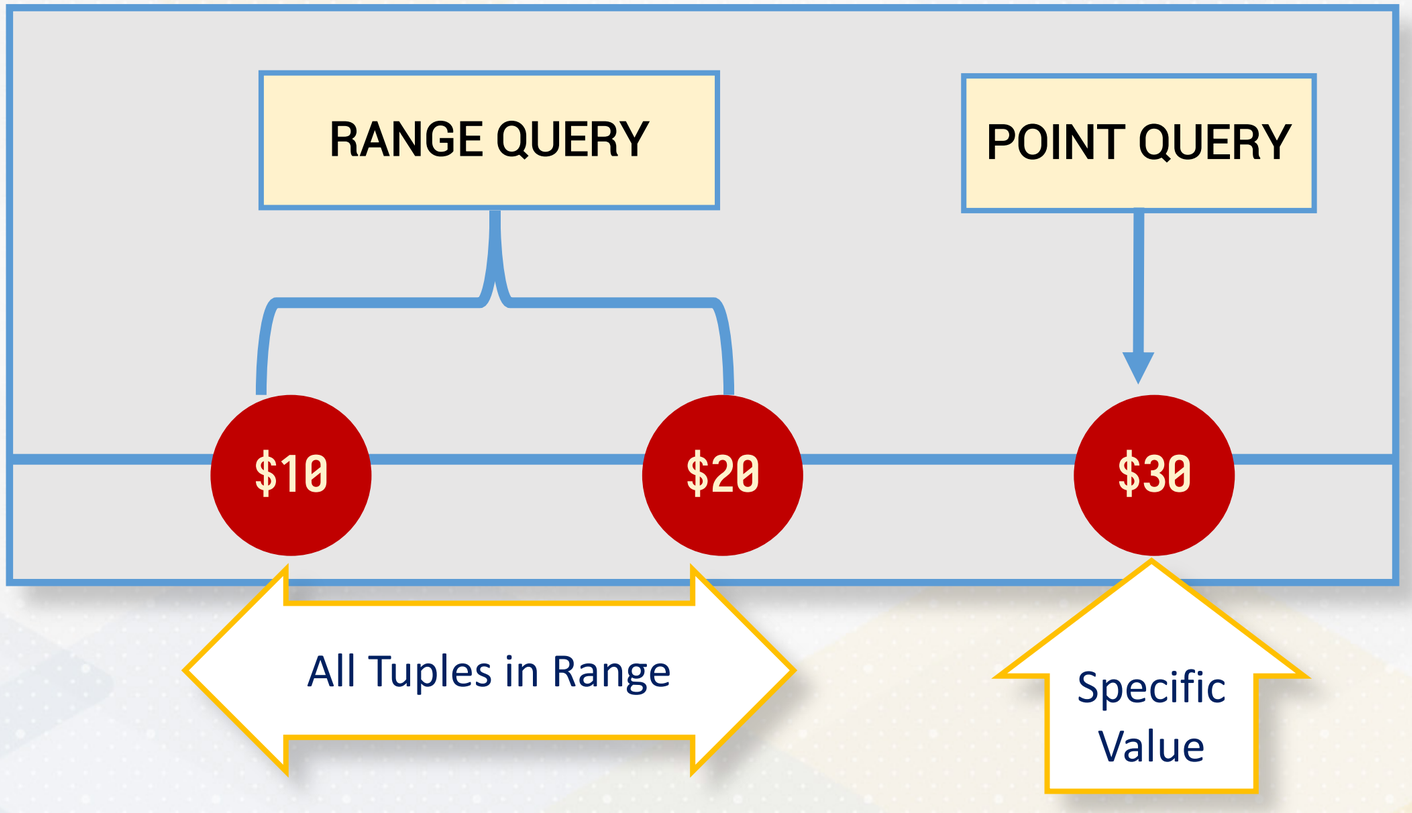
Products priced between $10 and $20

Point Query

Tuples where a column is equal to a Single Value

Products priced $30

Range Query vs Point Query



Point Query Using Hash Table Index

Look up a key in the hash table with O(1) average complexity.

```
std::vector<int> tuple_ids = productIndex.getValue(30);
```

POINT QUERY

0	1	2	3	4	5	6	7	8	9
$30		$12			$25		$17		
[103, 105]		[104]			[101]		[106, 102]		

Point Query Using Hash Table Index

Hash Table	Not ordered based on key	Iterate through entire table

RANGE QUERY

0	1	2	3	4	5	6	7	8	9
$30		$12			$25		$17		
[103, 105]		[104]			[101]		[106, 102]		

Point Query Using Hash Table Index

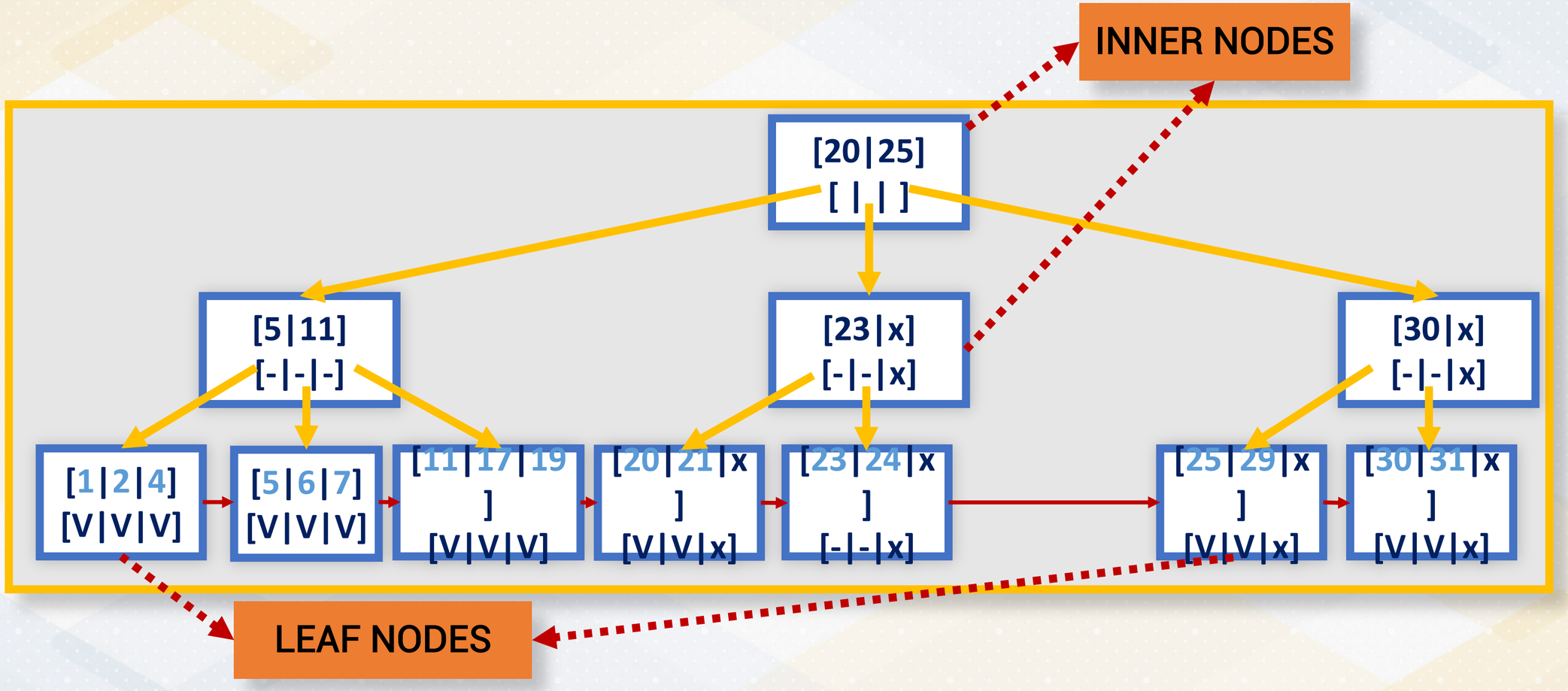
```
std::vector<int> rangeQuery(int lowerBound, int upperBound) const {
  std::vector<int> results;
  for (size_t i = 0; i < capacity; ++i) {
    if (hashTable[i].exists && hashTable[i].key >= lowerBound &&
        hashTable[i].key <= upperBound) {
      results.push_back(hashTable[i].value);
    }
  }
  return results;
}
```

Inefficient Range Query in BuzzDB

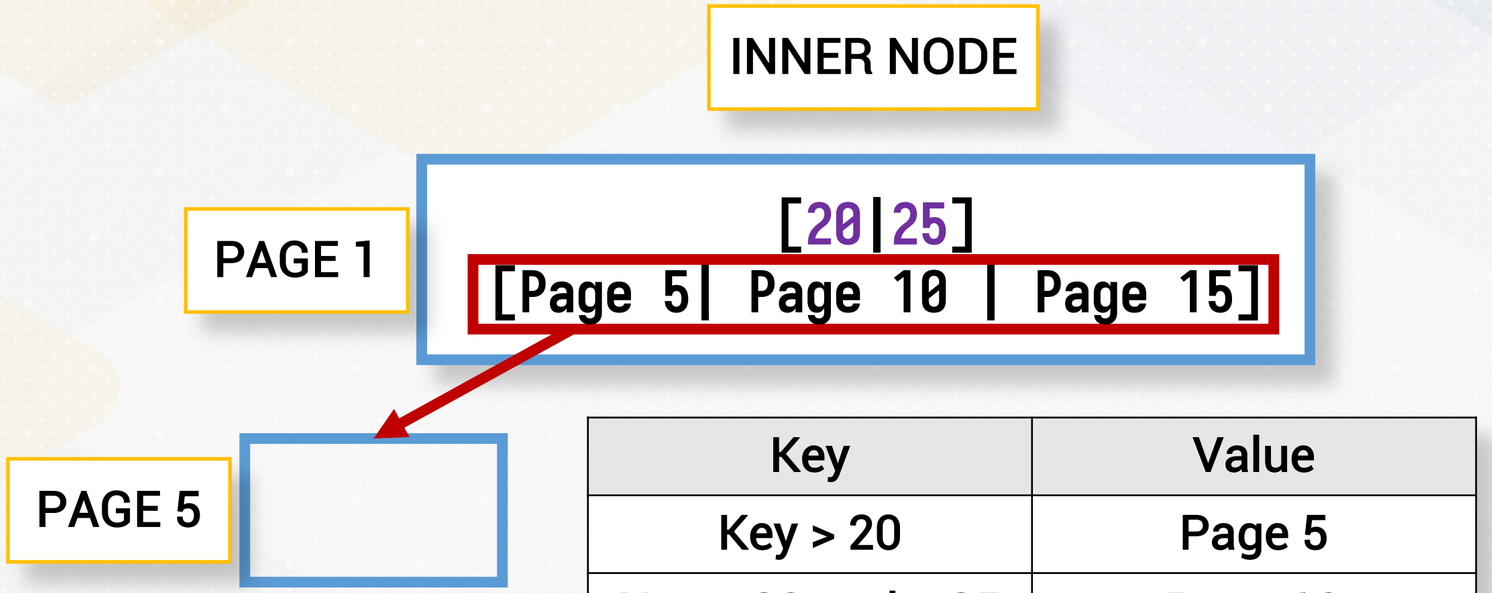

```
void performRangeQueryWithHashIndex(int lowerBound, int upperBound) {
  auto results = index.rangeQuery(lowerBound, upperBound);
  std::cout << "Found " << results.size() << " records in the range.\n";
}
```

B+Tree

B+Tree Structure

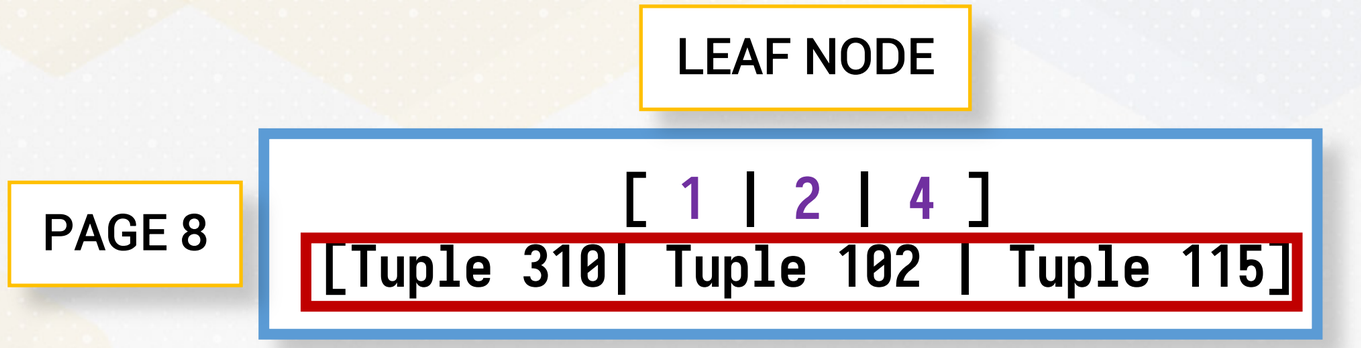


Inner Node



Key	Value
Key > 20	Page 5
Key = 20 and < 25	Page 10
Key > 25	Page 15

Leaf Node



Key	Value
Key = 1	Tuple 310
Key = 2	Tuple 102
Key = 4	Tuple 115

Key and Value Types

ENTRIES

Entry	Key	Value
Entry 1	“max_file_size"	“10 MB"
Entry 2	“default_language"	“English"
Entry 3	“session_timeout"	“30 minutes"

LEAF NODE

PAGE 8

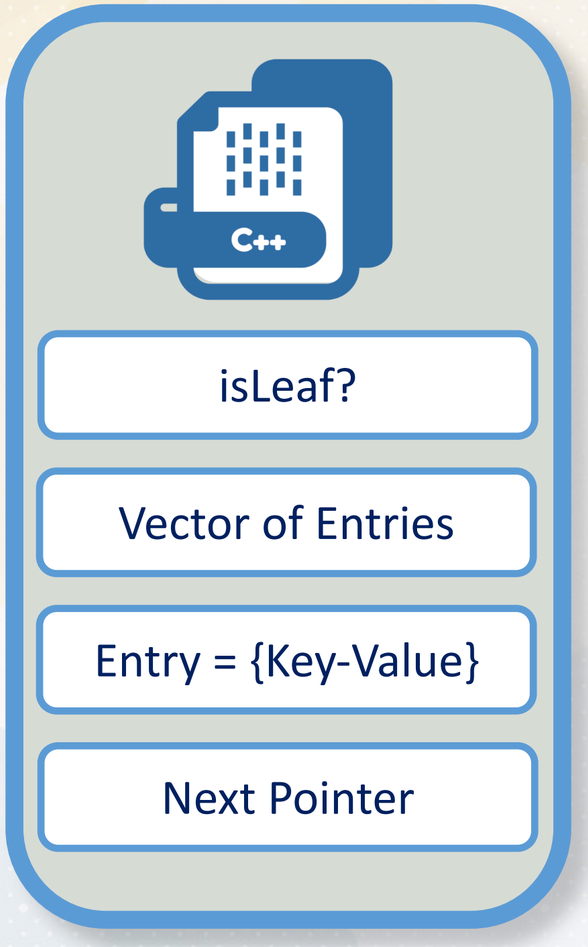
```
[ “max_file_size” | “default_language” | “session_timeout” ]
                [“10 MB” | “English” | “30 minutes”]
```

Entry in C++

```
struct Entry {
  Key key;
  Value value;  // Used only in leaf node entry
  NodePtr next; // Used only in inner node entry

  Entry(Key k, Value v) : key(k), value(v), next(nullptr) {}
};
```

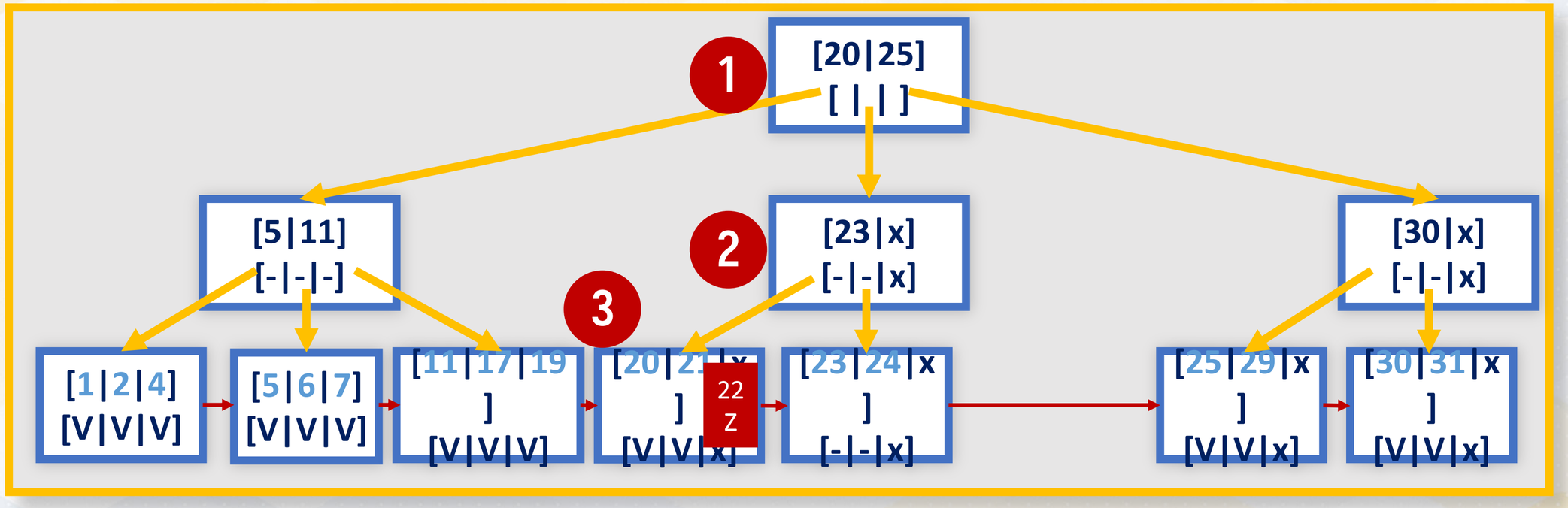
B+Tree Node in C++




```cpp
struct Node {
  bool isLeaf;
  std::vector<Entry> entries;
  std::unique_ptr<Node> next;

  Node(bool leaf) : isLeaf(leaf), next(nullptr) {}
};
```

Inserting a Key-Value Pair



Inserting a Key-Value Pair

Insertion process is recursive

```
void insertOrUpdate(const Key &key, const Value &value) {
    insertOrUpdateInternal(key, value, root);
}
```

Recursive Descent in an Inner Node

Find the appropriate child node by comparing keys.

```
void insertOrUpdateInternal(const Key &key, const Value &value, NodePtr &node) {
    ...
    else {
        bool found = false;
        for (auto it = node->entries.begin(); it != node->entries.end(); ++it) {
            if (key < it->key) {
                insertOrUpdateInternal(key, value, it->next);
                found = true;
                break;
            }
        }
    }
}
```

Insertion in a Leaf Node

If key does not exist in leaf node, insert it at the correct position to maintain sorted order.

```
void insertOrUpdateInternal(const Key &key, const Value &value, NodePtr &node) {
    if (node->isLeaf) {
      // Search for the key in the leaf node
      auto it = std::find_if(node->entries.begin(), node->entries.end(),
                   [&](const Entry &entry) { return entry.key == key; });
      // Key not found, proceed to insert in sorted order
      auto comp = [](const Entry &entry, const Key &k) { return entry.key < k;};
      it = std::lower_bound(node->entries.begin(), node->entries.end(), key,
comp);
      node->entries.insert(it, Entry(key, value));
    }
    ...
}
```

Binary Search using a Lambda Function

λ Function

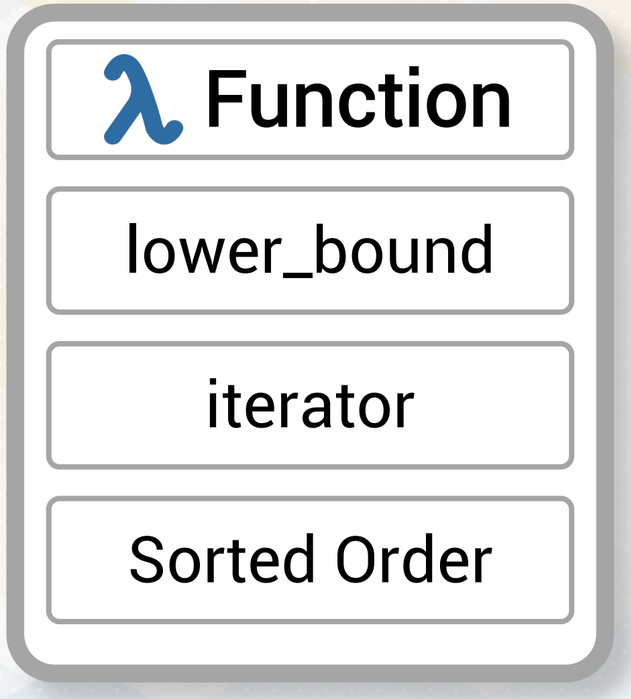
Anonymous

Short Functions

Comparator

```
// Key not found, proceed to insert in sorted order
auto comp = [](const Entry &entry, const Key &k)
                { return entry.key < k; };
it = std::lower_bound(node->entries.begin(),
                      node->entries.end(),
                      key, comp);
node->entries.insert(it, Entry(key, value));
```

Binary Search using a Lambda Function




```
// Key not found, proceed to insert in sorted order
auto comp = [](const Entry &entry, const Key &k)
                { return entry.key < k; };
it = std::lower_bound(node->entries.begin(),
                      node->entries.end(),
                      key, comp);
node->entries.insert(it, Entry(key, value));
```

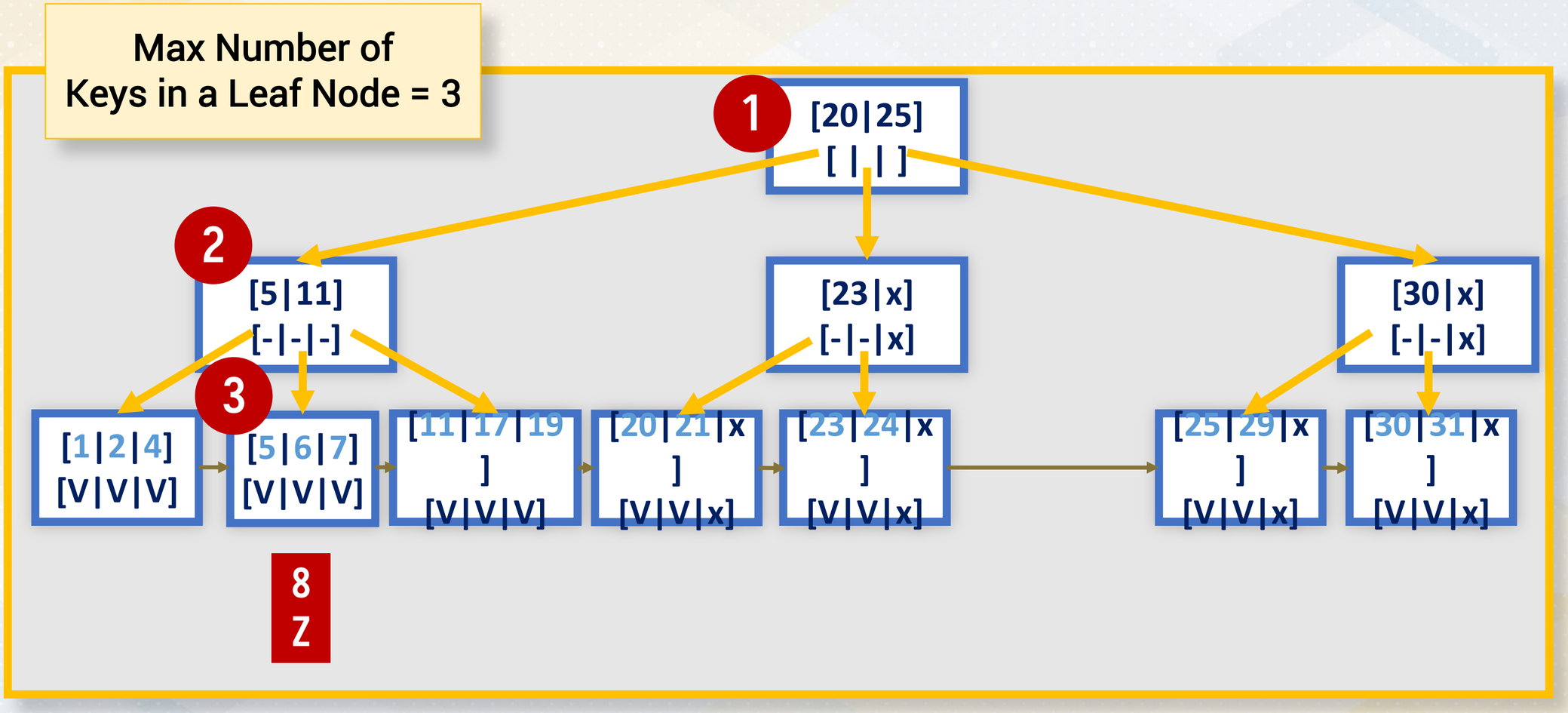
Node Split

Separate Vectors for Keys and Values

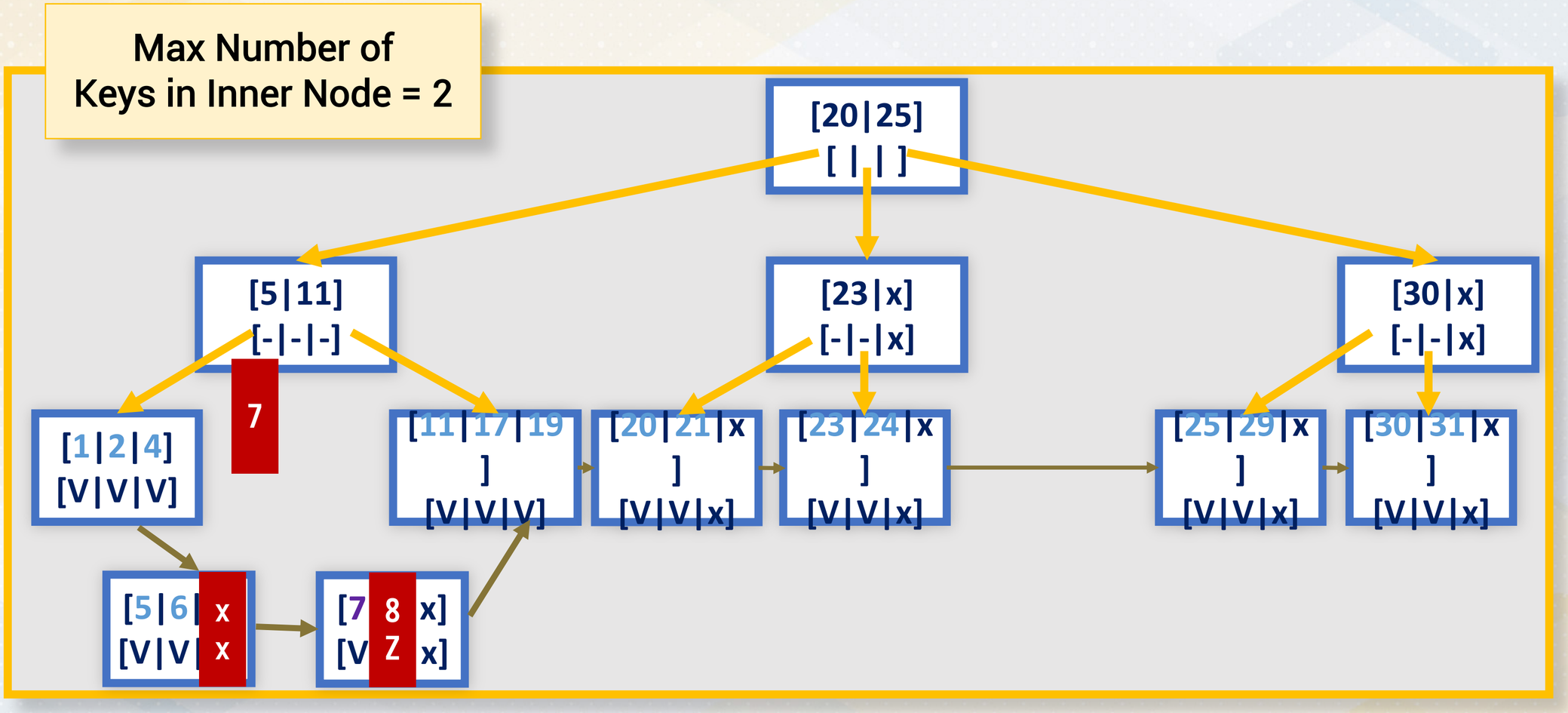
Node contains keys and values in separate vectors.

```
template <typename Key, typename Value> class BPlusTree {
  struct Node {
    std::vector<Key> keys;
    std::vector<Value> values;                    // Only used in leaf nodes
    std::vector<std::shared_ptr<Node>> children; // Only used in internal nodes
    std::shared_ptr<Node> next = nullptr;         // Next leaf node
    bool isLeaf = false;
    Node(bool leaf) : isLeaf(leaf) {}
  };
  size_t maxKeys; // Order of the tree
  std::shared_ptr<Node> root;
};
```

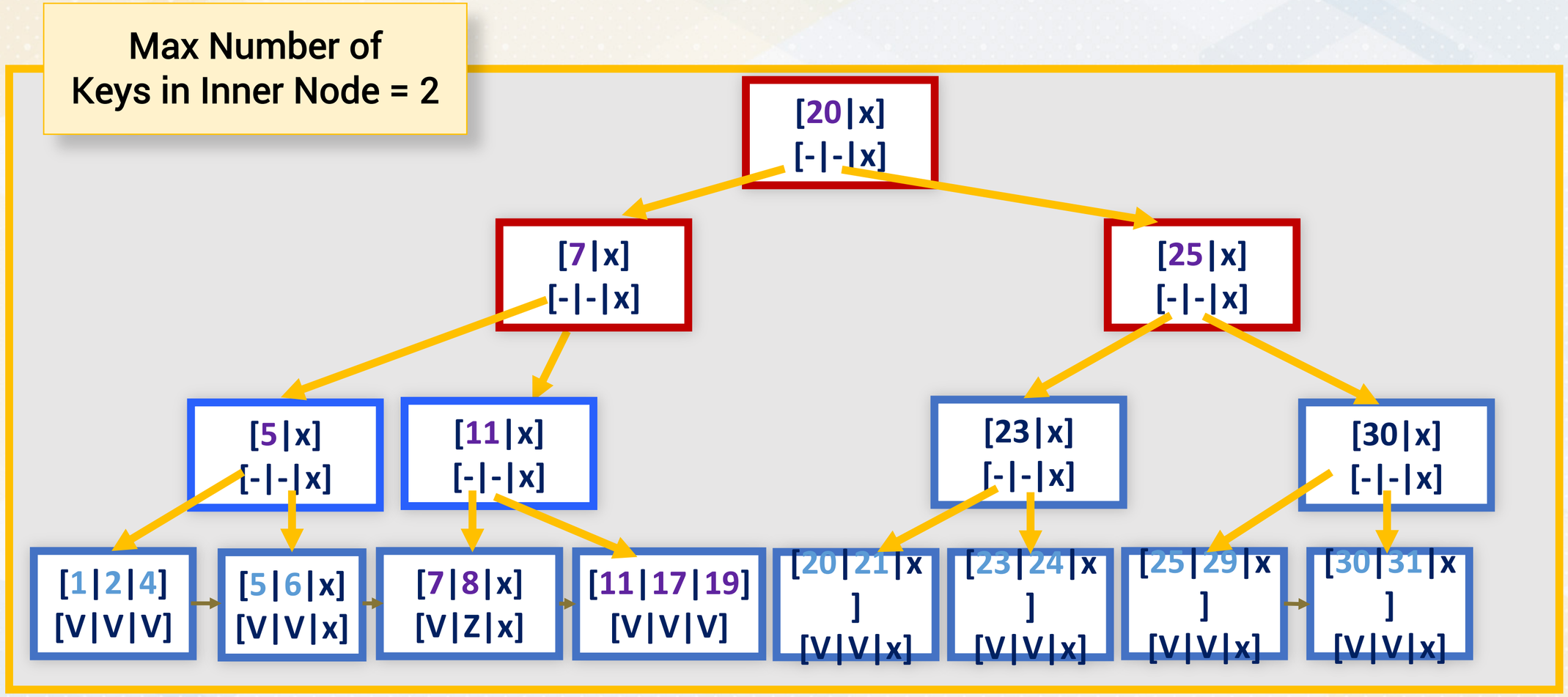
Node Split: Inserting [8, Z]



Node Split: Inserting [8, Z]



Node Split: Inserting [8, Z]



Node Split in C++

buzzDB

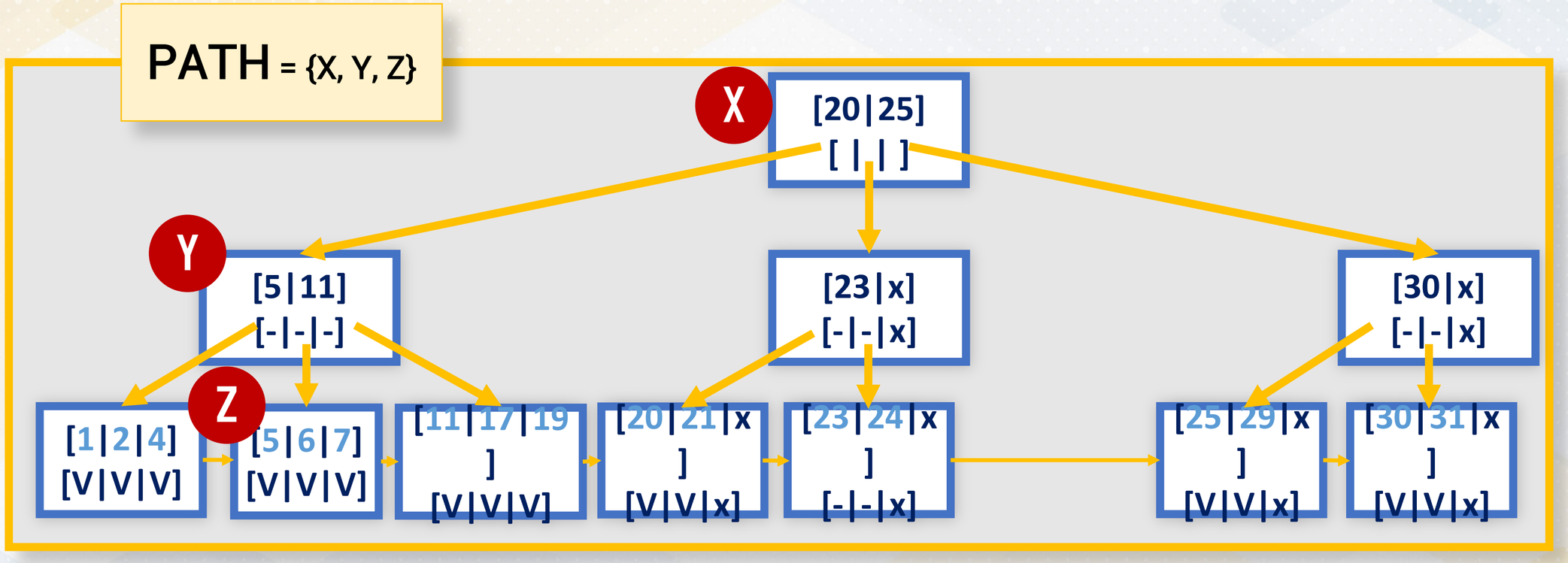
Node Exceeds Capacity → Split Node

```
if (node->keys.size() > maxKeys) {
  splitNode(path, node);
}
```

Insertion

```
auto node = root;
std::vector<std::shared_ptr<Node>> path; // Track the path for backtracking
while (!node->isLeaf) {
  path.push_back(node);
  auto it = std::upper_bound(node->keys.begin(), node->keys.end(), key);
  size_t index = it - node->keys.begin();
  node = node->children[index];
}
```

Path Tracking and Backtracking



Insertion in Leaf Node

```
auto it = std::lower_bound(node->keys.begin(), node->keys.end(), key);
if (it != node->keys.end() && *it == key) {
  size_t pos = std::distance(node->keys.begin(), it);
  node->values[pos] += value; // Update existing key's value
}
else {
  size_t pos = it - node->keys.begin();
  node->keys.insert(node->keys.begin() + pos, key);
  node->values.insert(node->values.begin() + pos, value);
}
```

Node Splitting in Leaf Node

```
if (node->isLeaf) {
  auto newNode = std::make_shared<Node>(true); // Create a new leaf node
  size_t mid = node->keys.size() / 2;          // Calculate the middle index
  // Move the second half of keys and values to the new node
  std::move(node->keys.begin() + mid, node->keys.end(),
            std::back_inserter(newNode->keys));
  std::move(node->values.begin() + mid, node->values.end(),
            std::back_inserter(newNode->values));
  newNode->next = node->next;
  node->next = newNode;
}
```

Node Splitting in Inner Node

```
else {                                                  // Handling internal nodes
  auto newNode = std::make_shared<Node>(false); // Create a new internal node
  size_t mid = node->keys.size() / 2;           // Middle index
  Key midKey = node->keys[mid]; // Key that will move to the parent
  // Move keys and children to the new node
  std::move(node->keys.begin() + mid + 1, node->keys.end(),
            std::back_inserter(newNode->keys));
  std::move(node->children.begin() + mid + 1, node->children.end(),
            std::back_inserter(newNode->children));
}
```

Backtracking to Parent Node

Path Vector

Determines Parent Node

Median Key Insertion

Parent Node May Exceed Capacity

Recursion

Can Travel Up to Root Node

```
else {
  auto parent = path.back(); // Get the parent node
  path.pop_back();           // Remove the last tracked node
  size_t pos = std::distance(
      parent->keys.begin(),
      std::lower_bound(parent->keys.begin(), parent->keys.end(), midKey));
  parent->keys.insert(parent->keys.begin() + pos, midKey); // Insert median key
  parent->children.insert(parent->children.begin() + pos + 1, newNode); // Insert
}
```

Backtracking to Root Node

Backtracking

- Empty Path Vector
- New Root Node
- Two Children
- Tree Height += 1

```
if (path.empty()) {
  // New root for the tree
  auto newRoot = std::make_shared<Node>(false);
  newRoot->keys.push_back(midKey);          // Add median key
  newRoot->children.push_back(node);        // Left child
  newRoot->children.push_back(newNode);     // Right child
  // Update root pointer
  root = newRoot;
}
```

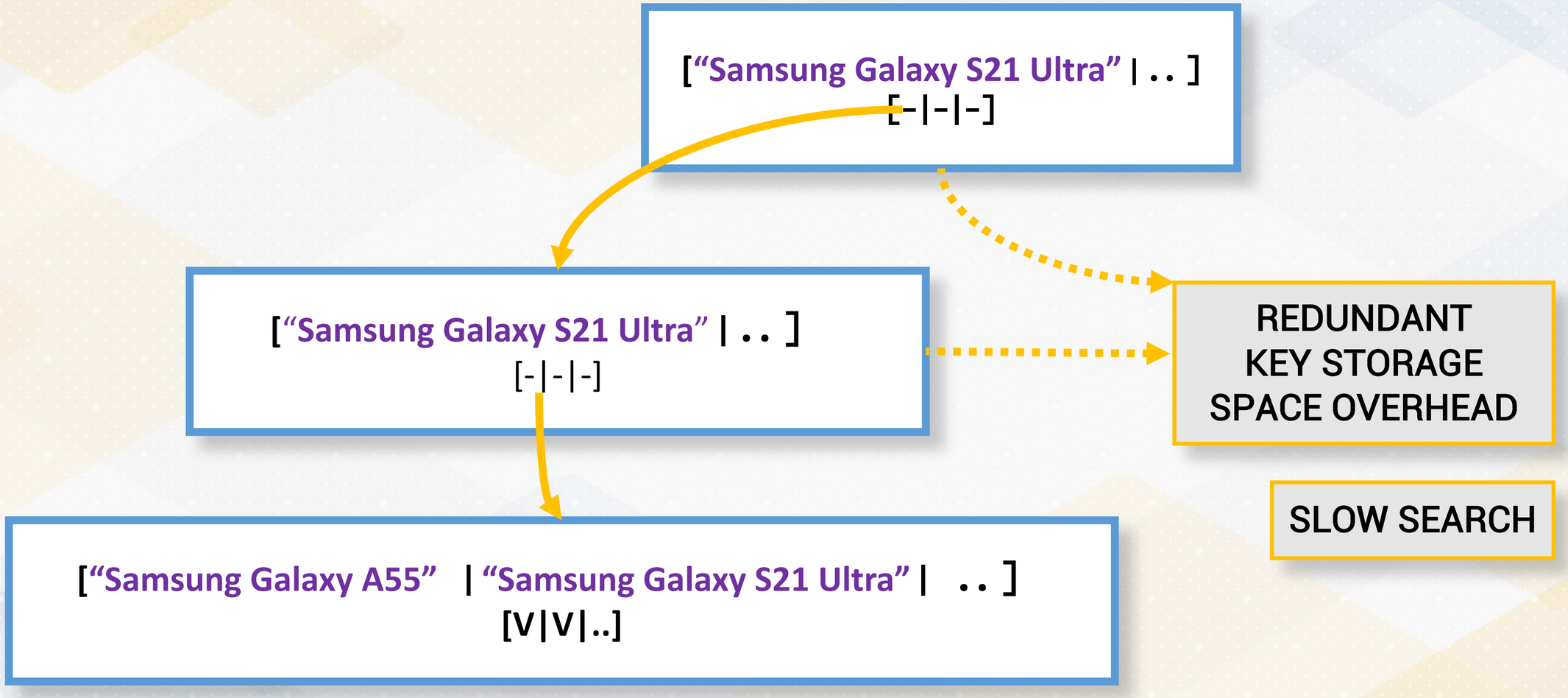
Trie

Product Catalog Search

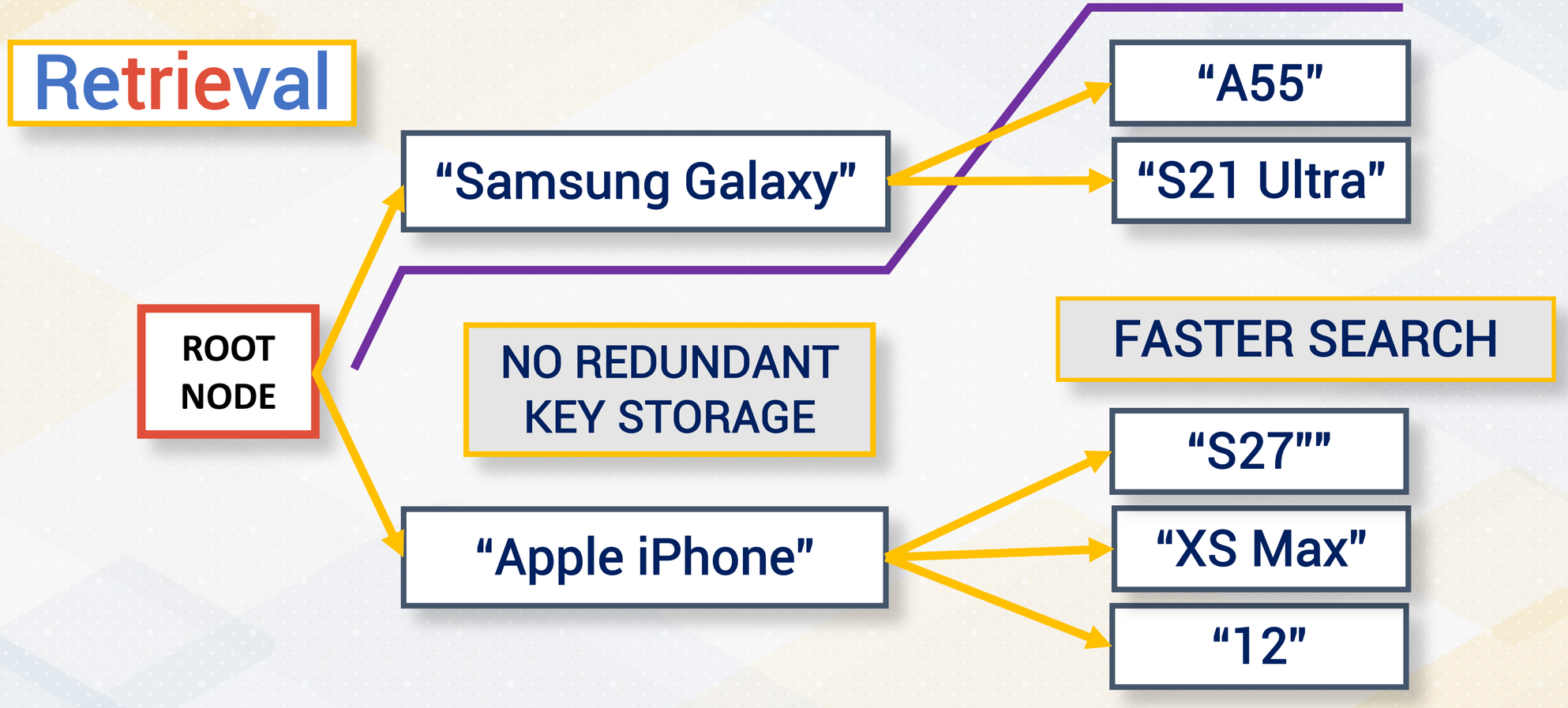
Product Name	Relevant or not
Samsung Galaxy S21 Ultra	Yes
Apple iPhone XS Max	No
Samsung Galaxy A55	Yes
Apple iPhone 12	No
Samsung Galaxy S27	Yes

```
// Find all Samsung Galaxy phones in the catalog
std::vector<std::string> galaxyPhones =
    productCatalog.startsWith("Samsung Galaxy");
```

Limitations of B+Tree



Trie

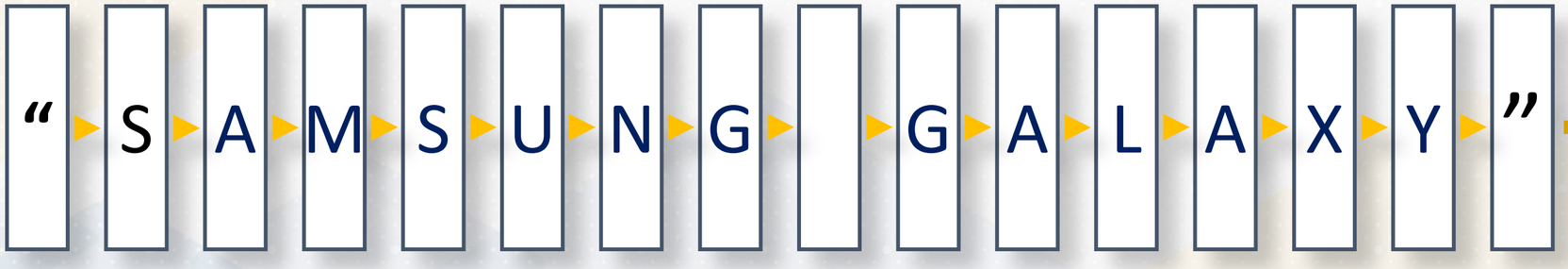

Retrieval
ROOT NODE
"Samsung Galaxy"
"Apple iPhone"
"A55"
"S21 Ultra"
"S27""
"XS Max"
"12"
NO REDUNDANT KEY STORAGE
FASTER SEARCH

Patricia Trie

Trie

Merge Single Character Nodes | Whole Strings

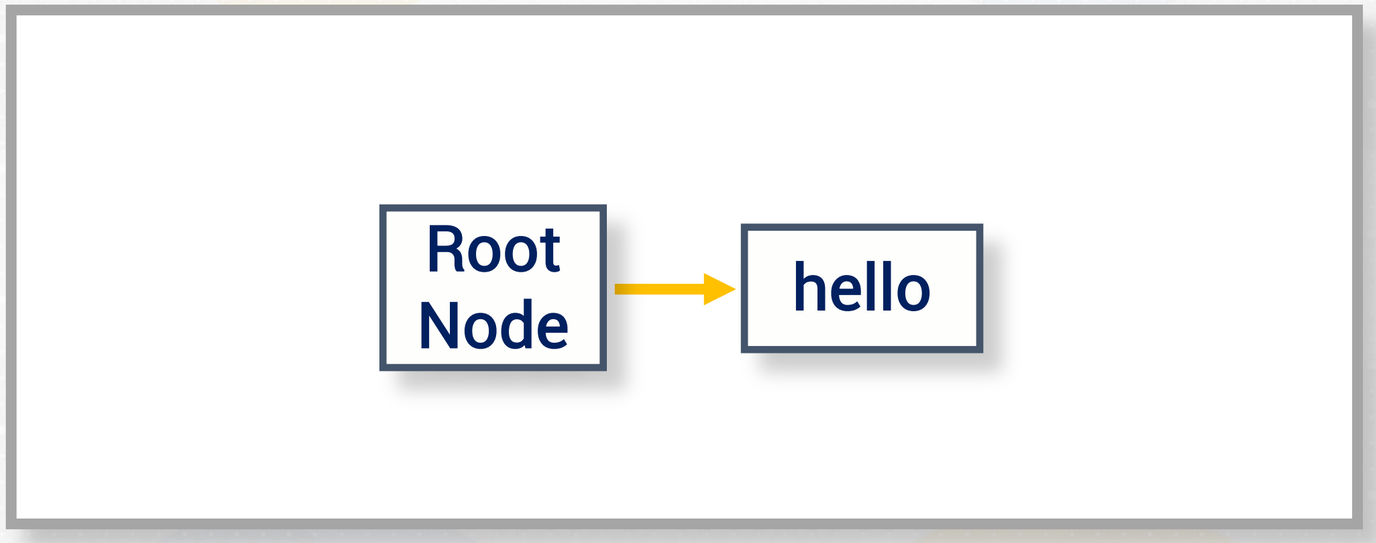
Basic Trie



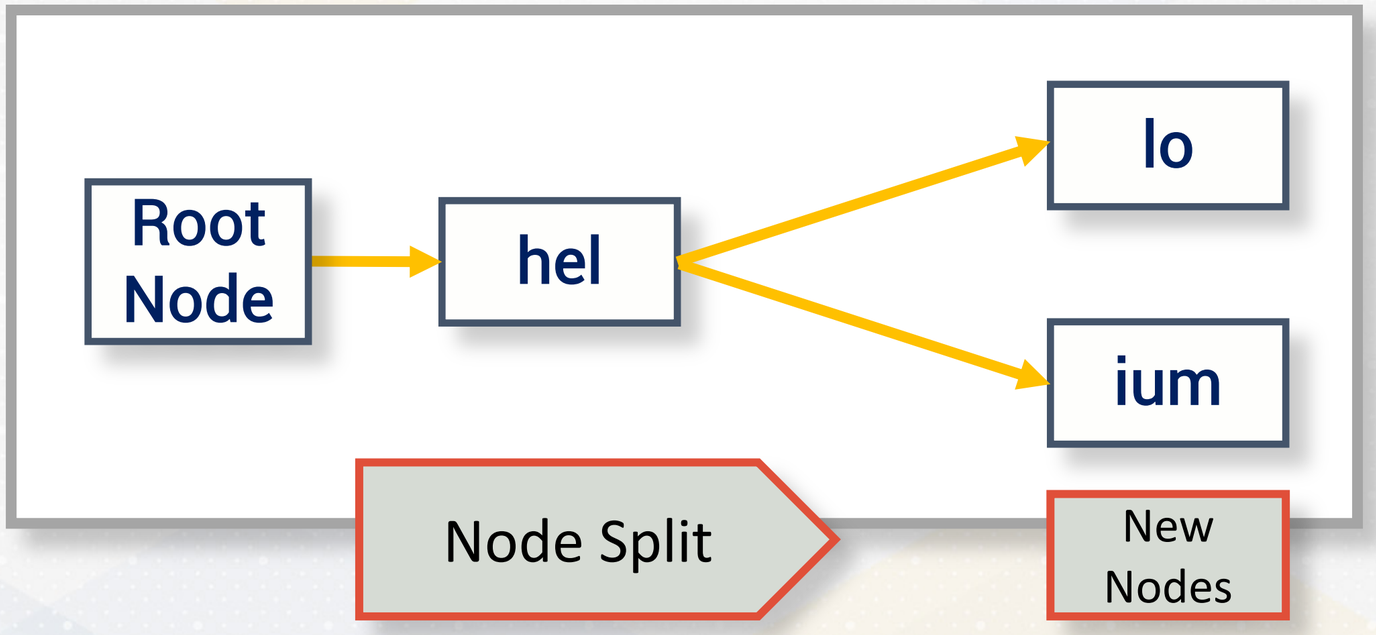
Patricia Trie

“Samsung Galaxy”

Insertion Example: “hello”



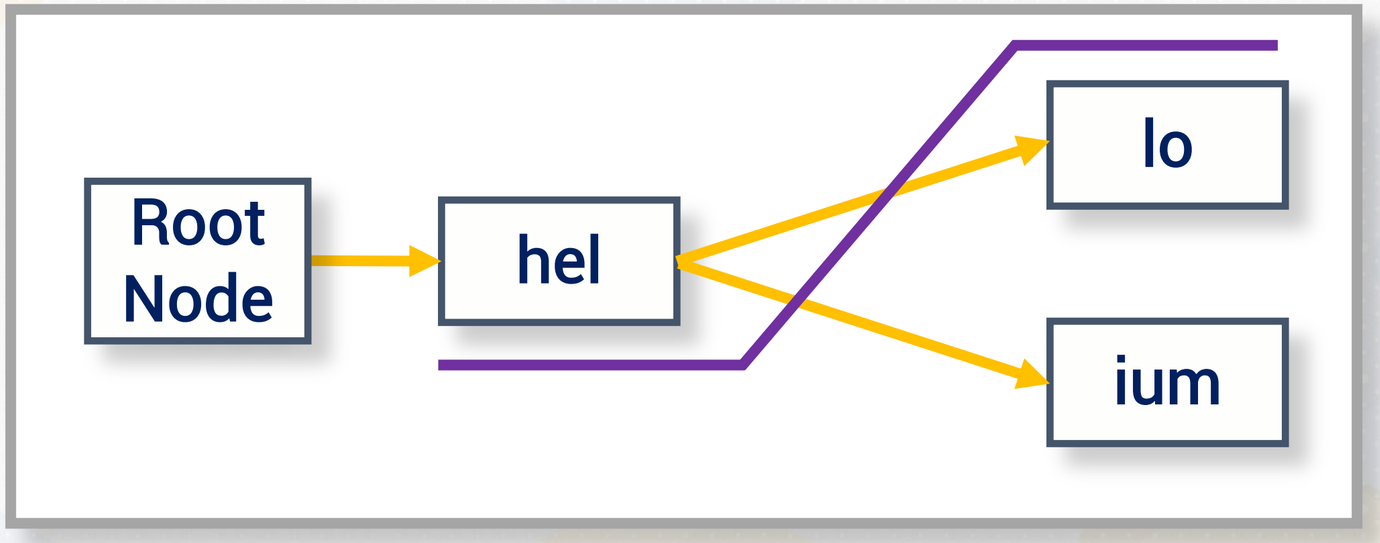
Insertion Example: “helium”



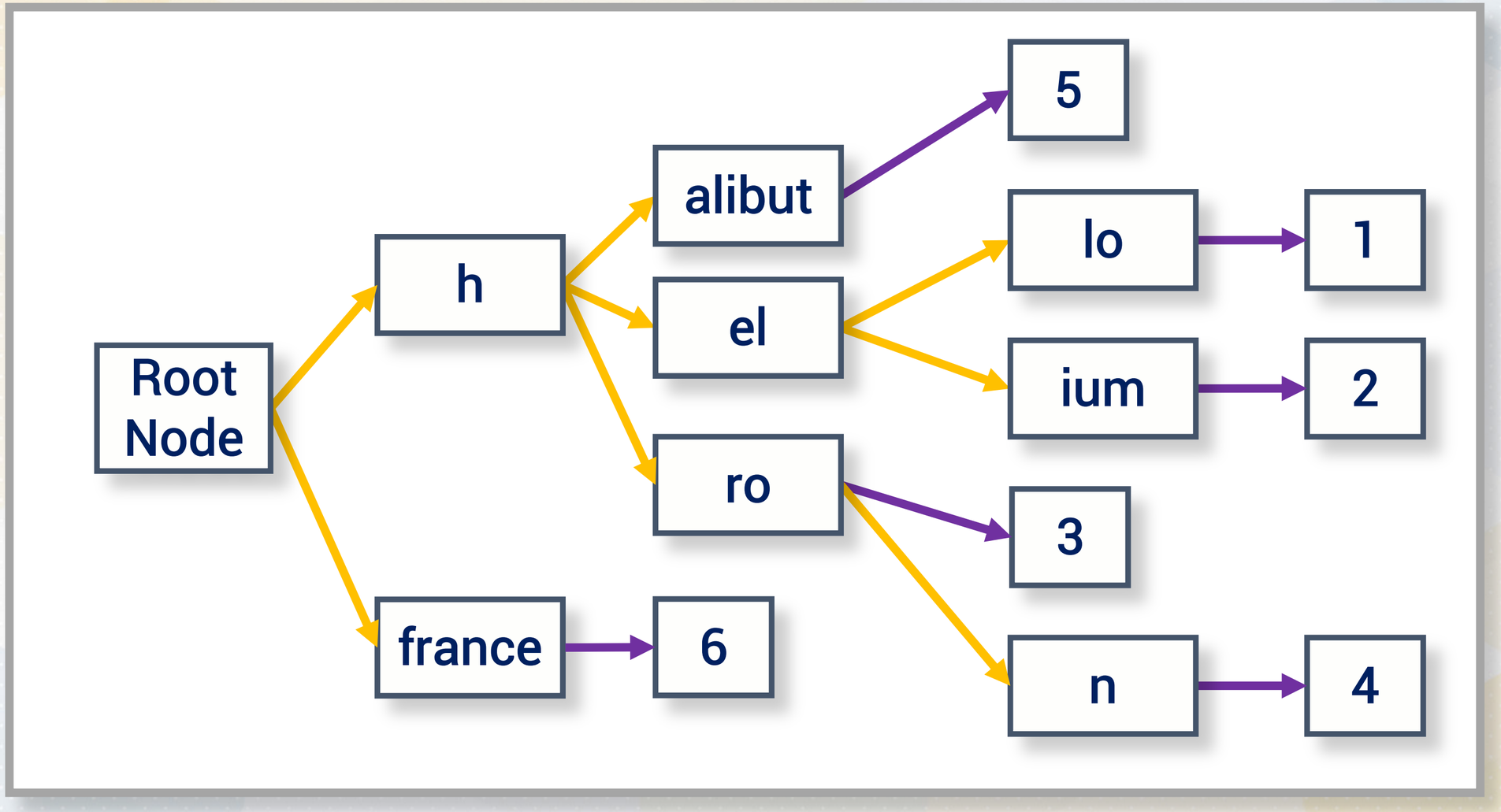
Retrieval: “hello” and “helicopter”

Navigate Root Node to Relevant Leaf Node

Each Character Narrows Down Search



More Complex Trie with Key-Value Pairs



Inverted Index

Inverted Index

Map words to the positions in which they appear in a document.

Word	Positions
russia	10, 20
napolean	25

Proximity Search

distance(“russia”, “napoleon”) less than ten words

Word	Positions
russia	10, 20
napoleon	25

Library Search

Map words to the documents in which they appear.

Word	Document ID, Position
russia	(1, 10), (1, 20)
napoleon	(1, 25), (2, 30), (3, 50)

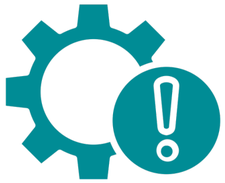
Why Inverted Index?

Inverted Index

Text Data

roximity Search

Internals



Hash Table Foundation

alues are Doc IDs, ositions

Structure of an Inverted Index

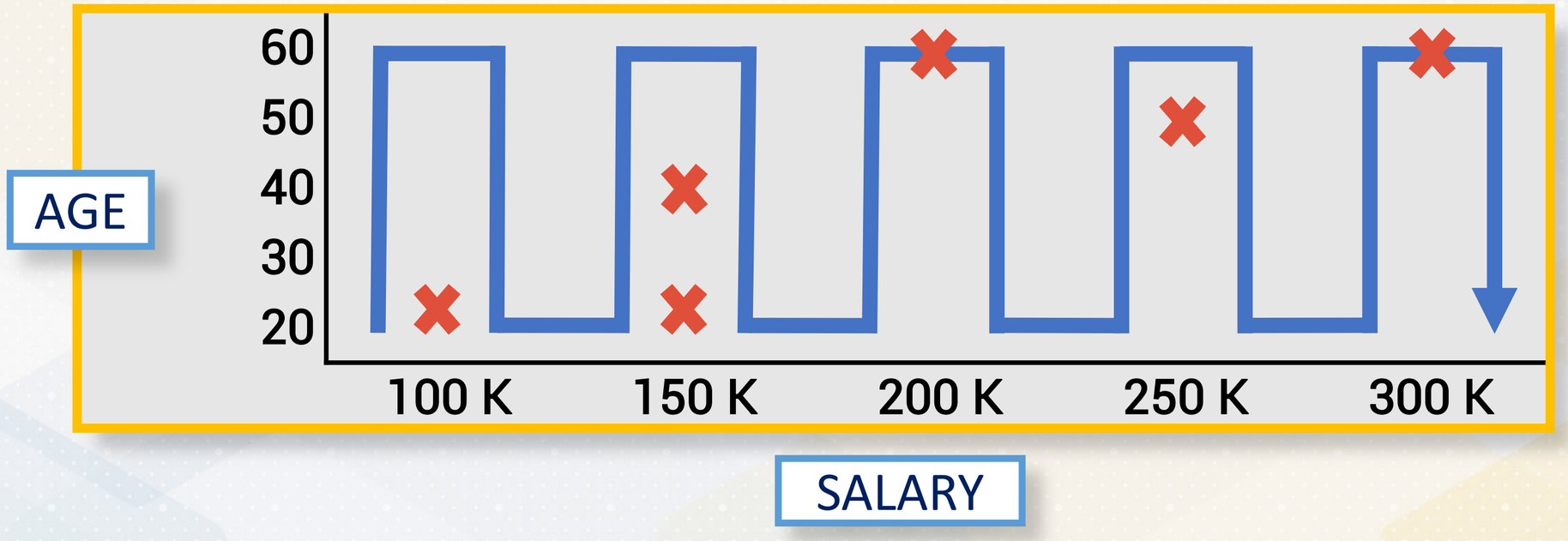
Key	“russia”

Value	{1: {5, 20}, 2: {10}}

```
std::unordered_map<
      std::string,
      std::unordered_map<int, std::vector<int>>
>
```

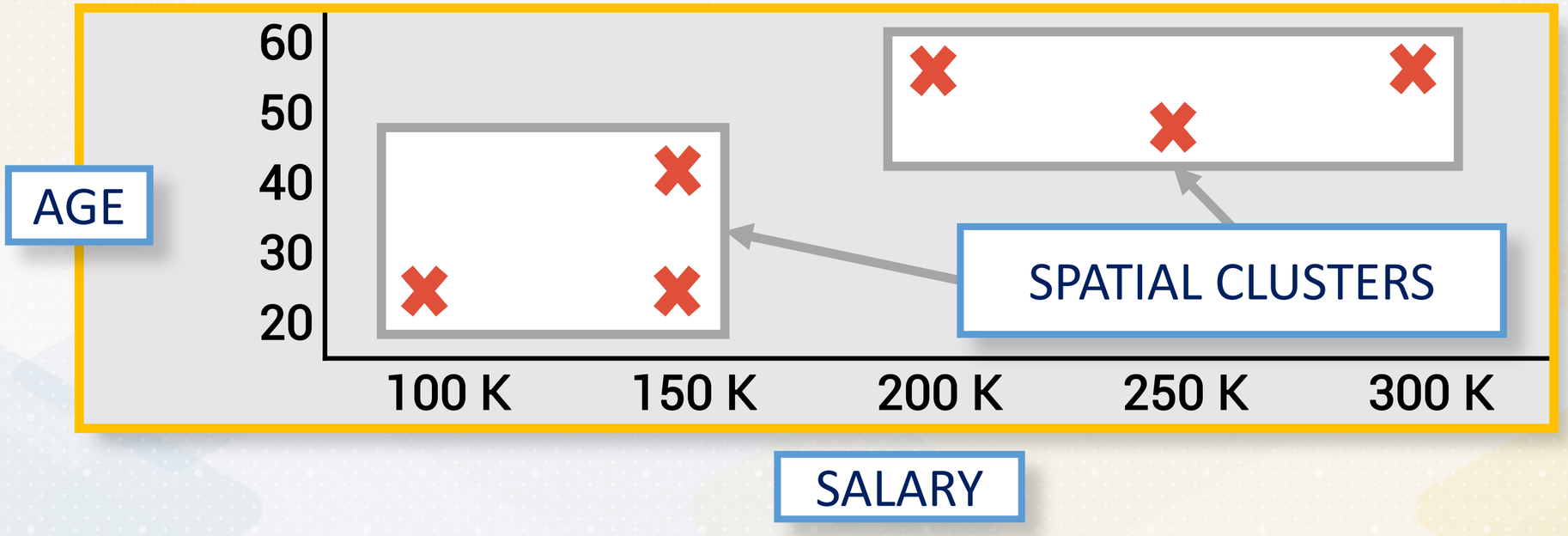
R Tree

Limitations of B+Tree



R-Tree: A Multidimensional Index

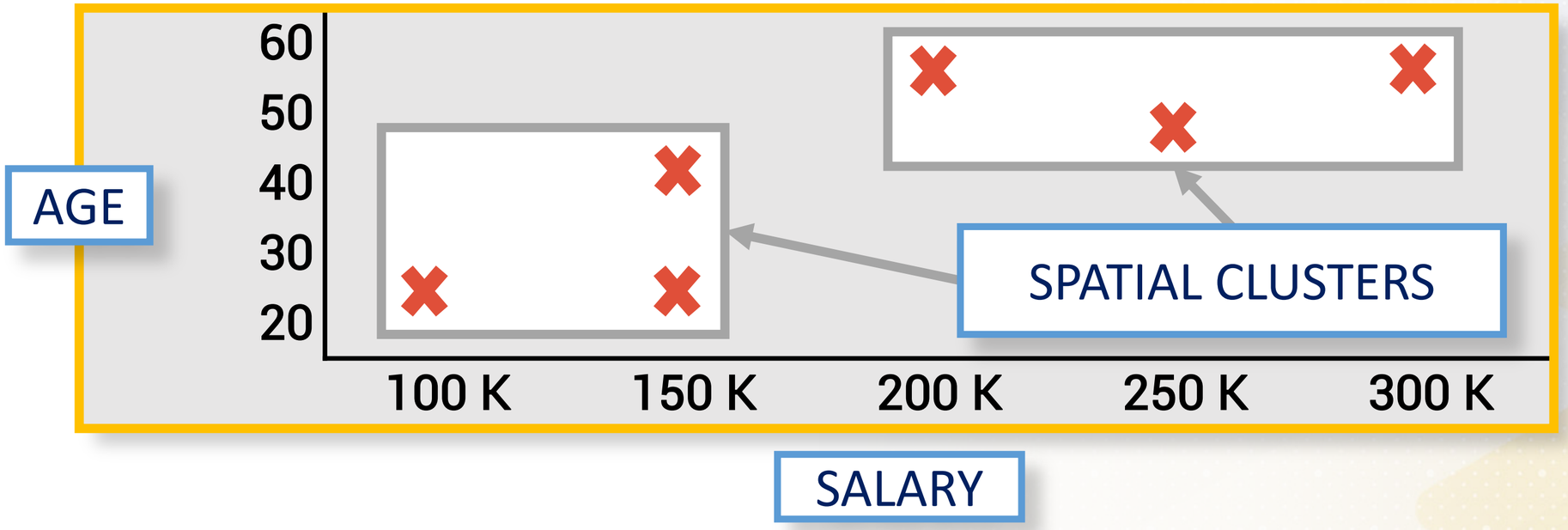
R-Tree groups multi-dimensional keys based on spatial proximity



R-Tree: A Multidimensional Index

(Salary, Age): Point

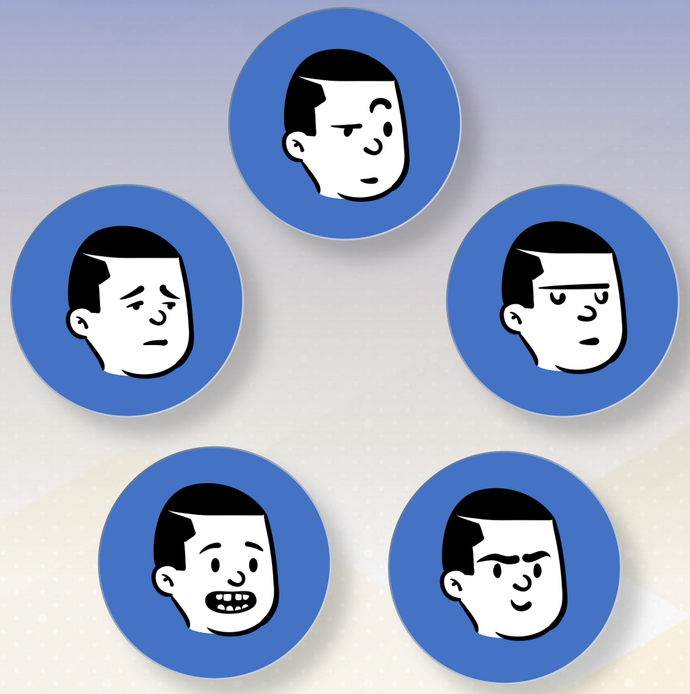
Clusters: Bounding Rectangles



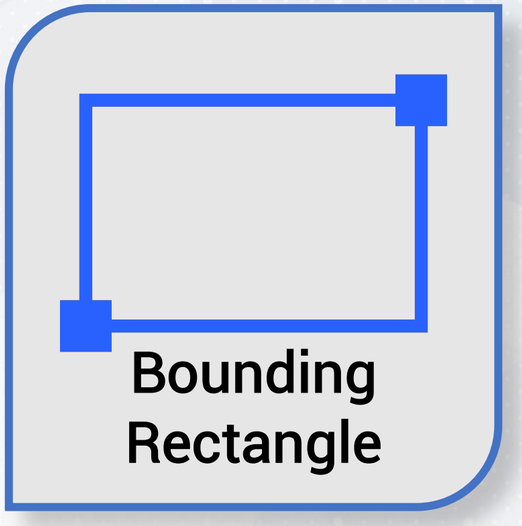
Supported Queries

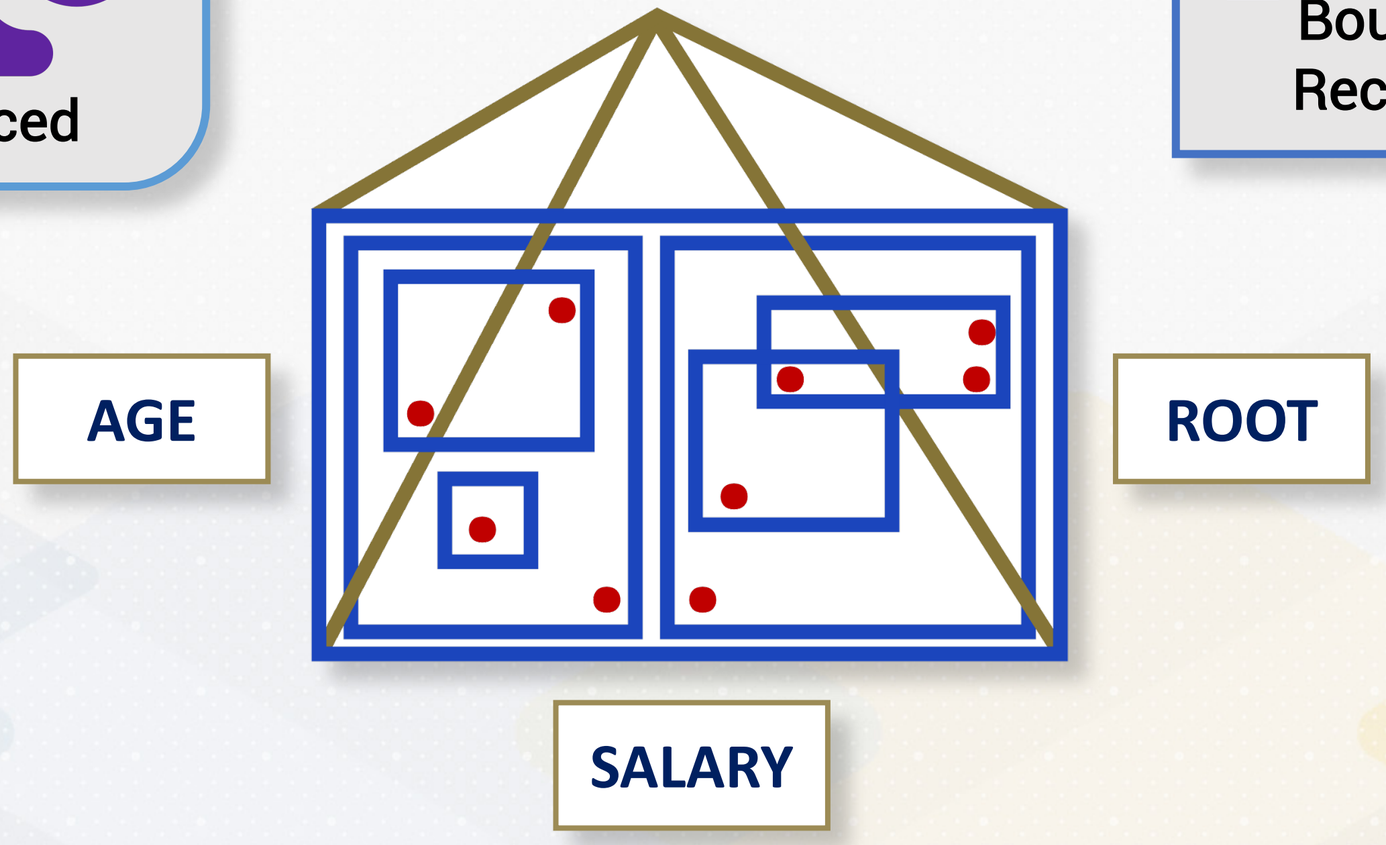
Spatial Queries

Nearest Neighbor Queries

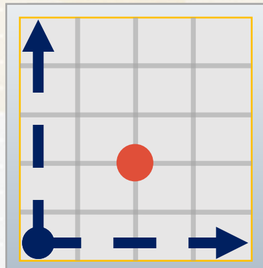


Key Characteristics



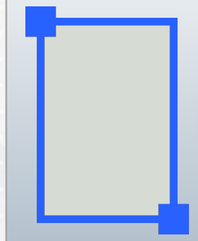


R-Tree Data Types



Point

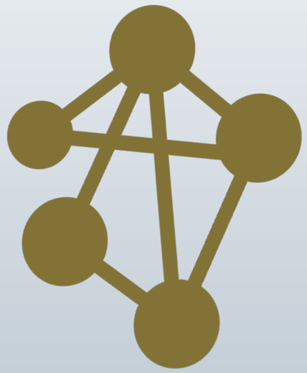
Specific location in space



Rectangle

loses points or other ectangles

R-Tree Nodes



R-Tree Node

Leaf Node

Contains Points; Lowest Level of Tree

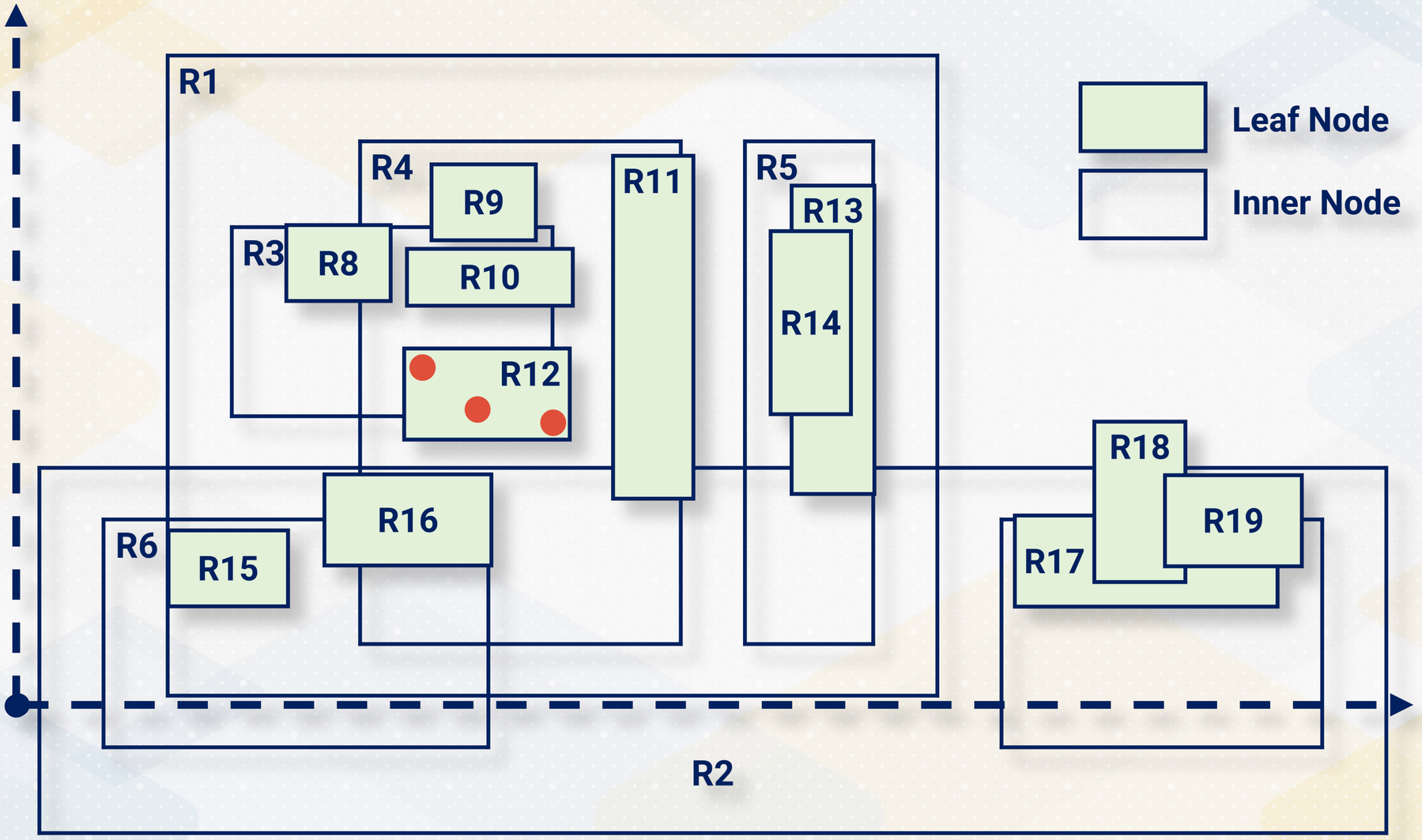
Inner Node

Contains Child Nodes and their Bounding Rectangles

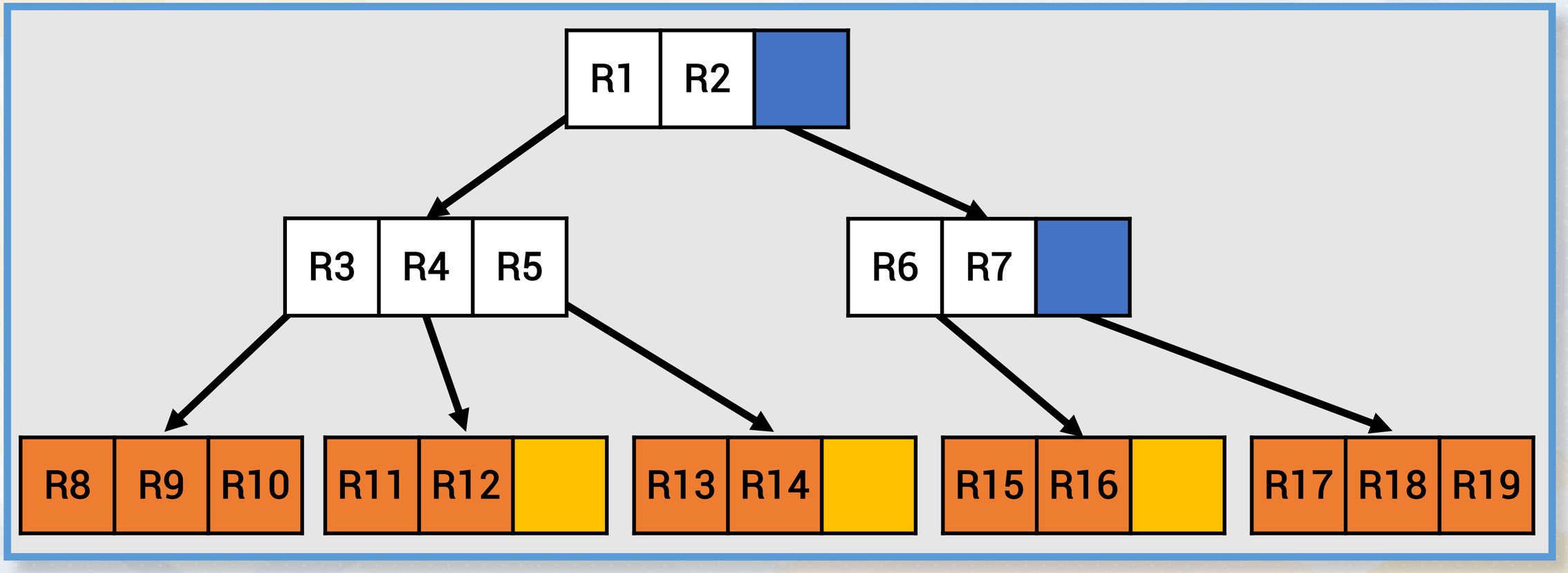
Root Node

Top-Most Inner Node; Entry Point

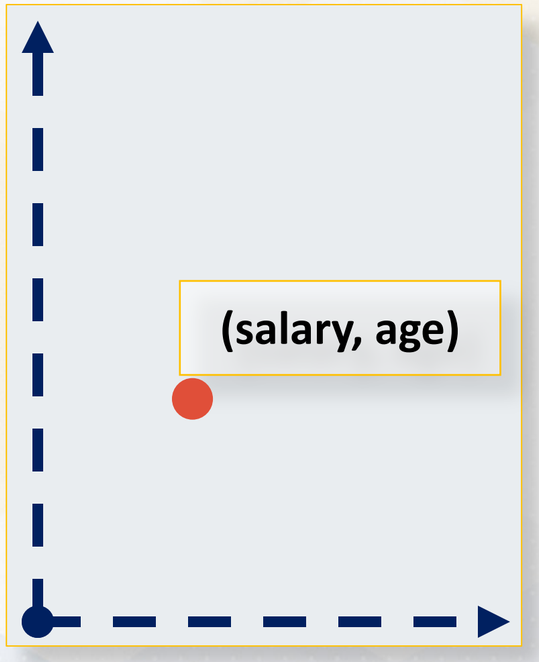
R-Tree Structure



R-Tree Structure



Point



(x, y) = (salary, age)

```
struct Point {
  float x, y;
  Point(float x, float y) : x(x), y(y) {}
};
```

Rectangle

Defines a bounding box using its minimum and maximum coordinates


```
struct Rectangle {
  float minX, minY, maxX, maxY;
  Rectangle(float minX, float minY, float maxX, float maxY)
      : minX(minX), minY(minY), maxX(maxX), maxY(maxY) {}

  bool contains(const Point &p);
  bool intersects(const Rectangle &other) const;
};
```

Conclusion

- Indexing
- Hash Table
- Double Hashing
- Range Query
- B+Tree
- Trie
- RTree