CS 4495 Computer Vision
Photometry

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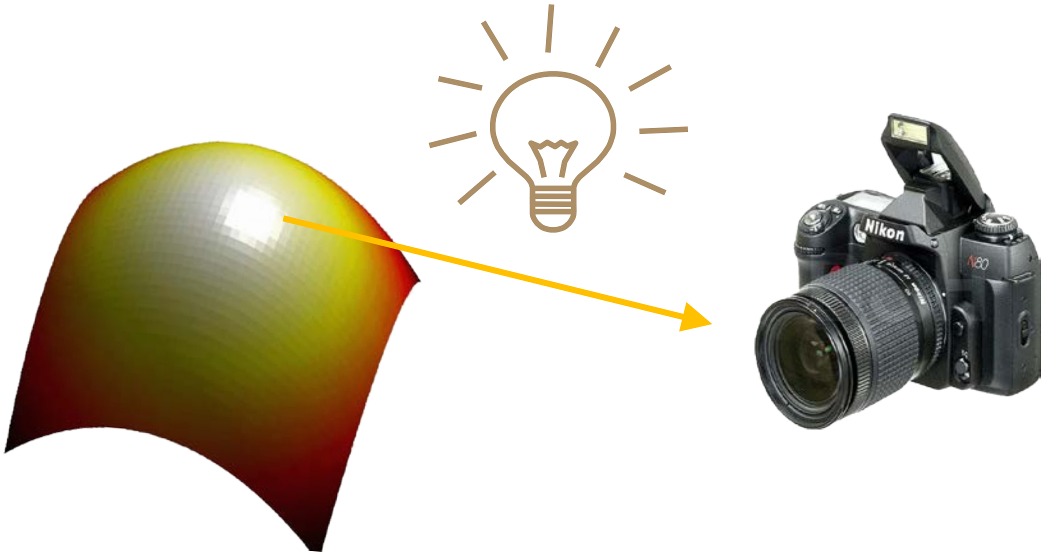
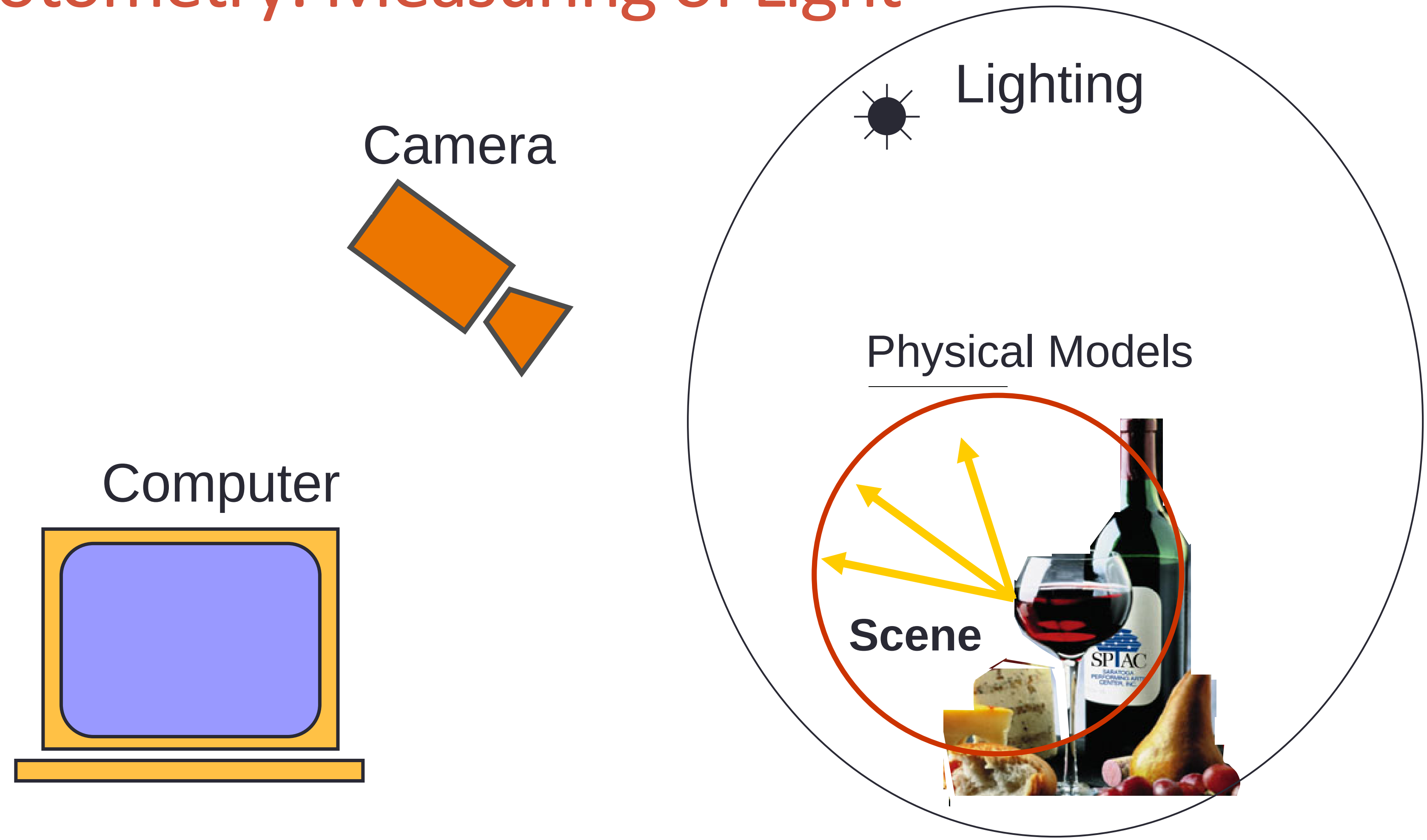
School of Interactive Computing



Thanks to Srinivasa Narasimhan, Shree Nayar, David Kreigman, Marc Pollefeys

Photometry: Measuring of Light



We need to understand the relation between the lighting, surface reflectance and medium and the image of the scene.

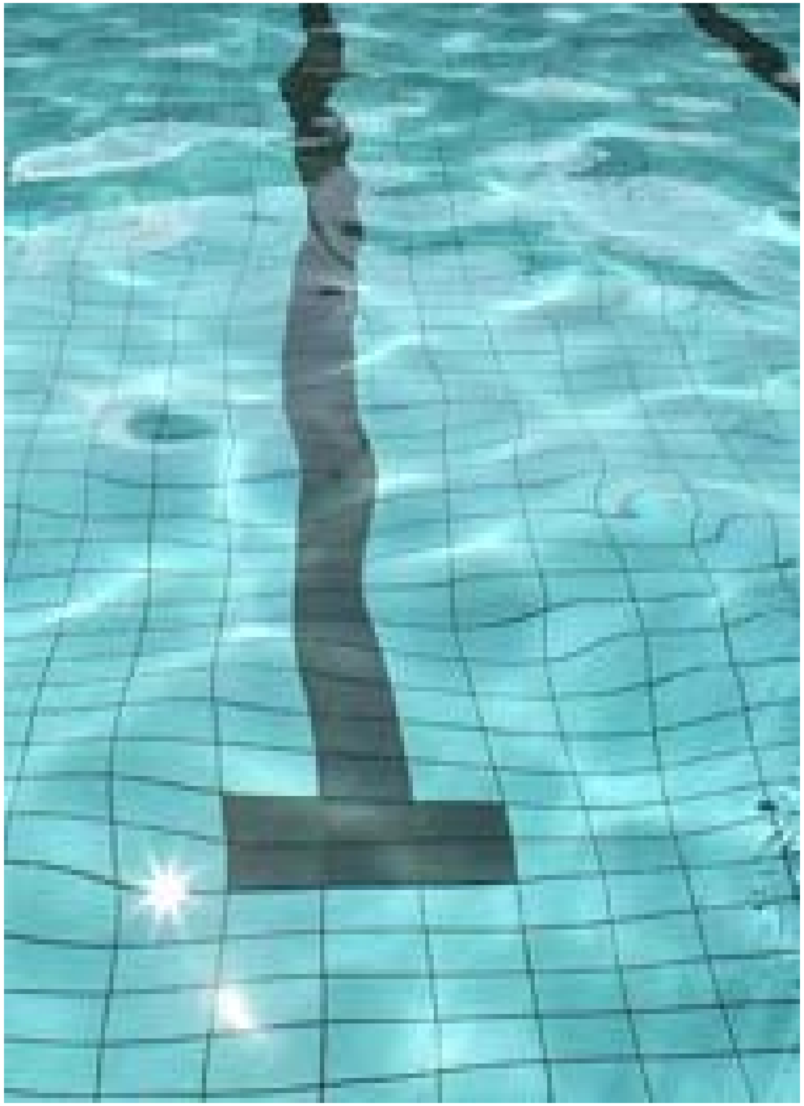
Why study the physics (optics) of the world?

Lets see some pictures!

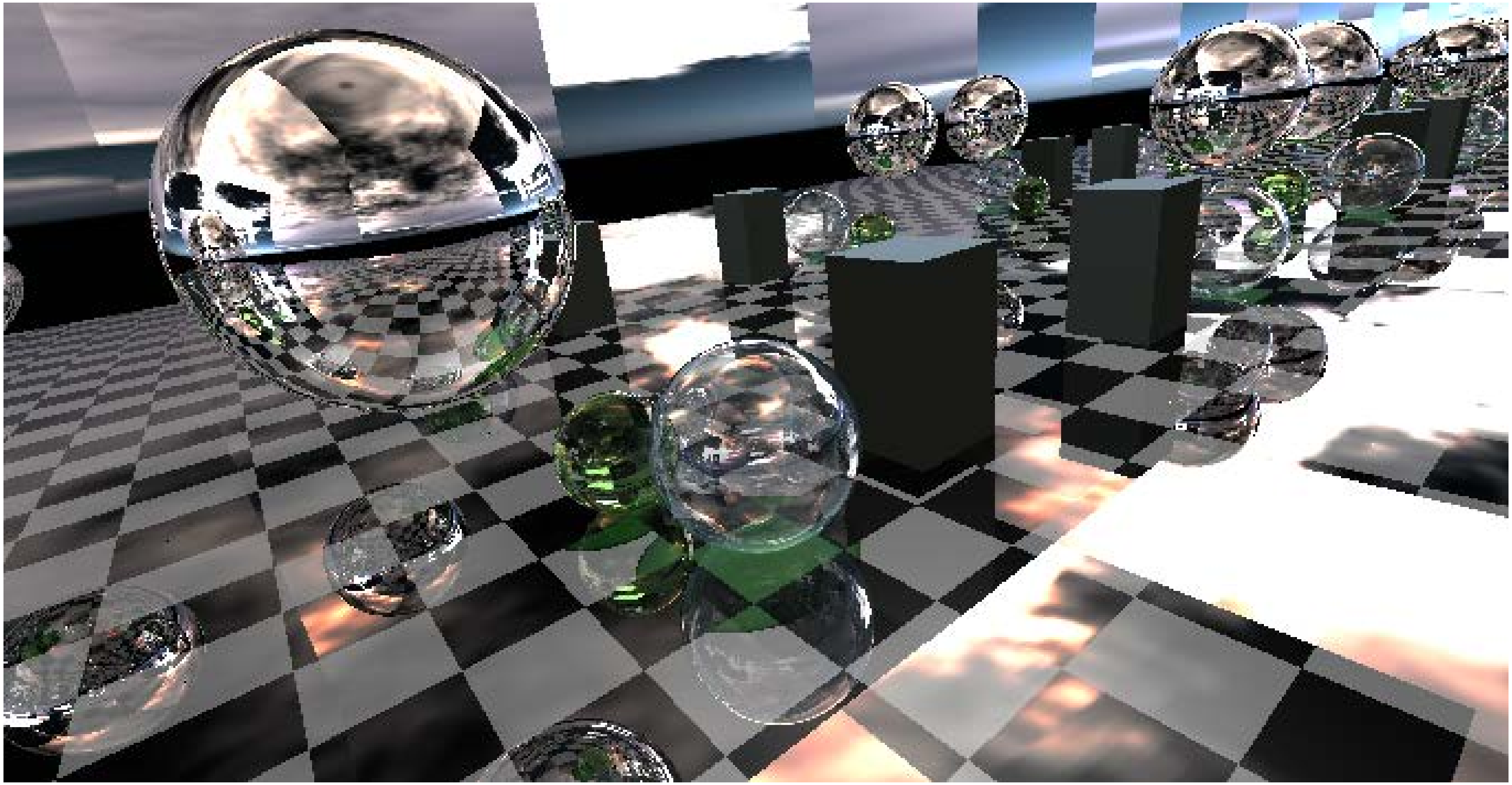
Lights and Shadows

Reflections

Refractions



Interreflections



Scattering

And many more

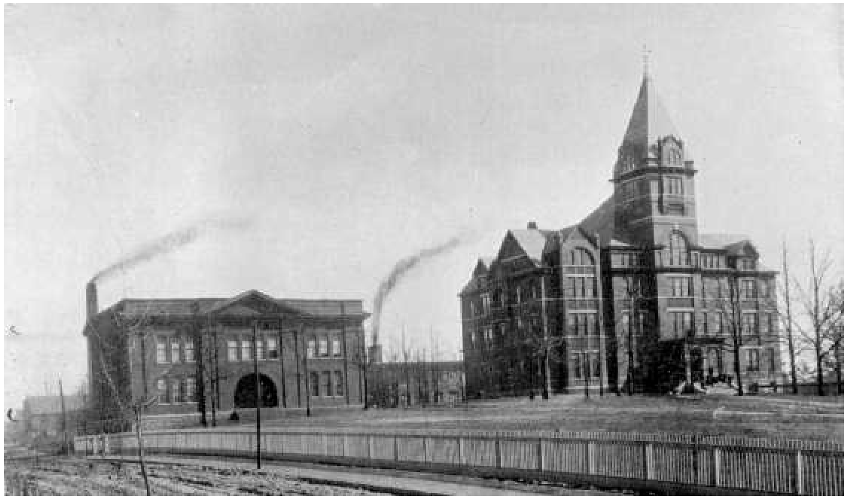




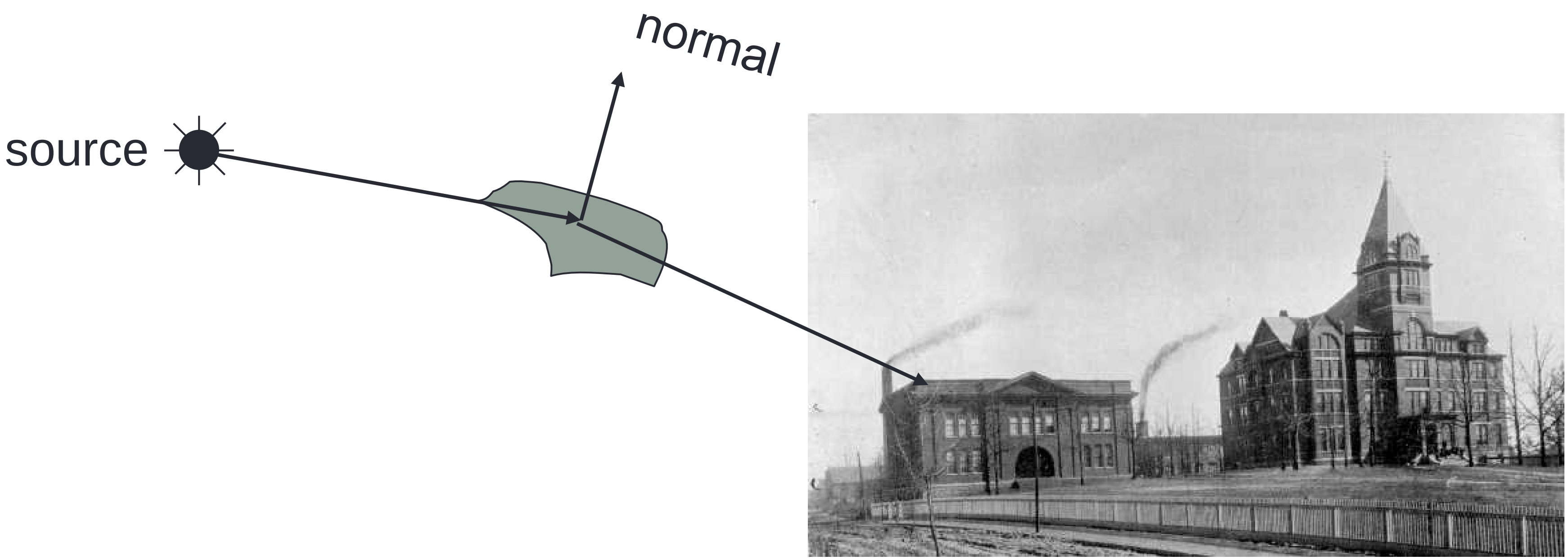




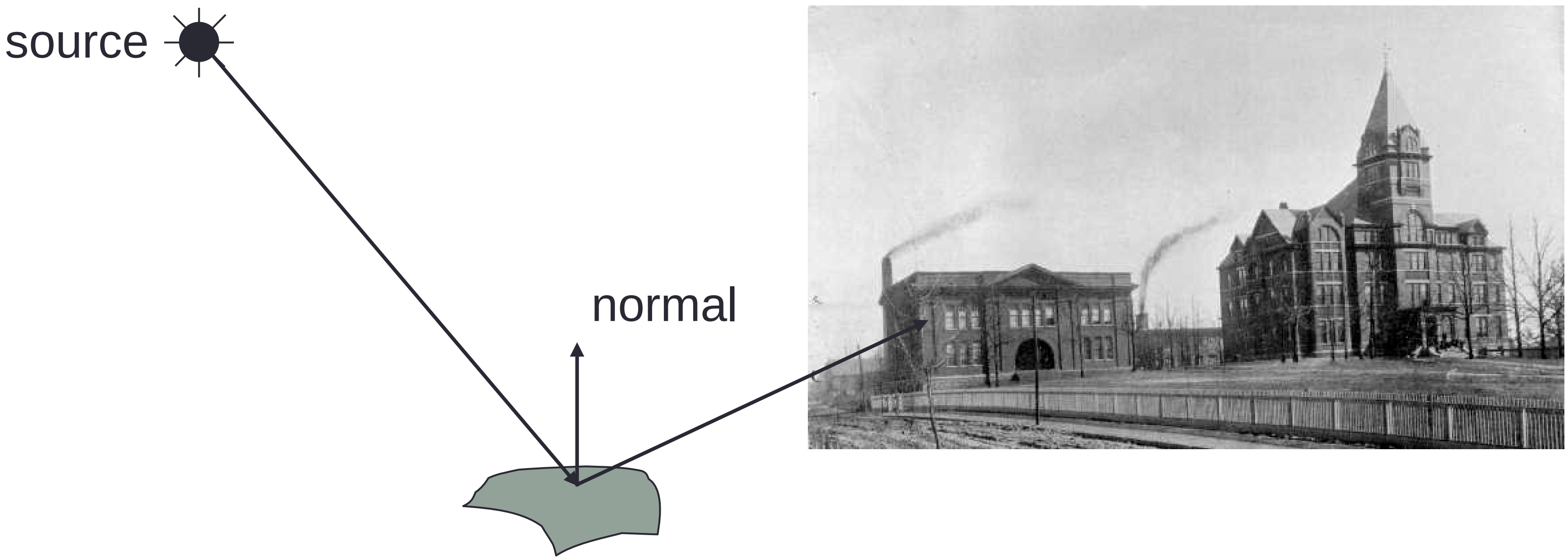
Surface Appearance



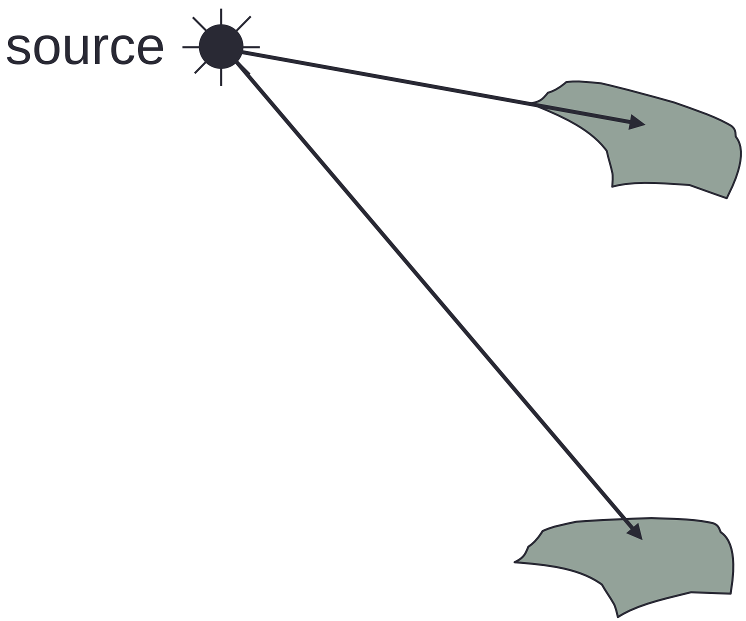
Surface Appearance

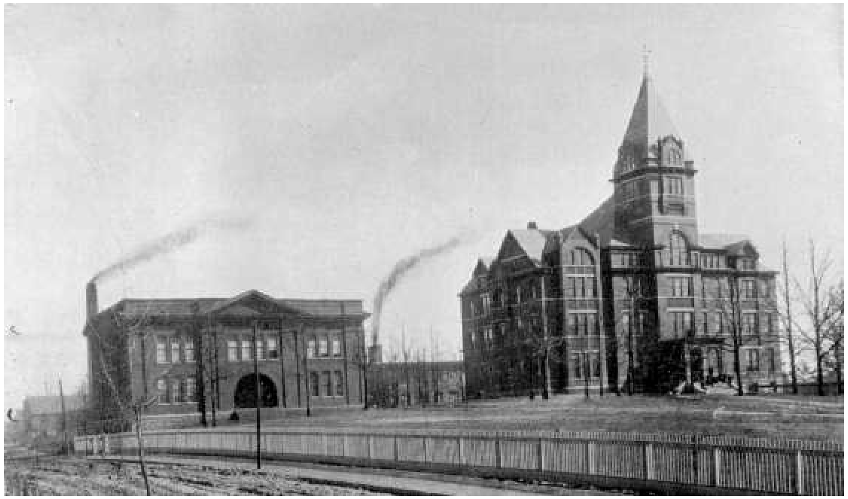


Surface Appearance



Surface Appearance

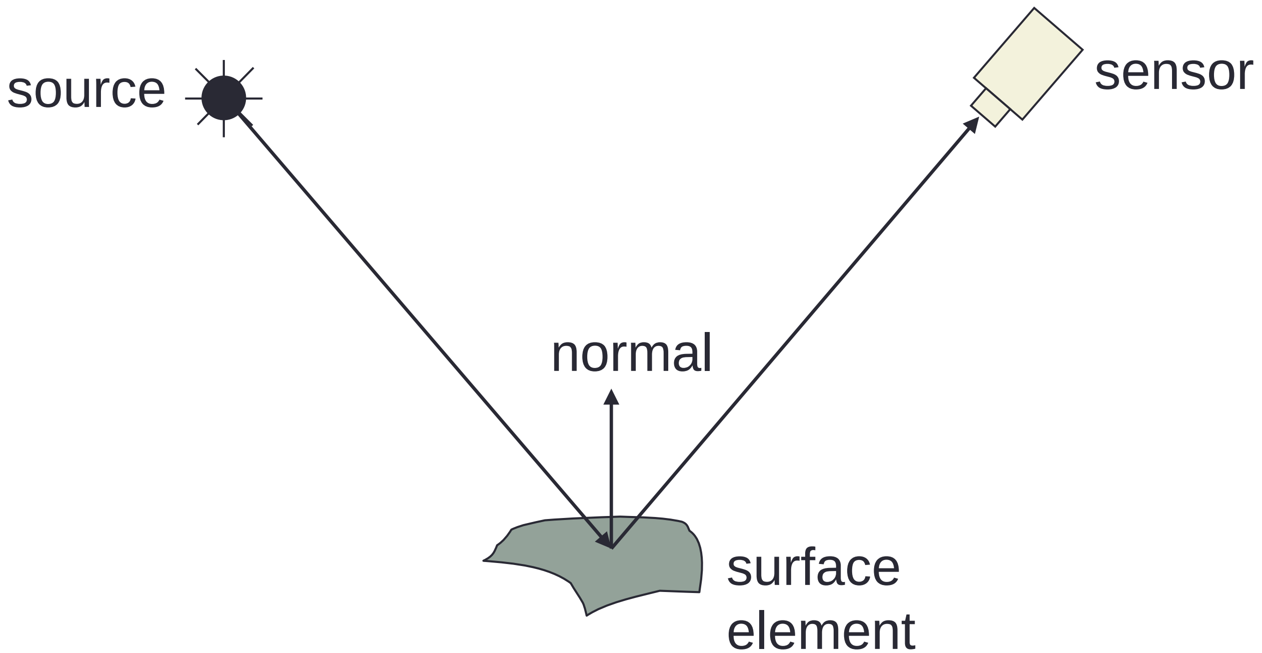




- Image intensities

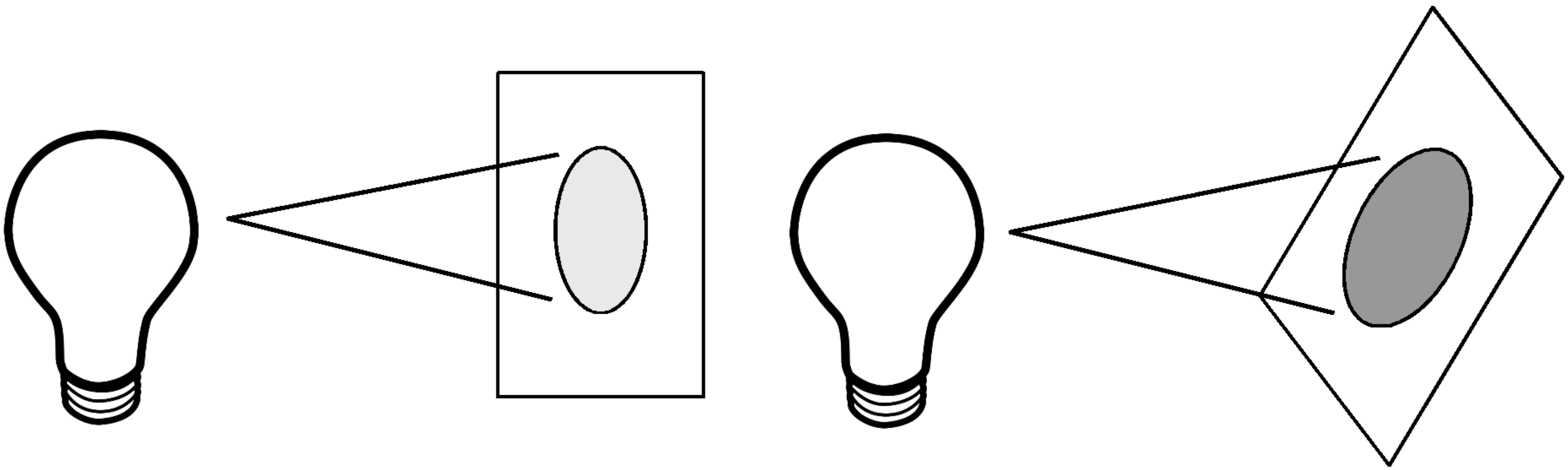
  = f (normal, surface reflectance, illumination)

Surface Appearance



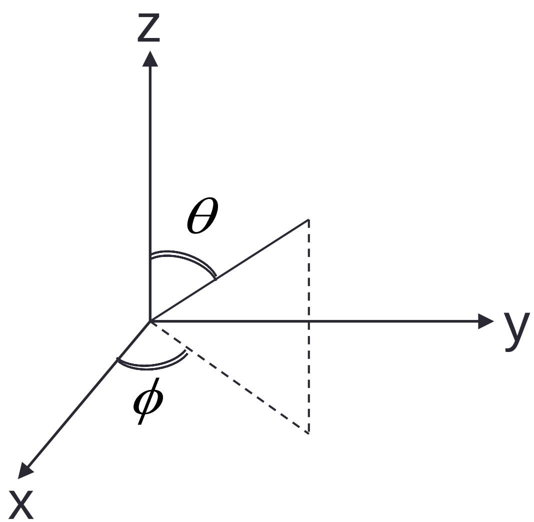
- Image intensities = f (normal, surface reflectance, illumination)
- Surface reflection depends on both the viewing and illumination directions.

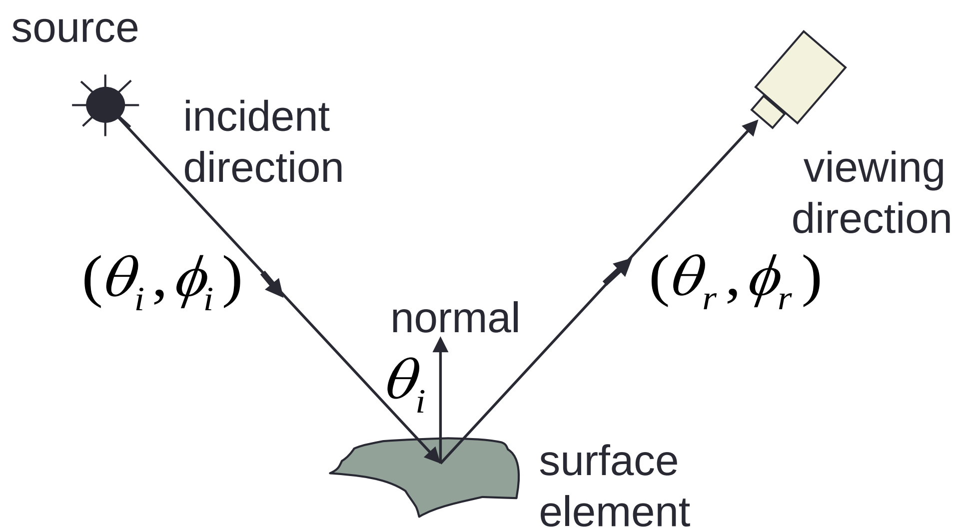
Foreshortening: A Simple Observation



As the surface tilts away from the light source the same light energy is spread over a larger area, making the surface darker

BRDF: Bidirectional Reflectance Distribution Function





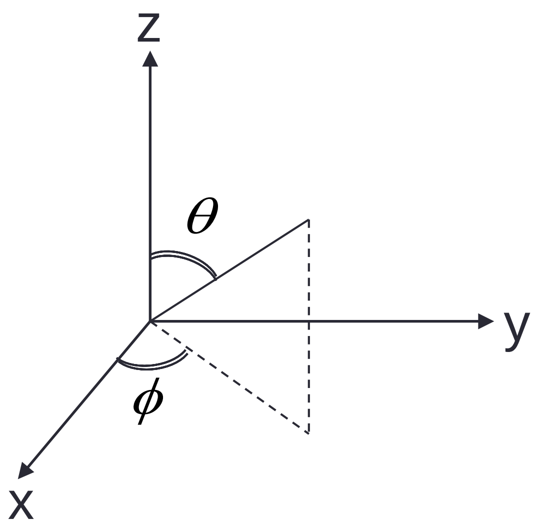
$E^{surface}(\theta_i, \phi_i)$ Irradiance at Surface in direction (θ_i, ϕ_i)

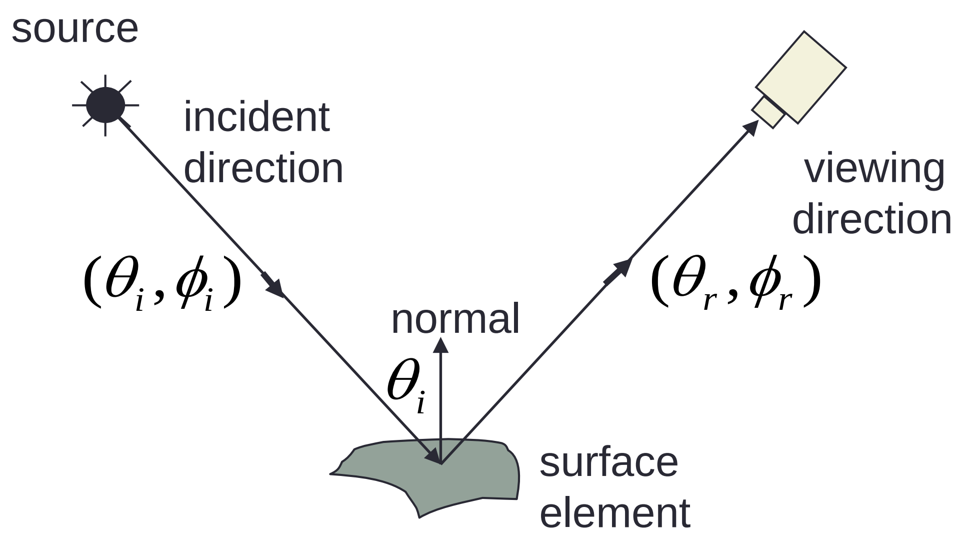
$L^{surface}(\theta_r, \phi_r)$ Radiance of Surface in direction (θ_r, ϕ_r)

Irradiance: power per unit area incident on the surface

Radiance: power from the surface from a given direction within a given solid angle

BRDF: Bidirectional Reflectance Distribution Function



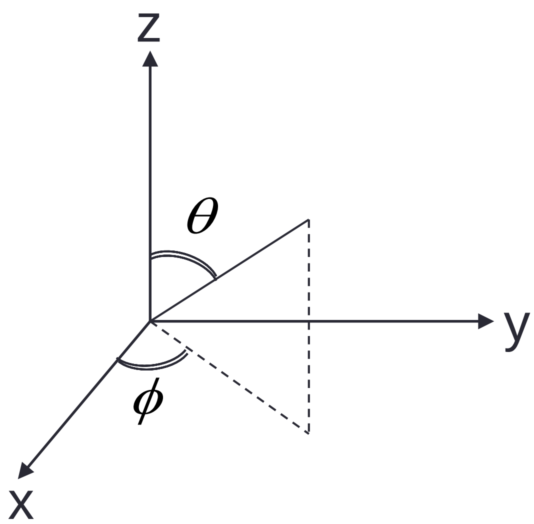


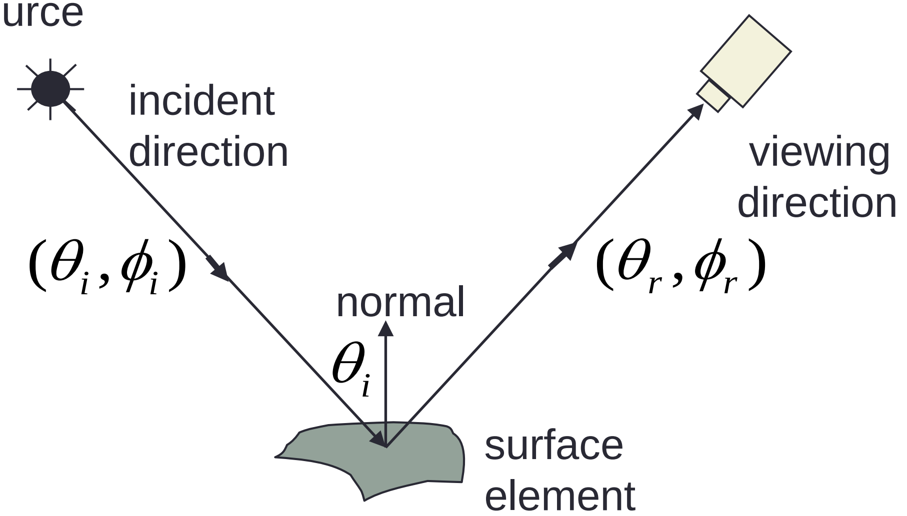
$E^{surface}(\theta_i, \phi_i)$ Irradiance at Surface in direction (θ_i, ϕ_i)

$L^{surface}(\theta_r, \phi_r)$ Radiance of Surface in direction (θ_r, ϕ_r)

$$E^{surface}(\theta_i, \phi_i) \sim \cos\theta_i \, L^{surface}(\theta_i, \phi_i)$$

BRDF: Bidirectional Reflectance Distribution Function



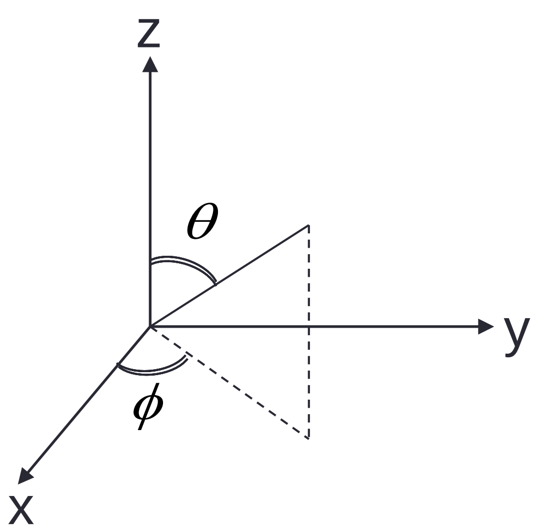


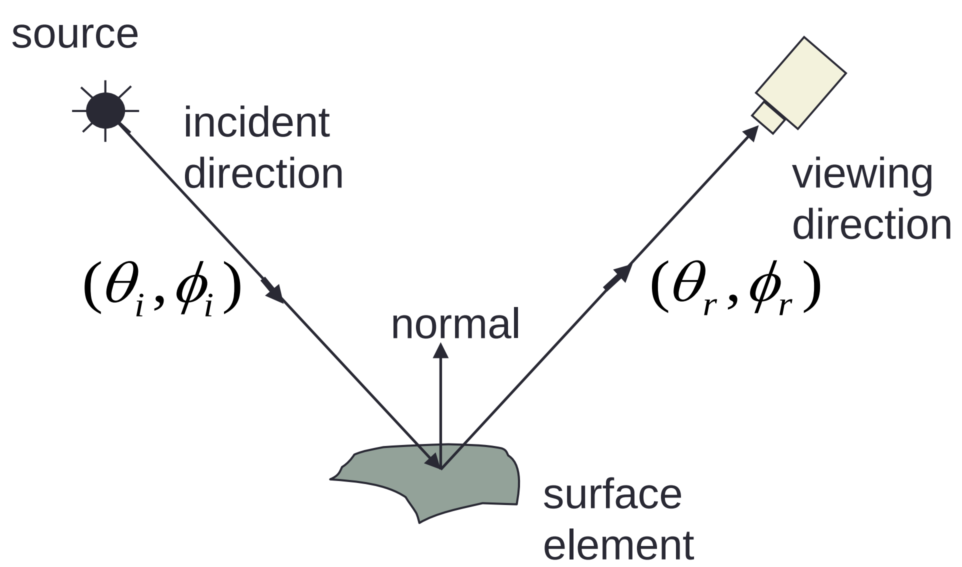
$E^{surface}(\theta_i, \phi_i)$ Irradiance at Surface in direction (θ_i, ϕ_i)

$L^{surface}(\theta_r, \phi_r)$ Radiance of Surface in direction (θ_r, ϕ_r)

$$\text{BRDF}: \; f(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{L^{surface}(\theta_r, \phi_r)}{E^{surface}(\theta_i, \phi_i)}$$

Important Properties of BRDFs



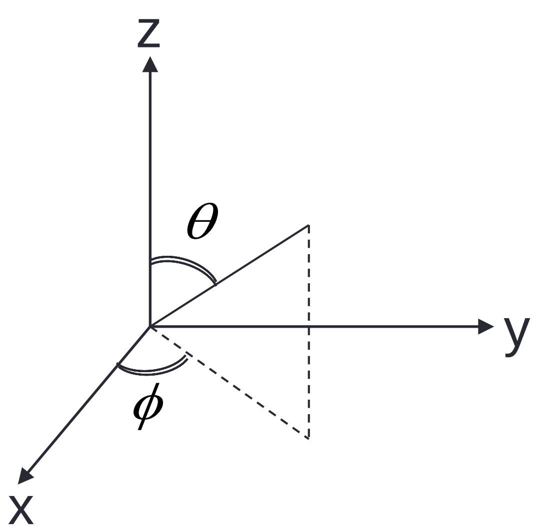


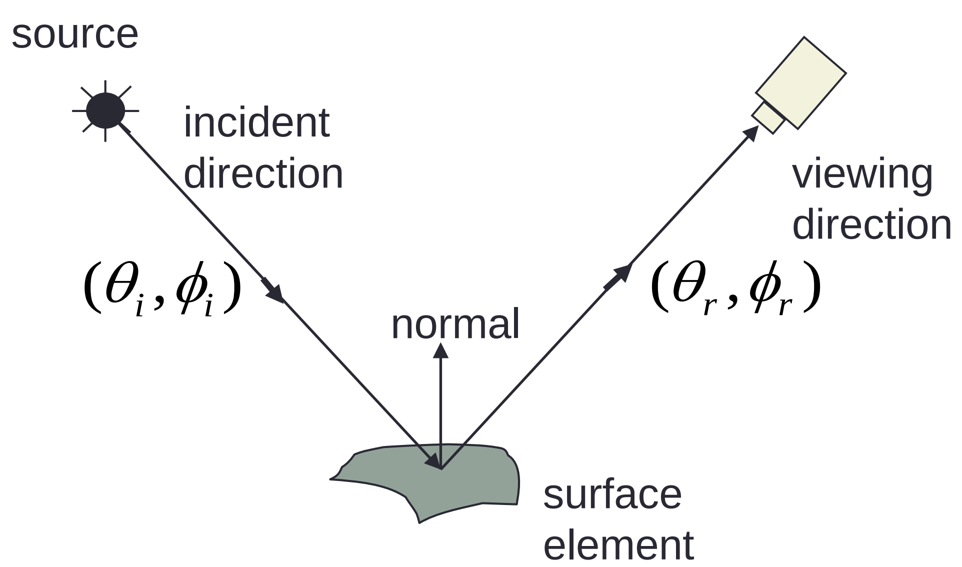
- Conservation of Energy:

$$\int_{hemisphere} f(\theta_i, \phi_i; \theta_r, \phi_r) \cos\theta_i d\omega_i \leq 1$$

Input power can not be larger than output power (surface not emitting light)

Important Properties of BRDFs



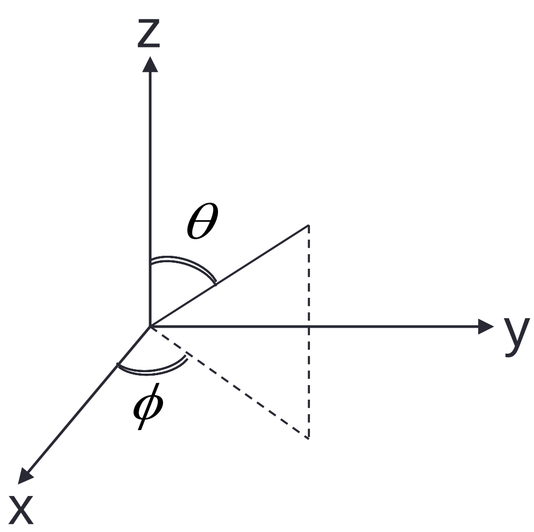


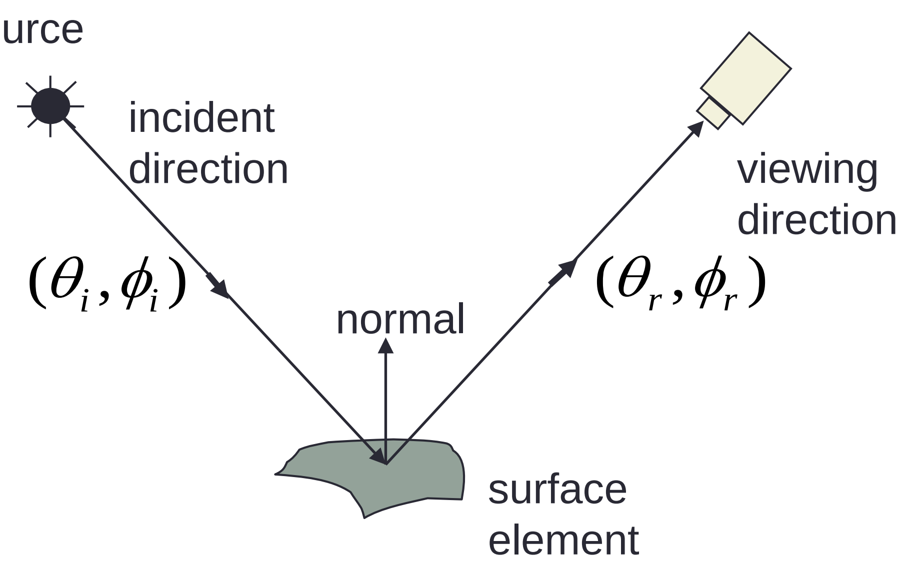
- Helmholtz Reciprocity: (follows from 2nd Law of Thermodynamics)

BRDF does not change when source and viewing directions are swapped.

$$f(\theta_i, \phi_i; \theta_r, \phi_r) \quad = \quad f(\theta_r, \phi_r; \theta_i, \phi_i)$$

Important Properties of BRDFs



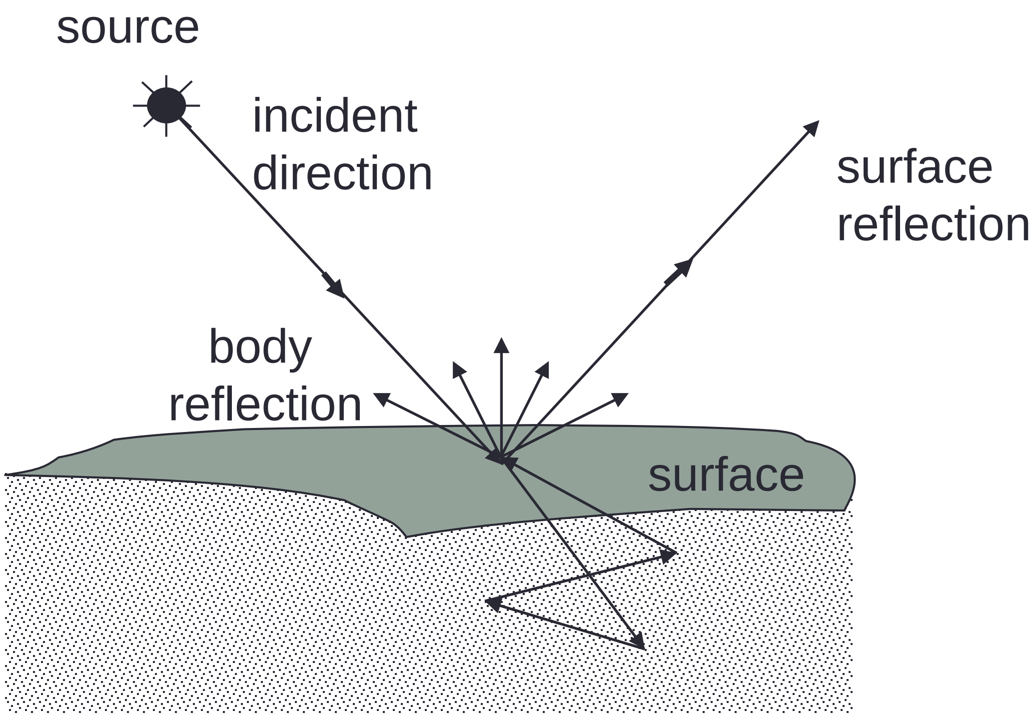


- Rotational Symmetry (Isotropy):

BRDF does not change when surface is rotated about the normal.

Can be written as a function of 3 variables : $f(\theta_i, \theta_r, \phi_i - \phi_r)$

Reflection!



- Body Reflection:

  Diffuse Reflection
  Matte Appearance
  Non-Homogeneous Medium
  Clay, paper, etc

- Surface Reflection:

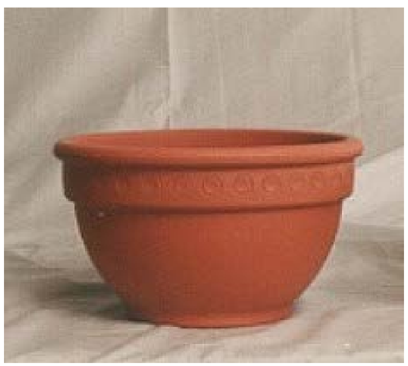
  Specular Reflection
  Glossy Appearance
  Highlights
  Dominant for Metals

Image Intensity = Body Reflection + Surface Reflection

Reflections: Example Surfaces

Body Reflection:

Diffuse Reflection
Matte Appearance
Non-Homogeneous Medium
Clay, paper, etc

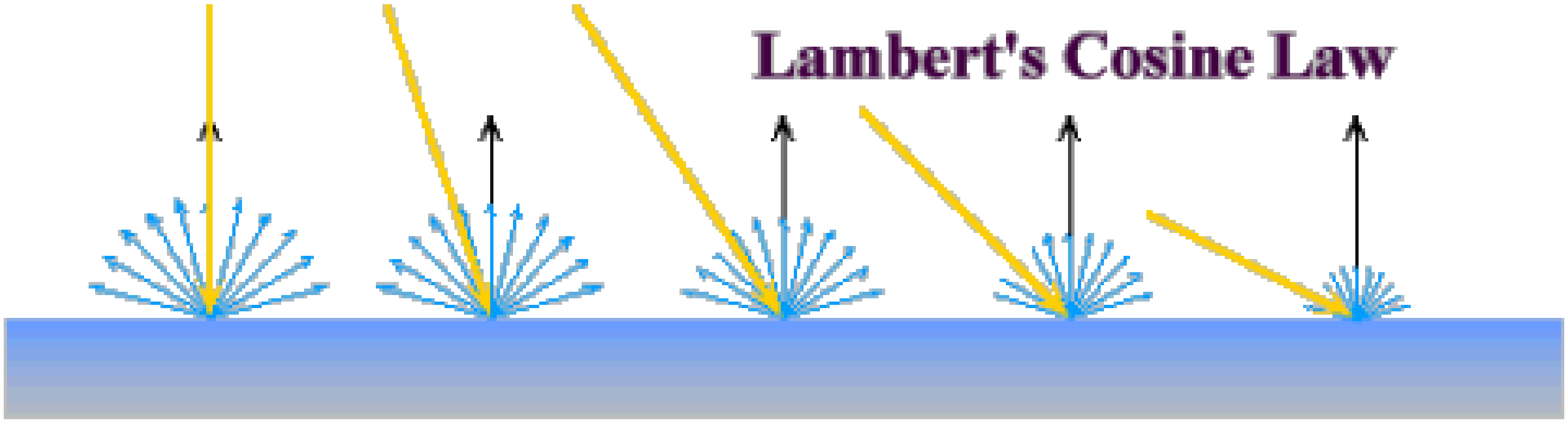


Surface Reflection:

Specular Reflection
Glossy Appearance
Highlights
Dominant for Metals

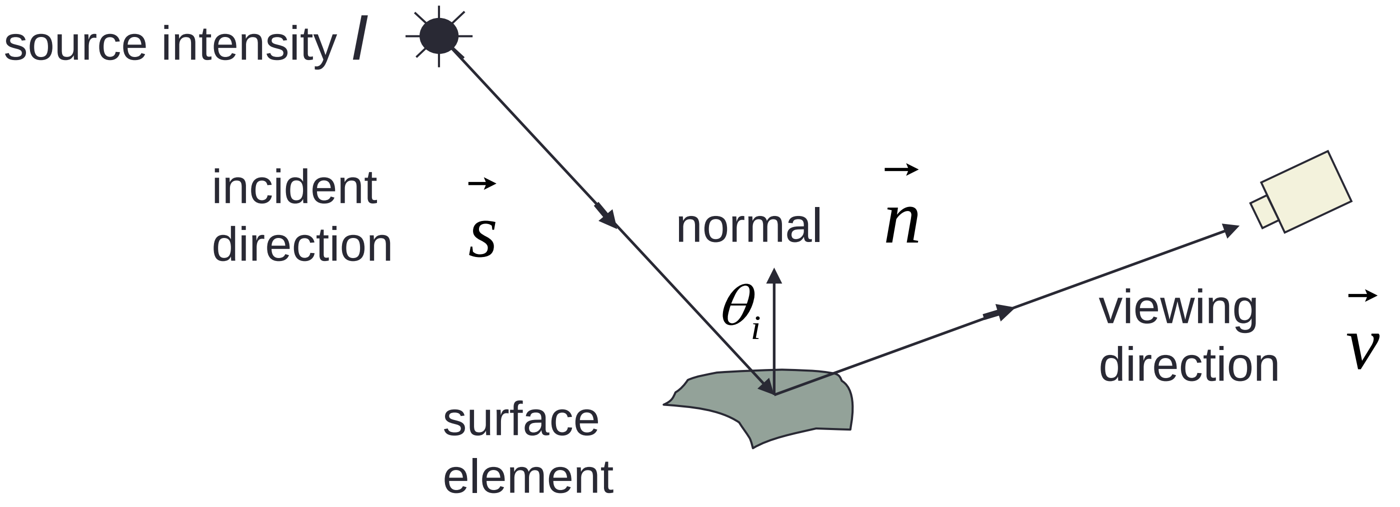
Many materials exhibit **both Reflections**:

Diffuse Reflection and Lambertian BRDF



- Only body reflection, and no specular reflection
- BRDF is independent of outgoing direction
- BRDF depends on indent direction (foreshortening)

Diffuse Reflection and Lambertian BRDF



- Surface appears equally bright from **ALL** directions! (independent of $\vec{v}$)
- Lambertian BRDF is simply a constant : $f(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{\rho_d}{\pi}$ Albedo "whiteness"
- Surface Radiance : $L = \frac{\rho_d}{\pi} I \cos\theta_i = \frac{\rho_d}{\pi} I \; \vec{n} \cdot \vec{s}$ source intensity
- Commonly used in Vision and Graphics!

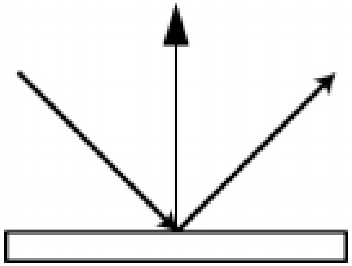
A Small Test

CAN’T perceive the shape of the snow covered terrain!

CAN perceive shape in regions lit by the street lamp!!

WHY?

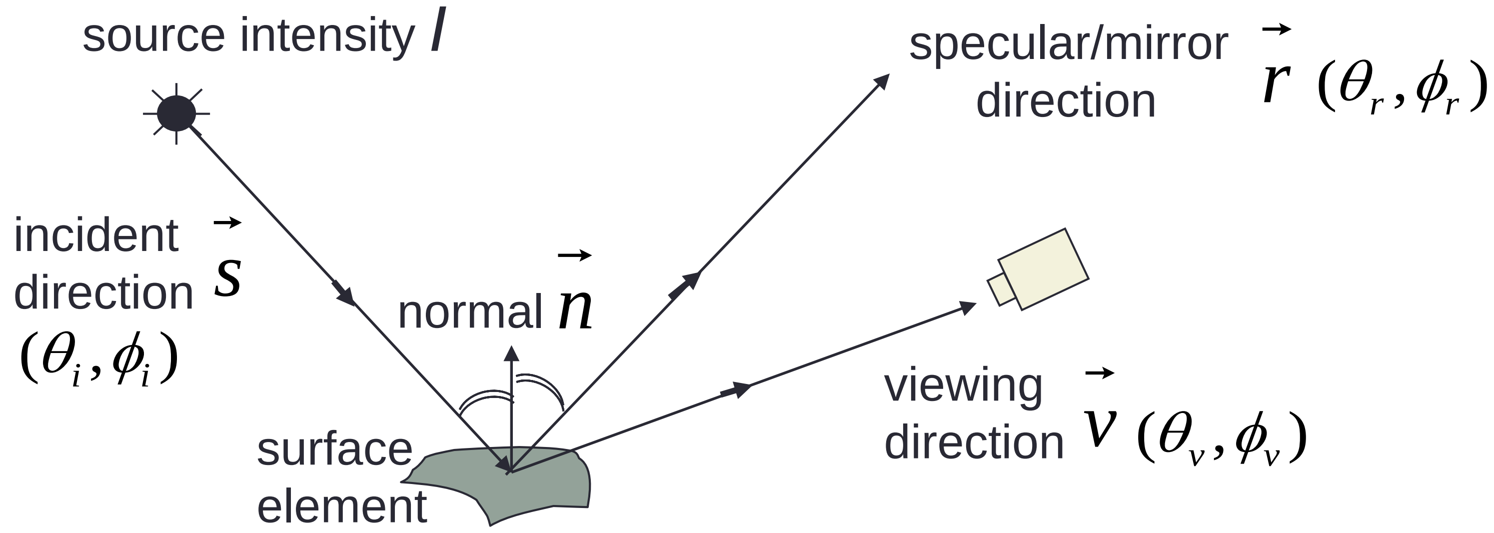
Specular Reflection and Mirror BRDF



How about a mirror?

Reflection **ONLY** at mirror angle

Specular Reflection and Mirror BRDF



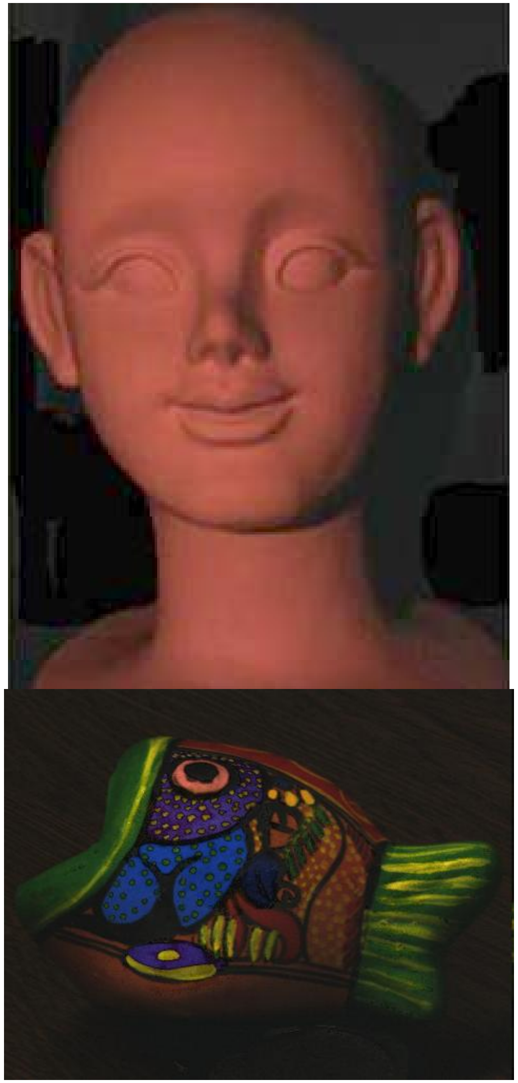
- Valid for very smooth surfaces.
- All incident light energy reflected in a **SINGLE** direction (only when $\vec{v} = \vec{r}$).
- Mirror BRDF is simply a double-delta function :

specular albedo

$$f(\theta_i, \phi_i; \theta_v, \phi_v) = \frac{\rho_s}{cos\theta_i} \delta(\theta_i - \theta_v)\, \delta(\phi_i + \pi - \phi_v)$$

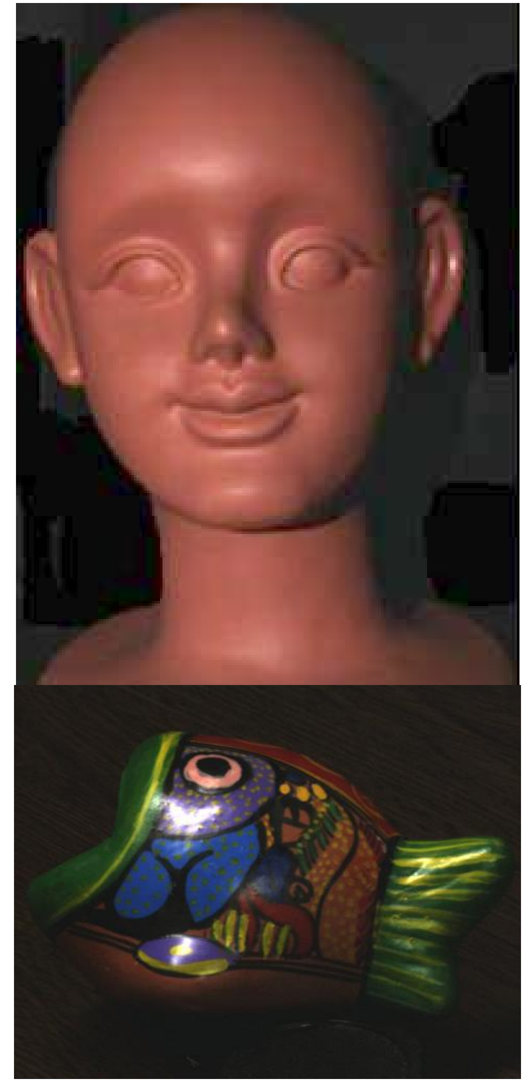
- Surface Radiance : $L = I\ \rho_s\ \delta(\theta_i - \theta_v)\, \delta(\phi_i + \pi - \phi_v)$

Diffusion + Specular Reflection



diffuse

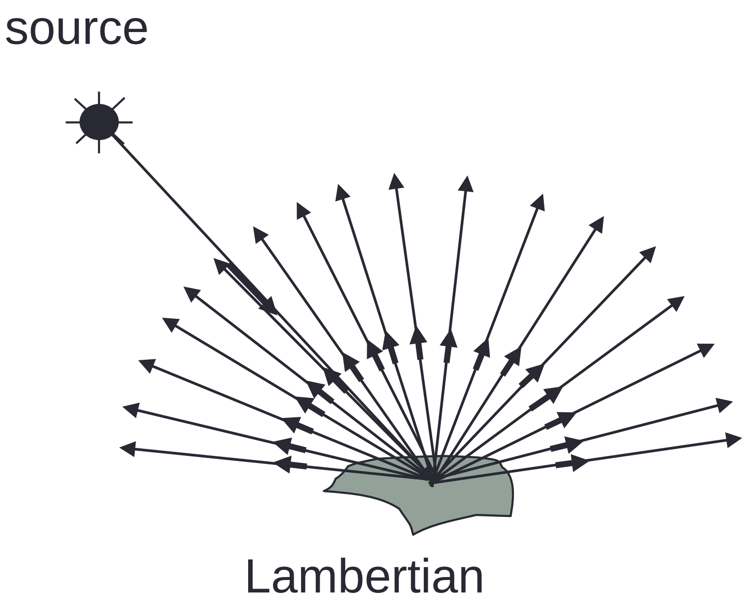
specular

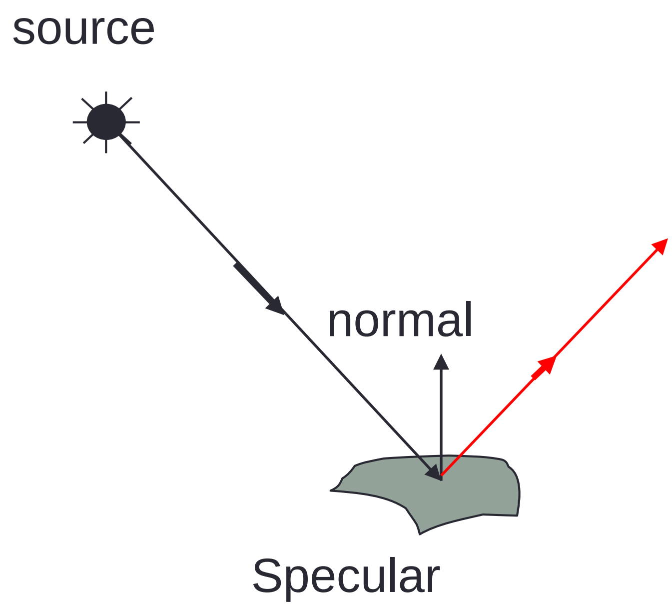


diffuse+specular

Phong Reflection Model

- The BRDF of many surfaces can be approximated by The Lambertian + Specular Model





Phong Reflection Model

- The BRDF of many surfaces can be approximated by
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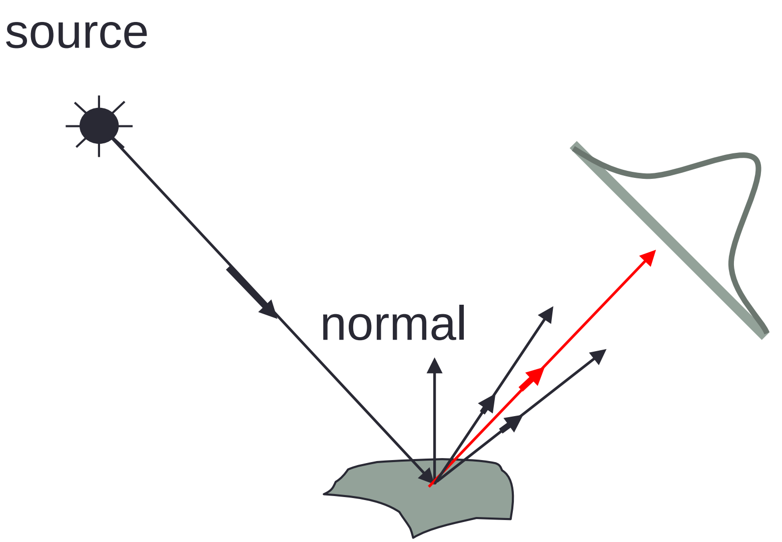
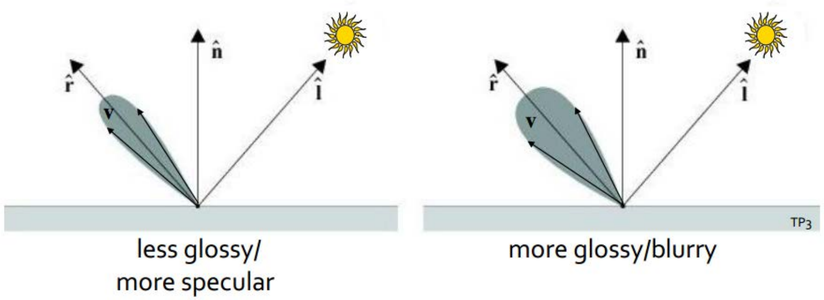
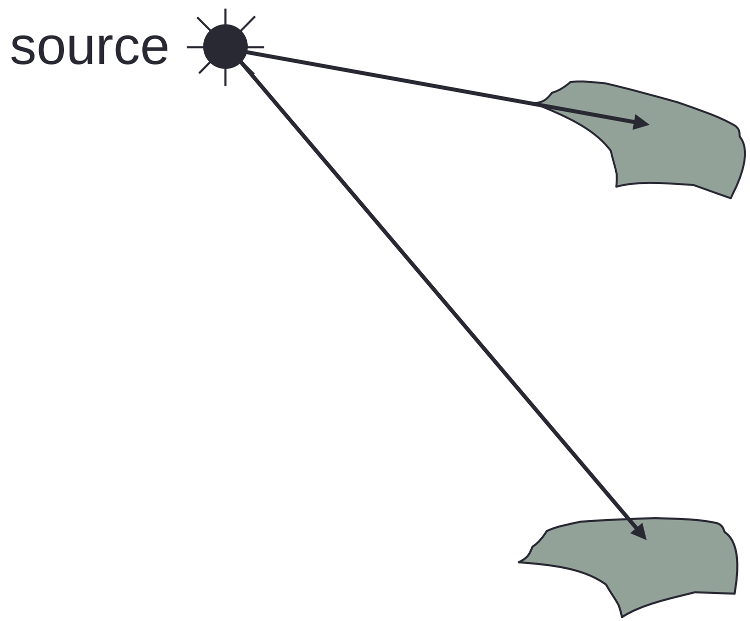
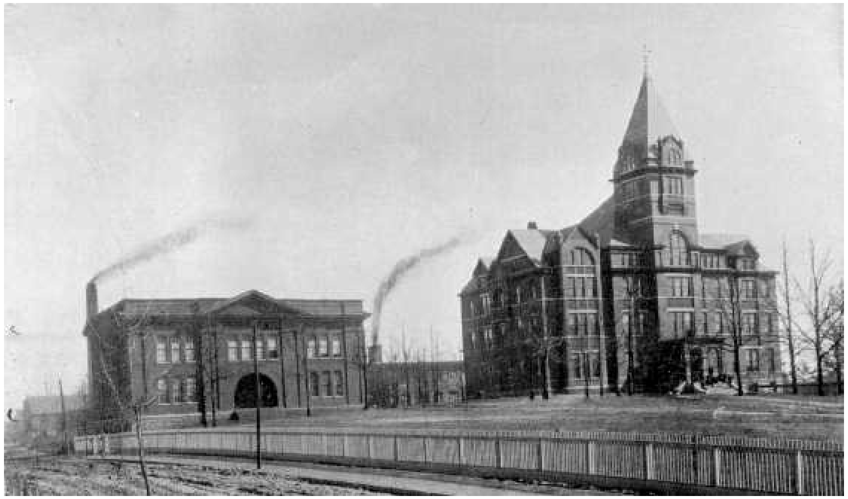


Lambertian + Specular Model



Surface Appearance

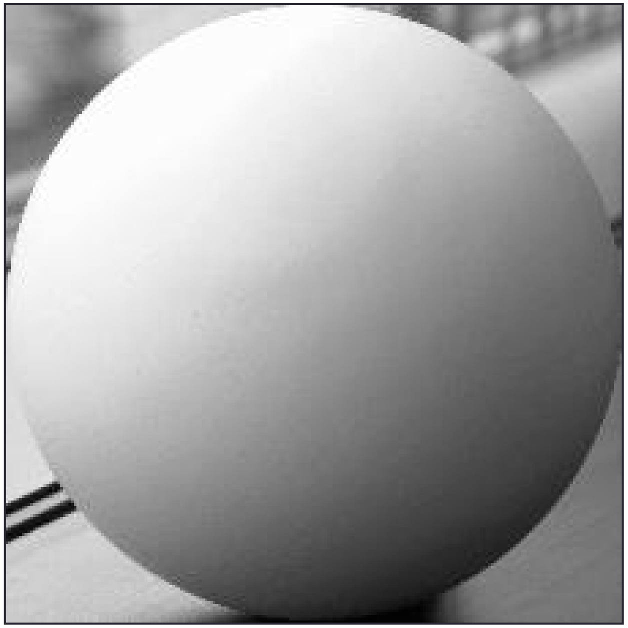




- Image intensities = f (normal, surface reflectance, illumination)

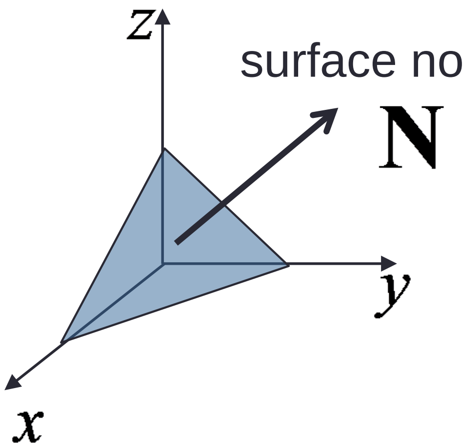
$\boxed{\text{Shape}}$ Reflectance Lighting

Shape from …



- Shading as a cue for shape reconstruction
- What is the relation between intensity and shape?
    - Reflectance Map

Surface Normal: A Bit More Math



Equation of plane

$$Ax + By + Cz + D = 0$$

or

$$\frac{A}{C}x + \frac{B}{C}y + z + \frac{D}{C} = 0$$

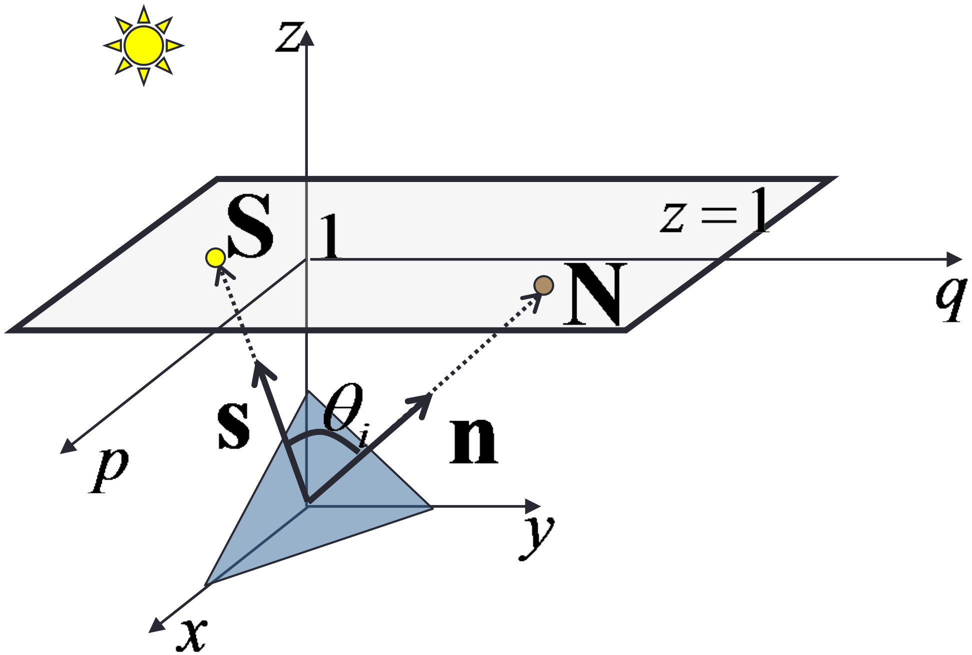
Let

$$-\frac{\partial z}{\partial x} = \frac{A}{C} = p \qquad -\frac{\partial z}{\partial y} = \frac{B}{C} = q$$

Surface normal

$$\boxed{\mathbf{N} = \left(\frac{A}{C}, \frac{B}{C}, 1\right) = (p, q, 1)}$$

Gradient Space (Tricky!)



Normal vector

$$\mathbf{n} = \frac{\mathbf{N}}{|\mathbf{N}|} = \frac{(p, q, 1)}{\sqrt{p^2 + q^2 + 1}}$$

Source vector

$$\mathbf{s} = \frac{\mathbf{S}}{|\mathbf{S}|} = \frac{(p_S, q_S, 1)}{\sqrt{p_S^2 + q_S^2 + 1}}$$

$$\cos\theta_i = \mathbf{n} \cdot \mathbf{s} = \frac{(pp_S + qq_S + 1)}{\sqrt{p^2 + q^2 + 1}\sqrt{p_S^2 + q_S^2 + 1}}$$

$z = 1$ plane is called the Gradient Space (*pq* plane)

- Every point on it corresponds to a particular surface orientation

Reflectance Map

- Relates image irradiance *I(x,y)* to surface orientation *(p,q)* for given source direction and surface reflectance
- Lambertian case:

k : source brightness

ρ: surface albedo (reflectance)

c : constant (optical system)

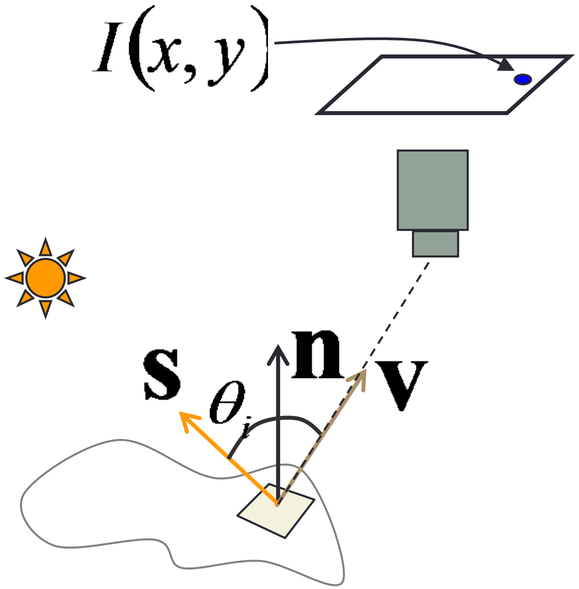


Image irradiance:

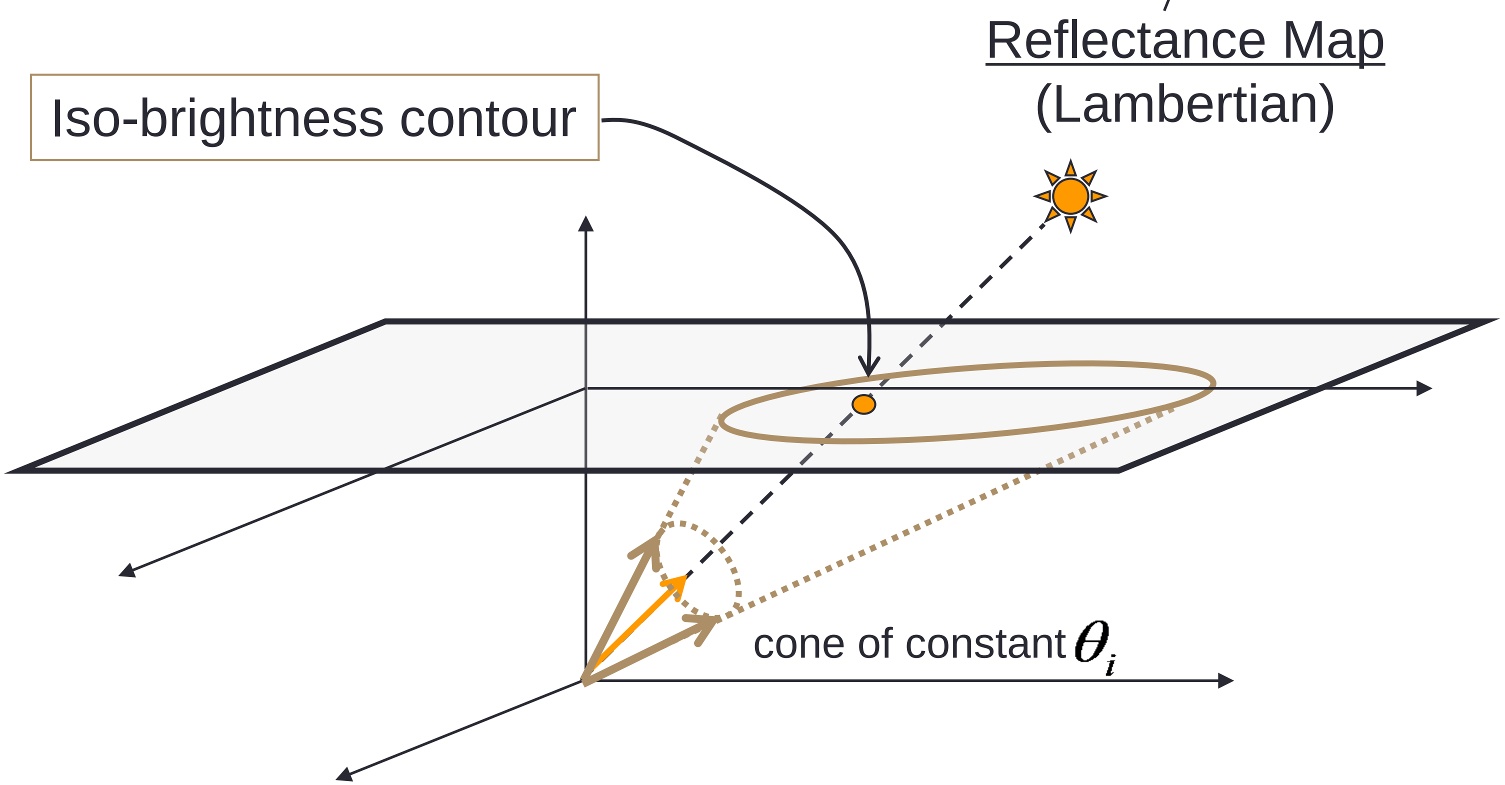
$$I = \frac{\rho}{\pi} kc \cos\theta_i = \frac{\rho}{\pi} kc\, \mathbf{n}\cdot\mathbf{s}$$

Let $\frac{\rho}{\pi} kc = 1$ then $\boxed{I = \cos\theta_i = \mathbf{n}\cdot\mathbf{s}}$

Reflectance Map

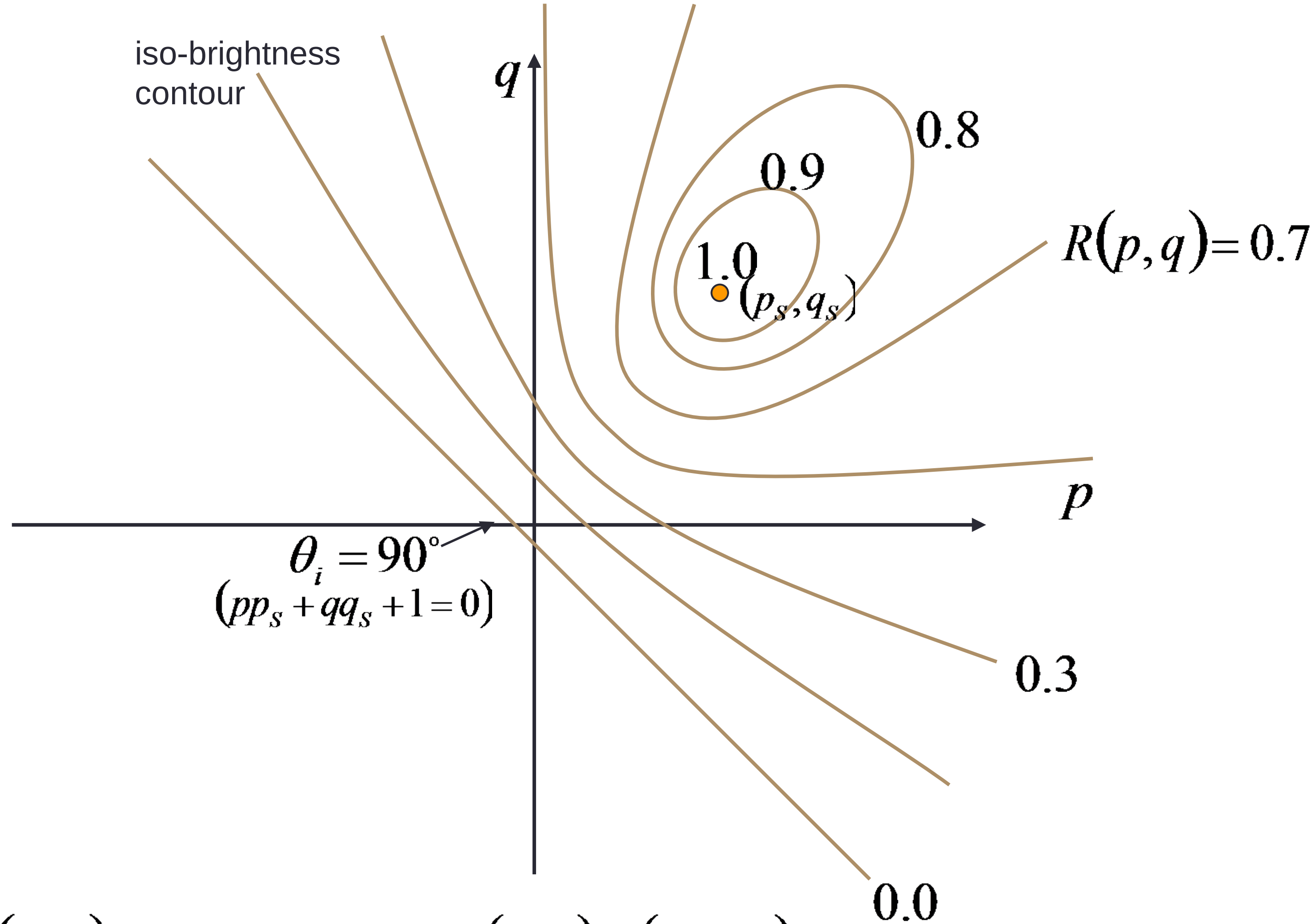
- Lambertian case

$$I = \cos\theta_i = \mathbf{n}\cdot\mathbf{s} = \frac{(pp_s + qq_s + 1)}{\sqrt{p^2+q^2+1}\sqrt{p_S^2+q_S^2+1}} = \boxed{R(p,q)}$$



Reflectance Map

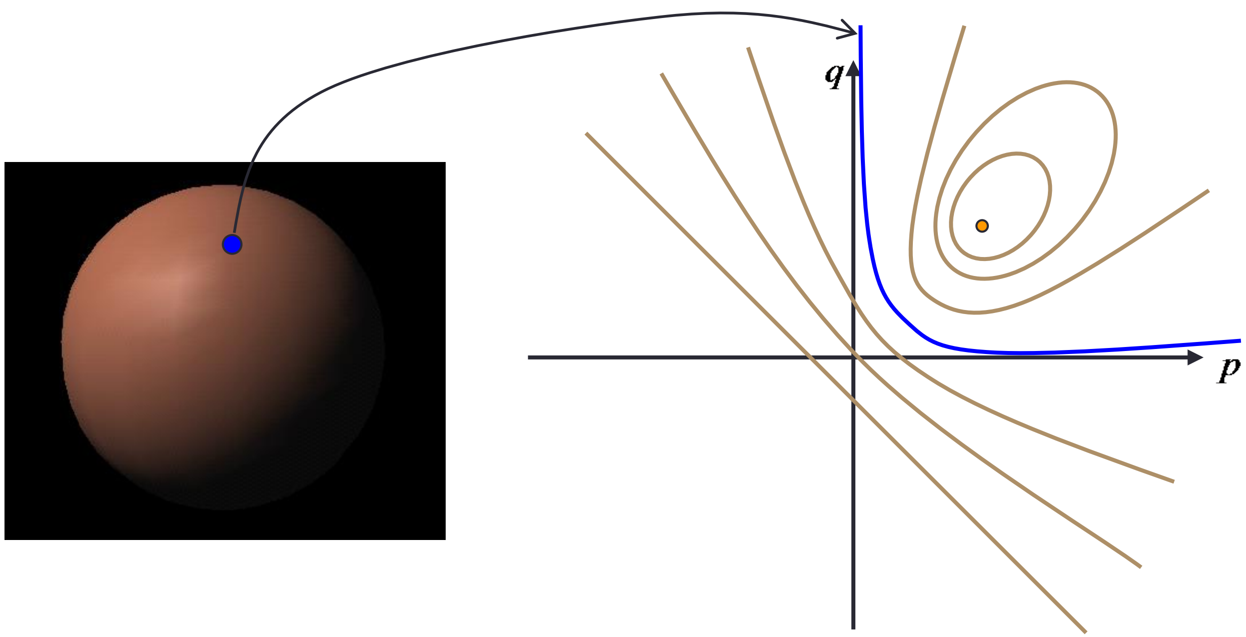
- Lambertian case



Note: $R(p,q)$ is maximum when $(p,q) = (p_S, q_S)$

Shape from A Single Image?

- Given a single image of an object with known surface reflectance taken under a known light source, can we recover the shape of the object?
- Given *R(p,q)* ((p_S,q_S) and surface reflectance) can we determine *(p,q)* uniquely for each image point?



NO

Shape from X

- Take more images
  - Photometric stereo

- Add more constraints
  - Shape-from-shading

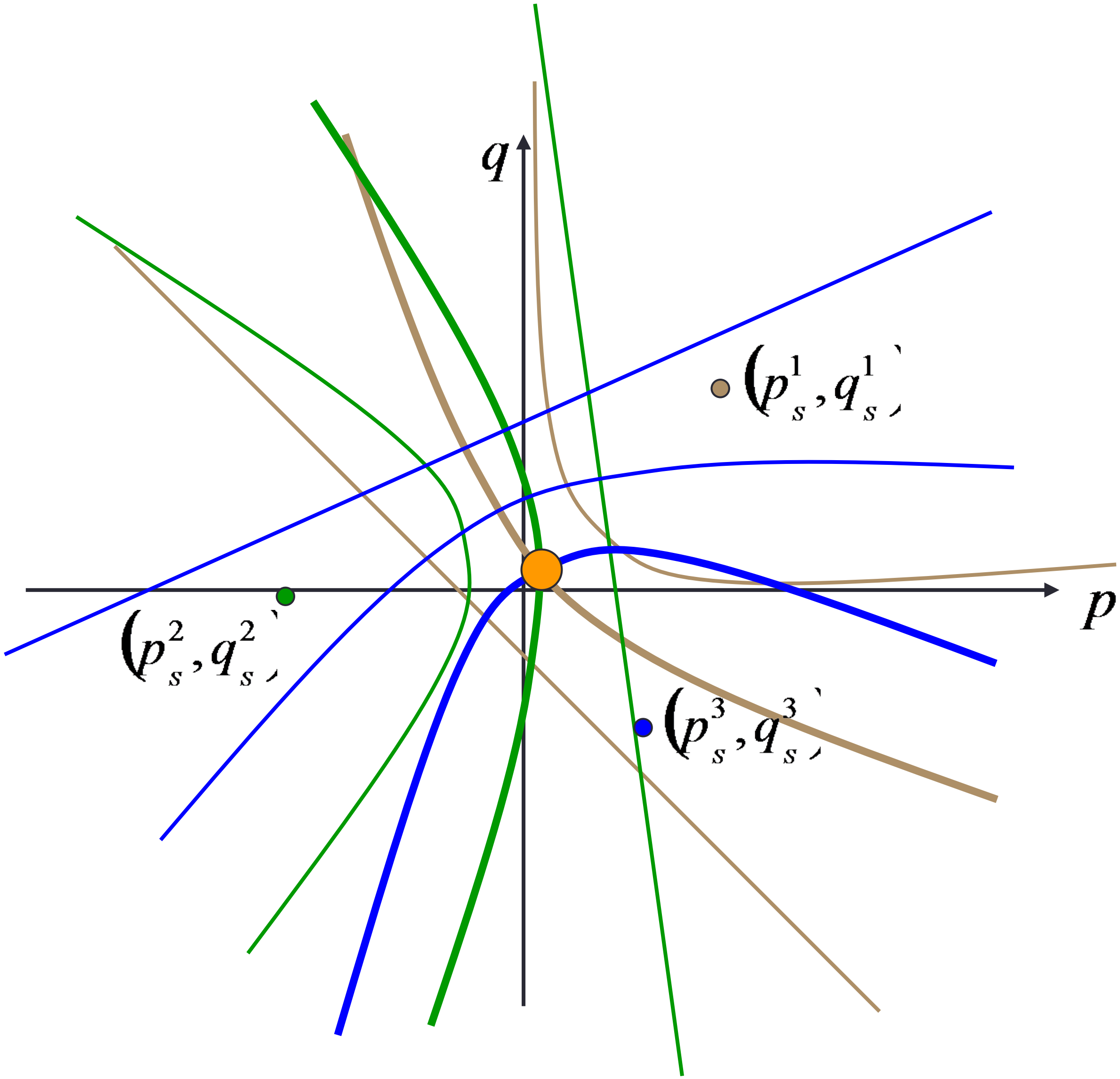
Photometric Stereo: Lambertian Toy




Surface Normals

Albedo

Photometric Stereo



Photometric Stereo

$$I = \frac{\rho}{\pi} kc \cos\theta_i = \rho \mathbf{n} \cdot \mathbf{s} \quad \left(\frac{kc}{\pi} = 1 \right)$$

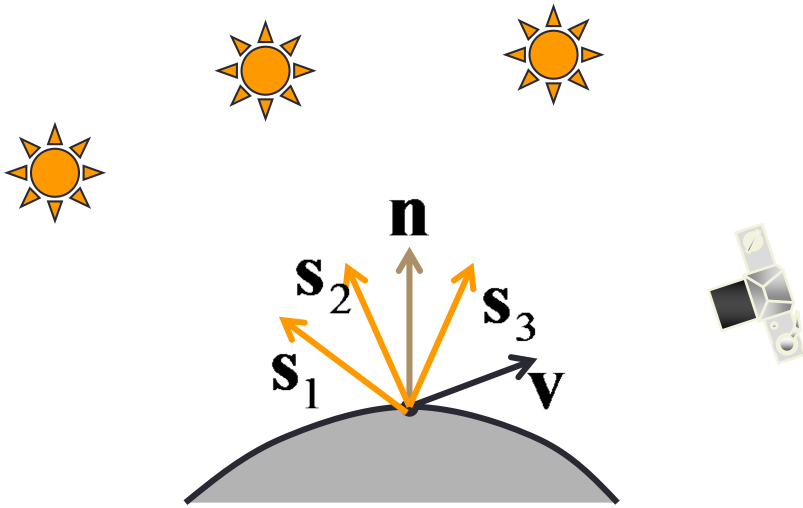


Image irradiance:

$$I_1 = \rho \mathbf{n} \cdot \mathbf{s}_1$$
$$I_2 = \rho \mathbf{n} \cdot \mathbf{s}_2$$
$$I_3 = \rho \mathbf{n} \cdot \mathbf{s}_3$$

- We can write this in matrix form:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_2 \end{bmatrix} = \rho \begin{bmatrix} \mathbf{s}_1^T \\ \mathbf{s}_2^T \\ \mathbf{s}_3^T \end{bmatrix} \mathbf{n}$$

Lambertian!

Solving the Equations: Least Square

- *I* and **s** are known

$$\underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_2 \end{bmatrix}}_{\mathbf{I}_{3\times 1}} = \underbrace{\begin{bmatrix} \mathbf{s}_1^T \\ \mathbf{s}_2^T \\ \mathbf{s}_3^T \end{bmatrix}}_{\mathbf{S}_{3\times 3}} \underbrace{\rho \mathbf{n}}_{\tilde{\mathbf{n}}_{3\times 1}}$$

$$\tilde{\mathbf{n}} = \mathbf{S}^{-1}\mathbf{I}$$

inverse

$$\rho = \left|\tilde{\mathbf{n}}\right|$$

$$\mathbf{n} = \frac{\tilde{\mathbf{n}}}{\left|\tilde{\mathbf{n}}\right|} = \frac{\tilde{\mathbf{n}}}{\rho}$$

Adding More Light Sources

- Get better results by using more (*N*) lights

$$\begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} \mathbf{s}_1^T \\ \vdots \\ \mathbf{s}_N^T \end{bmatrix} \rho \mathbf{n}$$

- Least squares solution:

$$\mathbf{I} = \mathbf{S}\tilde{\mathbf{n}} \longleftarrow N \times 1 = \underline{(N \times 3)}(3 \times 1)$$

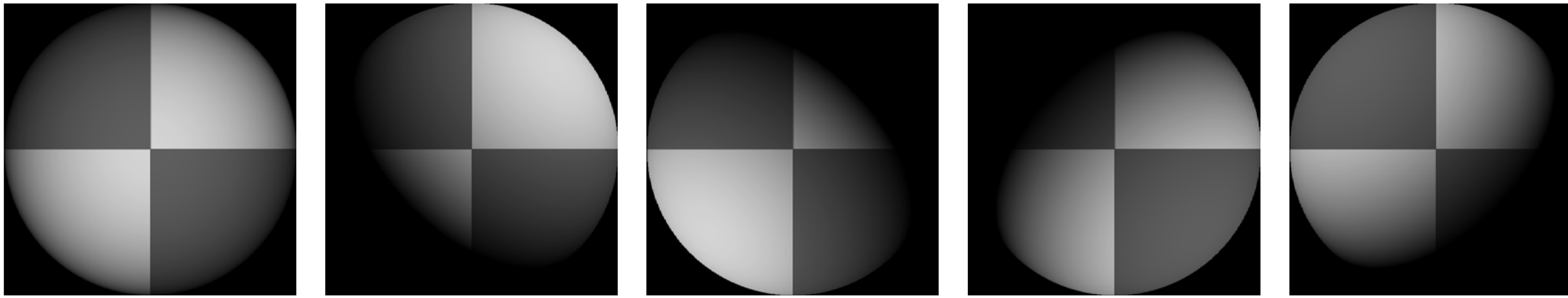
$$\mathbf{S}^T\mathbf{I} = \mathbf{S}^T\mathbf{S}\tilde{\mathbf{n}}$$

$$\tilde{\mathbf{n}} = \boxed{\left(\mathbf{S}^T\mathbf{S}\right)^{-1}\mathbf{S}^T}\mathbf{I} \qquad \min \|\mathrm{I} - \mathrm{S}\tilde{n}\|_2^2$$

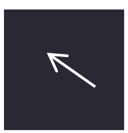
Moore-Penrose pseudo inverse

- Solve for $\rho, \mathbf{n}$ as before

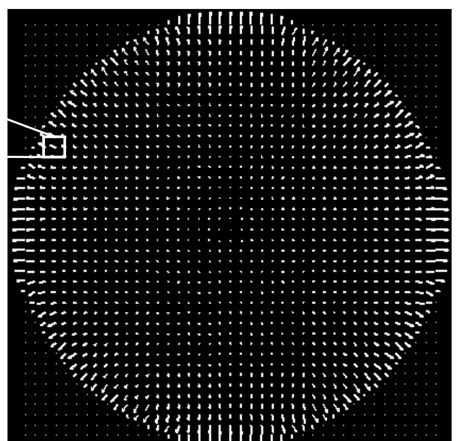
Results: Lambertian Sphere



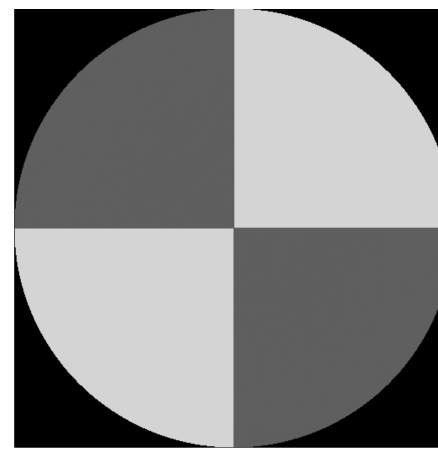
Input Images



Needles are projections of surface normals on image plane

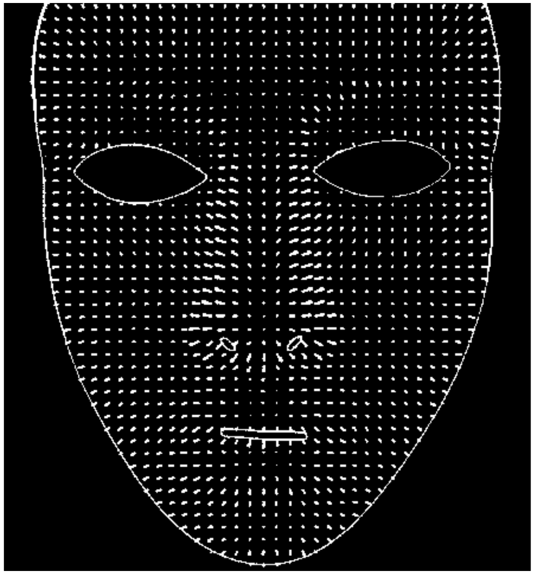


Estimated Surface Normals



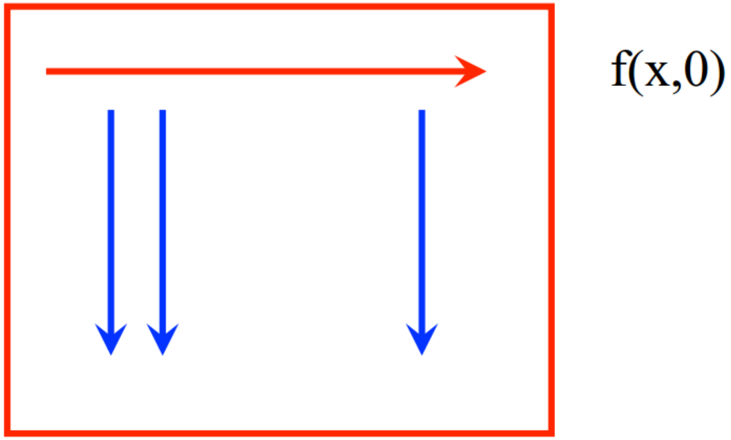
Estimated Albedo

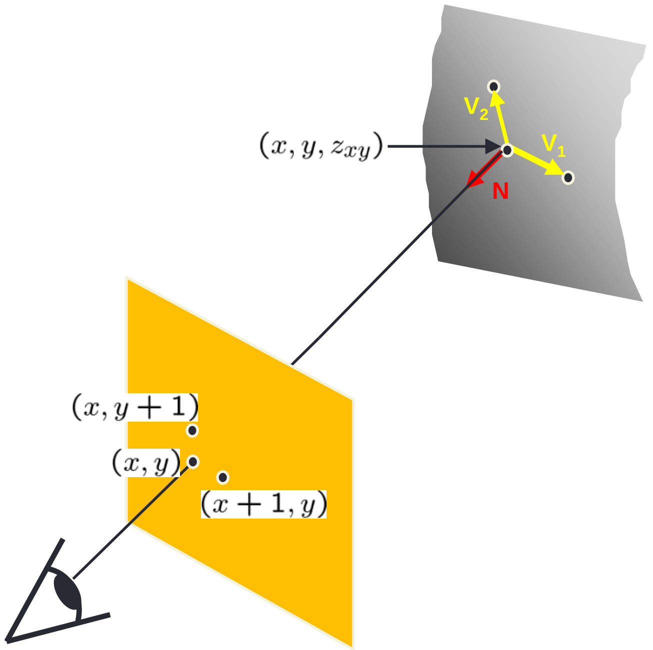
Results: Lambertian Mask



Final Step: From Normal to Surface

- Simplest method
  - Get surface normal field $n = (p,q,1)$
  - Integrate p = df/dx along a row (x, 0) to get f(x, 0)
  - Then integrate q = df/dx along each column (starting with value of the first row)





Limitations

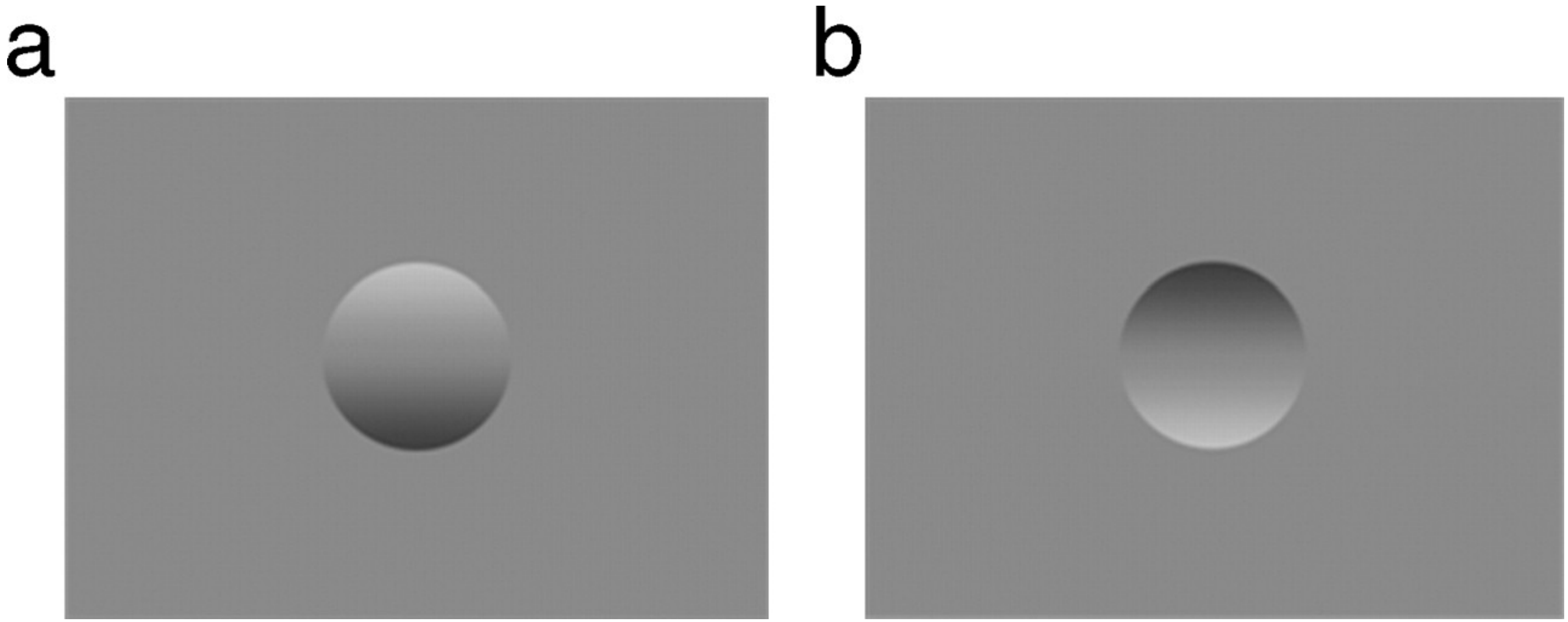
- Big problems
  - Doesn’t work for shiny things, semi-translucent things
  - Shadows, inter-reflections
- Smaller problems
  - Camera and lights have to be distant
  - Calibration requirements
    - measure light source directions, intensities
    - camera response function
- Check this cool APP from Grant Schindler

Shape from X

- Take more images
  - Photometric stereo
- Add more constraints
  - Shape-from-shading

Shape from Shading

Examples of the classic bump/dent stimuli used to test lighting assumptions when judging shape from shading, with shading orientations (a) 0° and (b) 180° from the vertical.



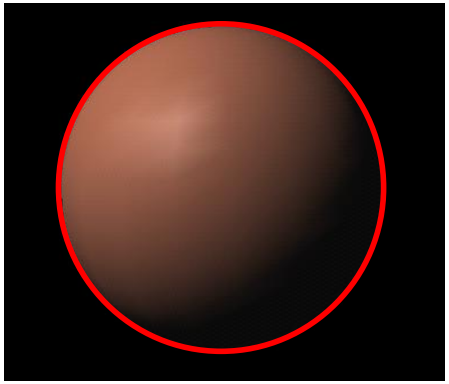
Thomas R et al. J Vis 2010;10:6

Journal of VISION

Also check Ramachandran's work on Shape from Shading by Humans
http://psy.ucsd.edu/chip/ramabio.html

Shape from Shading

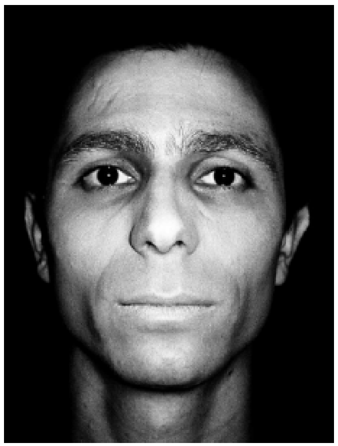
- Given a single image of an object with known surface reflectance taken under a known light source, can we recover the shape of the object?
- Given $R(p,q)$ ((p_S,q_S) and surface reflectance) can we determine (p,q) uniquely for each image point?

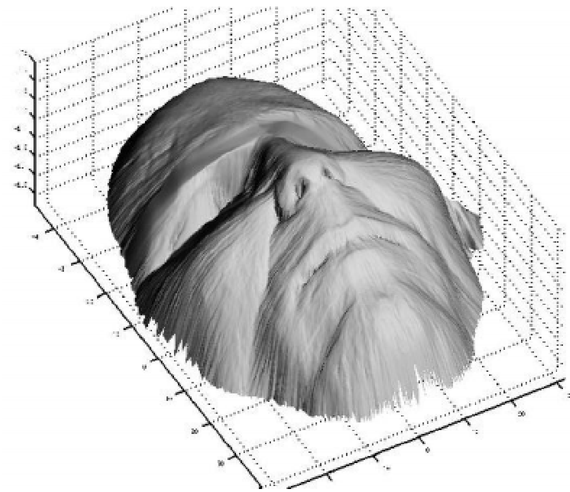


- Assume shape along the occluding boundary is known
- Constraints on neighboring normals—“integrability”
- Smoothness

Yes, you need to put them into a complicated optimization

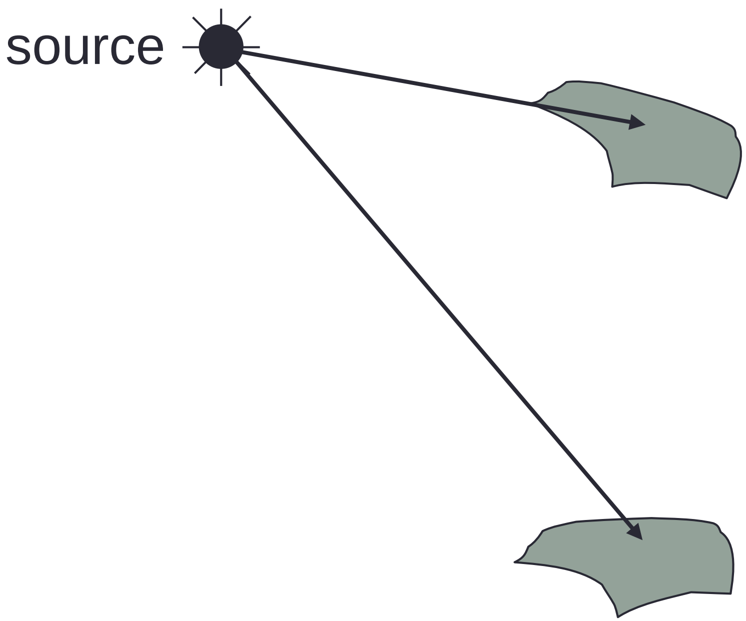
Shape from Shading: Results

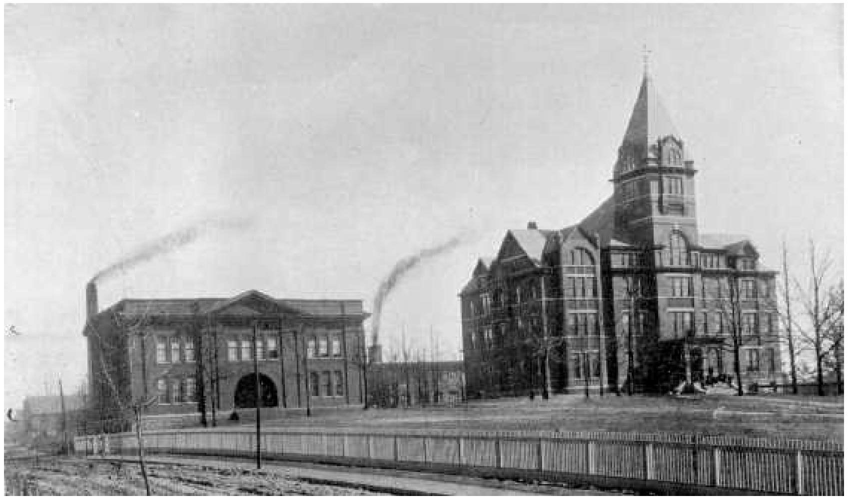




- It is not working well in practice
- Why? The assumptions are quite restrictive
- But this is recovery of **3D** from **single 2D** image

Surface Appearance



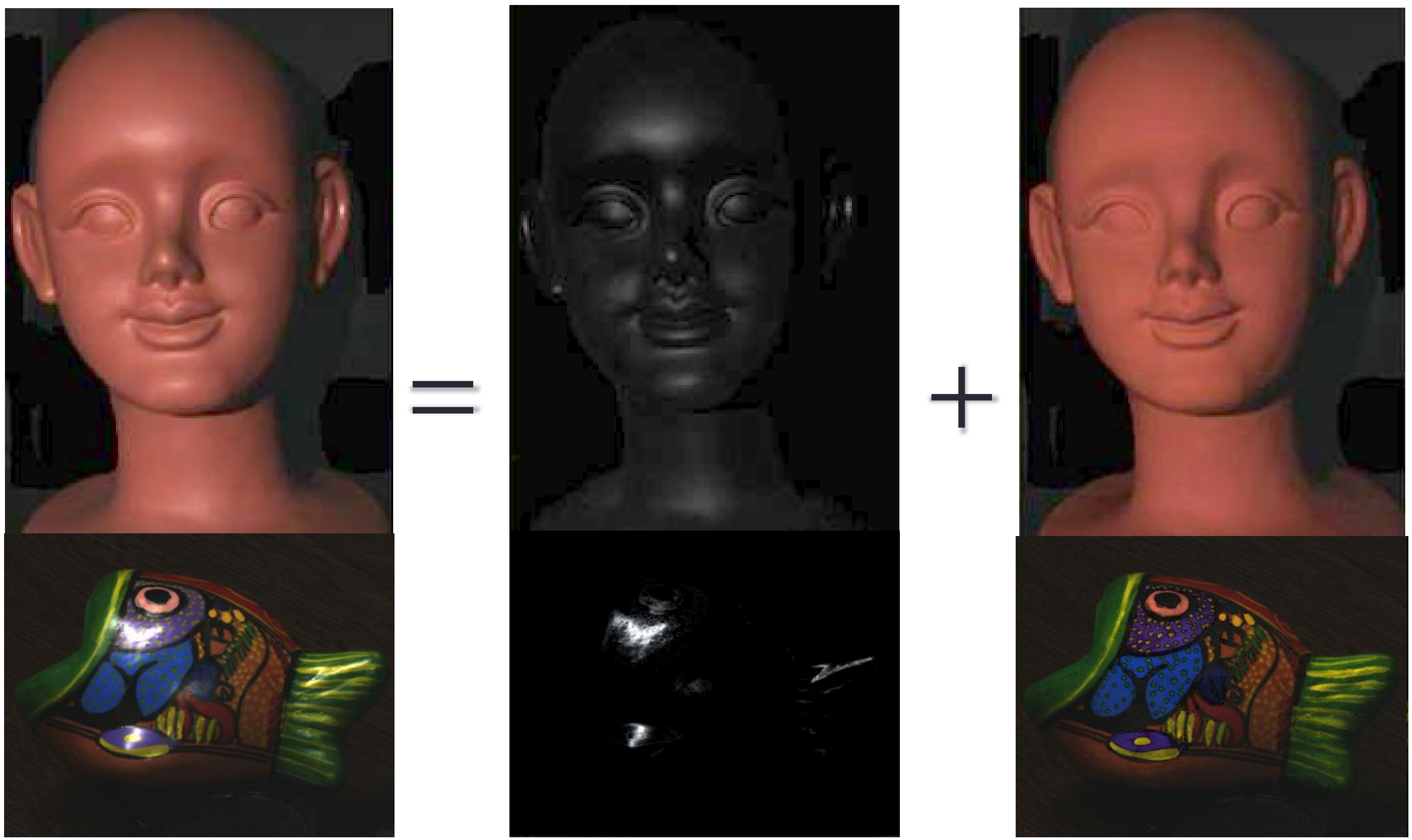


- Image intensities = f (normal, surface reflectance, illumination)

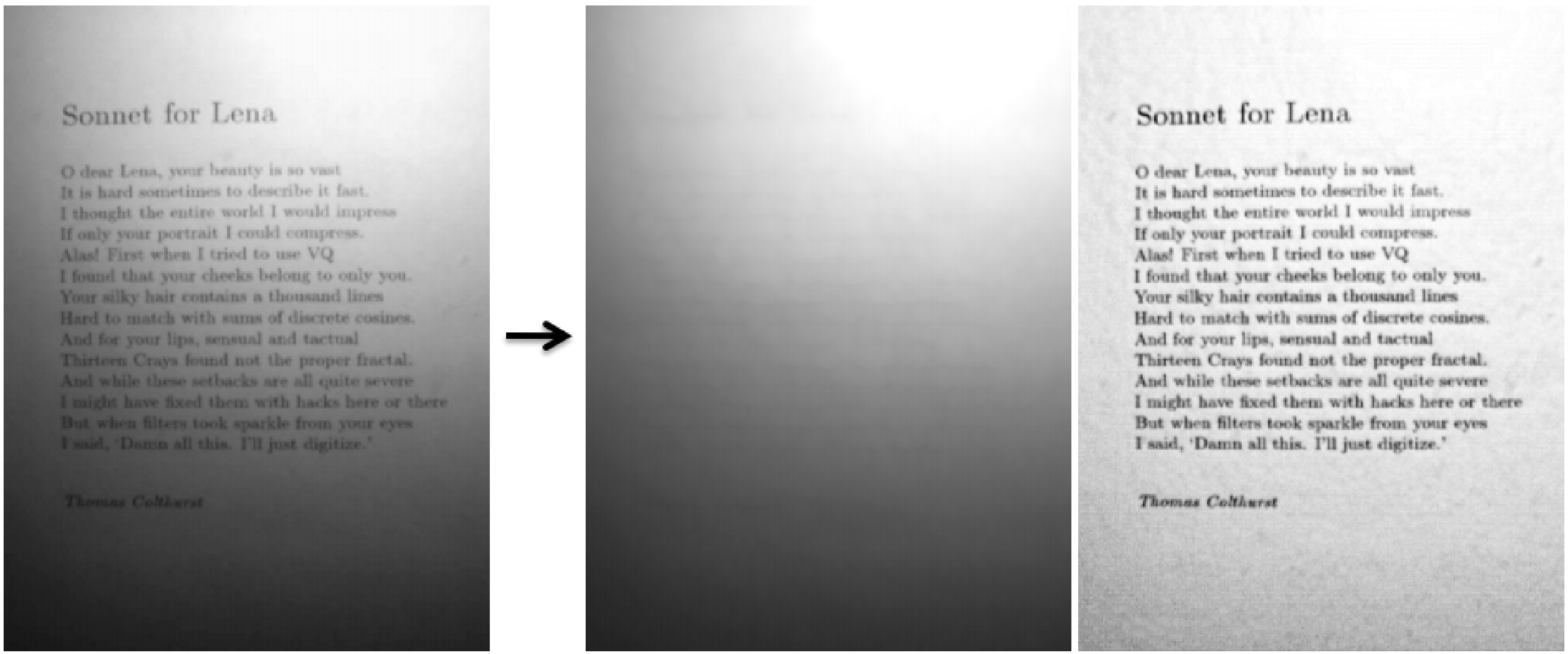
Shape

Reflectance Lighting

Intrinsic Image Decomposition



Intrinsic Image Decomposition



A "real" result assuming we know the shape …

Summary: Photometry

- Image intensities = f (normal, surface reflectance, illumination)
- BRDF captures the relationship between surface normal, surface reflectance and indent light
- Recovering shape from images/an image
  - Photometric Stereo
  - Shape from Shading
- Intrinsic Image: Decomposing the surface reflectance and the illumination