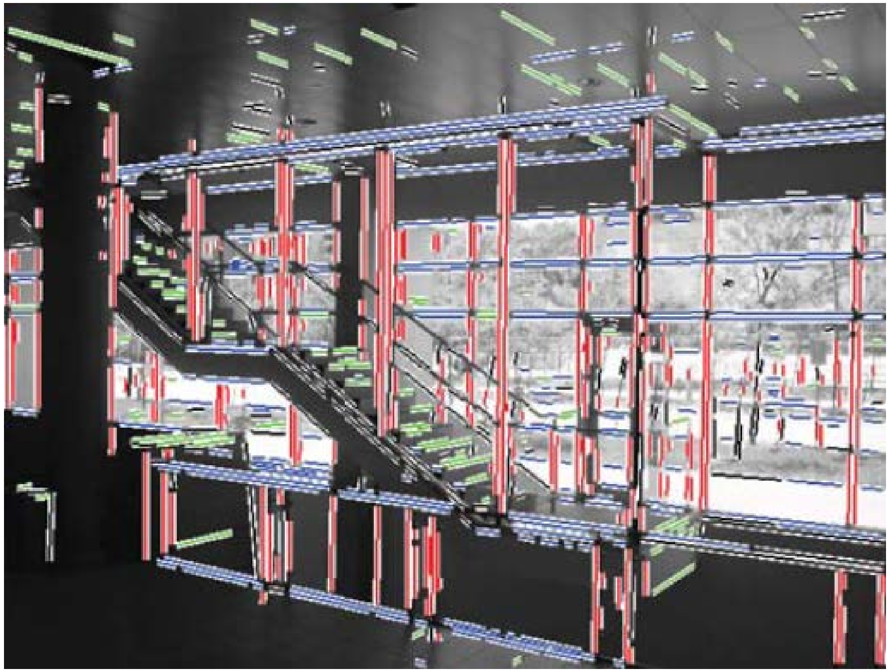
CS 4495 Computer Vision
RANdom **SA**mple **C**onsensus

Aaron Bobick

School of Interactive Computing

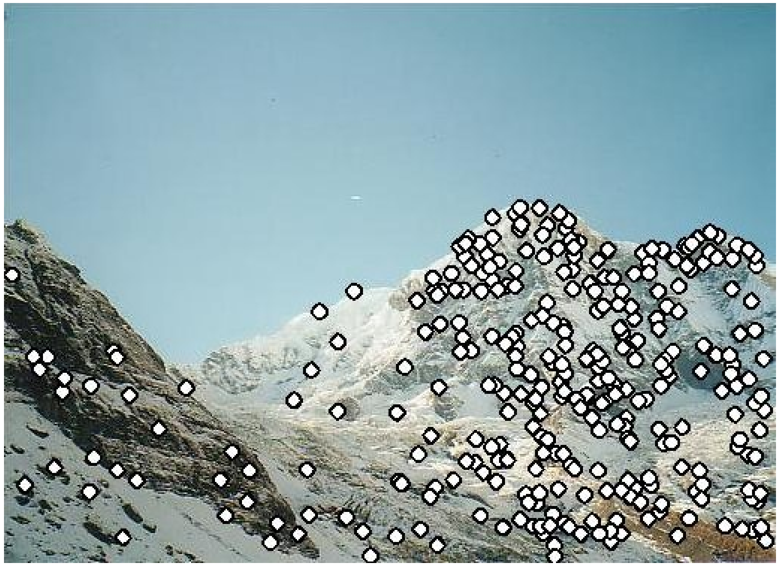


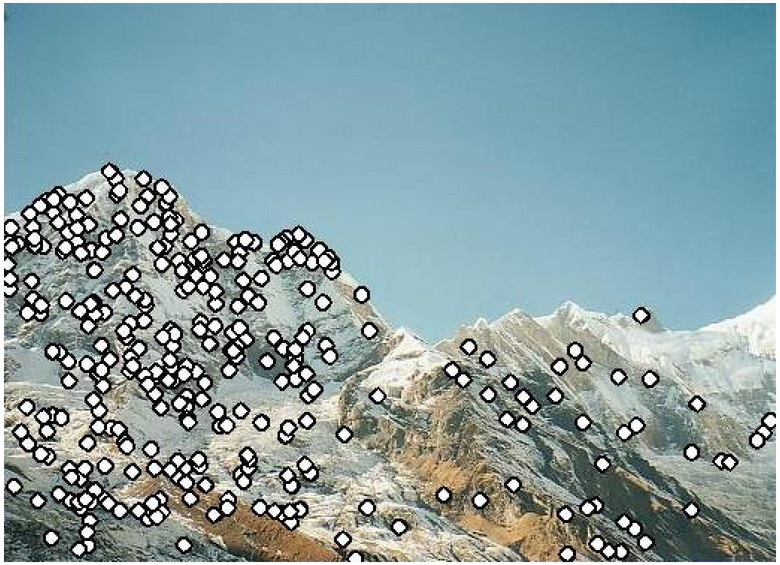
Administrivia

- PS 3:
  - Check Piazza - good conversations. In fact some pretty explicit solutions…
  - The F matrix: the actual numbers may vary quite a bit. But check the epipolar lines you get.
    - Normalization: read extra credit part. At least try removing the centroid. Since we're using homogenous coordinates (2D homogenous have 3 elements) it's easy to have a transformation matrix that subtracts off an offset.
  - Go back an recheck slides: A 3 vector in these projective geometry is both a point and a line.

Matching with Features

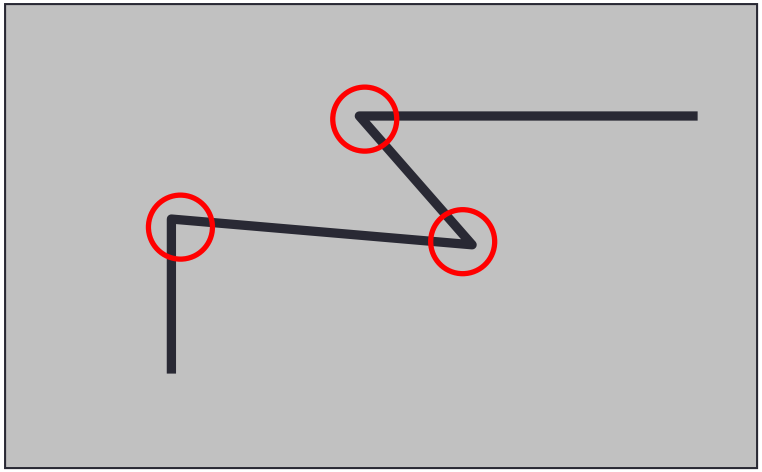
- Want to compute transformation from one image to the other
- Overall strategy:
  - Compute features
  - Match matching features (duh?)
  - Compute best transformation (translation, affine, homography) from matches





An introductory example:

Harris corner detector



C.Harris, M.Stephens. “A Combined Corner and Edge Detector”. 1988

Harris Detector: Mathematics

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Measure of corner response:

$$R = \det M - \alpha \left(\operatorname{trace} M \right)^2$$

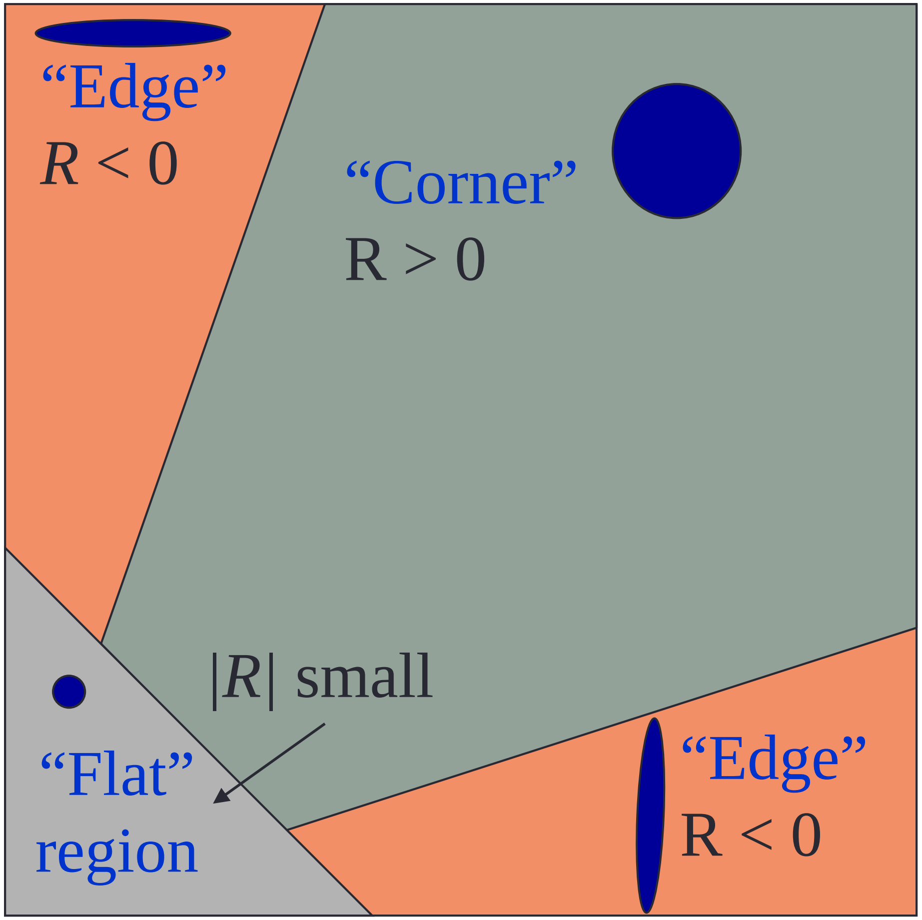
$$\det M = \lambda_1 \lambda_2$$
$$\operatorname{trace} M = \lambda_1 + \lambda_2$$

(α– empirical constant, typically 0.04 - 0.06)

Harris corner response function

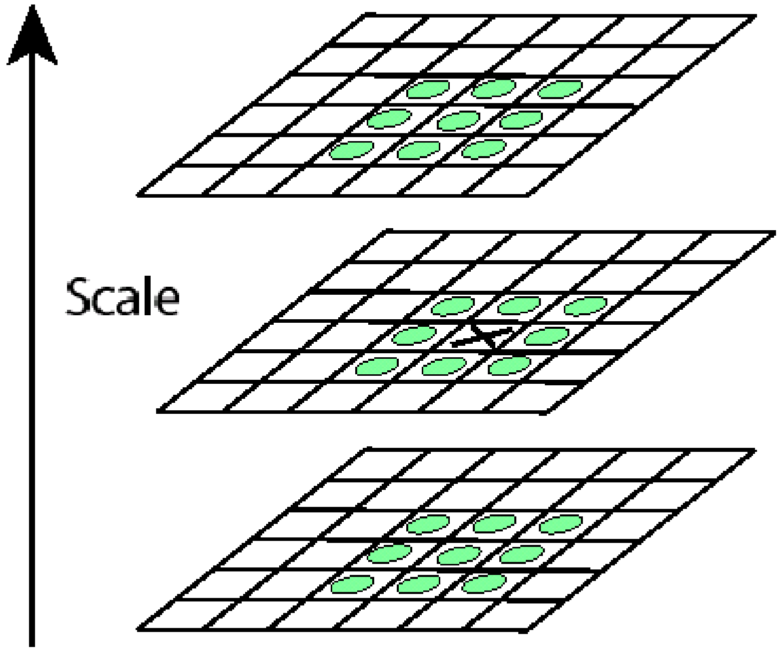
$$R = \det(M) - \alpha\,\text{trace}(M)^2 = \lambda_1\lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

- R depends only on eigenvalues of M, but don't compute them (no sqrt, so really fast!
- R is large for a corner
- R is negative with large magnitude for an edge
- $|R|$ is small for a flat region



Key point localization

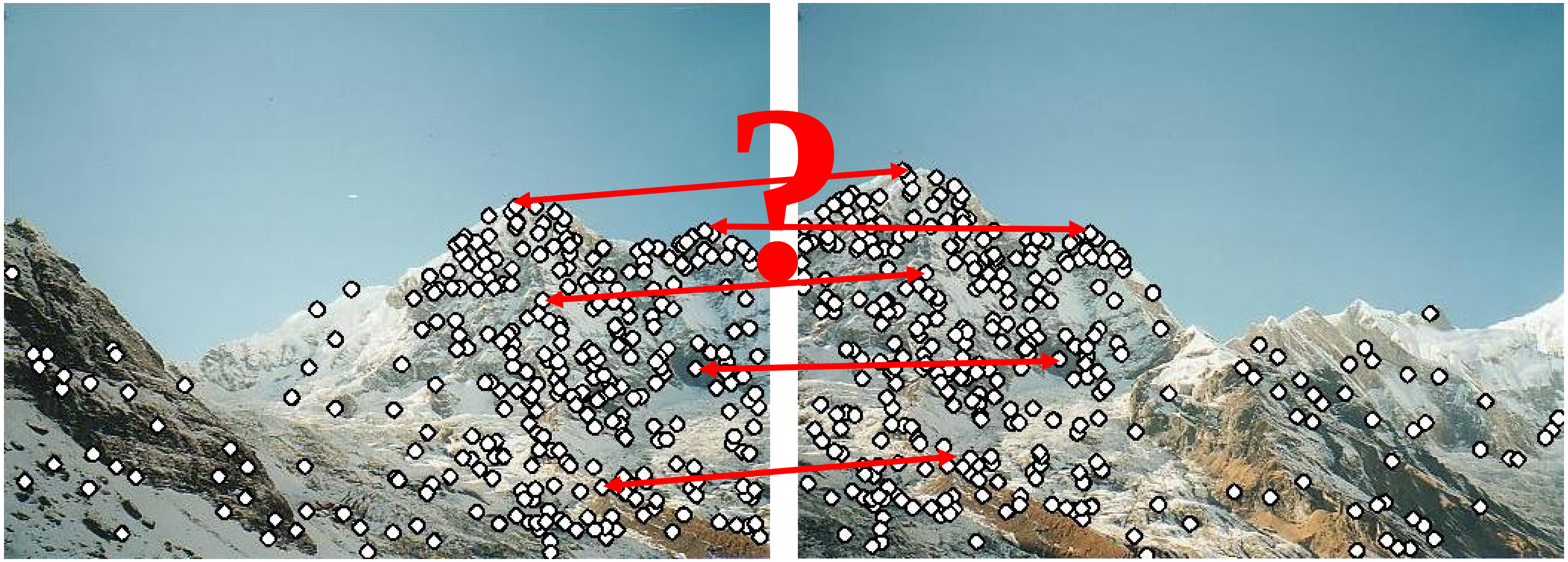
- General idea: find robust extremum (maximum or minimum) both in space and in scale.
- SIFT specific suggestion: use DoG pyramid to find maximum values (remember edge detection?) – then eliminate “edges” and pick only corners.
- More recent: use Harris detector to find maximums in space and then look at the Laplacian pyramid (we’ll do this later) for maximum in scale.



Each point is compared to its 8 neighbors in the current image and 9 neighbors each in the scales above and below.

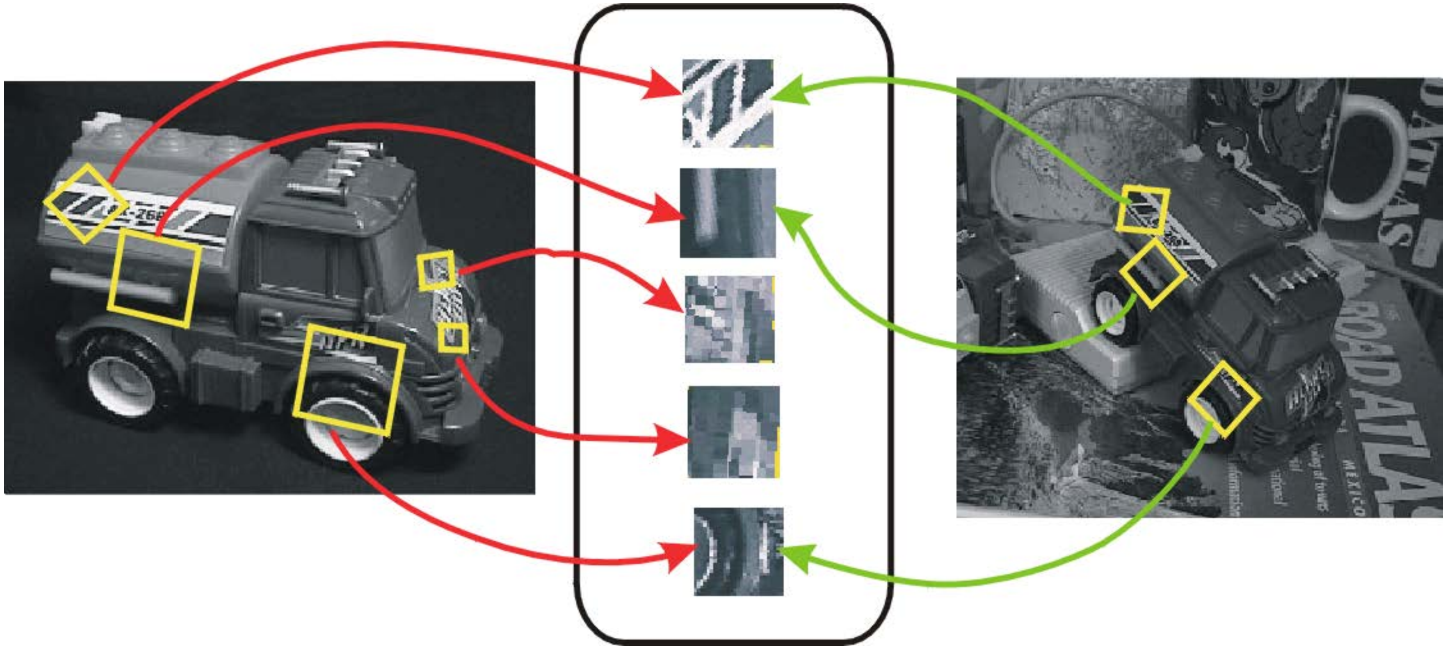
Point Descriptors

- We know how to detect points
- How to match them? Two parts:
  - Compute a descriptor for each and make the descriptor both as invariant and as distinctive as possible. (Competing goals) SIFT one example.



Idea of SIFT

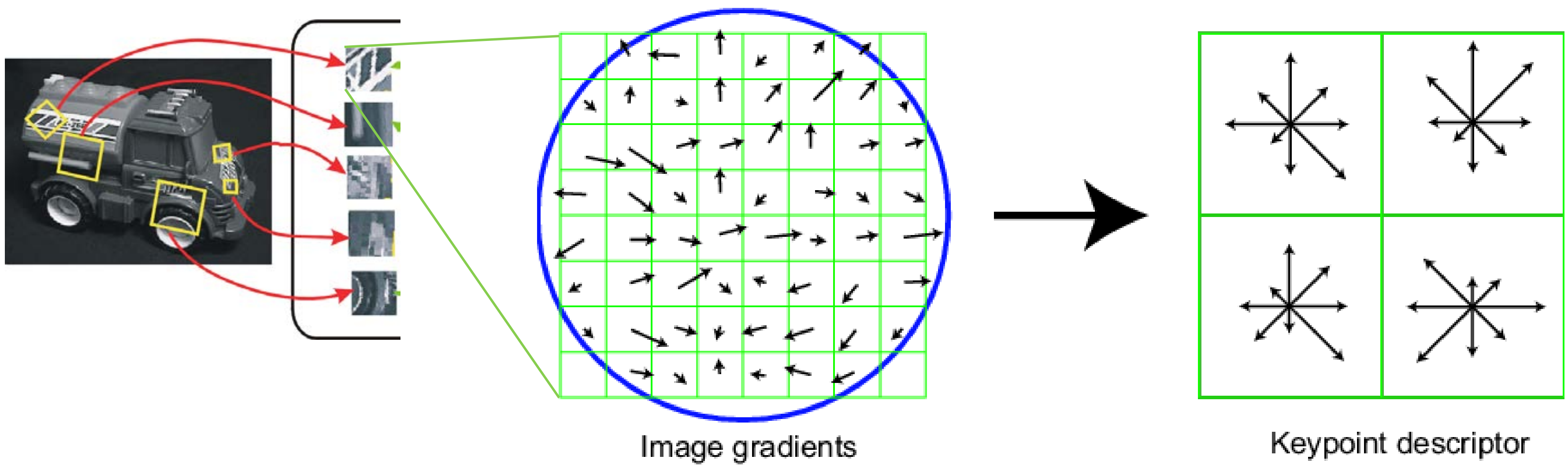
- Image content is transformed into local feature coordinates that are invariant to translation, rotation, scale, and other imaging parameters



SIFT Features

SIFT vector formation

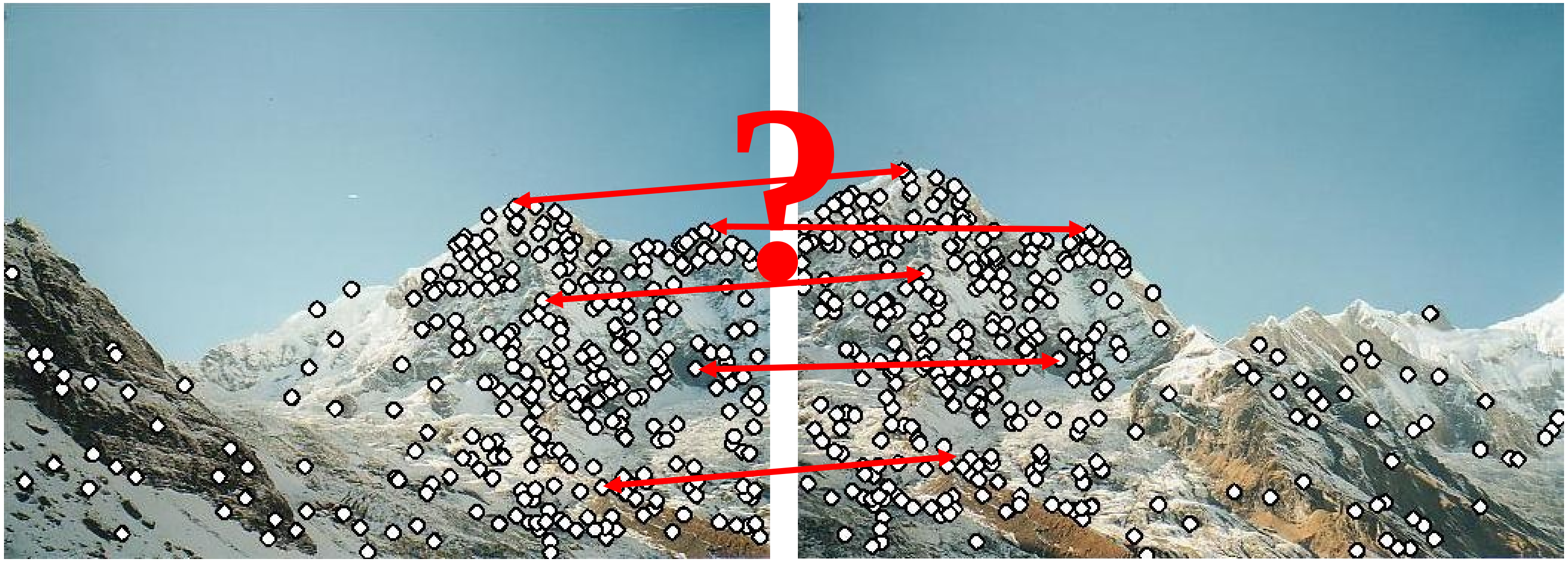
- 4x4 array of gradient orientation histograms over 4x4 pixels
  - not really histogram, weighted by magnitude
- 8 orientations x 4x4 array = 128 dimensions
- Motivation: some sensitivity to spatial layout, but not too much.



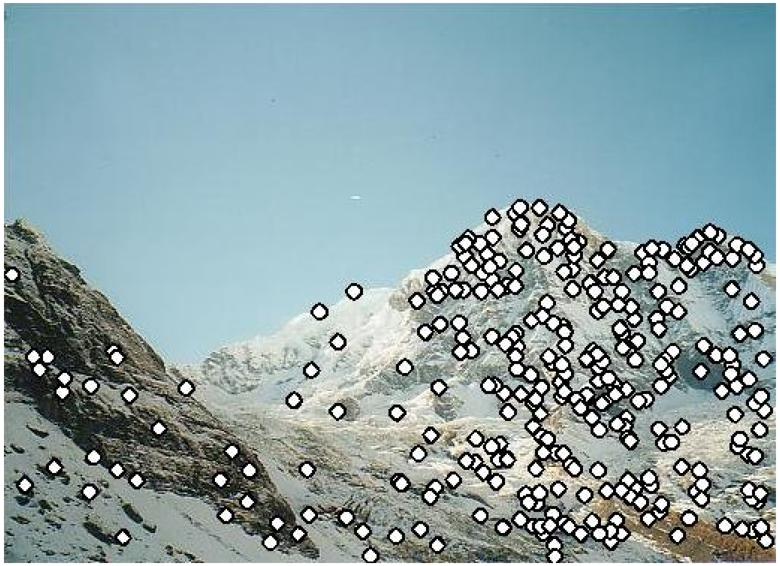
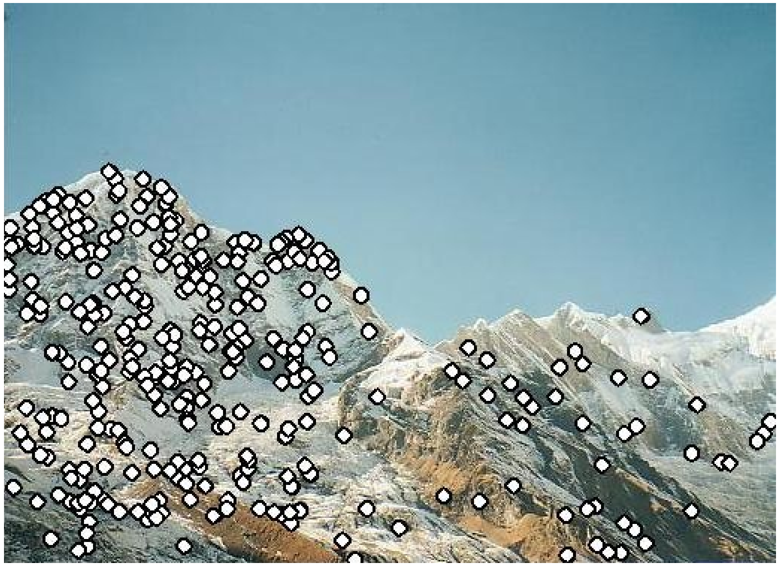
showing only 2x2 here but is 4x4

Point Descriptors

- We know how to detect points
- How to match them? Two parts:
  - Compute a descriptor for each and make the descriptor both as invariant and as distinctive as possible. (Competing goals) SIFT one example
  - ***Need to figure out which point matches which..***



Feature-based alignment outline

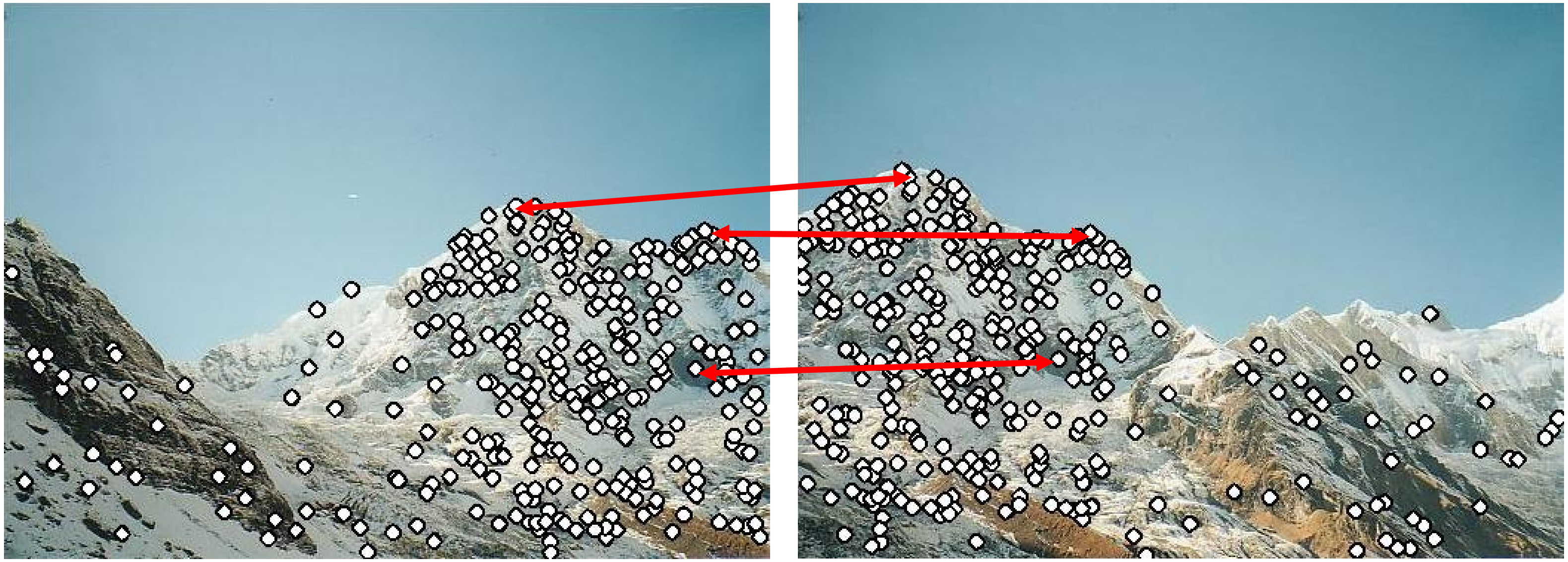



- Extract features

Feature-based alignment outline

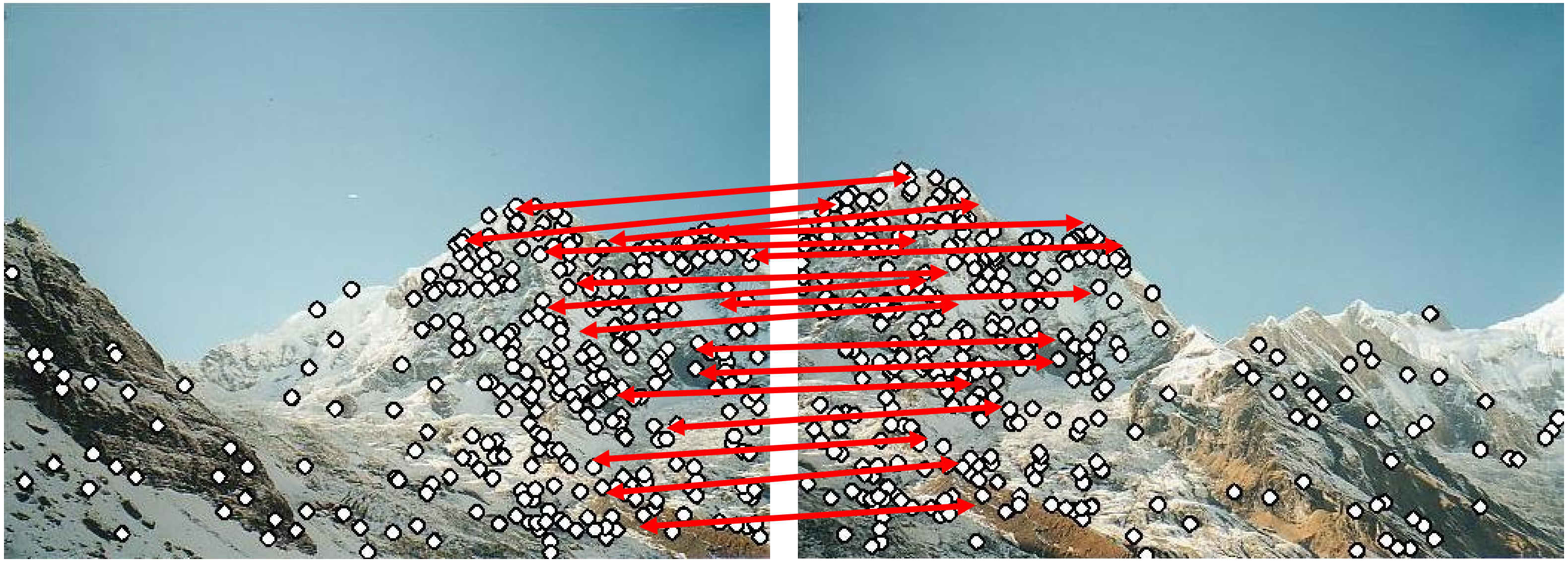
- Extract features
- Compute ***putative*** *matches* – e.g. "closest descriptor"

Feature-based alignment outline



- Extract features
- Compute ***putative*** *matches* – e.g. "closest descriptor"
- Loop:
  - *Hypothesize* transformation *T* from some matches

Feature-based alignment outline



- Extract features
- Compute ***putative*** *matches*– e.g. "closest descriptor"
- Loop:
  - *Hypothesize* transformation *T* from some matches
  - *Verify* transformation (search for other matches consistent with *T*)

Feature-based alignment outline

- Extract features
- Compute ***putative*** *matches*– e.g. “closest descriptor”
- Loop:
  - *Hypothesize* transformation *T* from some matches
  - *Verify* transformation (search for other matches consistent with *T*)
- Apply transformation

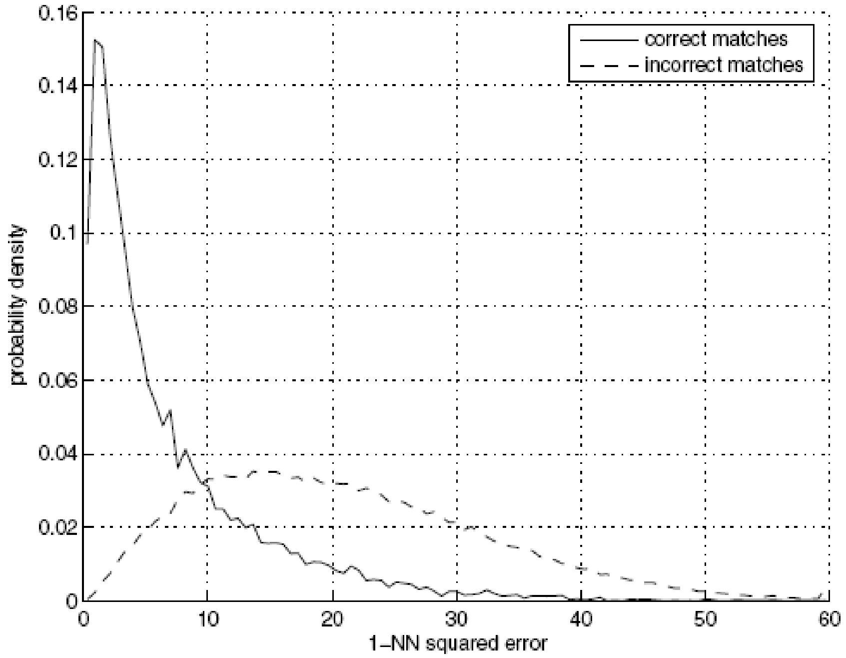
How to get “putative” matches?

Feature matching

- Exhaustive search
  - for each feature in one image, look at all the other features in the other image(s) – pick best one
- Hashing
  - compute a short descriptor from each feature vector, or hash longer descriptors (randomly)
- Nearest neighbor techniques
  - k-trees and their variants

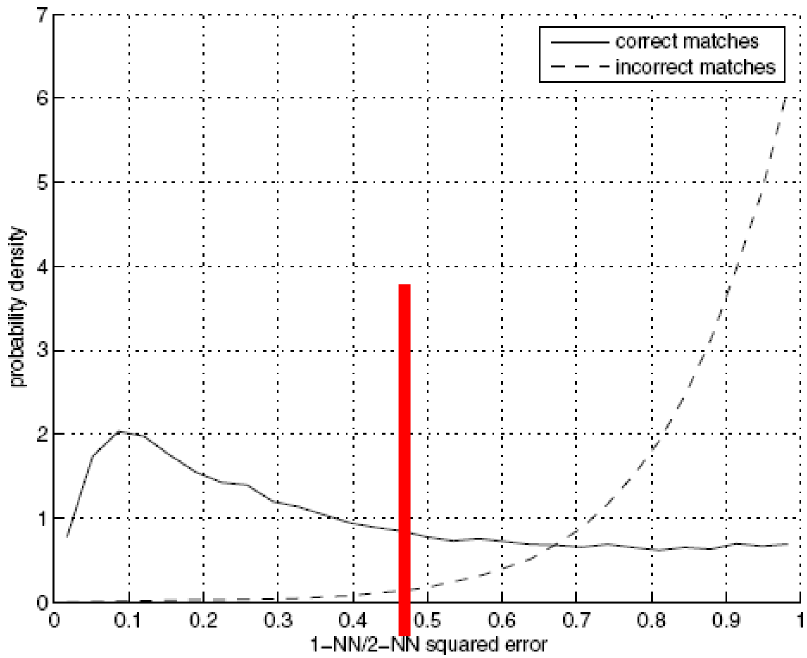
Feature-space outlier rejection

- Let’s not match all features, but only these that have “similar enough” matches?
- How can we do it?
  - SSD(patch1,patch2) < threshold
  - How to set threshold?



Feature-space outlier rejection

- A better way [Lowe, 1999]:
  - 1-NN: SSD of the closest match
  - 2-NN: SSD of the second-closest match
  - Look at how much better 1-NN is than 2-NN, e.g. 1-NN/2-NN
  - That is, is our best match much better than the next?



Feature matching

- Exhaustive search
  - for each feature in one image, look at *all* the other features in the other image(s) – pick best one
- Hashing
  - compute a short descriptor from each feature vector, or hash longer descriptors (randomly)
- Nearest neighbor techniques
  - *k*-trees and their variants
- But…
- Remember: distinctive vs invariant competition? Means:

- *Problem: Even when pick best match, still lots (and lots) of wrong matches – "outliers"*

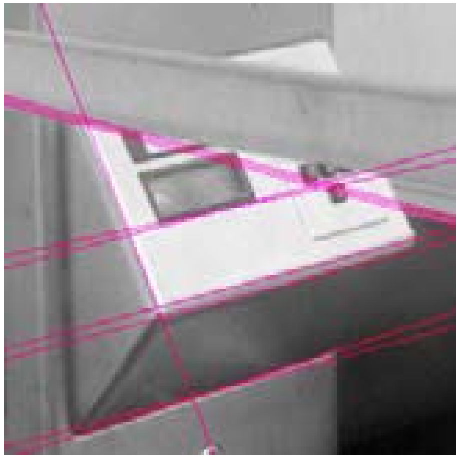
Another way to remove mistakes

- Why are we doing matching?
  - To compute a model of the relation between entities

- So this is really “model fitting”

Fitting

- Choose a parametric model to represent a set of features – *remember this???*



simple model: lines   simple model: circles

complicated model: car

Source: K. Grauman

Fitting: Issues

Case study: Line detection

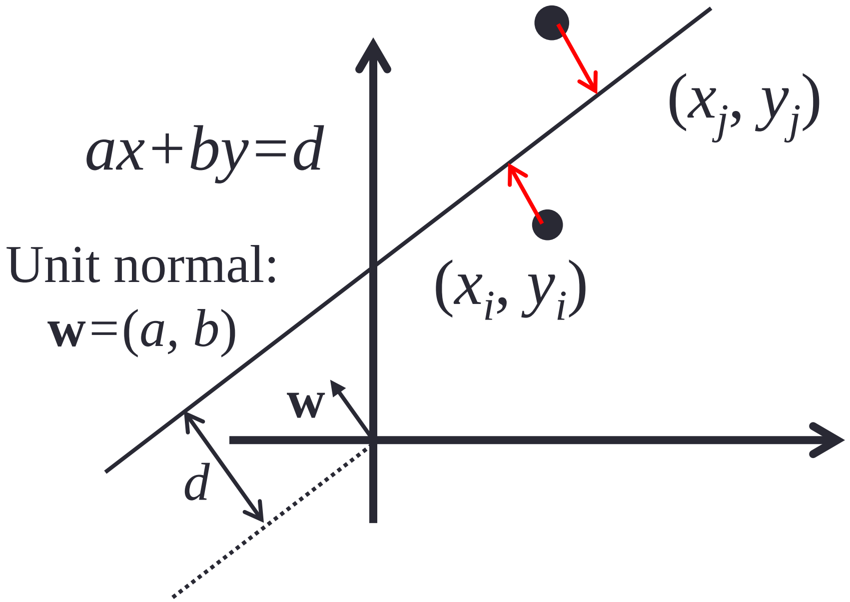
- **Noise** in the measured feature locations
- **Extraneous data:** clutter (outliers), multiple lines
- **Missing data:** occlusions

Slide: S. Lazebnik

Total least squares

- Distance between point (x_i, y_i) and line $ax+by=d$ $(a^2+b^2=1)$:

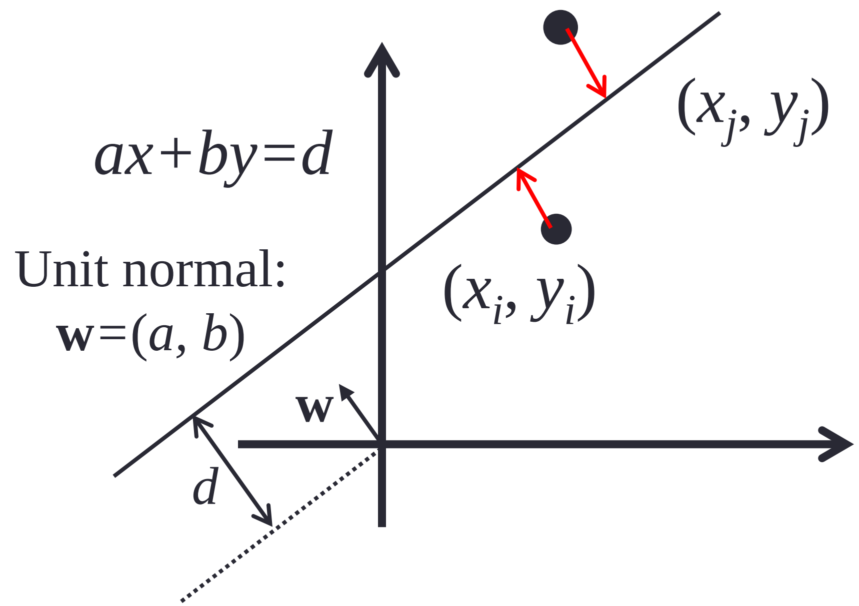
$$|ax_i + by_i - d|$$



Total least squares

- Distance between point (x_i, y_i) and line $ax+by=d$ $(a^2+b^2=1)$:

$$|ax_i + by_i - d|$$

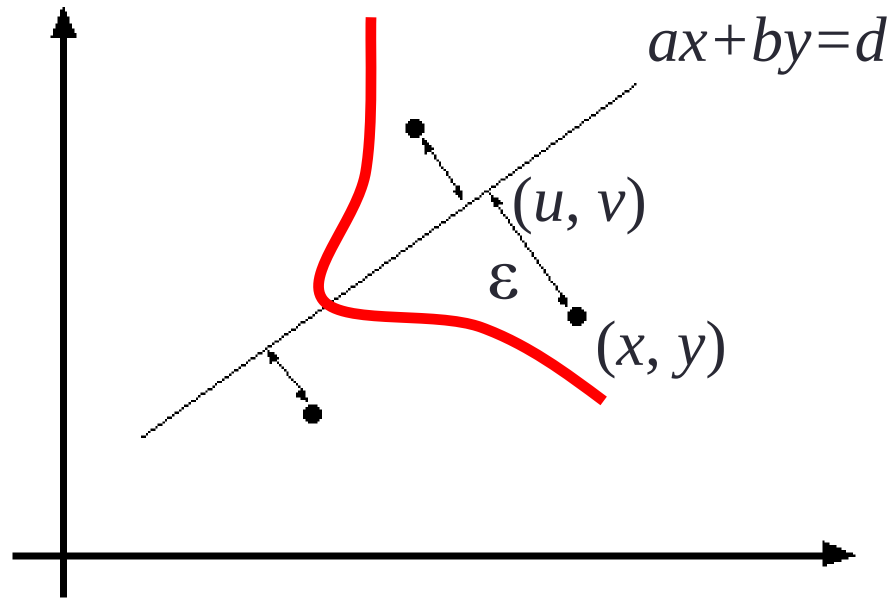


- Find (a, b, d) to minimize the sum of squared perpendicular distances

$$\boxed{E = \sum_{i=1}^{n}(ax_i + by_i - d)^2}$$

Least squares as likelihood maximization

- **Generative model**: line points are corrupted by Gaussian noise in the direction perpendicular to the line



$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} u \\ v \end{pmatrix} + \varepsilon \begin{pmatrix} a \\ b \end{pmatrix}$$

point on the line

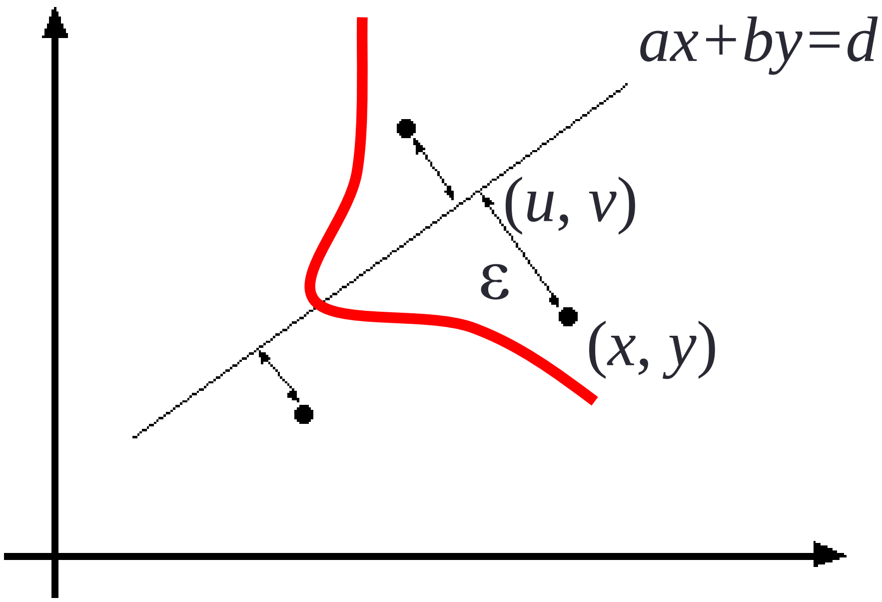
noise: sampled from zero-mean Gaussian with std. dev. σ

normal direction

Least squares as likelihood maximization

- **Generative model**: line points are corrupted by Gaussian noise in the direction perpendicular to the line

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} u \\ v \end{pmatrix} + \varepsilon \begin{pmatrix} a \\ b \end{pmatrix}$$



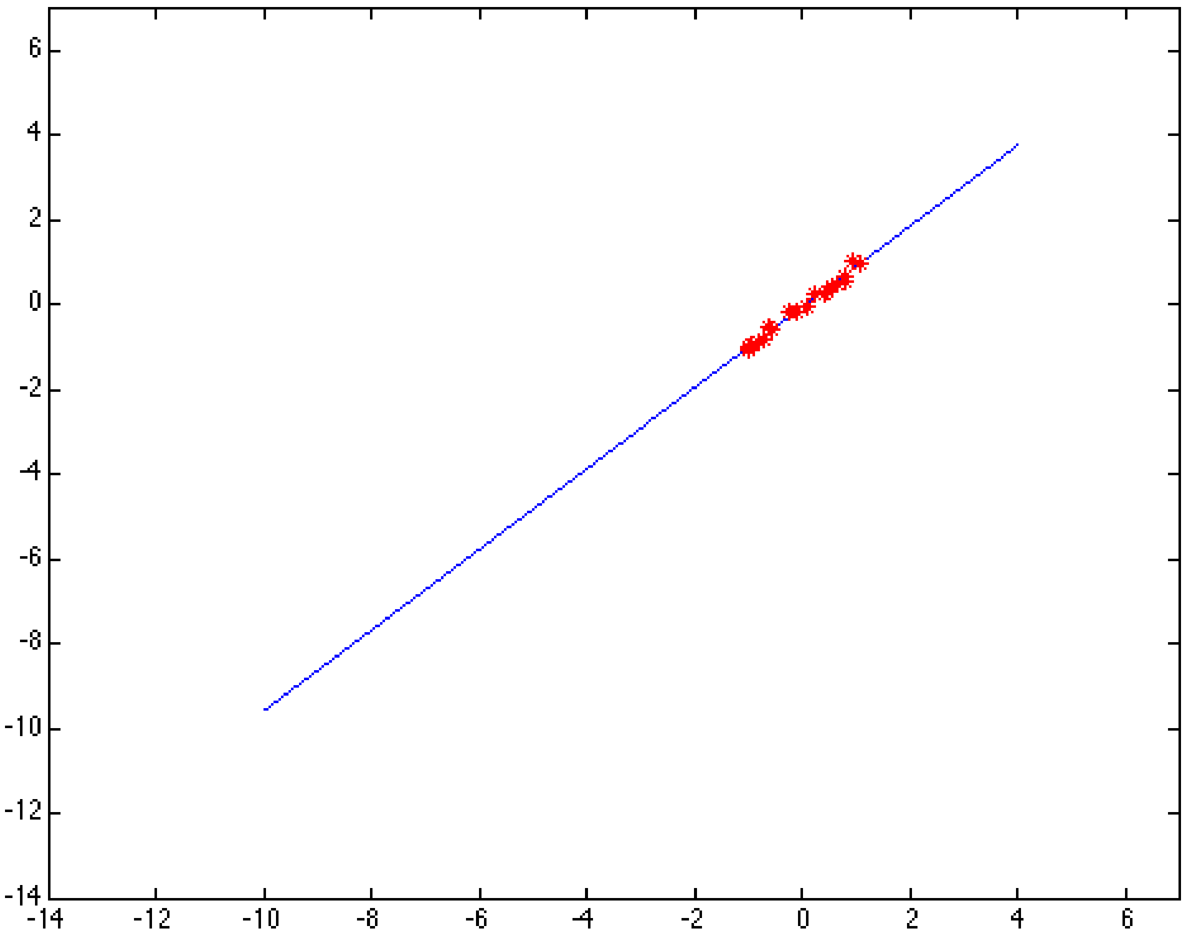
Likelihood of points given line parameters (*a*, *b*, *d*):

$$P(x_1, y_1, \ldots, x_n, y_n \mid a, b, d) = \prod_{i=1}^{n} P(x_i, y_i \mid a, b, d) \propto \prod_{i=1}^{n} \exp\left(-\frac{(ax_i + by_i - d)^2}{2\sigma^2} \right)$$

Log-likelihood: $L(x_1, y_1, \ldots, x_n, y_n \mid a, b, d) = -\frac{1}{2\sigma^2} \sum_{i=1}^{n} (ax_i + by_i - d)^2$

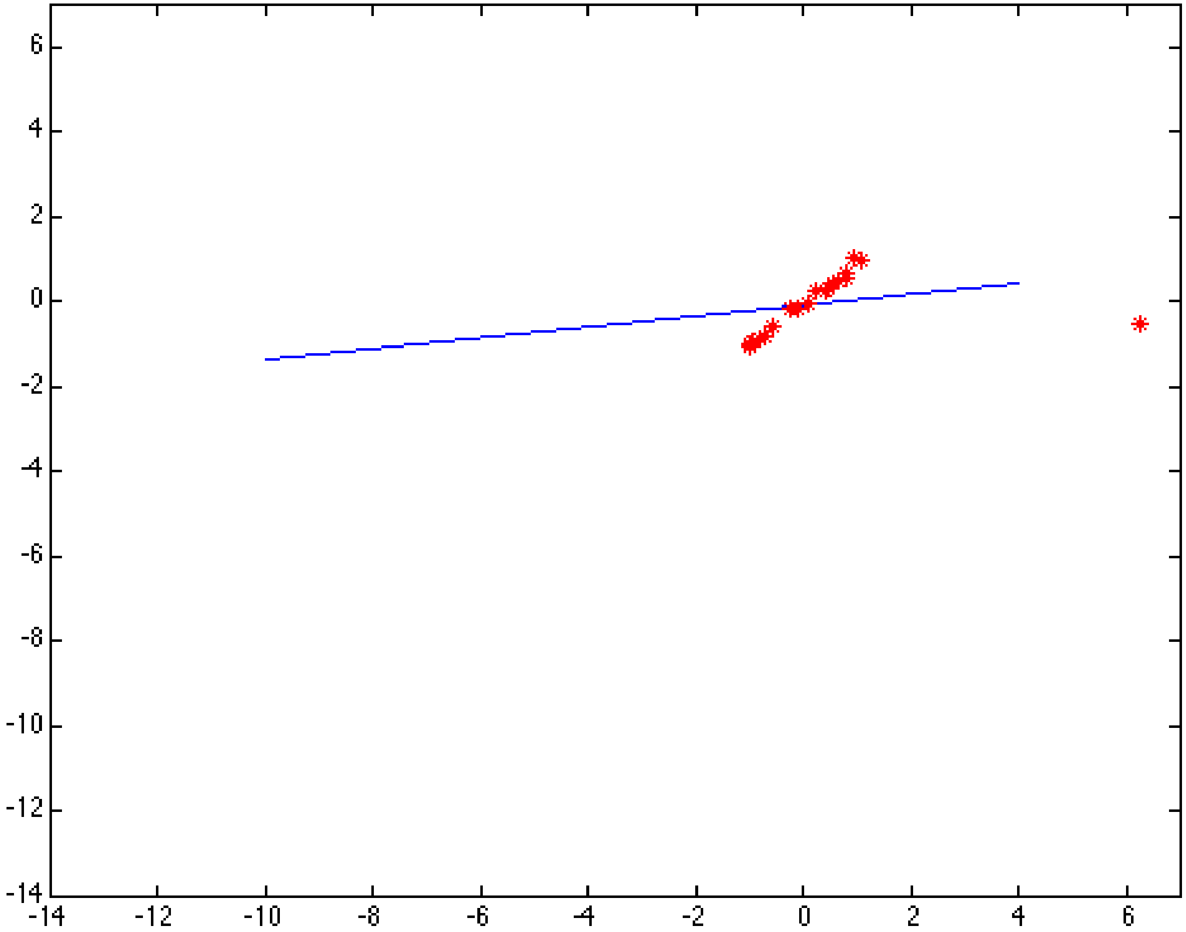
Least squares: Lack of robustness to (very) non-Gaussian noise

- Least squares fit to the red points:



Least squares: Lack of robustness to (very) non-Gaussian noise

- Least squares fit with an outlier:

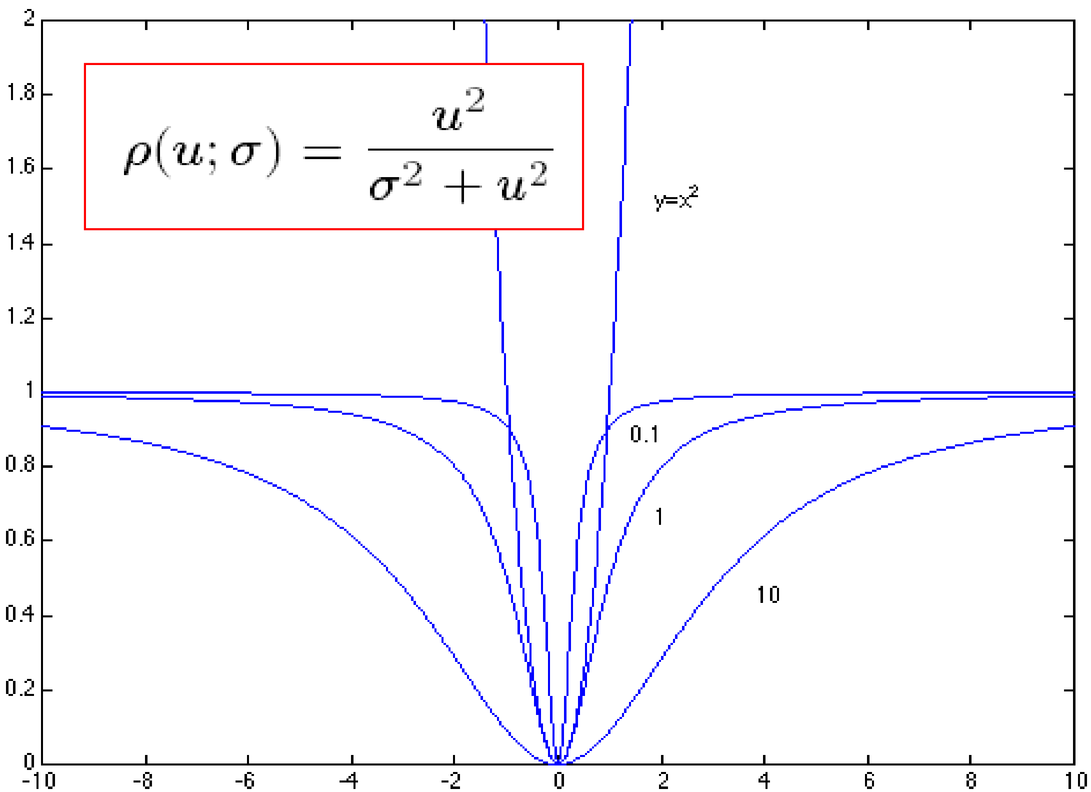


Problem: squared error heavily penalizes outliers

Robust estimators

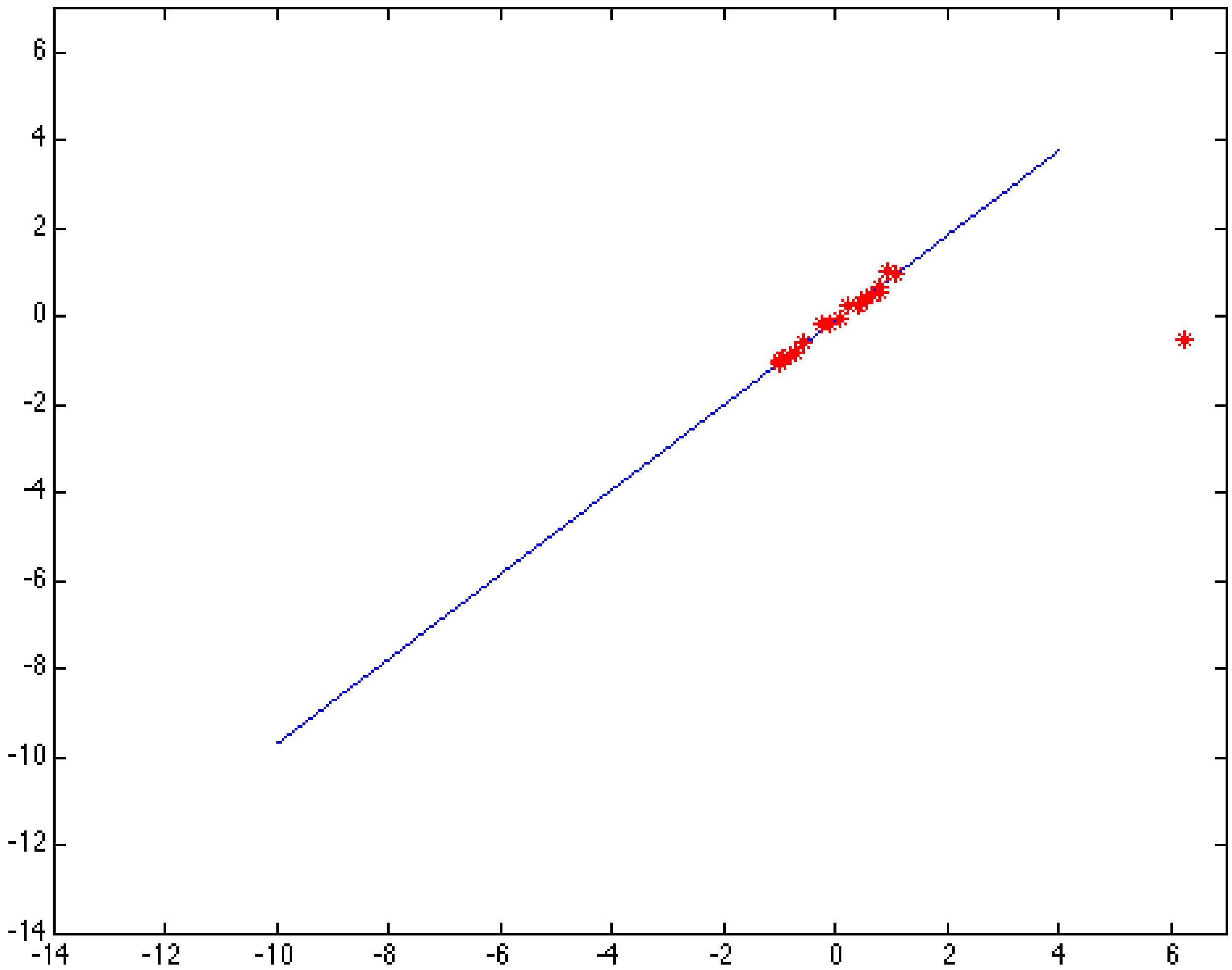
- General approach: minimize $$\sum_i \rho\left(r_i\left(x_i, \theta\right); \sigma\right)$$

$r_i\,(x_i, \theta)$ – residual of ith point w.r.t. model parameters θ
ρ – robust function with scale parameter σ



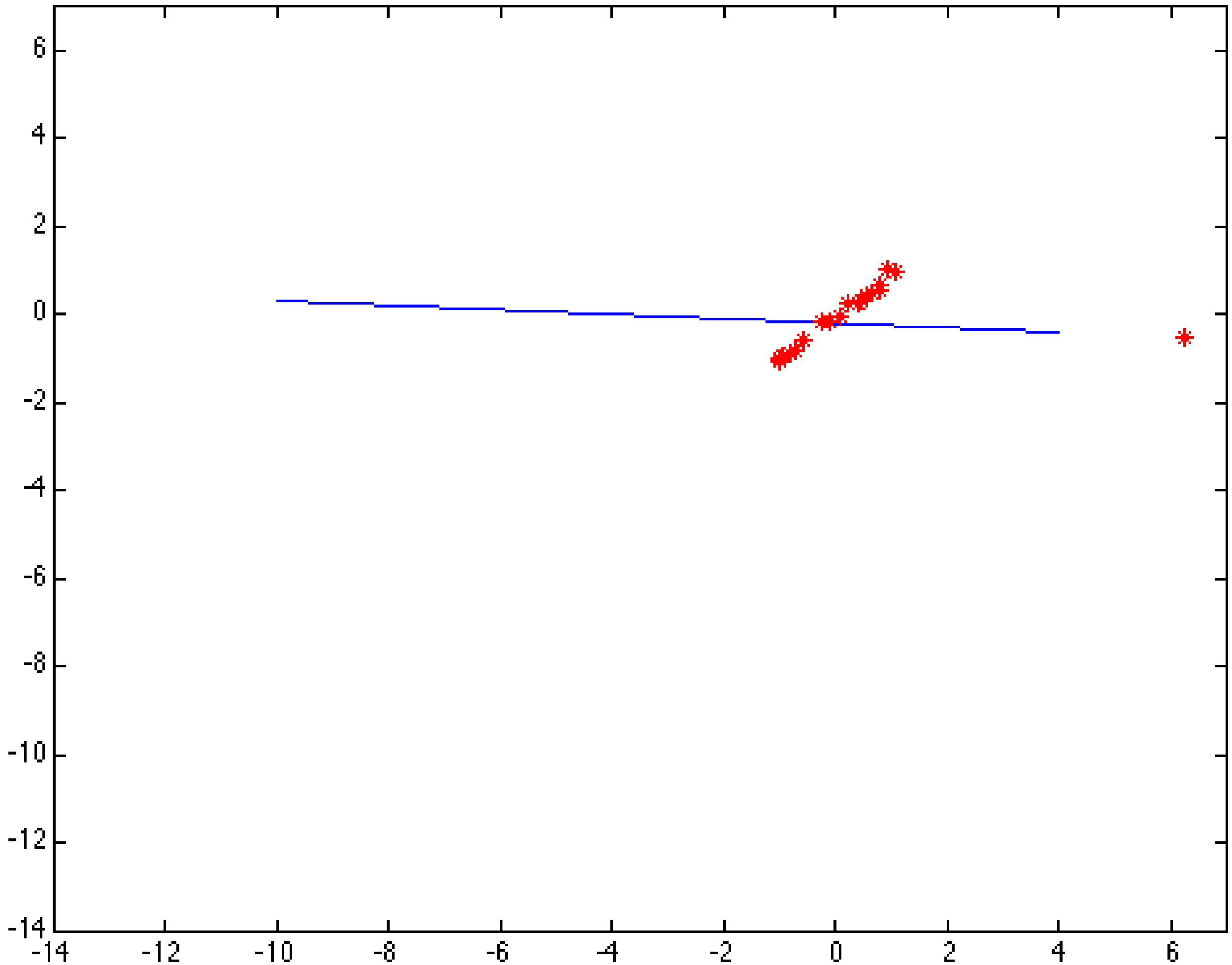
The robust function ρ behaves like squared distance for small values of the residual u but saturates for larger values of u

Choosing the scale: Just right



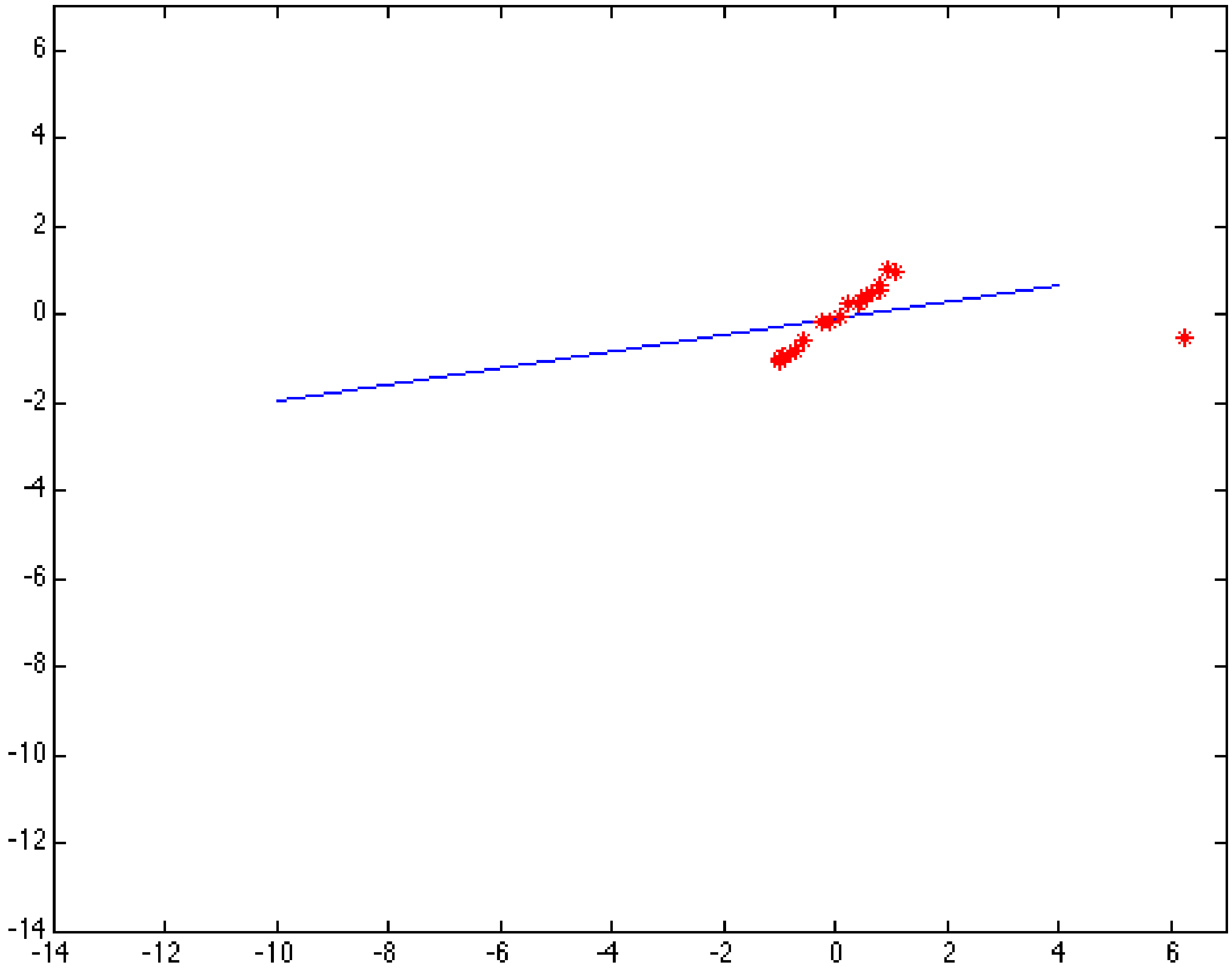
The effect of the outlier is minimized

Choosing the scale: Too small



The error value is almost the same for every point and the fit is very poor

Choosing the scale: Too large



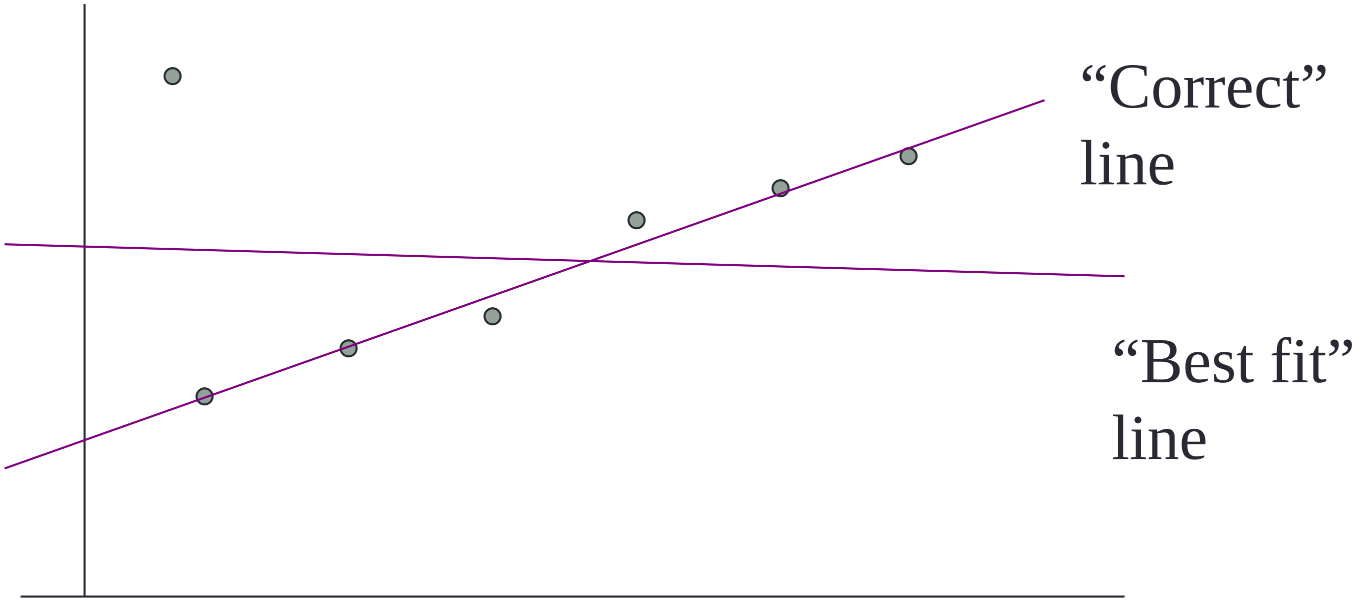
Behaves much the same as least squares

"Find consistent matches"???

- Some points (many points) are static in the world
- Some are not
- Need to find the right ones so can compute pose.
- Well tried approach:
  - *Random Sample Consensus (RANSAC)*

Simpler Example

- Fitting a straight line



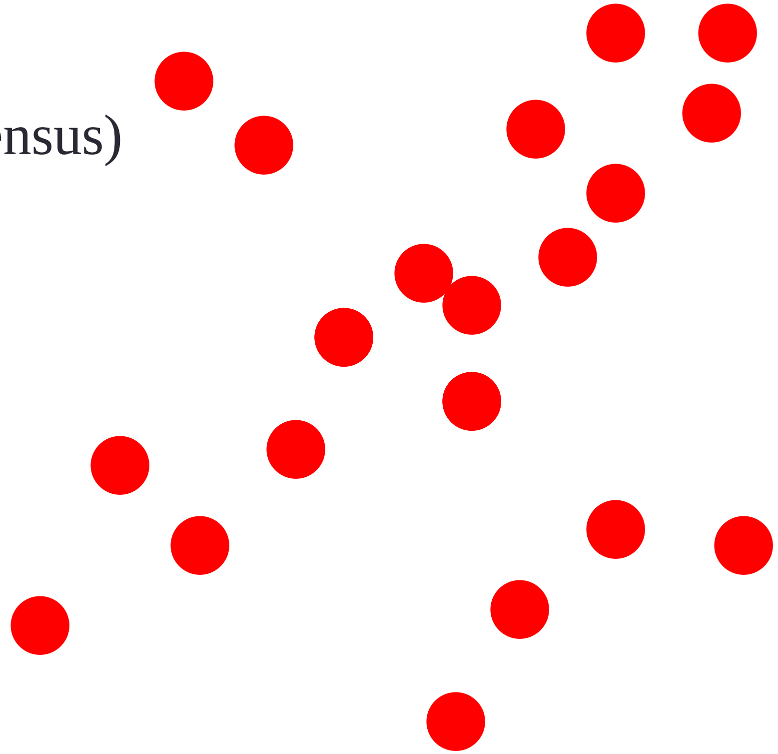
Discard Outliers

- Assume few real points with distance $d > \theta$
- RANSAC:
  - RANdom SAmple Consensus
  - Fischler & Bolles 1981
  - Copes with a large proportion of outliers

M. A. Fischler, R. C. Bolles. Random Sample Consensus: A Paradigm for Model Fitting with Applications to Image Analysis and Automated Cartography. Comm. of the ACM, Vol 24, pp 381-395, 1981.

RANSAC

(RANdom SAmple Consensus)



Algorithm:

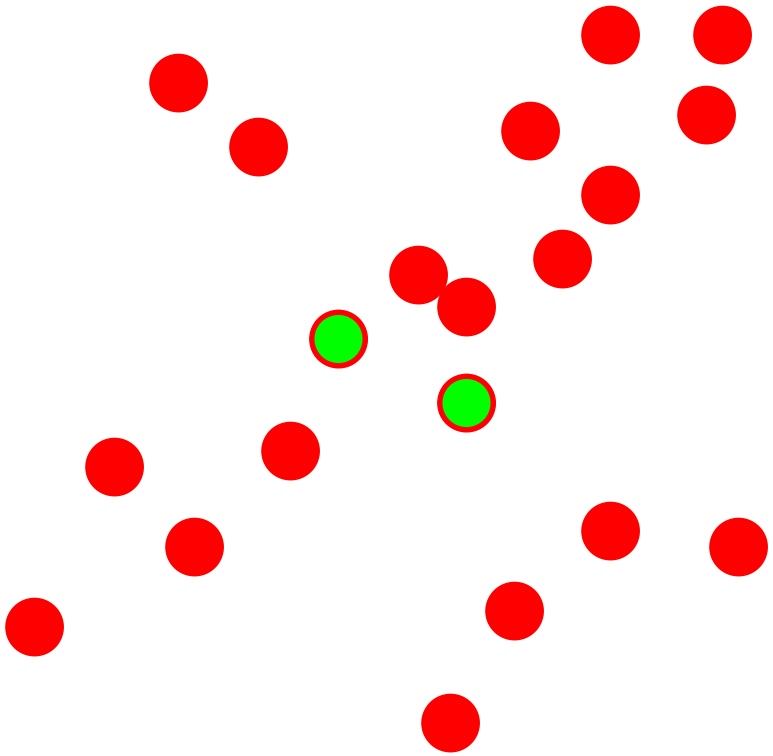
1. **Sample** (randomly) the number of points required to fit the model
2. **Solve** for model parameters using sample
3. **Score** by the fraction of ***inliers*** within a preset threshold of the model

Repeat 1-3 until the best model is found with high confidence

Illustration by Savarese

RANSAC

Line fitting example



Algorithm:

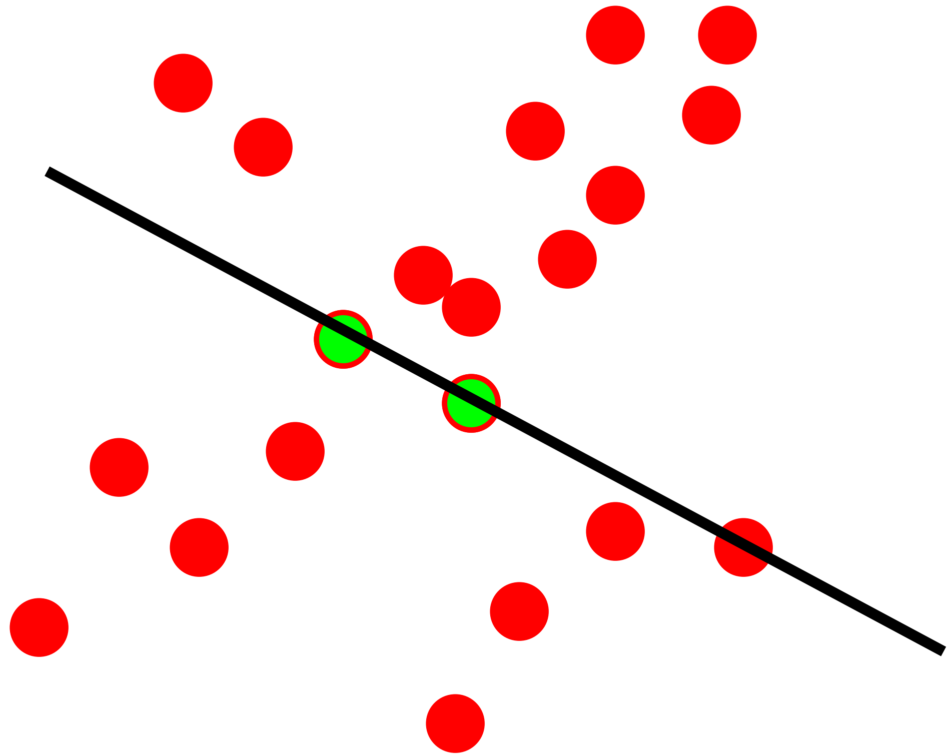
1. **Sample** (randomly) the number of points required to fit the model (#=2)
2. **Solve** for model parameters using sample
3. **Score** by the fraction of inliers within a preset threshold of the model

Repeat 1-3 until the best model is found with high confidence

Illustration by Savarese

RANSAC

Line fitting example



Algorithm:

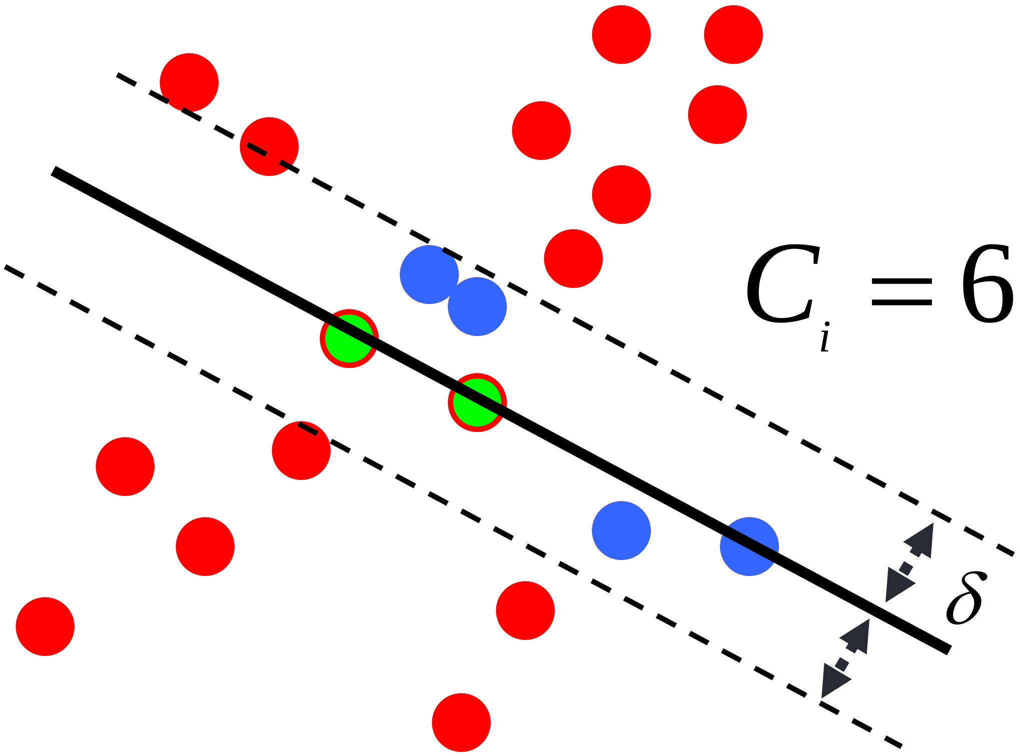
1. **Sample** (randomly) the number of points required to fit the model (#=2)
2. **Solve** for model parameters using sample
3. **Score** by the fraction of inliers within a preset threshold of the model

Repeat 1-3 until the best model is found with high confidence

Illustration by Savarese

RANSAC

Line fitting example



Algorithm:

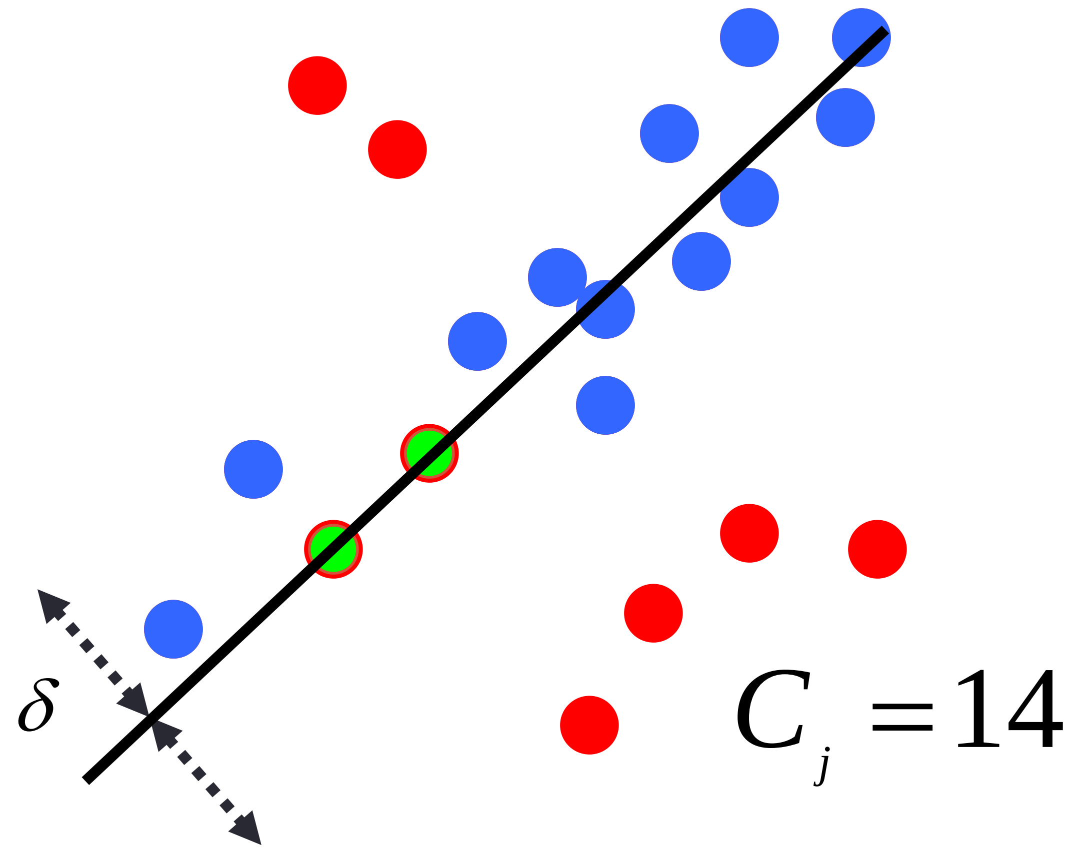
1. **Sample** (randomly) the number of points required to fit the model (#=2)
2. **Solve** for model parameters using the sample
3. **Score** by the fraction of inliers within a preset threshold of the model

Repeat 1-3 until the best model is found with high confidence

Illustration by Savarese

RANSAC

Line fitting example



Algorithm:

1. **Sample** (randomly) the number of points required to fit the model (#=2)
2. **Solve** for model parameters using sample
3. **Score** by the fraction of inliers within a preset threshold of the model

Repeat 1-3 until the best model is found with high confidence

Illustration by Savarese

The fundamental RANSAC assumption:

More support implies better fit..

RANSAC for general model

- A given model has a *minimal set* – the smallest number of samples from which the model can be computed.
  - Line: 2 points

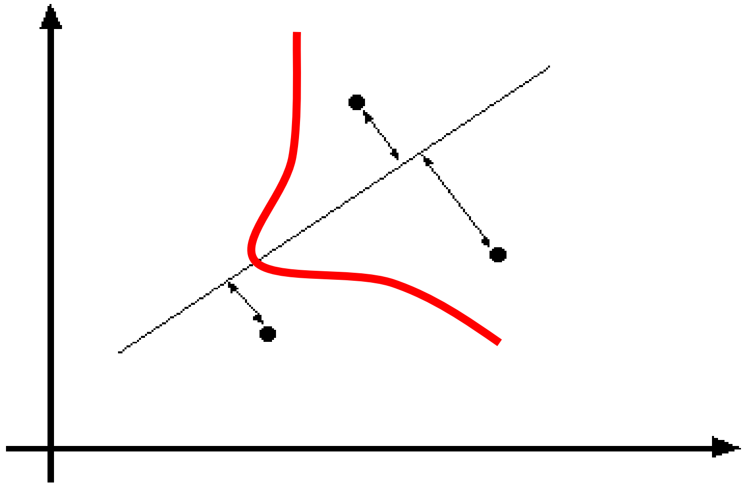
- Image transformations are models.
  Minimal set of *s* of point pairs/matches:
  - *Translation*: pick one point pair
  - *Homography* (for plane) – pick 4 point pairs
  - *Fundamental matrix* – pick 8 point pairs (really 7 but lets not go there)

- Algorithm
  - Randomly select s points (or point pairs) to form a sample
  - Instantiate a model
  - Get consensus set C_i
  - If $|C_i| > T$, terminate and return model
  - Repeat for N trials, return model with max $|C_i|$

Distance Threshold

- Let's assume **location** noise is Gaussian with σ^2
- Then the **distance** d has ***Chi*** distribution with k degrees of freedoms where k is the dimension of the Gaussian.
- If one dimension, e.g.distance off a line, then 1DOF

$$f(d) = \frac{\sqrt{2}e^{-(\frac{d^2}{2\sigma^2})}}{\sqrt{\pi}\sigma}, d \geq 0$$



Distance Threshold

For 95% cumulative threshold t when Gaussian with σ^2

$$t^2 = 3.84\sigma^2$$

That is: if $t^2 = 3.84\sigma^2$ then 95% probability that $d < t$ when point is inlier

But…

Usually set by "*empirically*"…

How many samples should we try?

- We want: at least one sample with all inliers

- With random samples we can't guarantee. But with probability p we can, e.g. $p = 0.99$

Choosing the parameters

- Initial number of points s
  - Typically minimum number needed to fit the model
- Distance threshold t
  - Choose t so probability for inlier is high (e.g. 0.95)
  - If zero-mean Gaussian noise with std. dev. σ: $t^2 = 3.84\sigma^2$
- Number of samples N
  - Choose N so that, with probability p, at least one random sample is free from outliers (e.g. $p = 0.99$)
  - Need to set N based on *outlier ratio*: e

Source: M. Pollefeys

Calculate N

1. s – number of points to compute solution
2. p – probability of success
3. e – proportion outliers, so % inliers = $(1-e)$
4. $P(sample\ set\ with\ all\ inliers) = (1-e)^s$
5. $P(sample\ set\ will\ have\ at\ least\ one\ outlier) =$
   $(1-(1-e)^s)$
6. $P(all\ N\ samples\ have\ outlier) = (1-(1-e)^s)^N$
7. We want $P(all\ N\ samples\ have\ outlier) < (1-p)$
8. So: $(1-(1-e)^s)^N < (1-p)$

$$\boxed{N > \log(1-p) / \log(1-(1-e)^s)}$$

N for probability p of at least one sample with only inliers

$$\boxed{N > \log(1-p)/\log(1-(1-e)^s)}$$

- Set p=0.99 – chance of getting good sample

$s = 2, e = 5\%$ => N=2

$s = 2, e = 50\%$ => N=17

$s = 4, e = 5\%$ => N=3

$s = 4, e = 50\%$ => N=72

$s = 8, e = 5\%$ => N=5

$s = 8, e = 50\%$ => N=1177

	proportion of outliers *e*						
s	5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

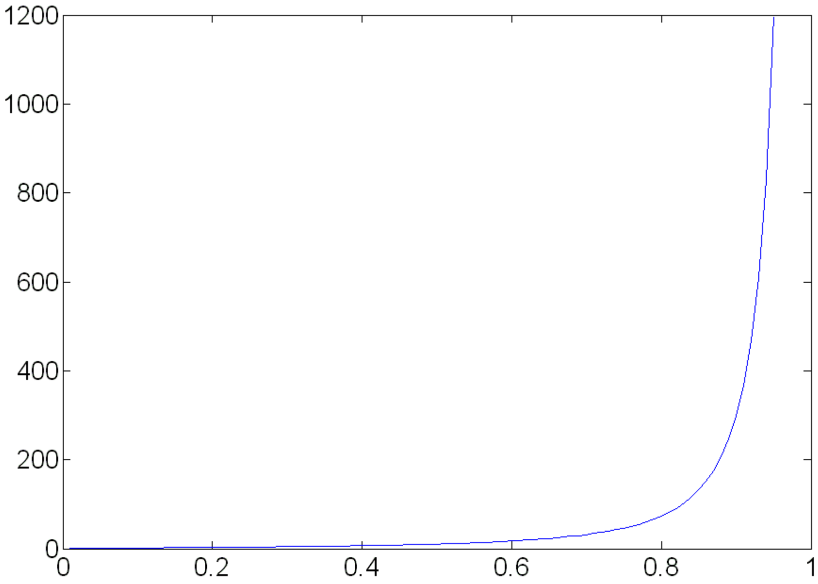
- N increases steeply with s

N for probability p of at least one sample with only inliers

$$\boxed{N > \log(1-p)/\log(1-(1-e)^s)}$$

- Set p=0.99 – chance of getting good sample

$s = 2, e = 5\%$	=> N=2
$s = 2, e = 50\%$	=> N=17
$s = 4, e = 5\%$	=> N=3
$s = 4, e = 50\%$	=> N=72
$s = 8, e = 5\%$	=> N=5
$s = 8, e = 50\%$	=> N=1177



- N increases steeply with s

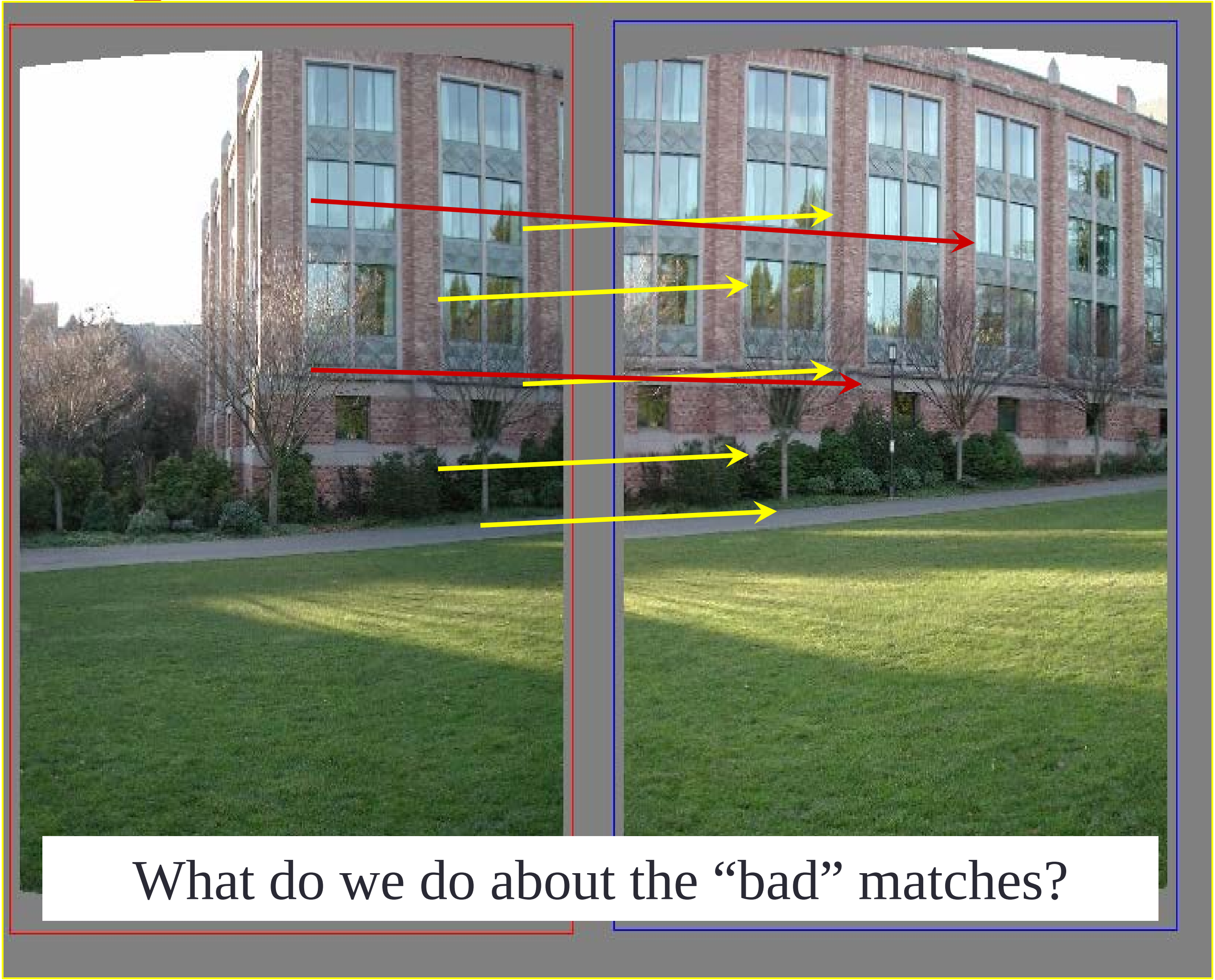
How big does N need to be?

$$\boxed{N > \log(1-p)/\log(1-(1-e)^s)}$$

- So $N = f(e,s,p)$
- What is N ***not a function of?***

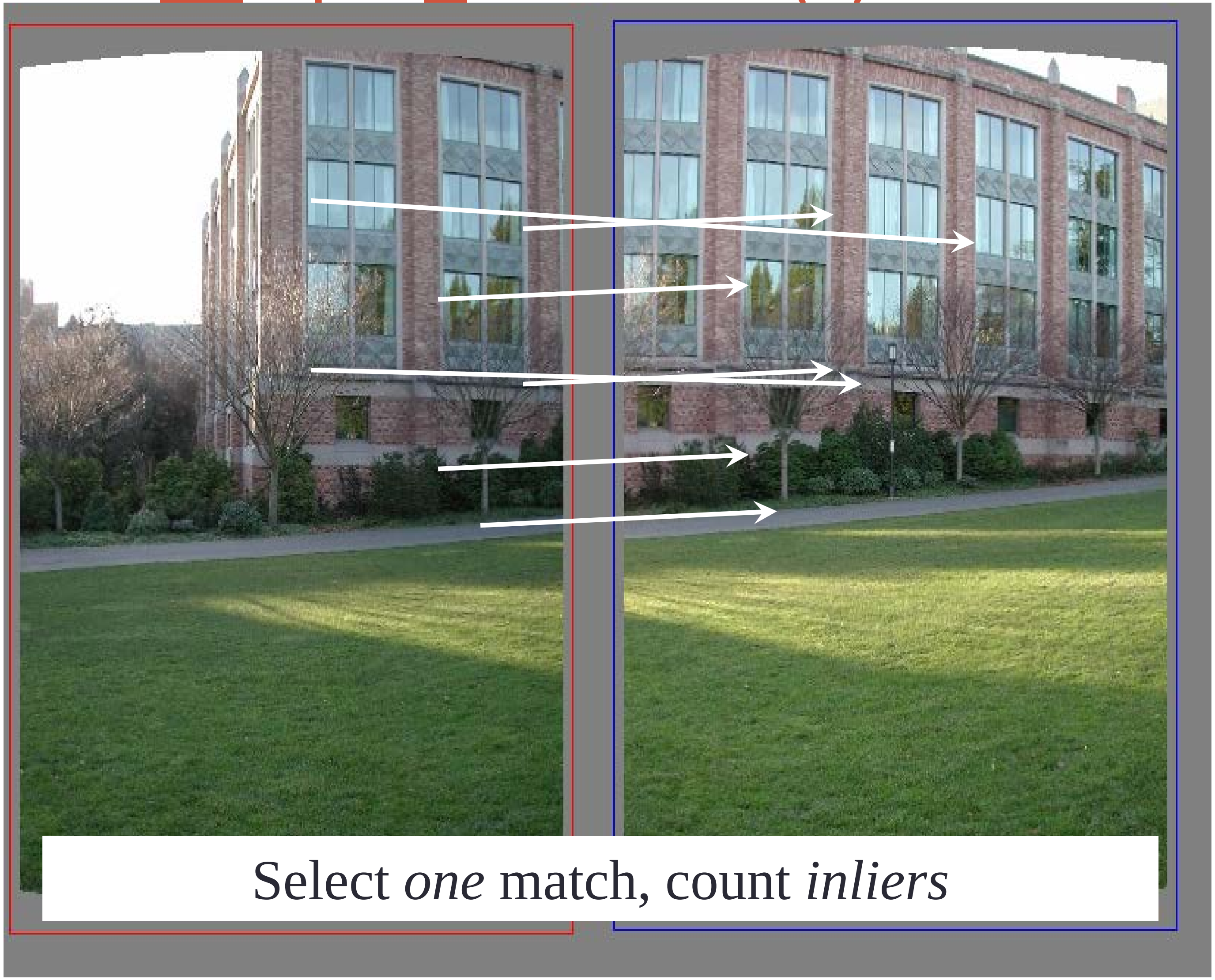
$N = f(e,s,p)$, *but not the number of points(matches)**!***

Matching features



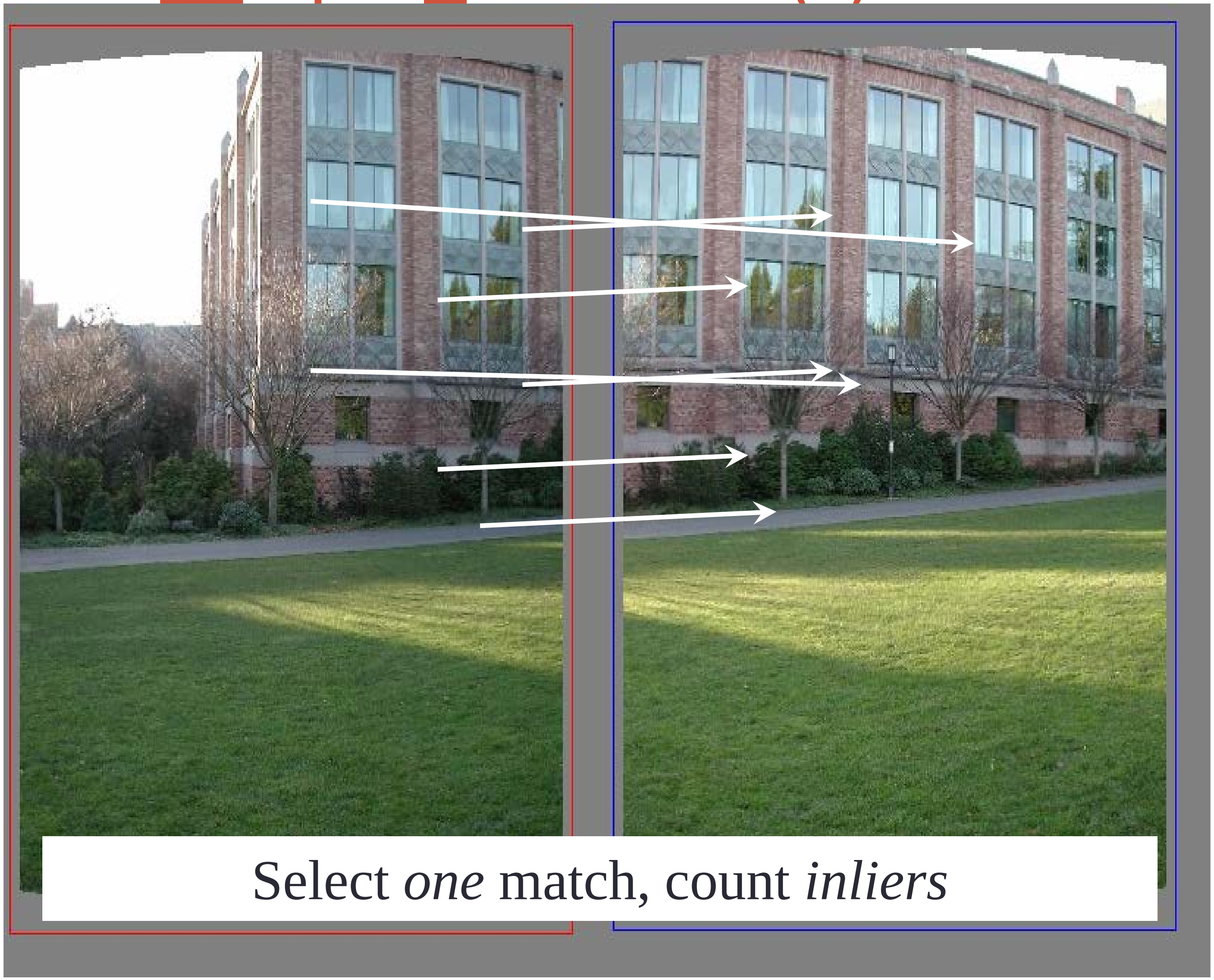
What do we do about the “bad” matches?

RAndom SAmple Consensus (1)



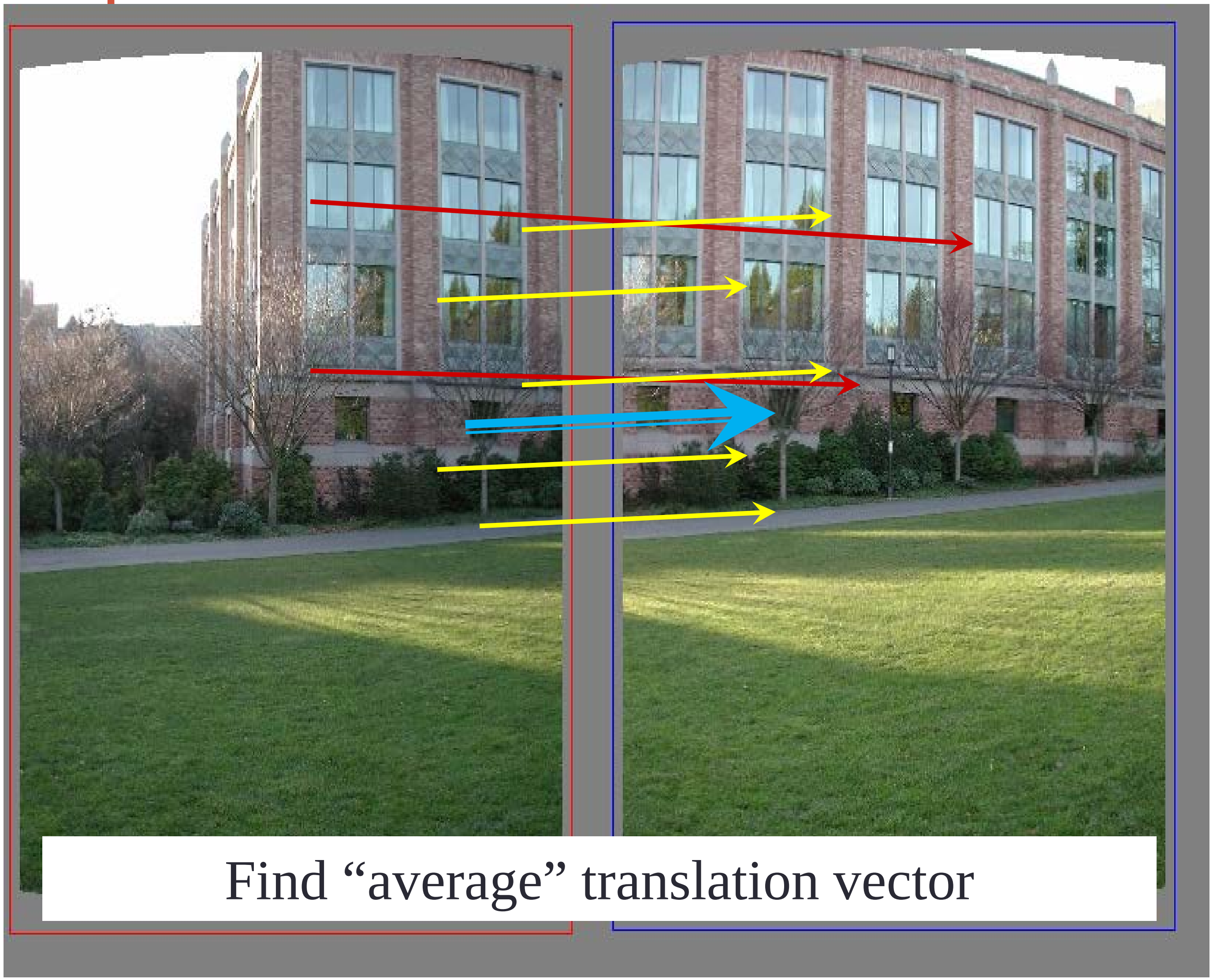
Select *one* match, count *inliers*

RAndom SAmple Consensus (2)



Select *one* match, count *inliers*

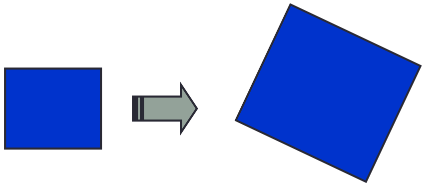
Least squares fit



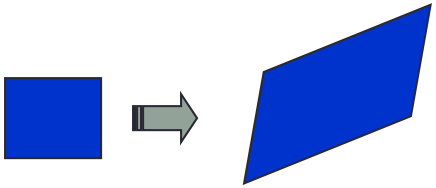
Find “average” translation vector

2D transformation models

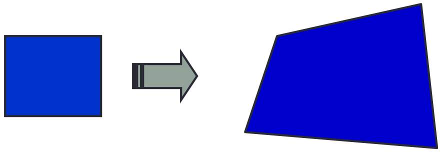
2 matches:
Similarity
(translation,
scale, rotation)



3 matches:
Affine



4 matches:
Projective
(homography)



Source: S. Lazebnik

RANSAC for estimating, say, homography

RANSAC loop:

1. Select four feature pairs (at random)
2. Compute homography $\boldsymbol{H}_{\mathrm{k}}$ (exact)
3. Compute inliers where $SSD(p_i', \boldsymbol{H}_{\mathrm{k}}\, pi) < \varepsilon$
4. Keep $\boldsymbol{H}_{\boldsymbol{k}}$, if C_k is the largest set of inliers
5. For a while go to 1
6. Re-compute least-squares $\boldsymbol{H}$ estimate on all of the C_k inliers

Adaptively determining the number of samples

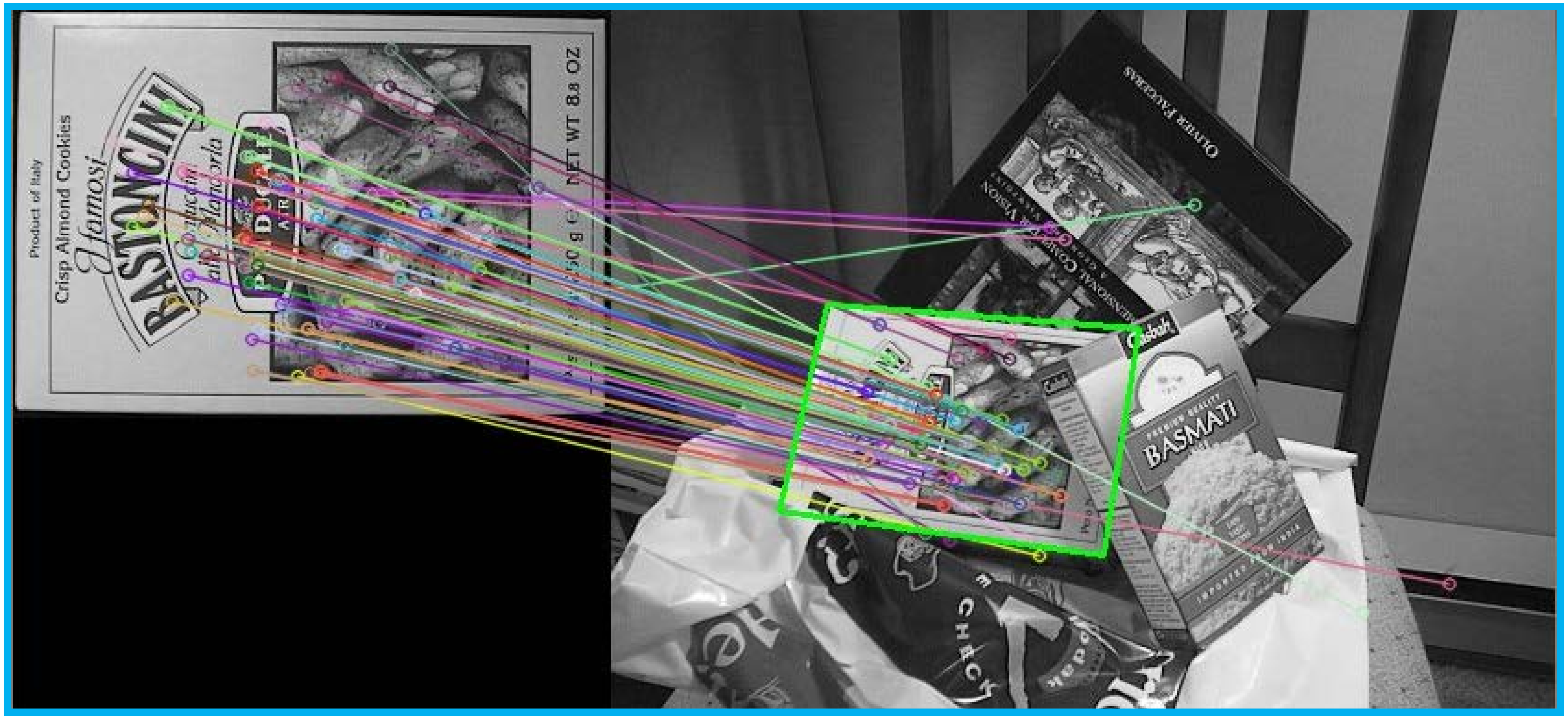
- Inlier ratio e is often unknown a priori, so pick worst case, e.g. 50%, and adapt if more inliers are found, e.g. 80% would yield e=0.2
- Adaptive procedure:
  - N= ∞, sample_count =0, $e = 1.0$
  - While N >sample_count
    - Choose a sample and count the number of inliers
    - Set $e_0 = 1 - (number\ of\ inliers)/(total\ number\ of\ points)$
    - If $e_0 < e$ Set $e = e_0$ and recompute N from e:

$$N = \log(1-p)/\log\left(1-(1-e)^s\right)$$

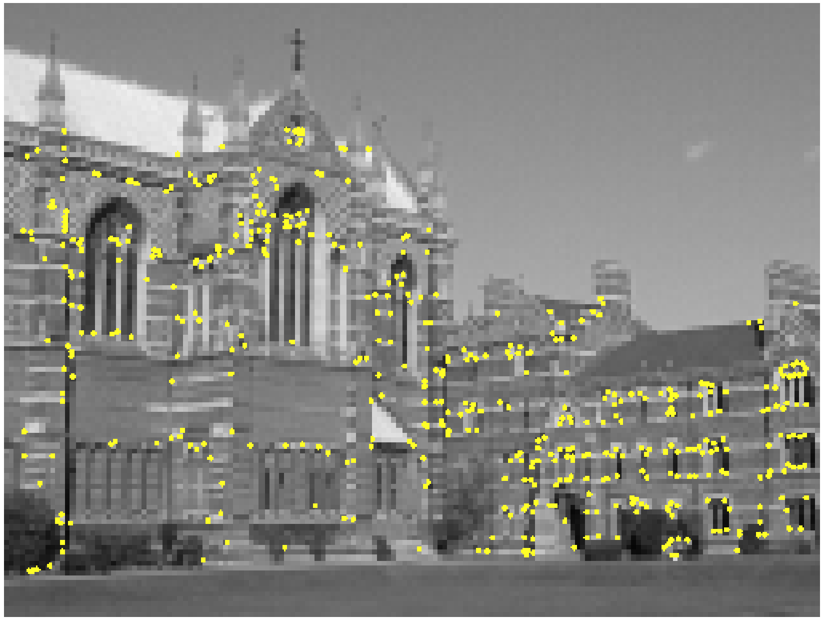
    - Increment the sample_count by 1

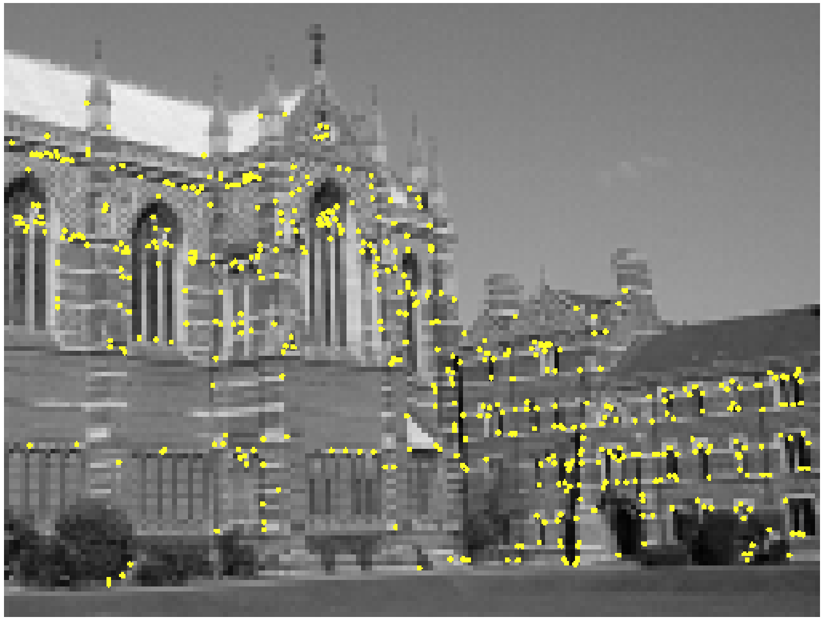
Source: M. Pollefeys

RANSAC for recognition

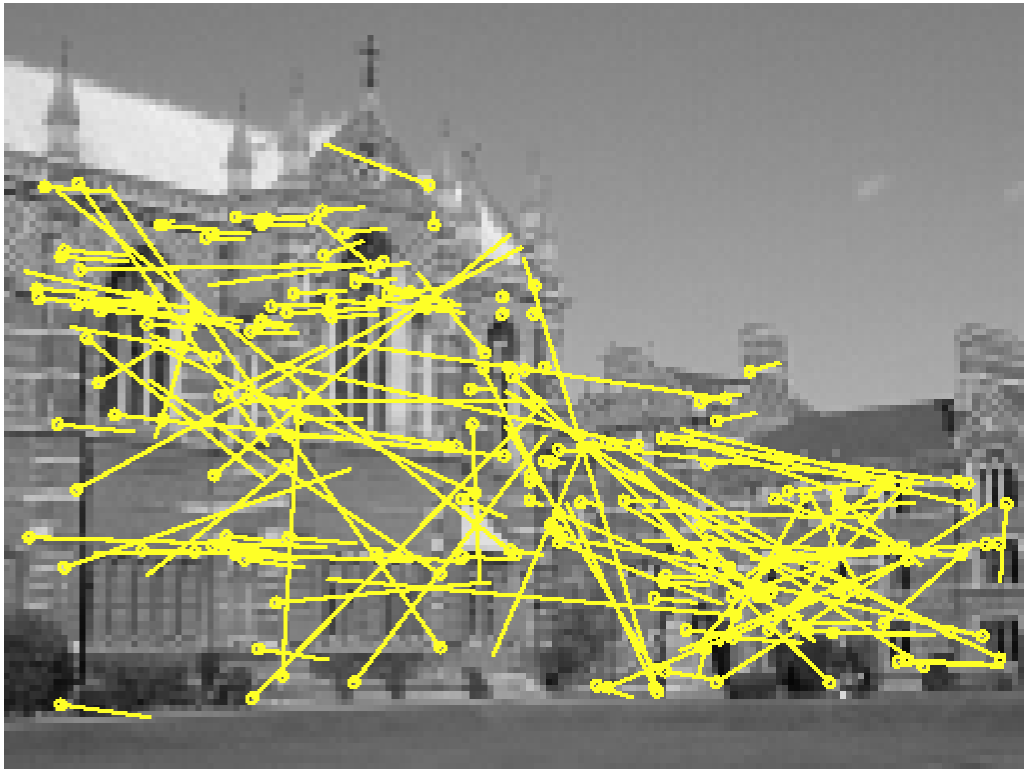


RANSAC for fundamental matrix



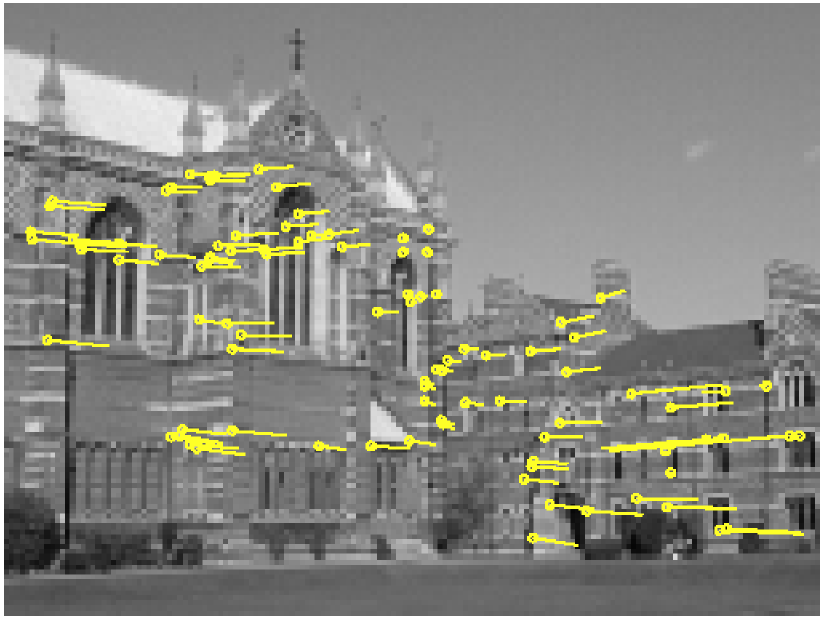


Putative matches (motion) by cross-correlation (188)

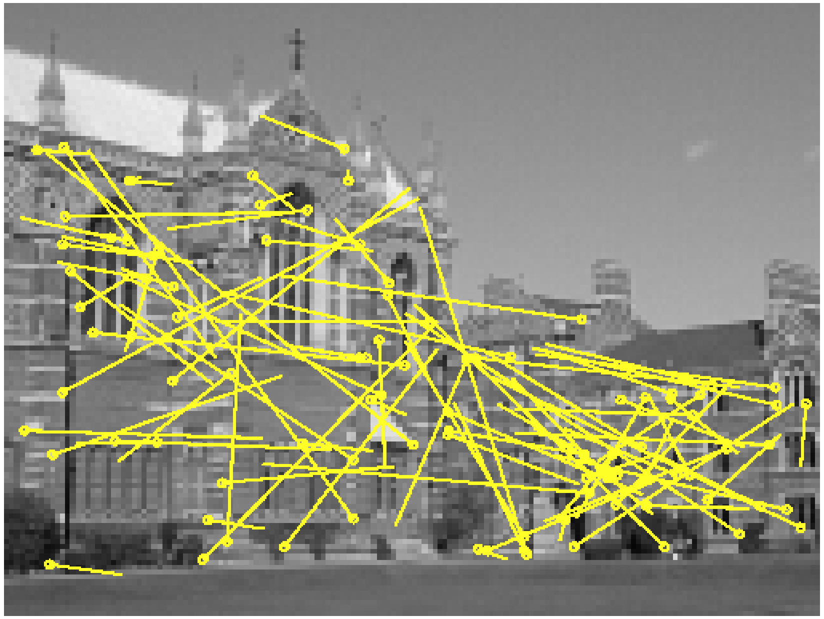


RANSAC for fundamental matrix

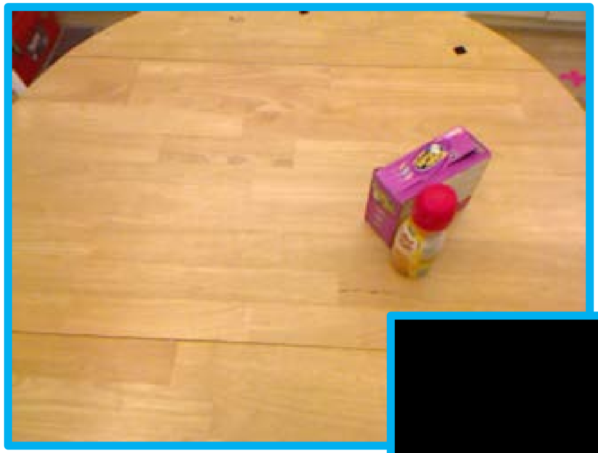
Inliers (99)

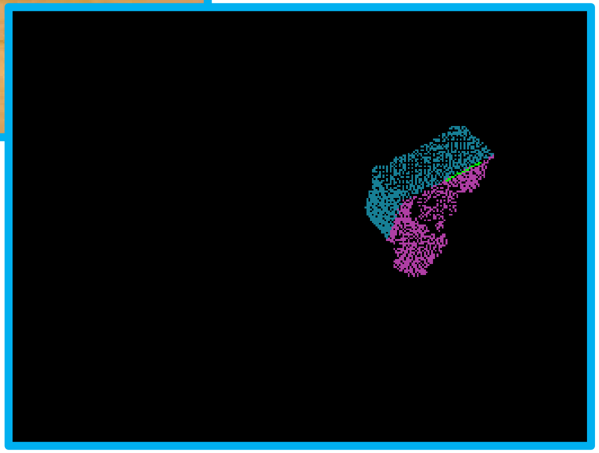


Outliers (89)



Point cloud planes

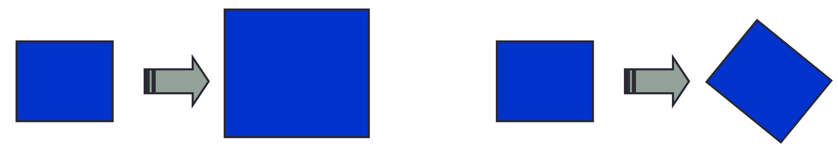


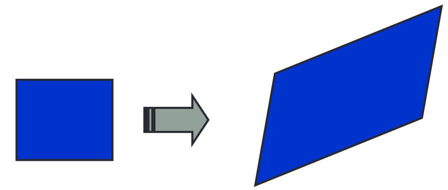


Find the plane and object in realtime

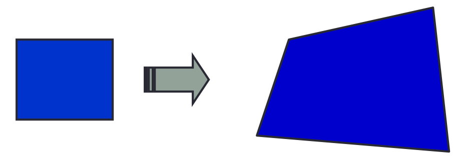
2D transformation models

- Similarity (translation, scale, rotation)





- Affine



- Projective (homography)

Source: S. Lazebnik

RANSAC conclusions

The good…

- Simple and general
- Applicable to many different problems, often works well in practice
- *Robust to large numbers of outliers*
- Applicable for larger number of parameters than Hough transform
- Parameters are easier to choose than Hough transform

RANSAC conclusions

The not-so-good…

- Computational time grows quickly with the number of model parameters
- *Sometimes problematic for approximate models*

RANSAC conclusions

Common applications

- Computing a homography (e.g., image stitching) or other image transform
- Estimating fundamental matrix (relating two views)
- *Pretty much every problem in robot vision*