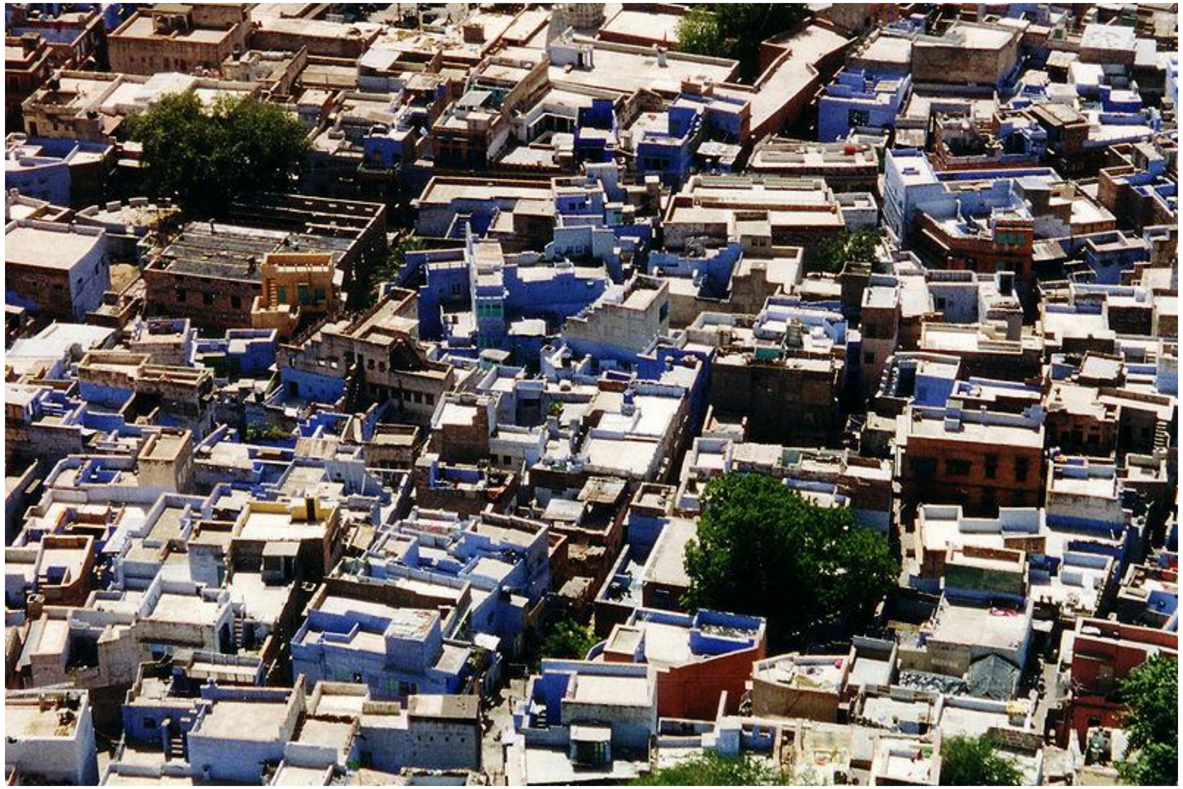
Miniature faking

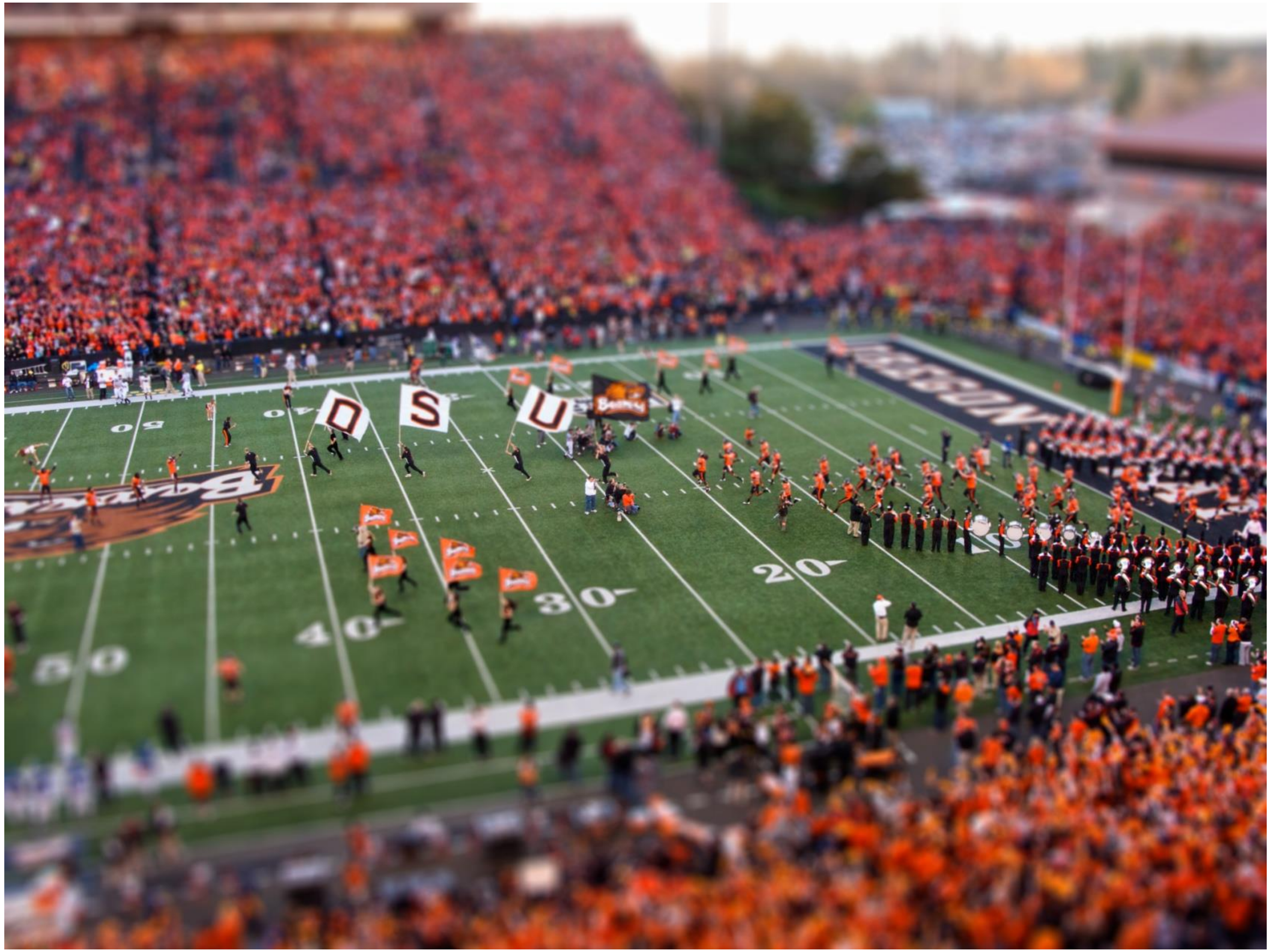
In close-up photo, the depth of field is limited.

http://en.wikipedia.org/wiki/File:Jodhpur_tilt_shift.jpg

Miniature faking



Miniature faking

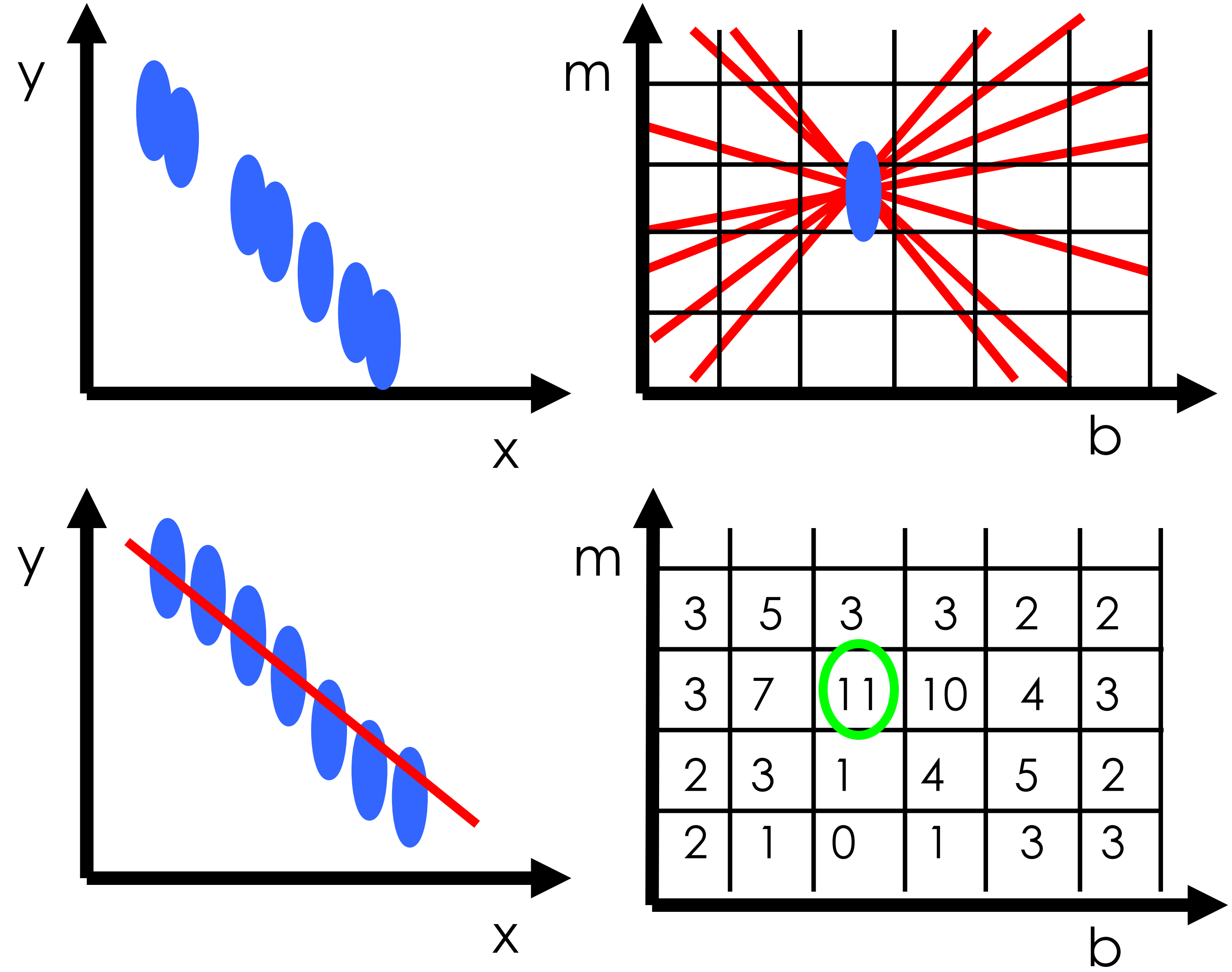


http://en.wikipedia.org/wiki/File:Oregon_State_Beavers_Tilt-Shift_Miniature_Greg_Keene.jpg

Review

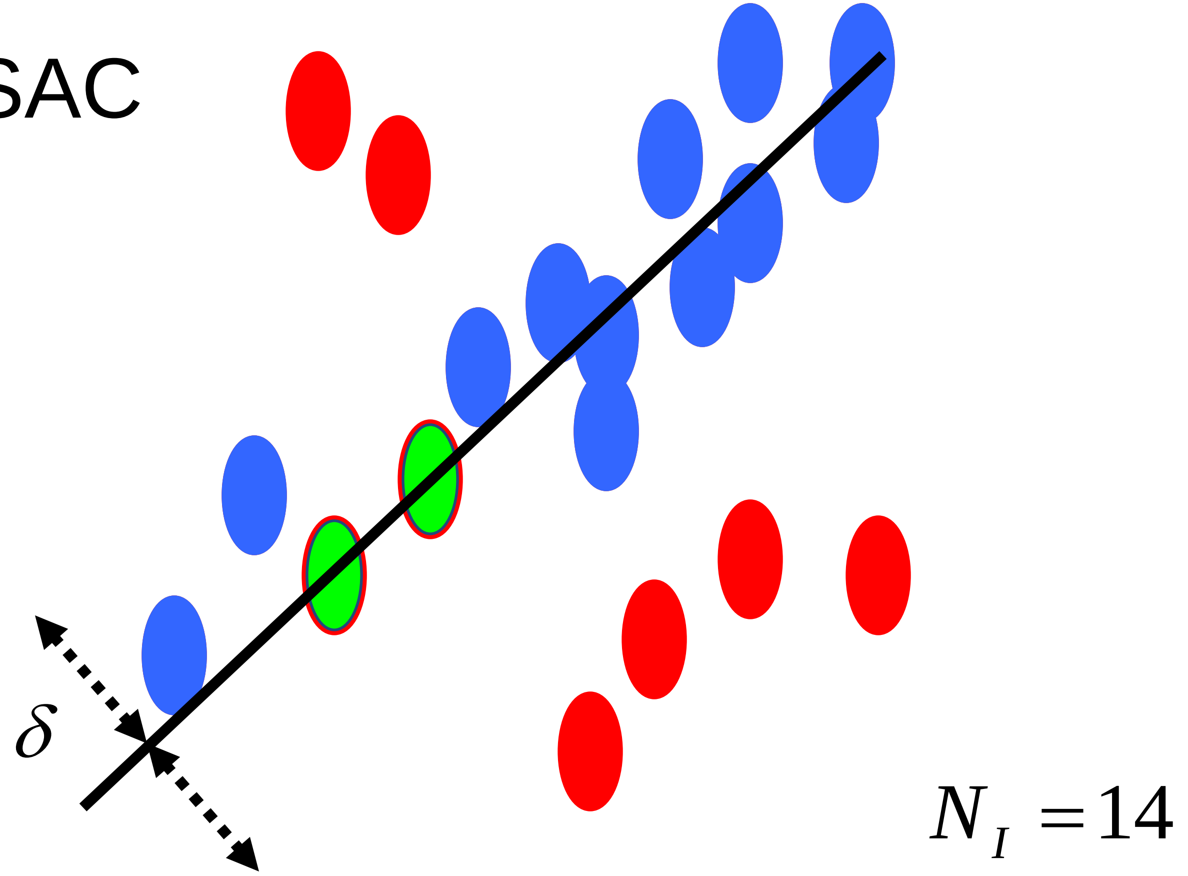
- Previous section:
  - Model fitting and outlier rejection

Review: Hough transform



Slide from S. Savarese

Review: RANSAC

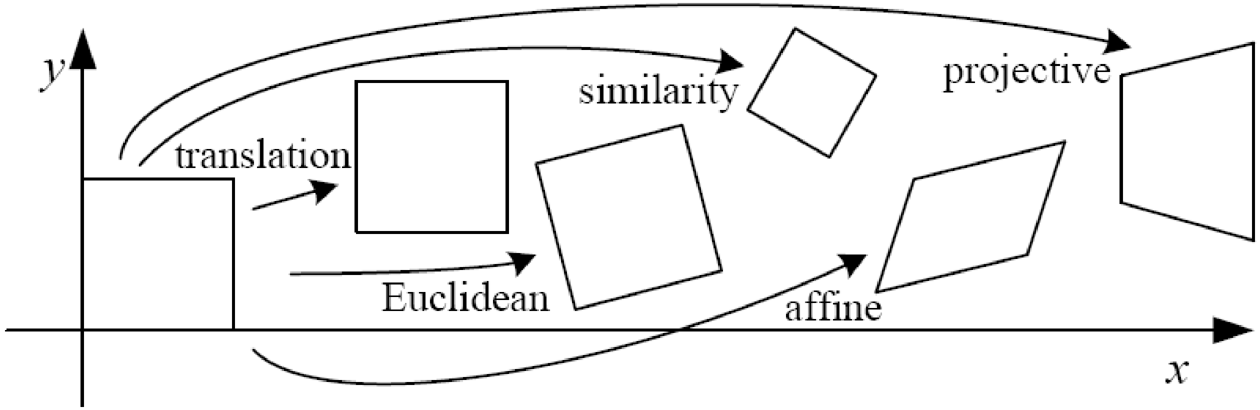


Algorithm:

1. **Sample** (randomly) the number of points required to fit the model (#=2)
2. **Solve** for model parameters using samples
3. **Score** by the fraction of inliers within a preset threshold of the model

Repeat 1-3 until the best model is found with high confidence

Review: 2D image transformations



Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$\begin{bmatrix} \boldsymbol{I} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$	2	orientation $+ \cdots$	
rigid (Euclidean)	$\begin{bmatrix} \boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$	3	lengths $+ \cdots$	
similarity	$\begin{bmatrix} s\boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$	4	angles $+ \cdots$	
affine	$\begin{bmatrix} \boldsymbol{A} \end{bmatrix}_{2\times 3}$	6	parallelism $+ \cdots$	
projective	$\begin{bmatrix} \tilde{\boldsymbol{H}} \end{bmatrix}_{3\times 3}$	8	straight lines	

Szeliski 2.1

Rough count of mentions in recent literature

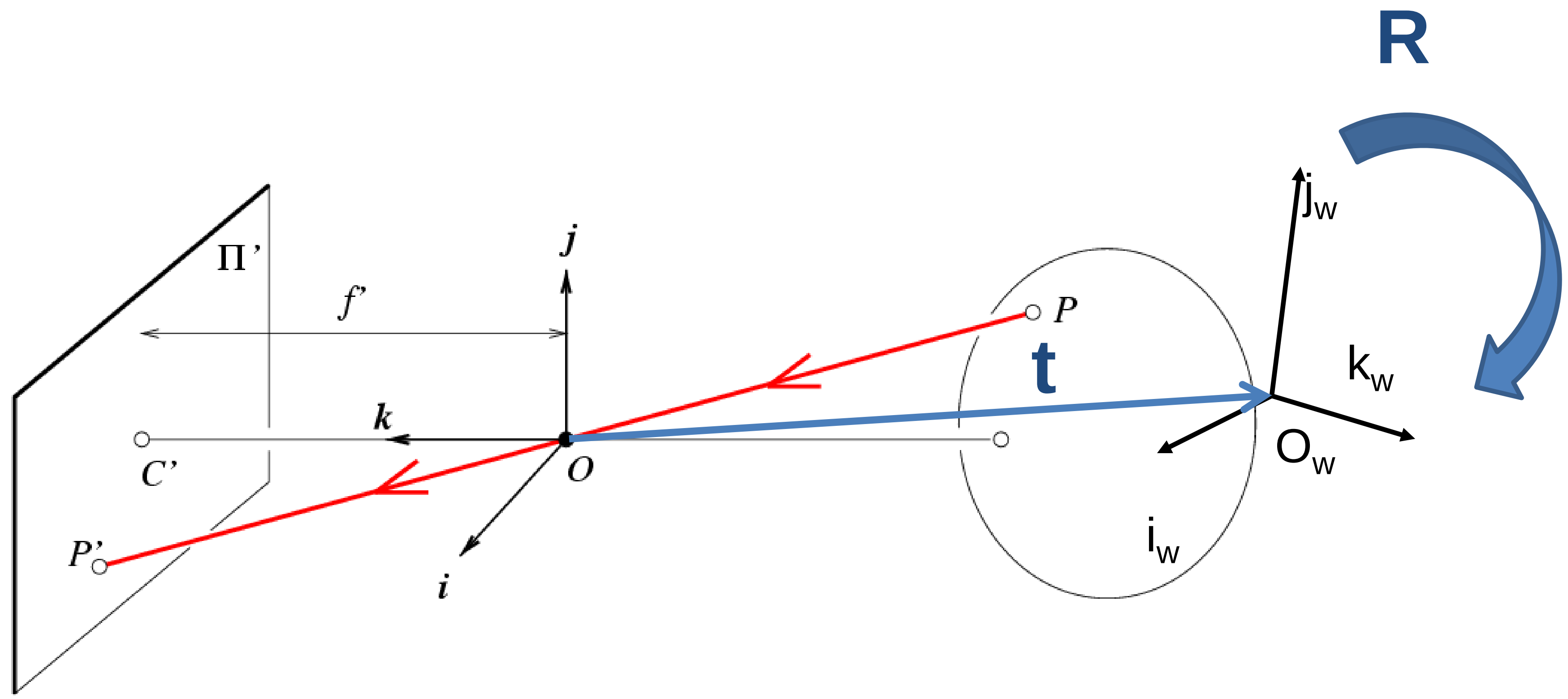
- “interest point” 958 mentions
- SIFT 3,660 mentions
- “Least Squares” 3,050 mentions
- Hough: 1,230 mentions
- RANSAC: 2,250 mentions
- ICP: 1,360 mentions
- Affine 4,090 mentions
- ResNet: 15,100 mentions
- ViT: 6,070 mentions
- Convolution: 40,500 mentions

Google search for site:https://openaccess.thecvf.com [term]
Seems to find results since 2013.

This section – multiple views

- Today – Camera Calibration. Intro to multiple views and Stereo.
- Next Lecture – Epipolar Geometry and Fundamental Matrix. Stereo Matching (if there is time).
- Both lectures are relevant for project 2.

Recap: Oriented and Translated Camera



Recap: Degrees of freedom

$$\mathbf{x} = \mathbf{K}\left[\mathbf{R} \quad \mathbf{t}\right]\mathbf{X}$$

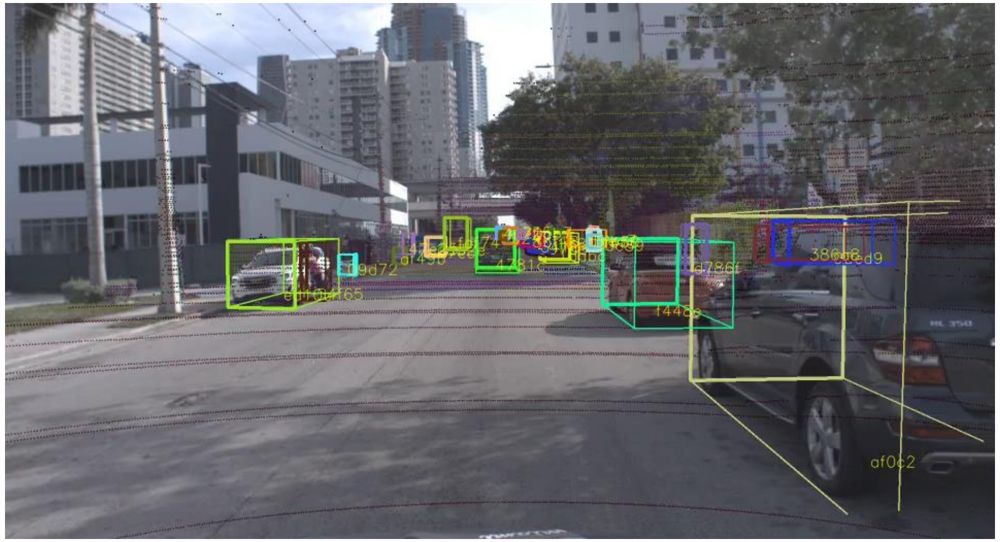
$$w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \overset{5}{\begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}} \overset{6}{\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix}} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

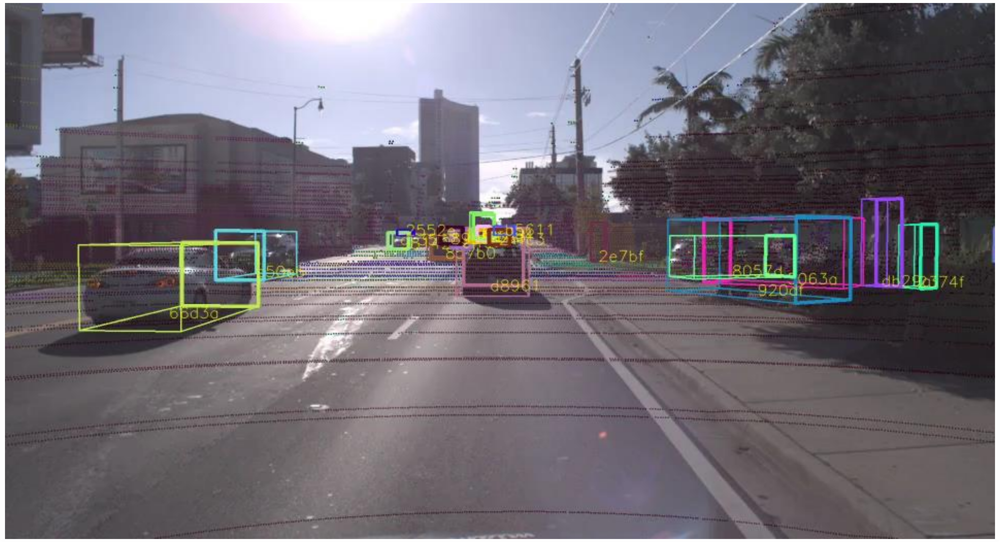
This Lecture: How to calibrate the camera?

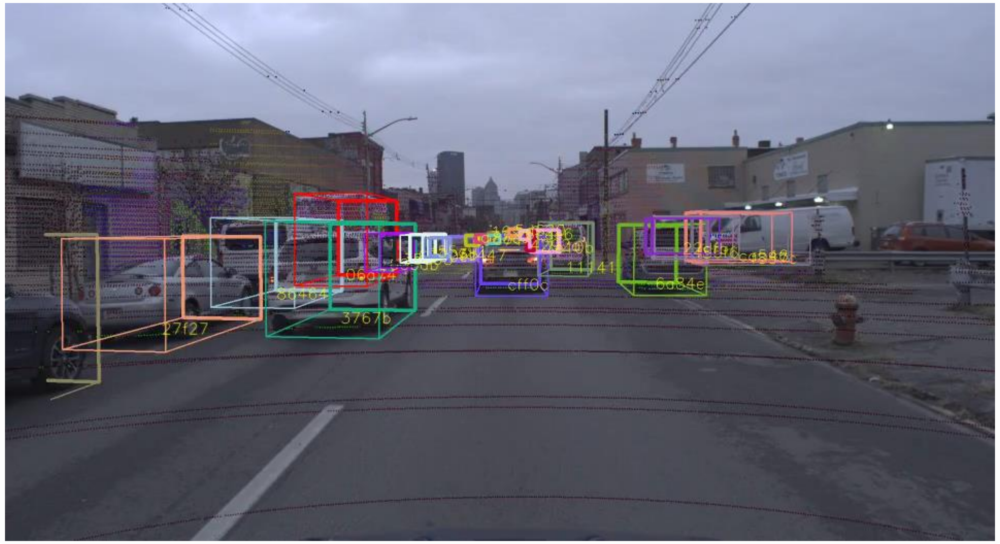
$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

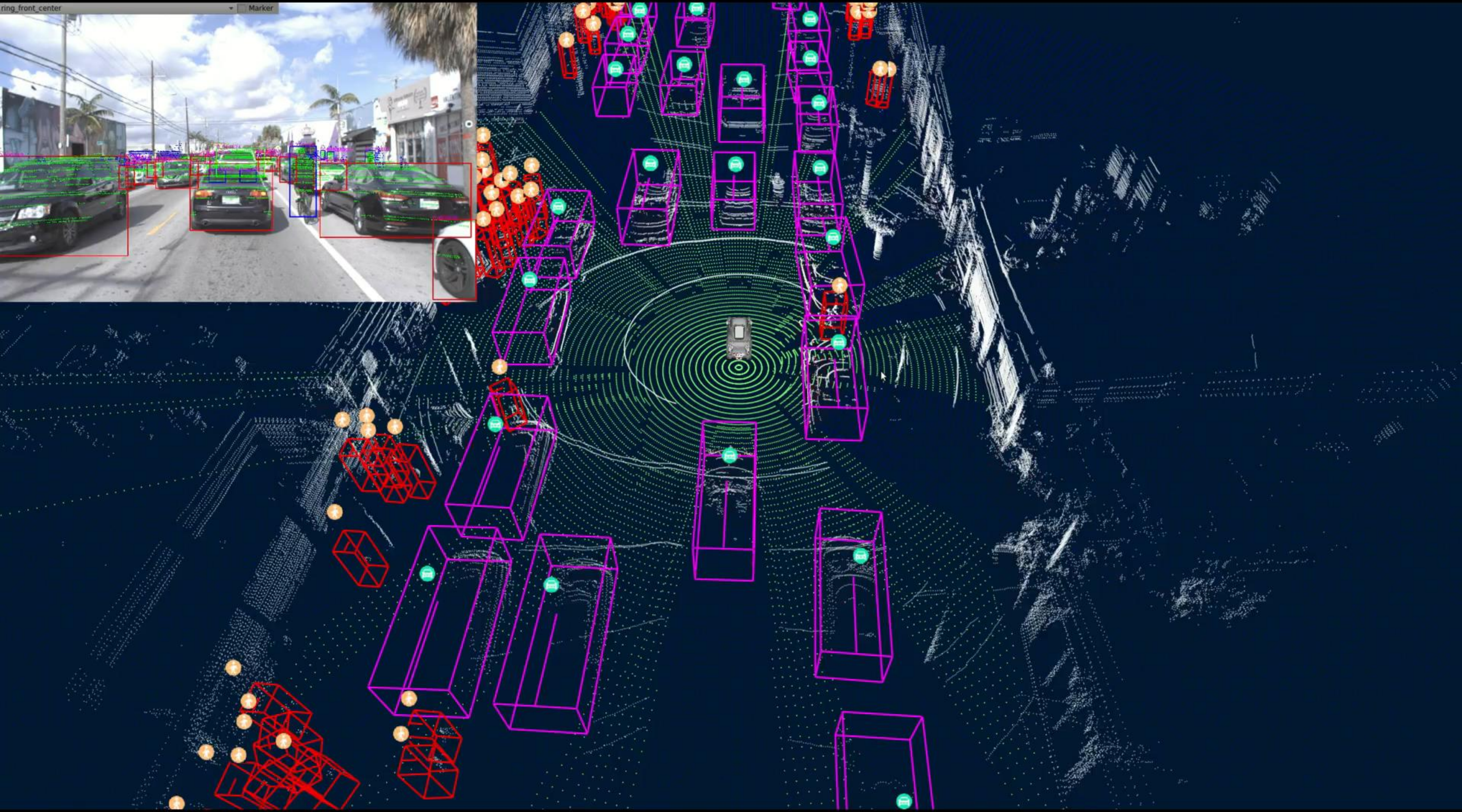
$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

What can we do with camera calibration?

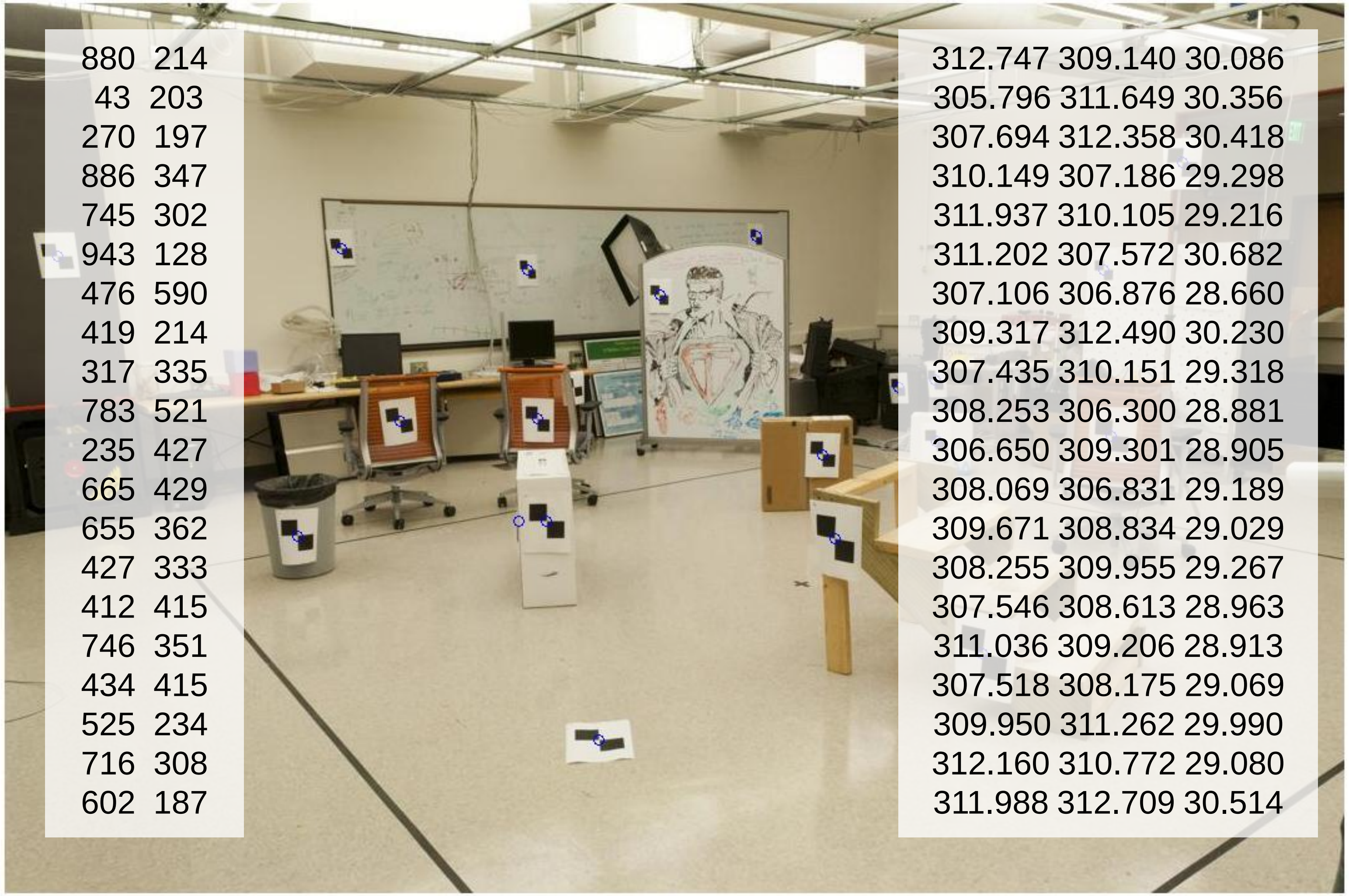




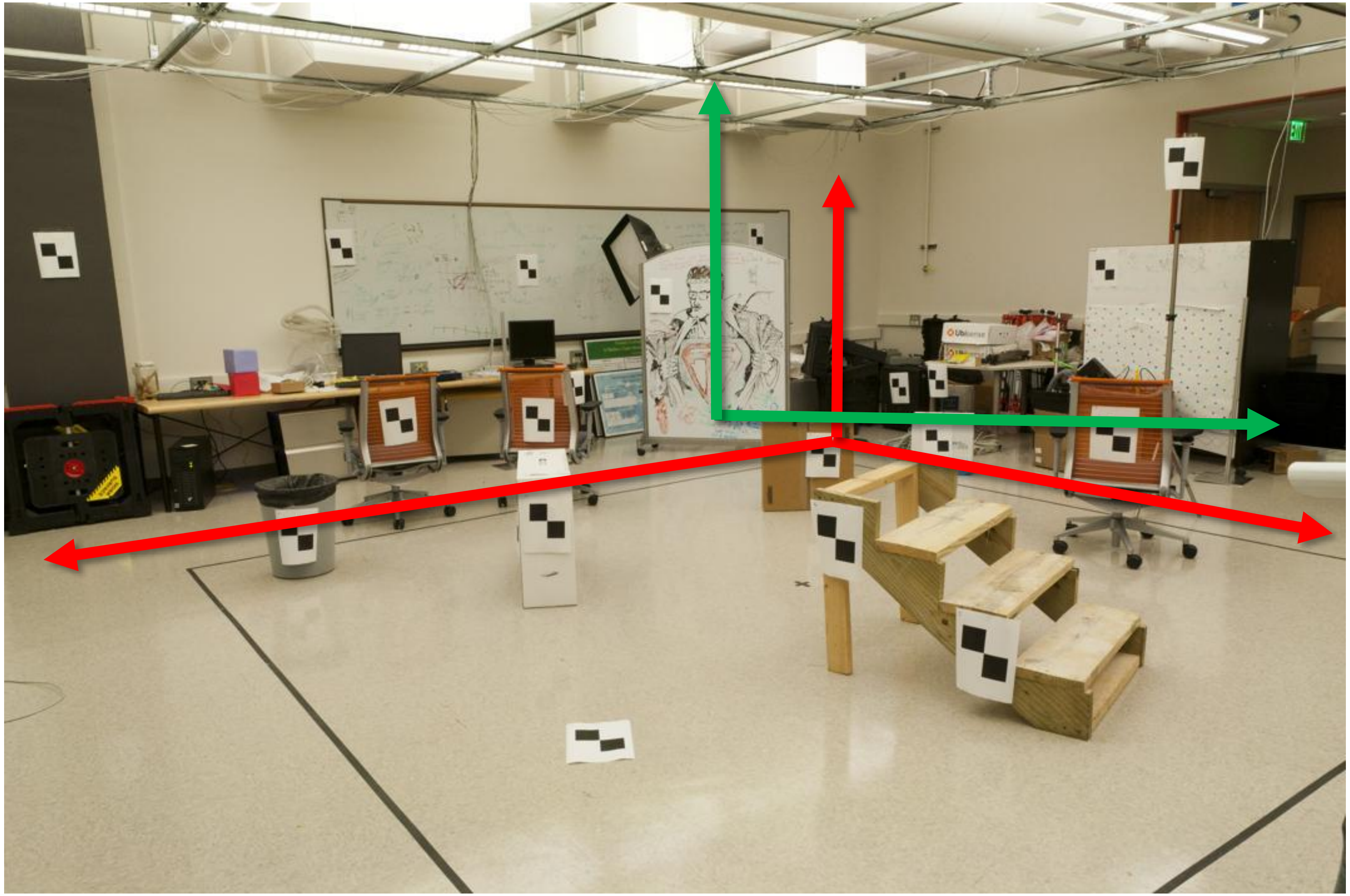



ring_front_center
Marker

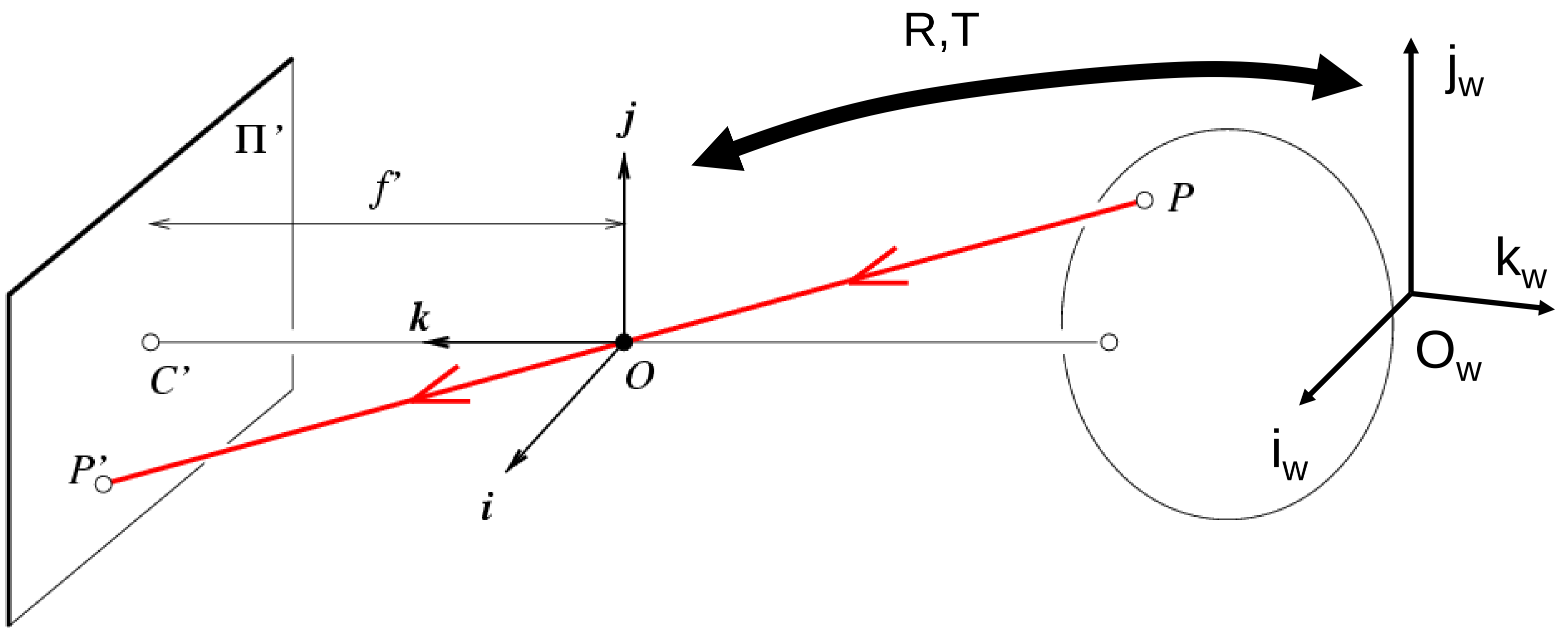
How do we calibrate a camera?



World vs Camera coordinates



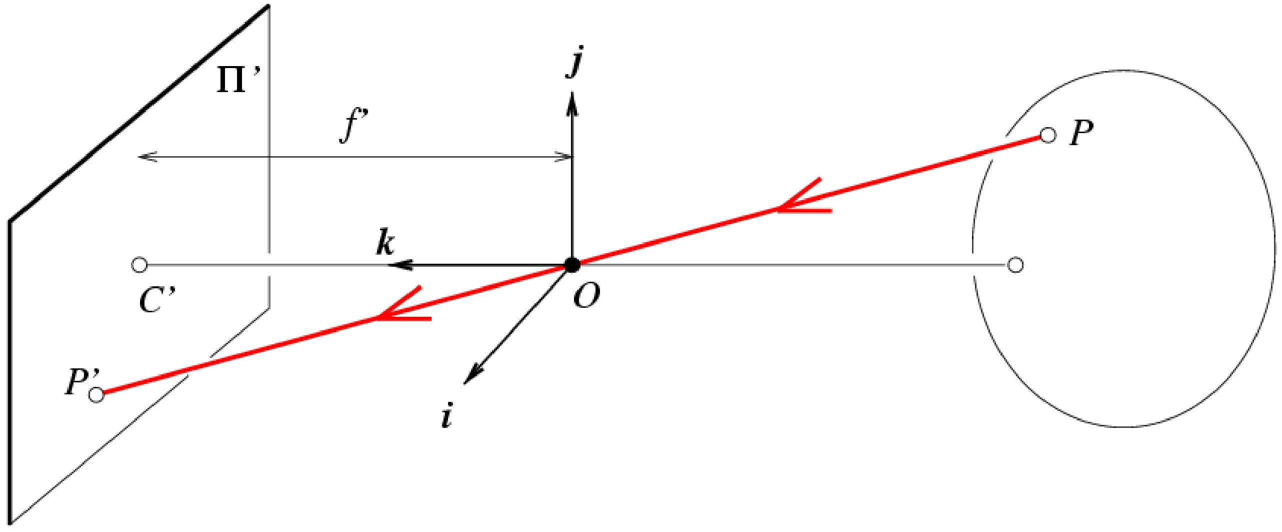
Projection matrix



$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

x: Image Coordinates: (u,v,1)
K: Intrinsic Matrix (3x3)
R: Rotation (3x3)
t: Translation (3x1)
X: World Coordinates: (X,Y,Z,1)

Projection matrix



Intrinsic Assumptions

- Unit aspect ratio
- Optical center at (0,0)
- No skew

Extrinsic Assumptions

- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{X} \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \boxed{\begin{matrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{matrix}}^{\,\mathbf{K}} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Slide Credit: Saverese

Remove assumption: known optical center

Intrinsic Assumptions

- Unit aspect ratio
- No skew

Extrinsic Assumptions

- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{X} \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & u_0 & 0 \\ 0 & f & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Remove assumption: square pixels

Intrinsic Assumptions
- No skew

Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{X} \Rightarrow w\begin{bmatrix}u\\v\\1\end{bmatrix} = \begin{bmatrix}\alpha & 0 & u_0 & 0\\0 & \beta & v_0 & 0\\0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}$$

Remove assumption: non-skewed pixels

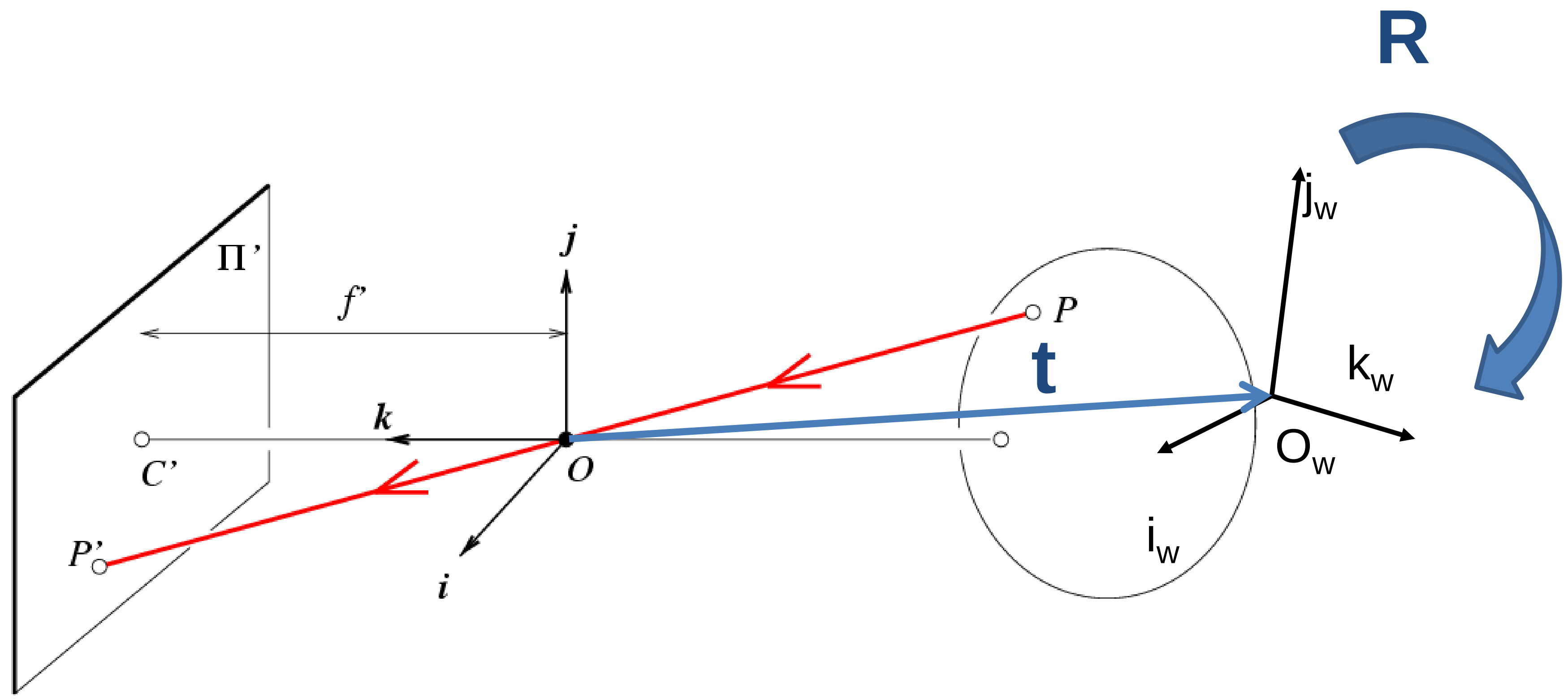
Intrinsic Assumptions

Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{X} \Rightarrow w\begin{bmatrix}u \\ v \\ 1\end{bmatrix} = \begin{bmatrix}\alpha & s & u_0 & 0 \\ 0 & \beta & v_0 & 0 \\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x \\ y \\ z \\ 1\end{bmatrix}$$

Note: different books use different notation for parameters

Oriented and Translated Camera



Allow camera translation

Intrinsic Assumptions

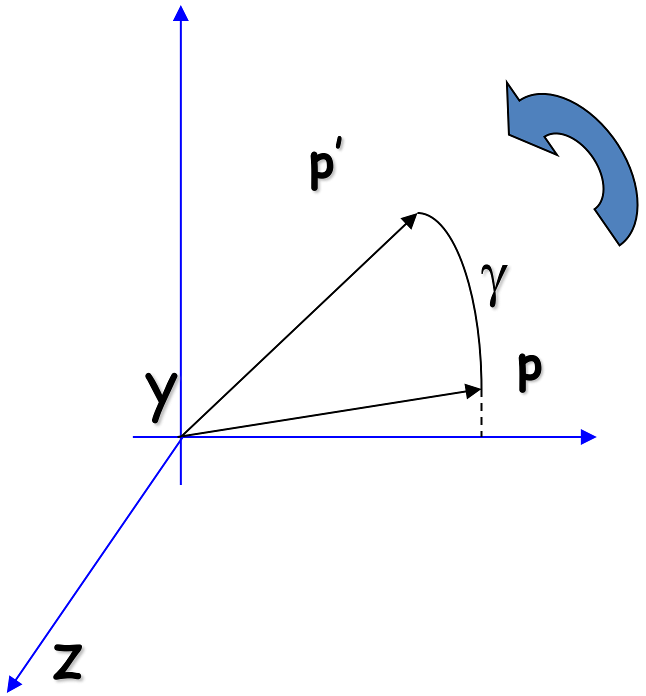
Extrinsic Assumptions
- No rotation

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{t}\end{bmatrix}\mathbf{X} \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Slide Credit: Saverese

3D Rotation of Points

Rotation around the coordinate axes, counter-clockwise:



$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

$$R_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

$$R_z(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Allow camera rotation

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

$$w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

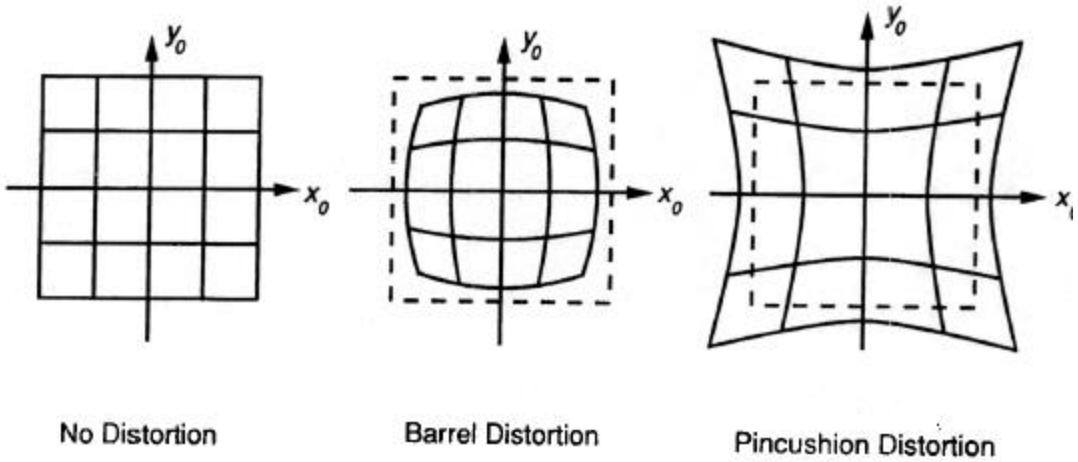
Degrees of freedom

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

$$w\begin{bmatrix}u \\ v \\ 1\end{bmatrix} = \overset{5}{\begin{bmatrix}\alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1\end{bmatrix}} \overset{6}{\begin{bmatrix}r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z\end{bmatrix}} \begin{bmatrix}x \\ y \\ z \\ 1\end{bmatrix}$$

Beyond Pinholes: Radial Distortion

- Common in wide-angle lenses or for special applications (e.g., security)
- Creates non-linear terms in projection
- Usually handled by through solving for non-linear terms and then correcting image



Corrected Barrel Distortion

Image from Martin Habbecke

How to calibrate the camera?

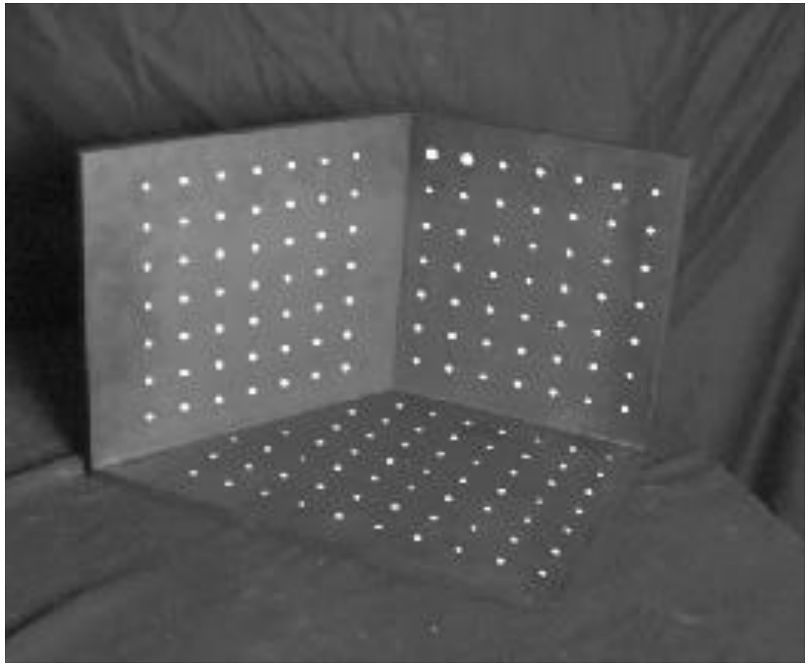
$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Calibrating the Camera

Use a scene with known geometry

- Correspond image points to 3d points
- Get least squares solution (or non-linear solution)



Known 2d image coords

Known 3d locations

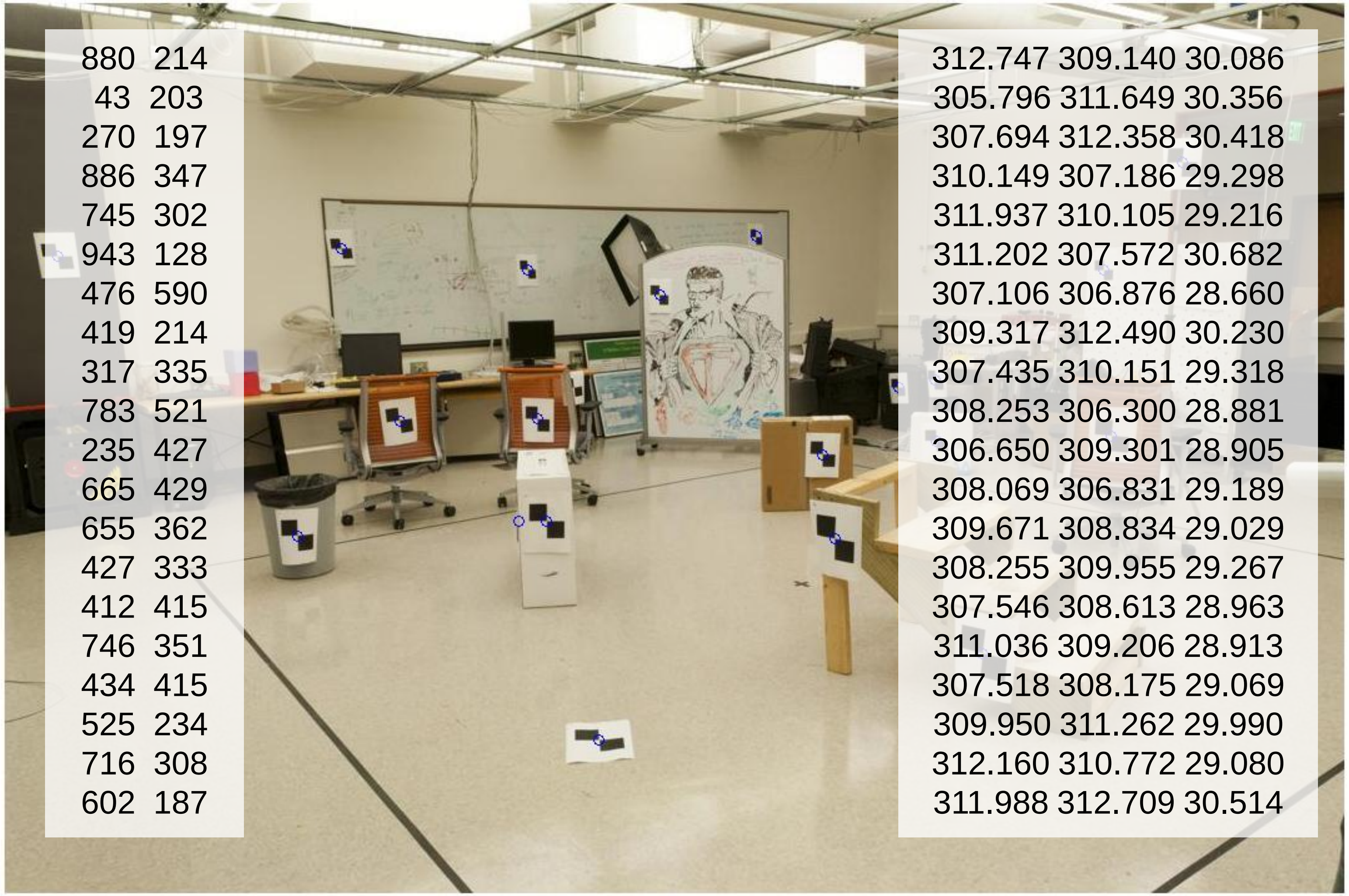
$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Unknown Camera Parameters

How do we calibrate a camera?

Known 2d image coords

Known 3d locations



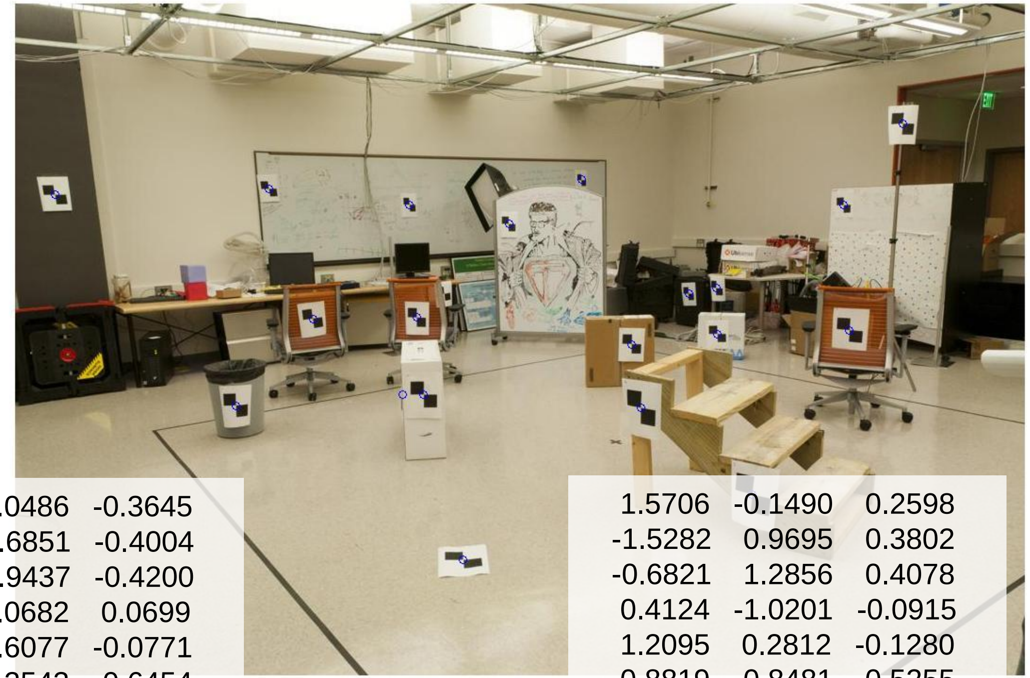
Known 2d image coords:

880	214
43	203
270	197
886	347
745	302
943	128
476	590
419	214
317	335
783	521
235	427
665	429
655	362
427	333
412	415
746	351
434	415
525	234
716	308
602	187

Known 3d locations:

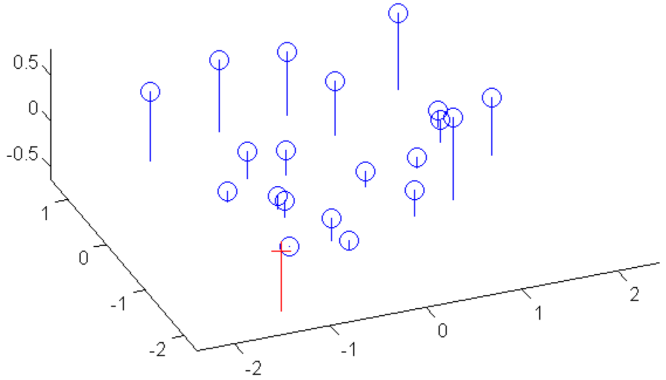
312.747	309.140	30.086
305.796	311.649	30.356
307.694	312.358	30.418
310.149	307.186	29.298
311.937	310.105	29.216
311.202	307.572	30.682
307.106	306.876	28.660
309.317	312.490	30.230
307.435	310.151	29.318
308.253	306.300	28.881
306.650	309.301	28.905
308.069	306.831	29.189
309.671	308.834	29.029
308.255	309.955	29.267
307.546	308.613	28.963
311.036	309.206	28.913
307.518	308.175	29.069
309.950	311.262	29.990
312.160	310.772	29.080
311.988	312.709	30.514

Estimate of camera center



1.0486	-0.3645
-1.6851	-0.4004
-0.9437	-0.4200
1.0682	0.0699
0.6077	-0.0771
1.2543	-0.6454
-0.2709	0.8635
-0.4571	-0.3645
-0.7902	0.0307
0.7318	0.6382
-1.0580	0.3312
0.3464	0.3377
0.3137	0.1189
-0.4310	0.0242
-0.4799	0.2920
0.6109	0.0830
-0.4081	0.2920
-0.1109	-0.2992
0.5129	-0.0575
0.1406	-0.4527

1.5706	-0.1490	0.2598
-1.5282	0.9695	0.3802
-0.6821	1.2856	0.4078
0.4124	-1.0201	-0.0915
1.2095	0.2812	-0.1280
0.8819	-0.8481	0.5255
-0.9442	-1.1583	-0.3759
0.0415	1.3445	0.3240
-0.7975	0.3017	-0.0826
-0.4329	-1.4151	-0.2774
-1.1475	-0.0772	-0.2667
-0.5149	-1.1784	-0.1401
0.1993	-0.2854	-0.2114
-0.4320	0.2143	-0.1053
-0.7481	-0.3840	-0.2408
0.8078	-0.1196	-0.2631
-0.7605	-0.5792	-0.1936
0.3237	0.7970	0.2170
1.3089	0.5786	-0.1887
1.2323	1.4421	0.4506



Unknown Camera Parameters

Known 2d image coords

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Known 3d locations

$$su = m_{11}X + m_{12}Y + m_{13}Z + m_{14}$$

$$sv = m_{21}X + m_{22}Y + m_{23}Z + m_{24}$$

$$s = m_{31}X + m_{32}Y + m_{33}Z + m_{34}$$

$$(m_{31}X + m_{32}Y + m_{33}Z + m_{34})u = m_{11}X + m_{12}Y + m_{13}Z + m_{14}$$

$$(m_{31}X + m_{32}Y + m_{33}Z + m_{34})v = m_{21}X + m_{22}Y + m_{23}Z + m_{24}$$

$$m_{31}uX + m_{32}uY + m_{33}uZ + m_{34}u = m_{11}X + m_{12}Y + m_{13}Z + m_{14}$$

$$m_{31}vX + m_{32}vY + m_{33}vZ + m_{34}v = m_{21}X + m_{22}Y + m_{23}Z + m_{24}$$

Unknown Camera Parameters

Known 2d image coords

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Known 3d locations

$$m_{31}uX + m_{32}uY + m_{33}uZ + m_{34}u = m_{11}X + m_{12}Y + m_{13}Z + m_{14}$$
$$m_{31}vX + m_{32}vY + m_{33}vZ + m_{34}v = m_{21}X + m_{22}Y + m_{23}Z + m_{24}$$

$$0 = m_{11}X + m_{12}Y + m_{13}Z + m_{14} - m_{31}uX - m_{32}uY - m_{33}uZ - m_{34}u$$
$$0 = m_{21}X + m_{22}Y + m_{23}Z + m_{24} - m_{31}vX - m_{32}vY - m_{33}vZ - m_{34}v$$

Unknown Camera Parameters

Known 2d image coords

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Known 3d locations

$$0 = m_{11}X + m_{12}Y + m_{13}Z + m_{14} - m_{31}uX - m_{32}uY - m_{33}uZ - m_{34}u$$

$$0 = m_{21}X + m_{22}Y + m_{23}Z + m_{24} - m_{31}vX - m_{32}vY - m_{33}vZ - m_{34}v$$

- Method 1 – homogeneous linear system. Solve for m's entries using linear least squares

$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1X_1 & -u_1Y_1 & -u_1Z_1 & -u_1 \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1X_1 & -v_1Y_1 & -v_1Z_1 & -v_1 \\ & & & & & & \vdots & & & & & \\ X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -u_nX_n & -u_nY_n & -u_nZ_n & -u_n \\ 0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -v_nX_n & -v_nY_n & -v_nZ_n & -v_n \end{bmatrix} \begin{bmatrix} m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{31} \\ m_{32} \\ m_{33} \\ m_{34} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

```
[U, S, V] = svd(A);
M = V(:,end);
M = reshape(M,[],3)';
```

For python, see numpy.linalg.svd

Unknown Camera Parameters

Known 2d image coords

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Known 3d locations

- Method 2 – nonhomogeneous linear system. Solve for m's entries using linear least squares

Ax=b form

$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1X_1 & -u_1Y_1 & -u_1Z_1 \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1X_1 & -v_1Y_1 & -v_1Z_1 \\ & & & & & & \vdots & & & & \\ X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -u_nX_n & -u_nY_n & -u_nZ_n \\ 0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -v_nX_n & -v_nY_n & -v_nZ_n \end{bmatrix} \begin{bmatrix} m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{31} \\ m_{32} \\ m_{33} \end{bmatrix} = \begin{bmatrix} u_1 \\ v_1 \\ \vdots \\ u_n \\ v_n \end{bmatrix}$$

```
M = A\Y;
M = [M;1];
M = reshape(M,[],3)';
```

For python, see numpy.linalg.lstsq

Calibration with linear method

- Advantages
  - Easy to formulate and solve
  - Provides initialization for non-linear methods

- Disadvantages
  - Doesn’t directly give you human-interpretable camera parameters
  - Doesn’t model radial distortion
  - Can’t impose constraints, such as known focal length

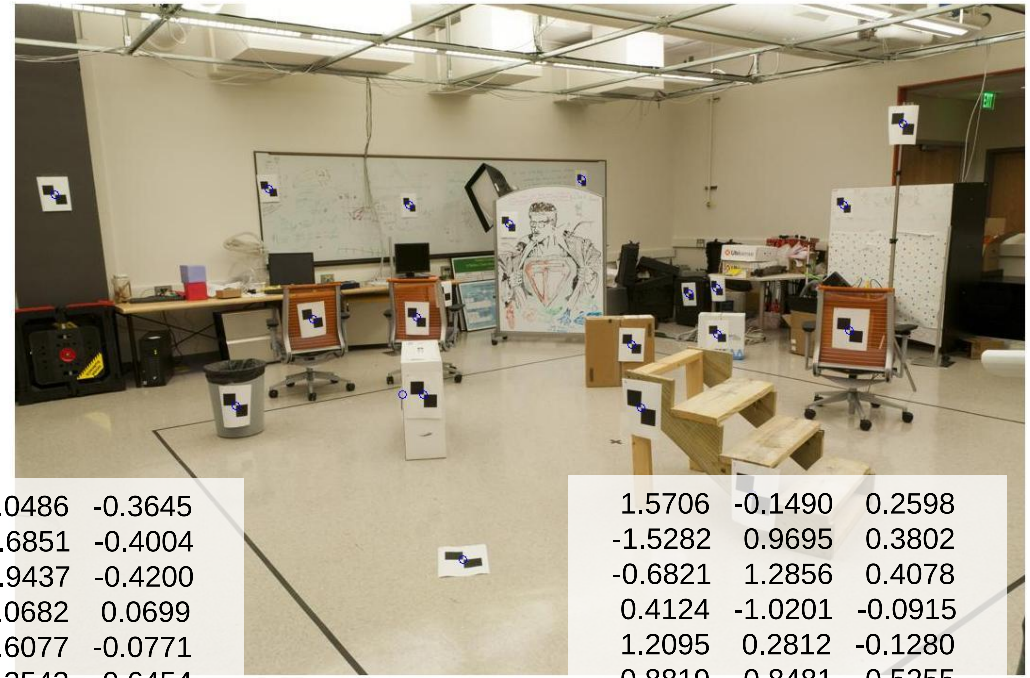
- Non-linear methods are preferred
  - Define error as difference between projected points and measured points
  - Minimize error using Newton’s method or other non-linear optimization

Can we factorize M back to K [R | T]?

- Yes!
- You can use *RQ* factorization (note – not the more familiar *QR* factorization). *R* (right diagonal) is K, and *Q* (orthogonal basis) is R. T, the last column of [R | T], is inv(K) * last column of M.
  - But you need to do a bit of post-processing to make sure that the matrices are valid. See http://ksimek.github.io/2012/08/14/decompose/

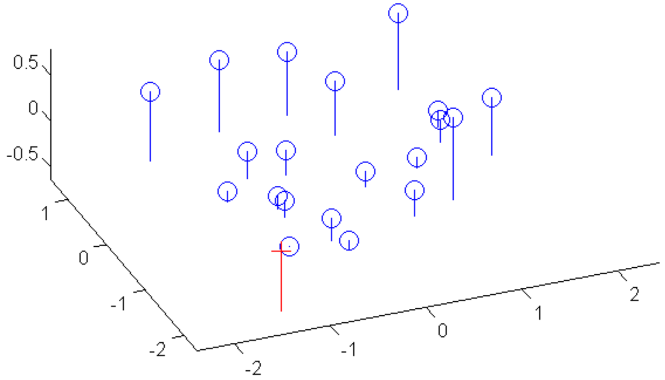
For project 3, we want the camera center

Estimate of camera center

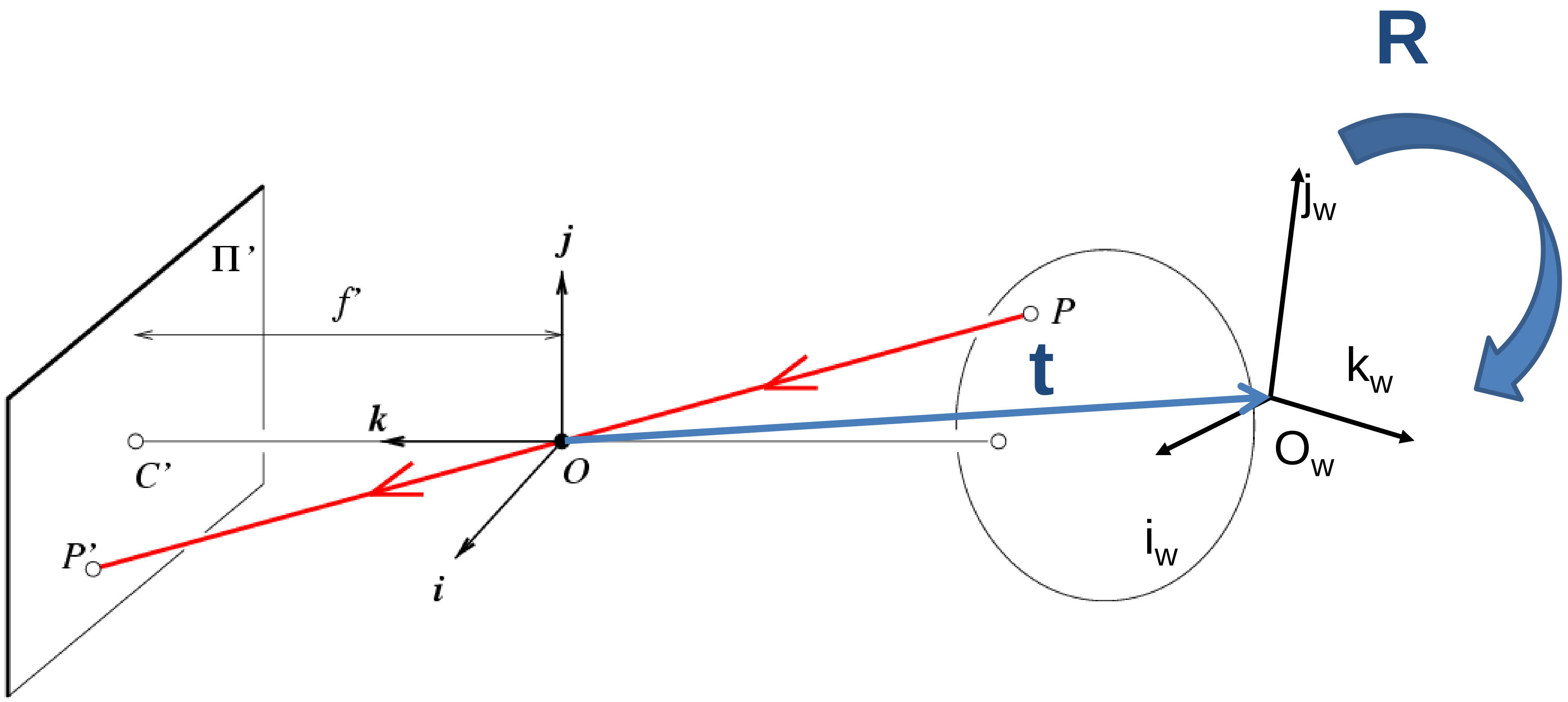


1.0486	-0.3645
-1.6851	-0.4004
-0.9437	-0.4200
1.0682	0.0699
0.6077	-0.0771
1.2543	-0.6454
-0.2709	0.8635
-0.4571	-0.3645
-0.7902	0.0307
0.7318	0.6382
-1.0580	0.3312
0.3464	0.3377
0.3137	0.1189
-0.4310	0.0242
-0.4799	0.2920
0.6109	0.0830
-0.4081	0.2920
-0.1109	-0.2992
0.5129	-0.0575
0.1406	-0.4527

1.5706	-0.1490	0.2598
-1.5282	0.9695	0.3802
-0.6821	1.2856	0.4078
0.4124	-1.0201	-0.0915
1.2095	0.2812	-0.1280
0.8819	-0.8481	0.5255
-0.9442	-1.1583	-0.3759
0.0415	1.3445	0.3240
-0.7975	0.3017	-0.0826
-0.4329	-1.4151	-0.2774
-1.1475	-0.0772	-0.2667
-0.5149	-1.1784	-0.1401
0.1993	-0.2854	-0.2114
-0.4320	0.2143	-0.1053
-0.7481	-0.3840	-0.2408
0.8078	-0.1196	-0.2631
-0.7605	-0.5792	-0.1936
0.3237	0.7970	0.2170
1.3089	0.5786	-0.1887
1.2323	1.4421	0.4506



Oriented and Translated Camera



Recovering the camera center

$$\mathbf{x} = \mathbf{K}[\mathbf{R} \quad \mathbf{t}]\mathbf{X}$$

$$w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & \boxed{t_x} \\ r_{21} & r_{22} & r_{23} & \boxed{t_y} \\ r_{31} & r_{32} & r_{33} & \boxed{t_z} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

This is not the camera center -C. It is –RC (because a point will be rotated before t_x, t_y, and t_z are added)

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} \boxed{\begin{matrix} * & * & * \\ * & * & * \\ * & * & * \end{matrix}} & \boxed{\begin{matrix} * \\ * \\ * \end{matrix}} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Q

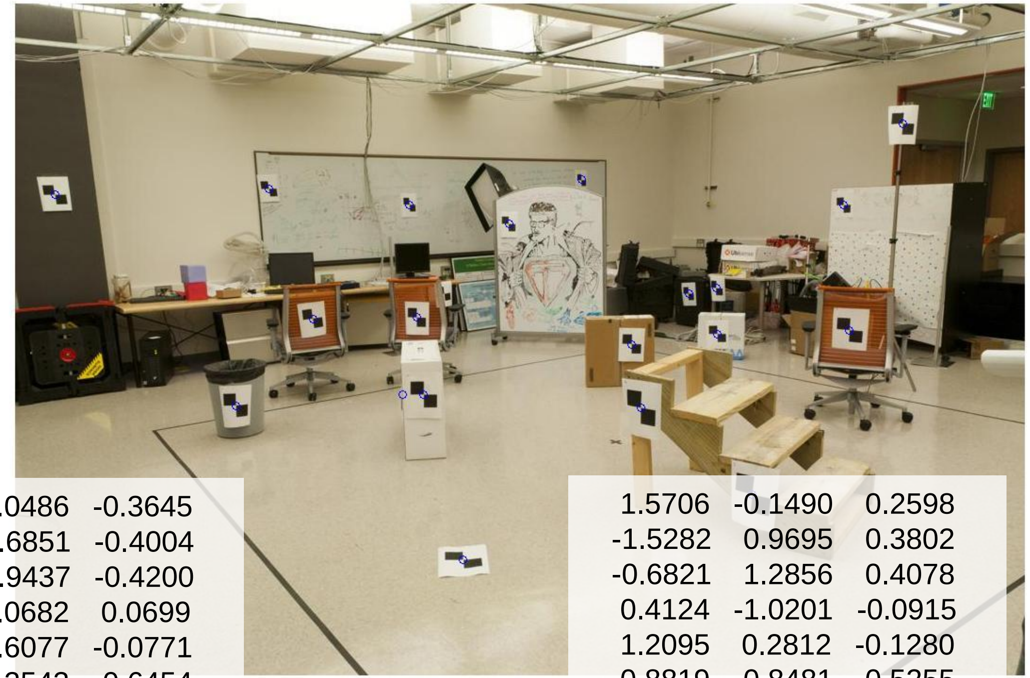
This, m_4, is K * **t**

So K^{-1} m_4 is **t**

So we need
$-R^{-1} K^{-1} m_4$ to get C

Q is K * R. So we just need $-Q^{-1} m_4$

Estimate of camera center



1.0486	-0.3645
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-0.4329	-1.4151	-0.2774
-1.1475	-0.0772	-0.2667
-0.5149	-1.1784	-0.1401
0.1993	-0.2854	-0.2114
-0.4320	0.2143	-0.1053
-0.7481	-0.3840	-0.2408
0.8078	-0.1196	-0.2631
-0.7605	-0.5792	-0.1936
0.3237	0.7970	0.2170
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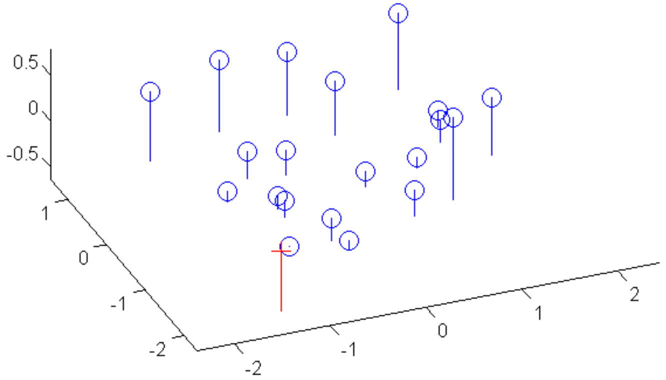
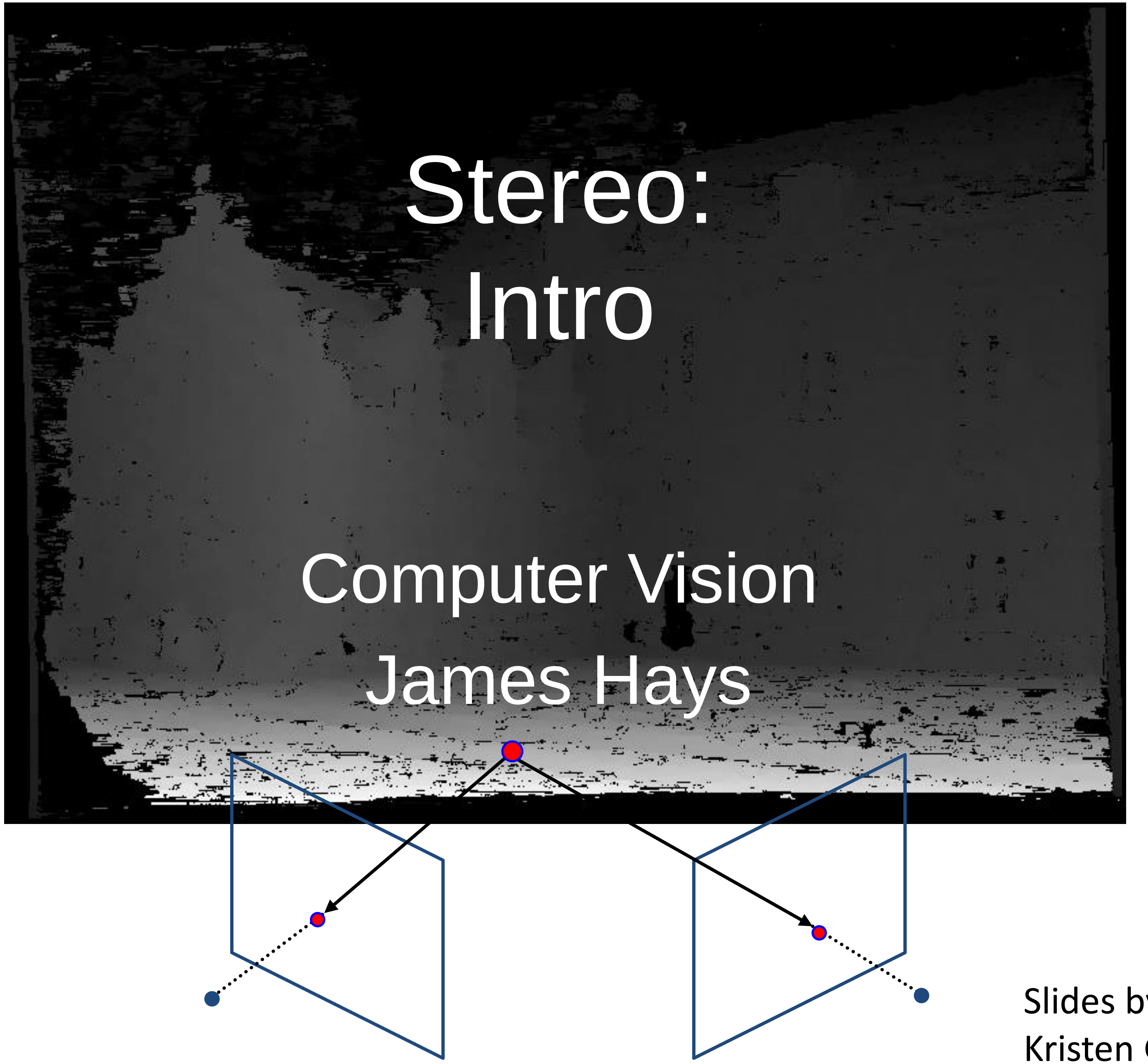


Stereo: Intro

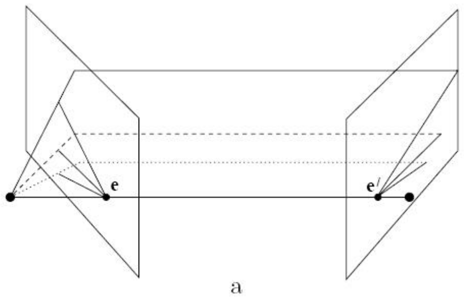
Computer Vision

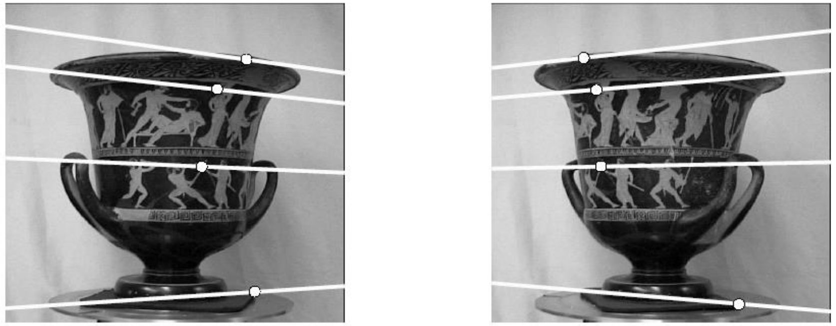
James Hays



Slides by
Kristen Grauman

Multiple views





Hartley and Zisserman

stereo vision
structure from motion
optical flow

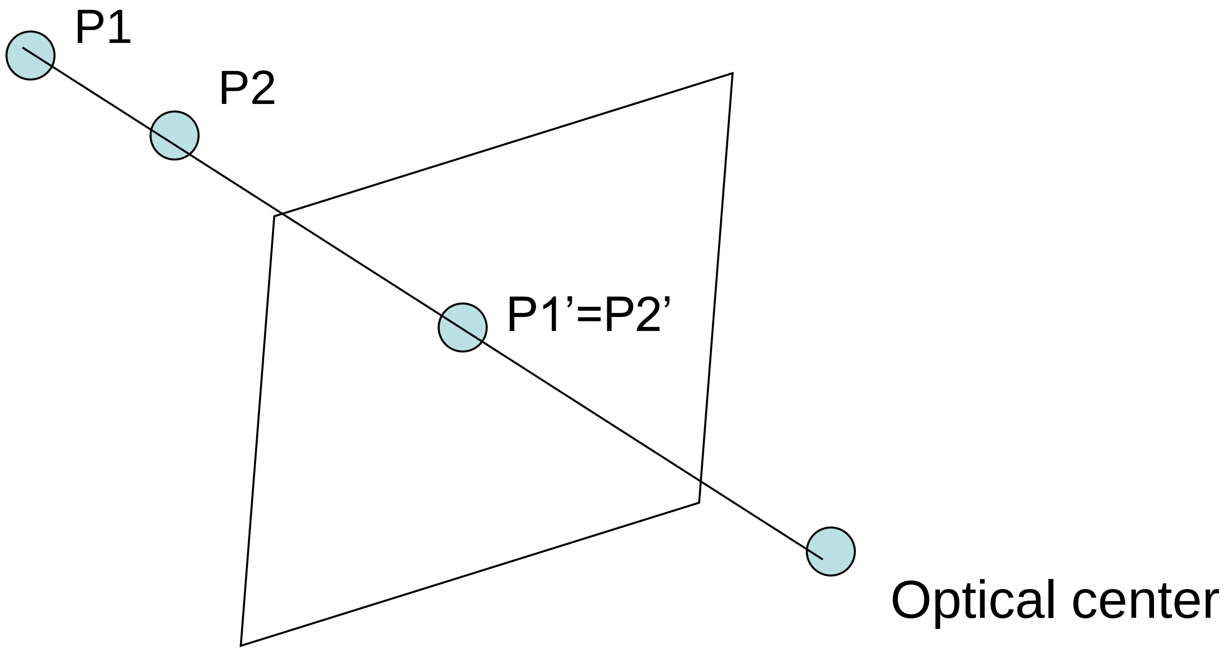
Why multiple views?

- Structure and depth are inherently ambiguous from single views.

Images from Lana Lazebnik

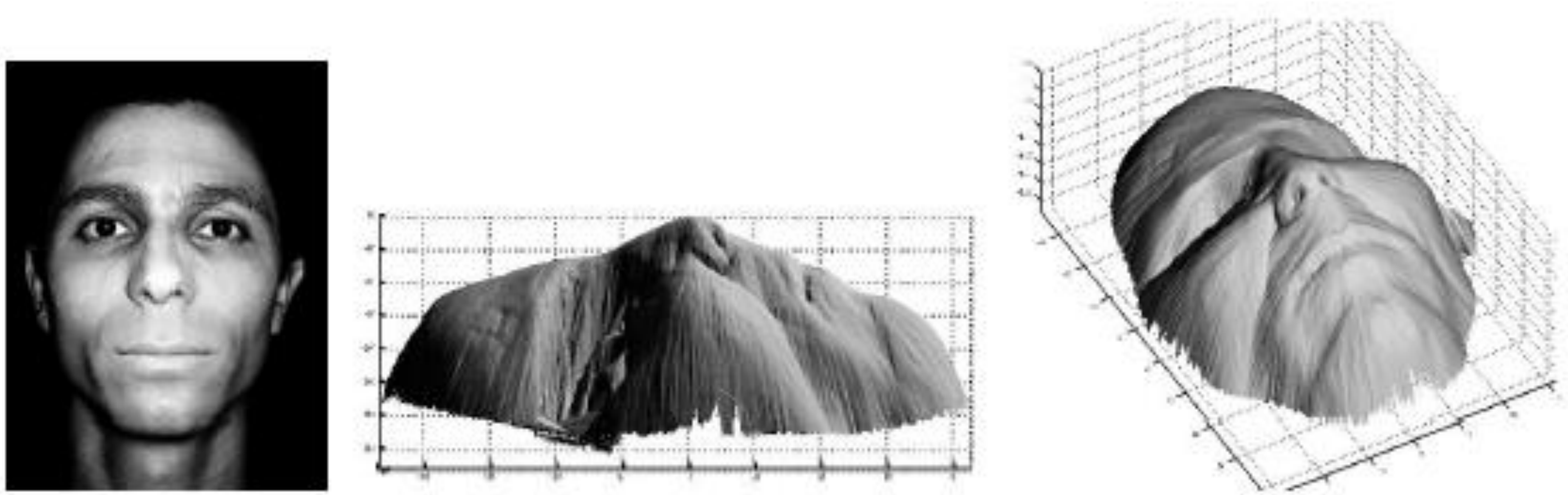
Why multiple views?

- Structure and depth are inherently ambiguous from single views.



- What cues help us to perceive 3d shape and depth?

Shading



a) b) c)

[Figure from Prados & Faugeras 2006]

Shading from multiple light sources: Photometric stereo

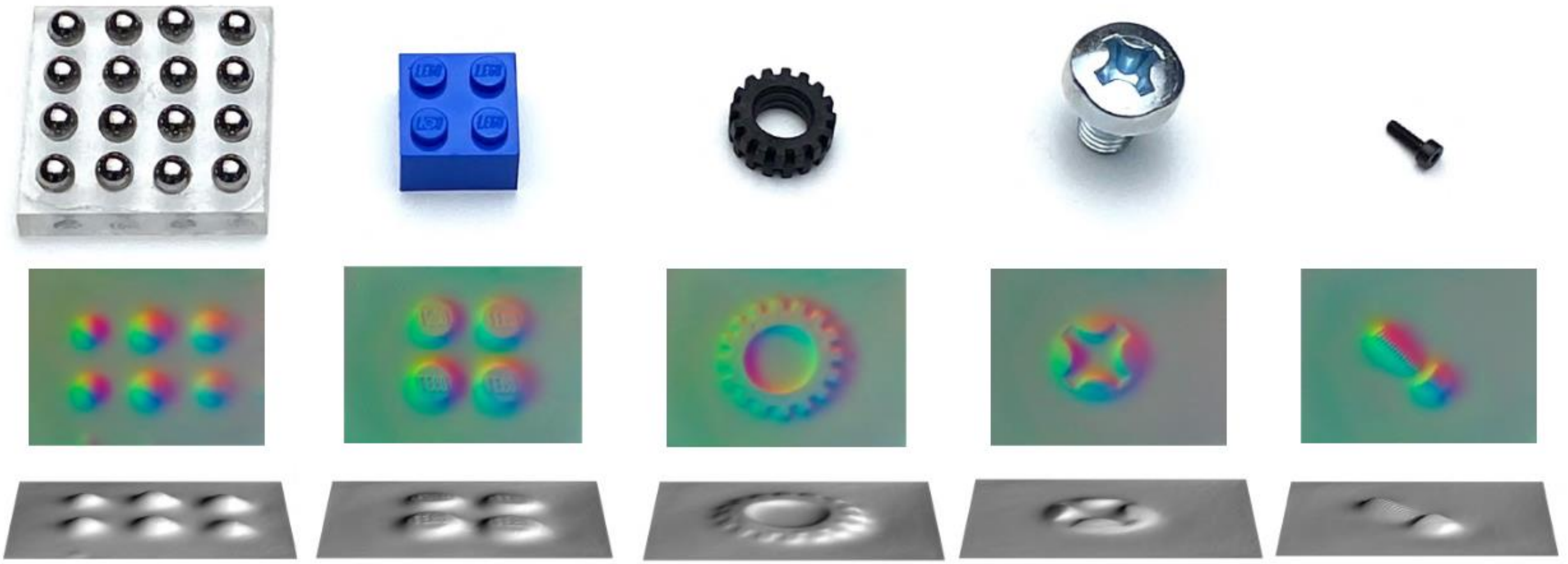
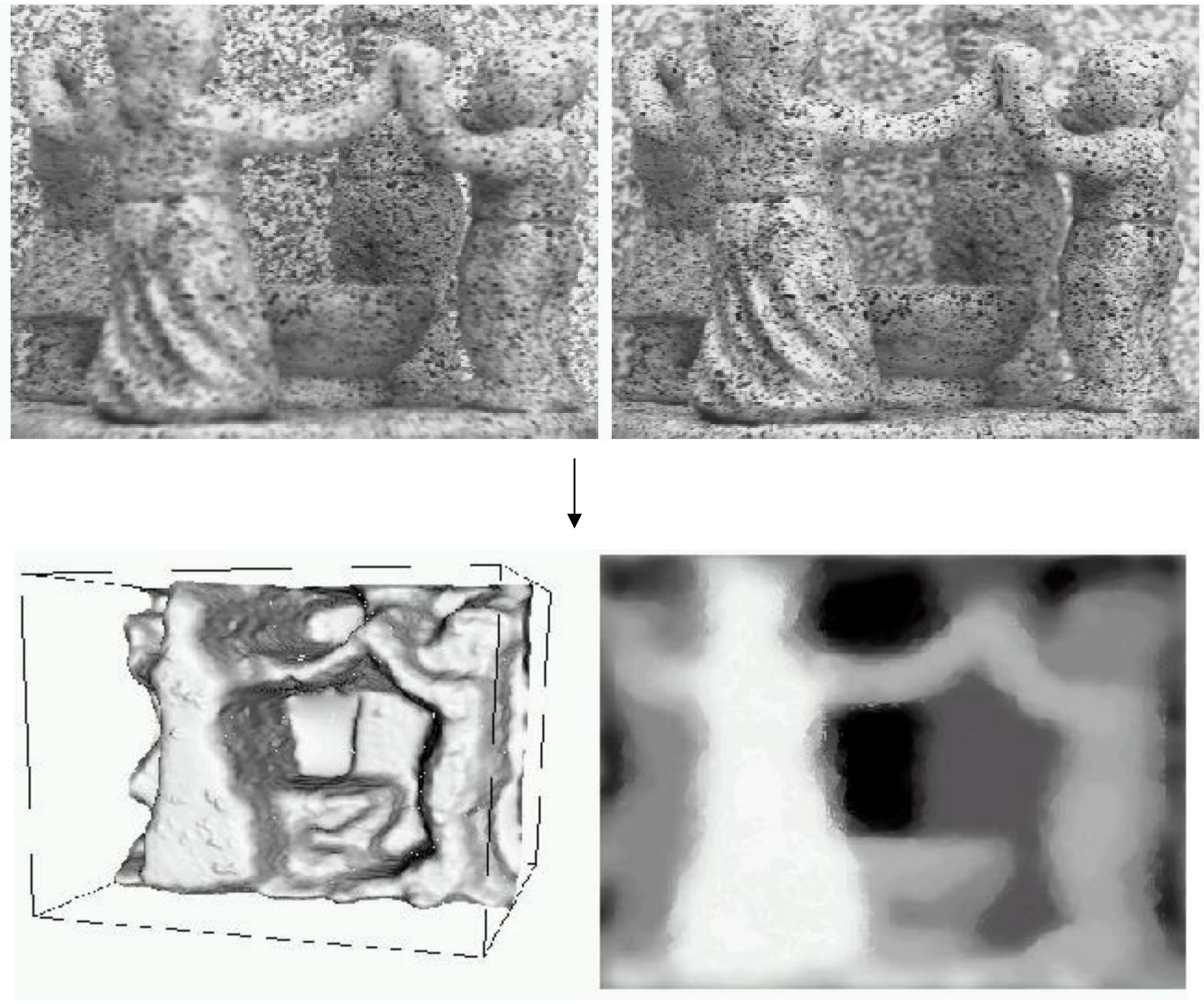


Fig. 7. From top row to bottom: visual images, GelSight imprints, and inferred depth of a ball array, a Lego block, a rubber tyre, a screw cap, a M2 screw.

Focus/defocus

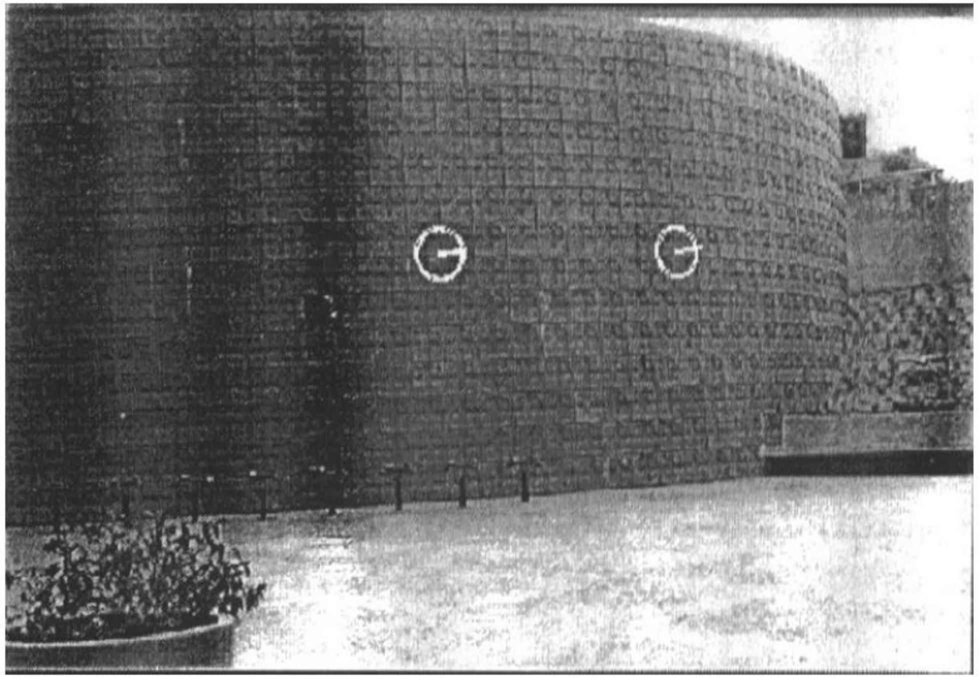


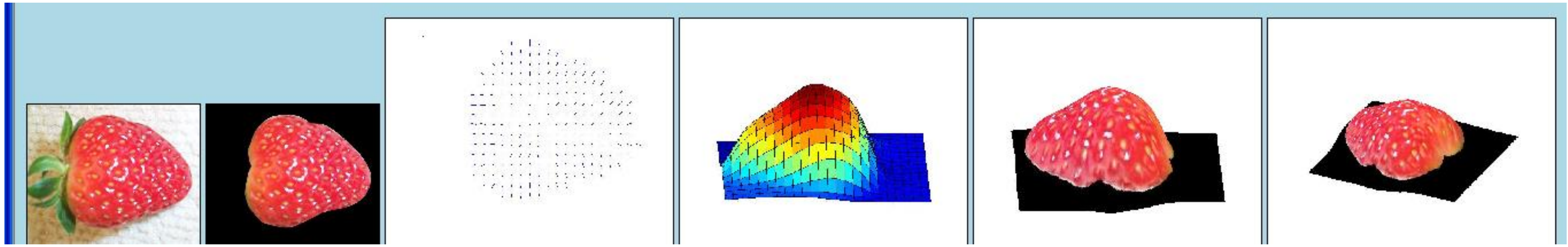
Images from same point of view, different camera parameters

3d shape / depth estimates

[figs from H. Jin and P. Favaro, 2002]

Texture



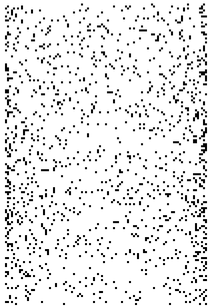


[From A.M. Loh. The recovery of 3-D structure using visual texture patterns. PhD thesis]

Perspective effects

Image credit: S. Seitz

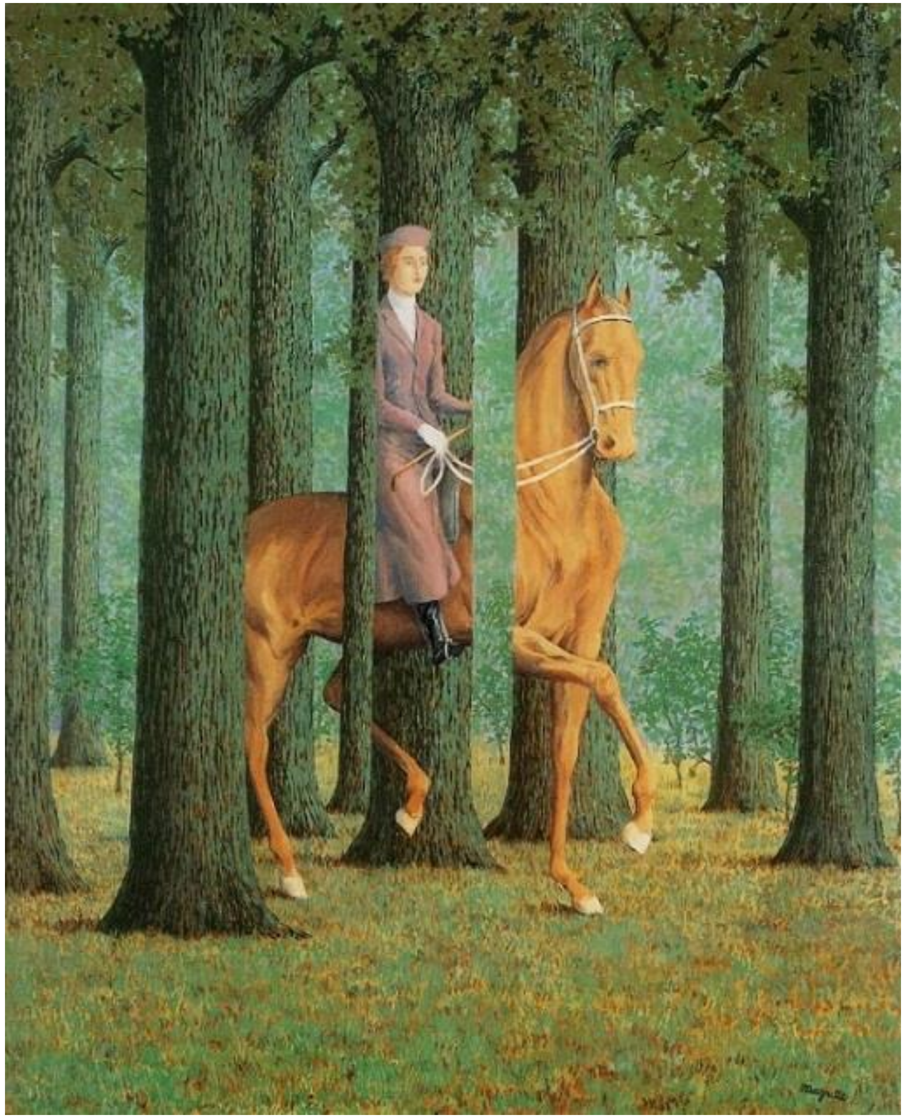
Motion



Figures from L. Zhang

http://www.brainconnection.com/teasers/?main=illusion/motion-shape

Occlusion

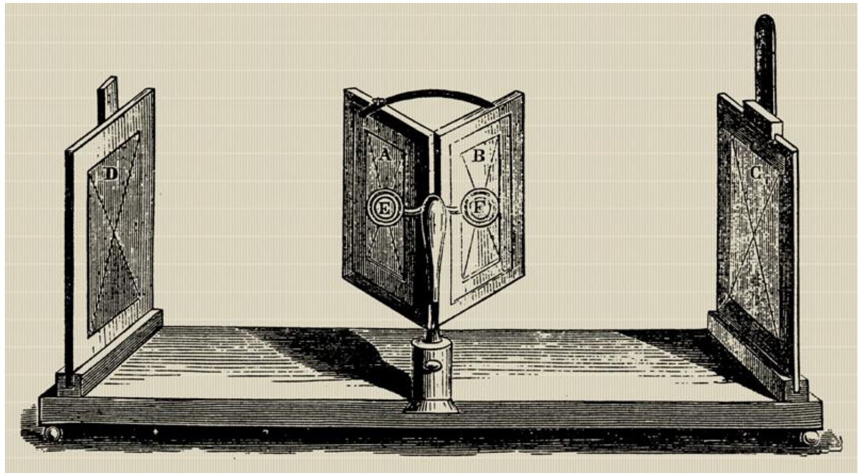


Rene Magritt'e famous painting *Le Blanc-Seing* (literal translation: "The Blank Signature") roughly translates as "free hand“. 1965

If stereo were critical for depth perception, navigation, recognition, etc., then this would be a problem

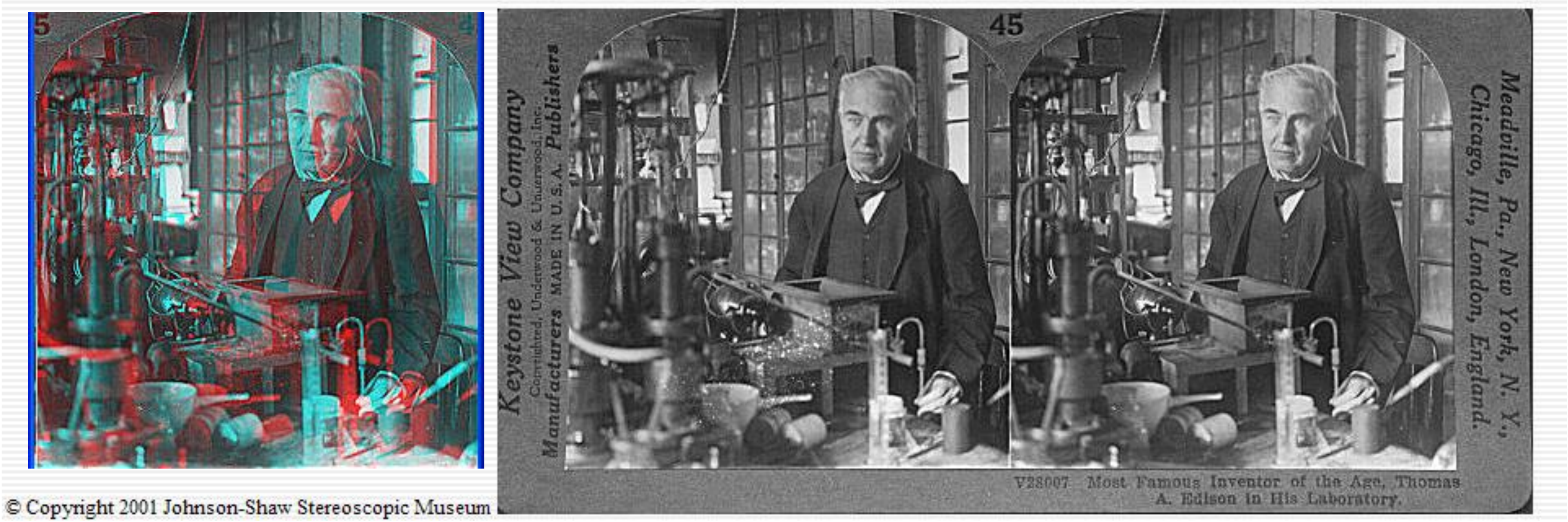
Stereo photography and stereo viewers

Take two pictures of the same subject from two slightly different viewpoints and display so that each eye sees only one of the images.

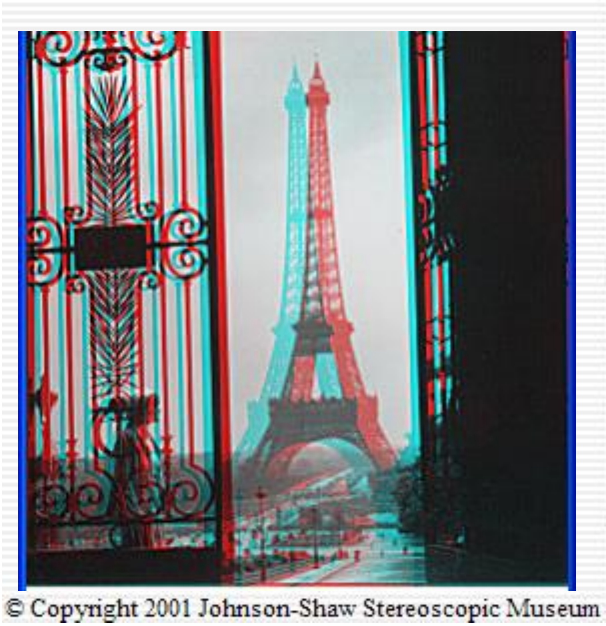


Invented by Sir Charles Wheatstone, 1838

Image from fisher-price.com



http://www.johnsonshawmuseum.org



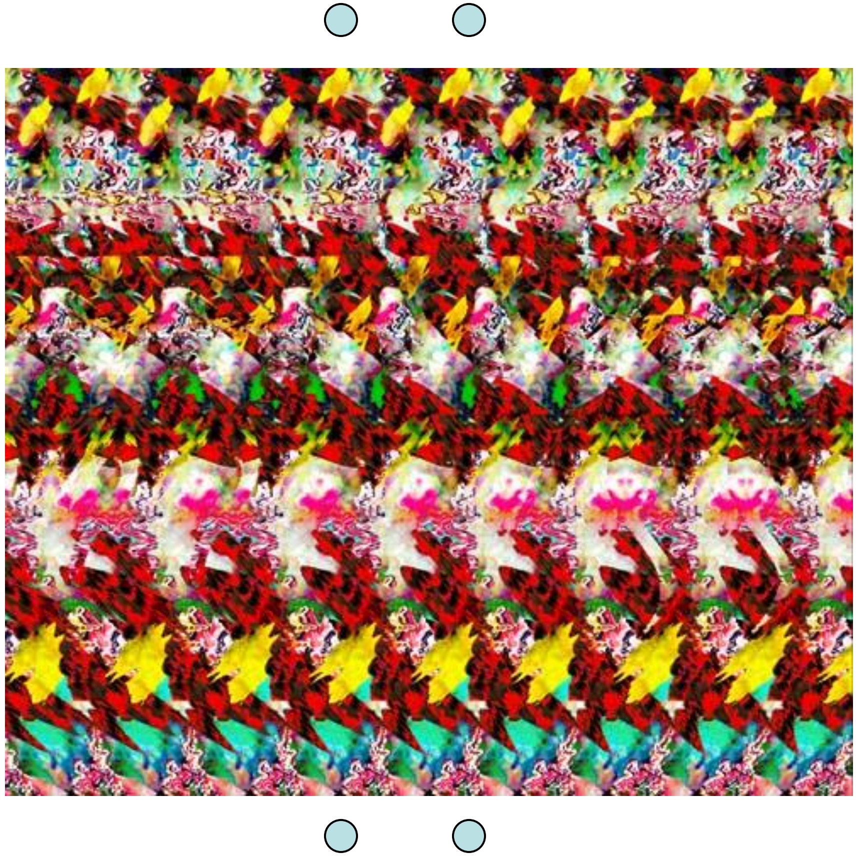
http://www.johnsonshawmuseum.org

Public Library, Stereoscopic Looking Room, Chicago, by Phillips, 1923

http://www.well.com/~jimg/stereo/stereo_list.html

http://www.well.com/~jimg/stereo/stereo_list.html

Autostereograms



Exploit disparity as depth cue using single image.

(Single image random dot stereogram, Single image stereogram)

Images from magiceye.com

Autostereograms

Images from magiceye.com

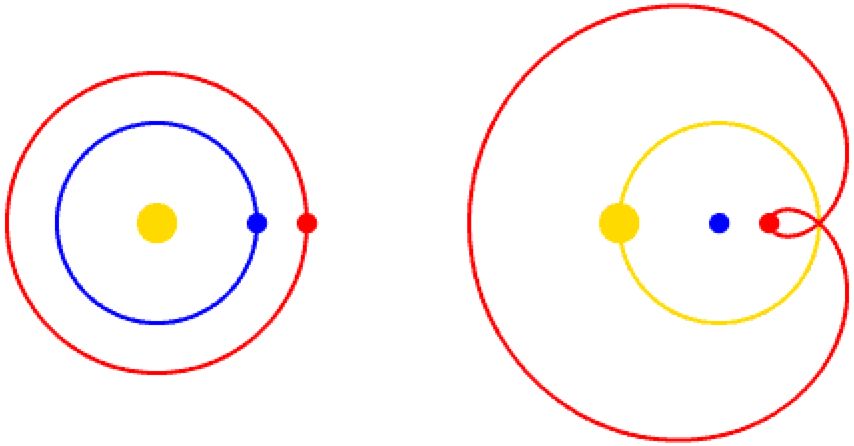
Parallax and our universe

Look again at that dot. That's here. That's home. That's us. On it everyone you love, everyone you know, everyone you ever heard of, every human being who ever was, lived out their lives. The aggregate of our joy and suffering, thousands of confident religions, ideologies, and economic doctrines, every hunter and forager, every hero and coward, every creator and destroyer of civilization, every king and peasant, every young couple in love, every mother and father, hopeful child, inventor and explorer, every teacher of morals, every corrupt politician, every "superstar," every "supreme leader," every saint and sinner in the history of our species lived there--on a mote of dust suspended in a sunbeam.

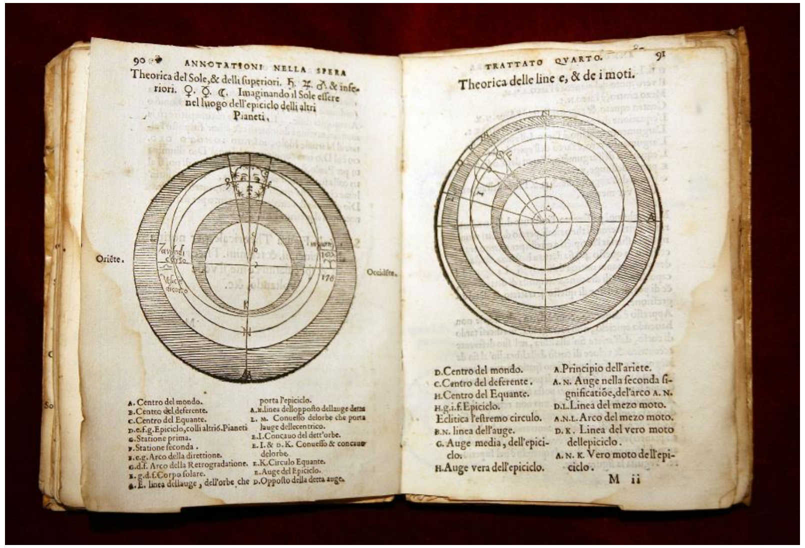
— Carl Sagan

https://en.wikipedia.org/wiki/Pale_Blue_Dot

Nicolaus Copernicus



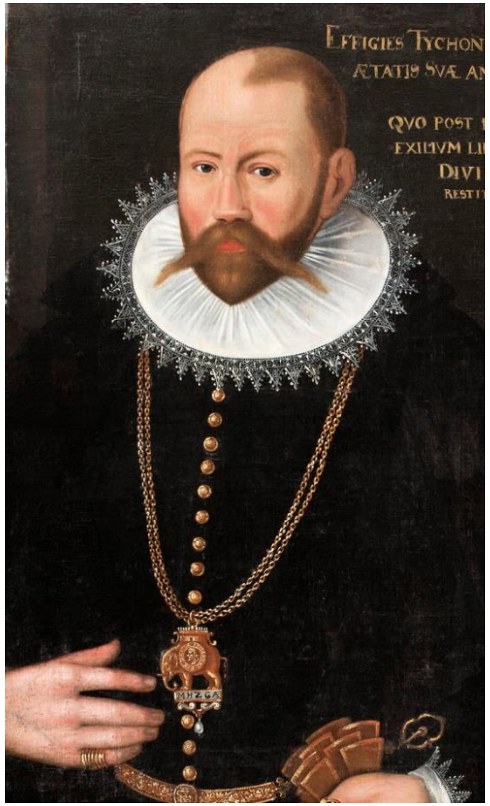
Motion of Sun (yellow), Earth (blue), and Mars (red). At left, Copernicus' heliocentric motion. At right, traditional geocentric motion, including the retrograde motion of Mars.



geocentric model (often exemplified specifically by the **Ptolemaic system**)

https://en.wikipedia.org/wiki/Heliocentrism

If the apparent motion of the planets is caused by parallax, why aren’t we seeing parallax for stars?

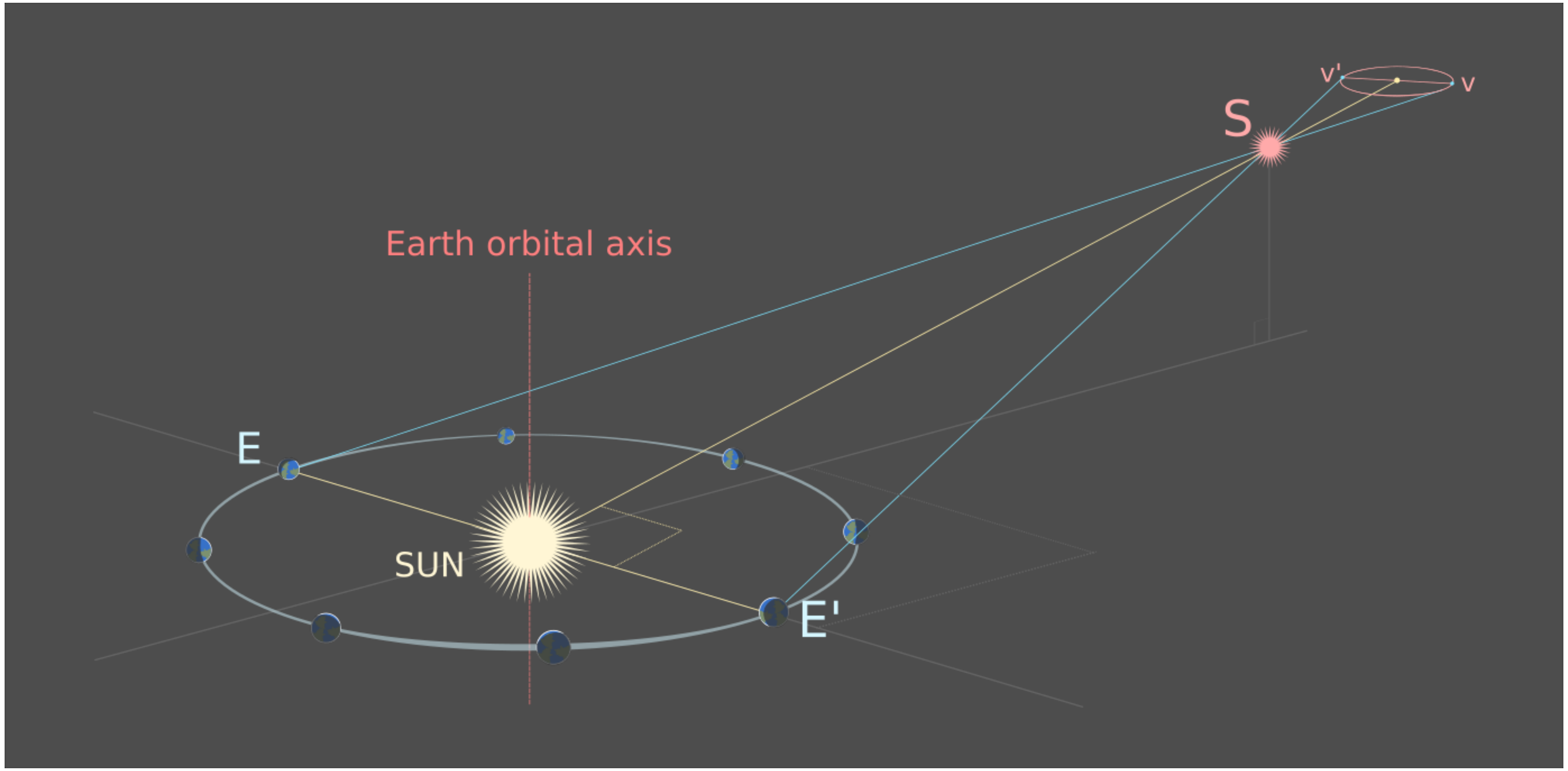


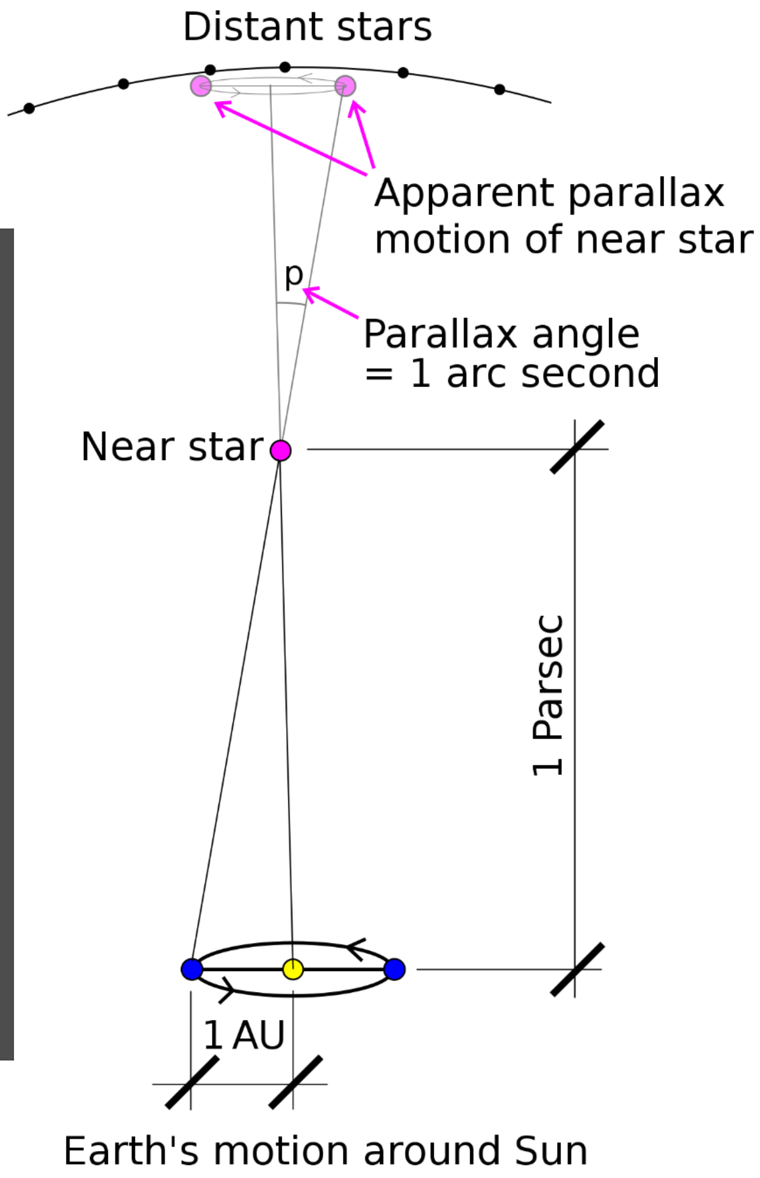
Tycho Brahe

It was one of Tycho Brahe's principal objections to Copernican heliocentrism that for it to be compatible with the lack of observable stellar parallax, there would have to be an enormous and unlikely void between the orbit of Saturn and the eighth sphere (the fixed stars).

The angles involved in these calculations are very small and thus difficult to measure. The nearest star to the Sun (and also the star with the largest parallax), Proxima Centauri, has a parallax of 0.7685 ± 0.0002 arcsec.[1] This angle is approximately that subtended by an object 2 centimeters in diameter located 5.3 kilometers away. First reliable measurements of parallax were not made until 1838, by Friedrich Bessel

https://en.wikipedia.org/wiki/Stellar_parallax





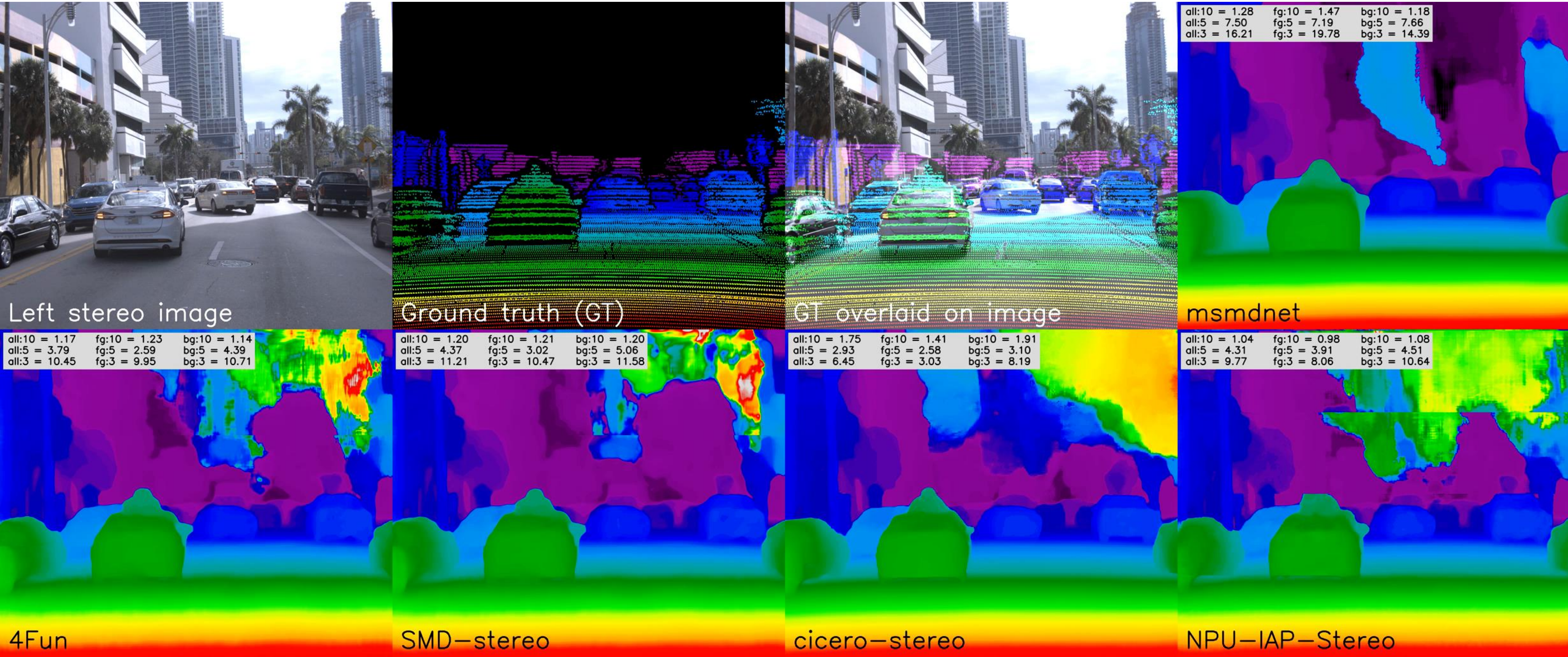
https://en.wikipedia.org/wiki/Stellar_parallax

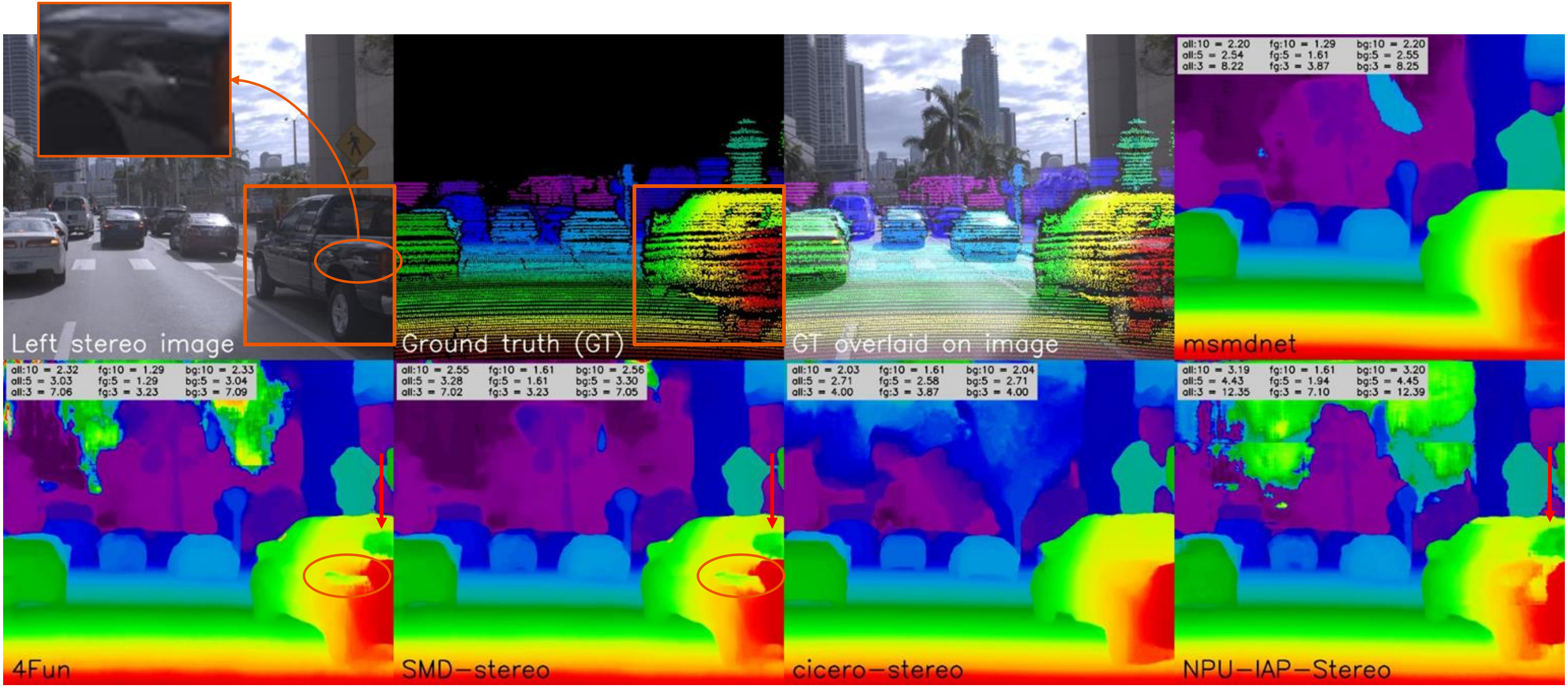
Stereo vision

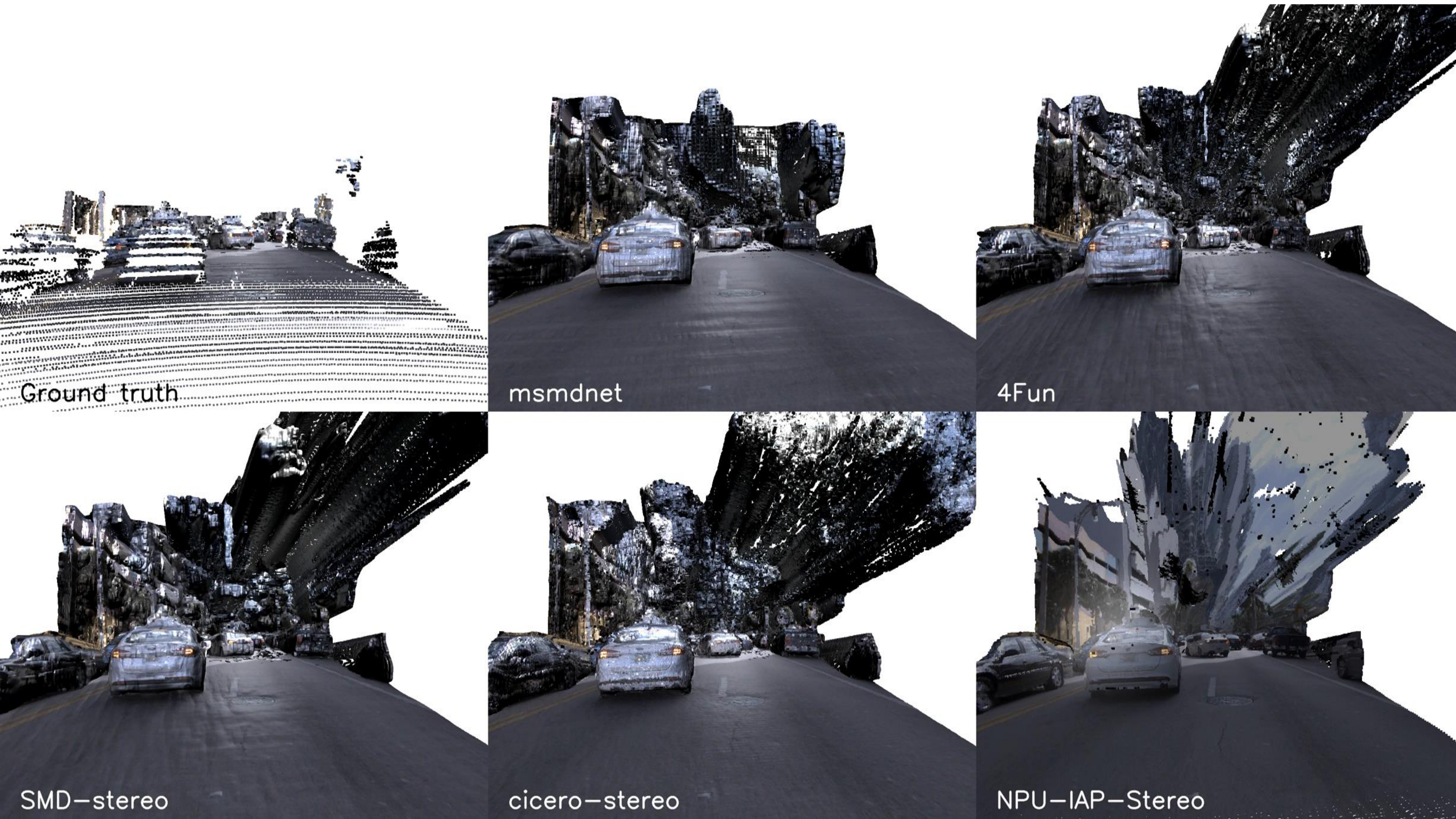
Two cameras, simultaneous views

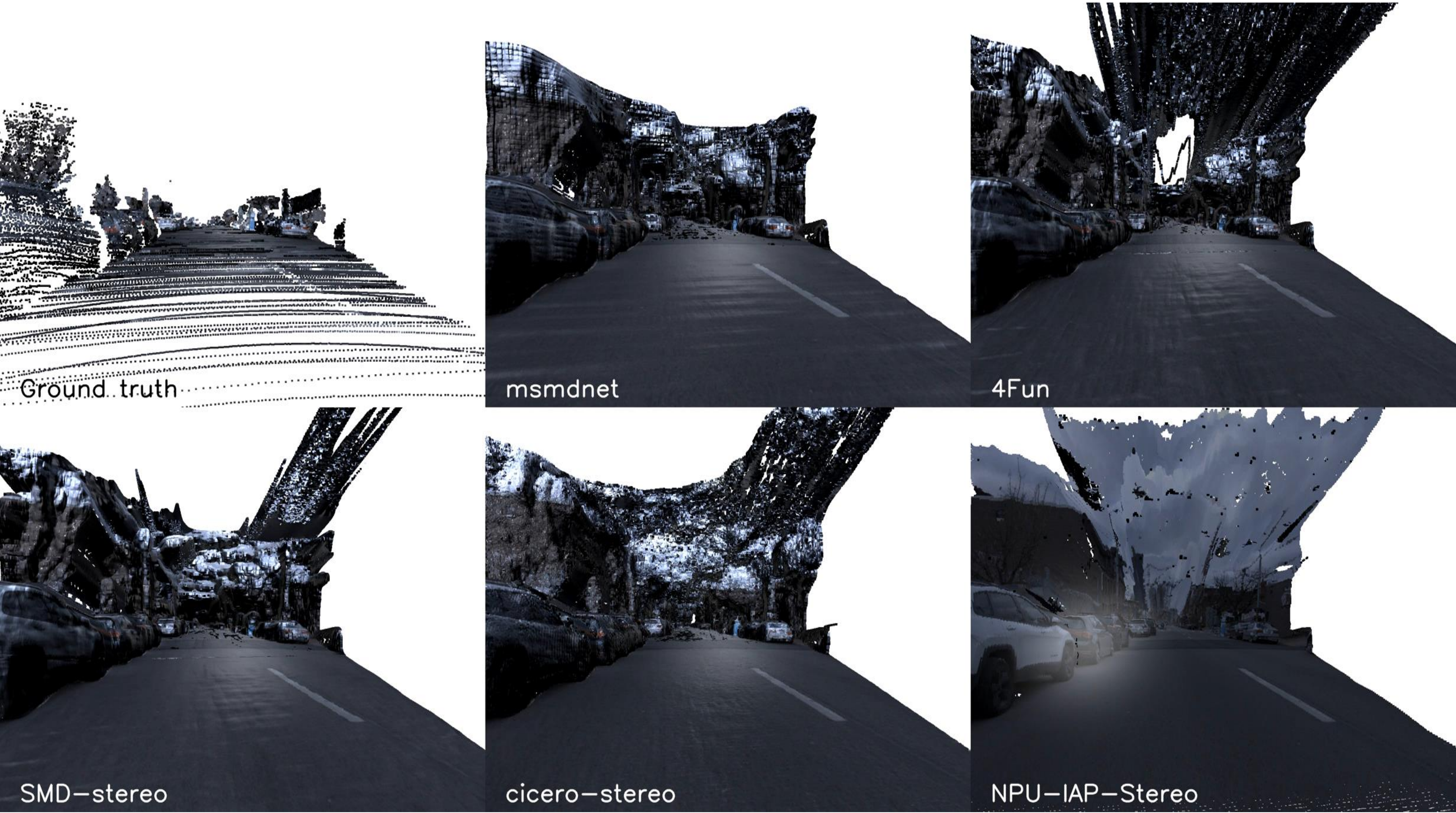
Single moving camera and static scene

Modern stereo depth estimation example



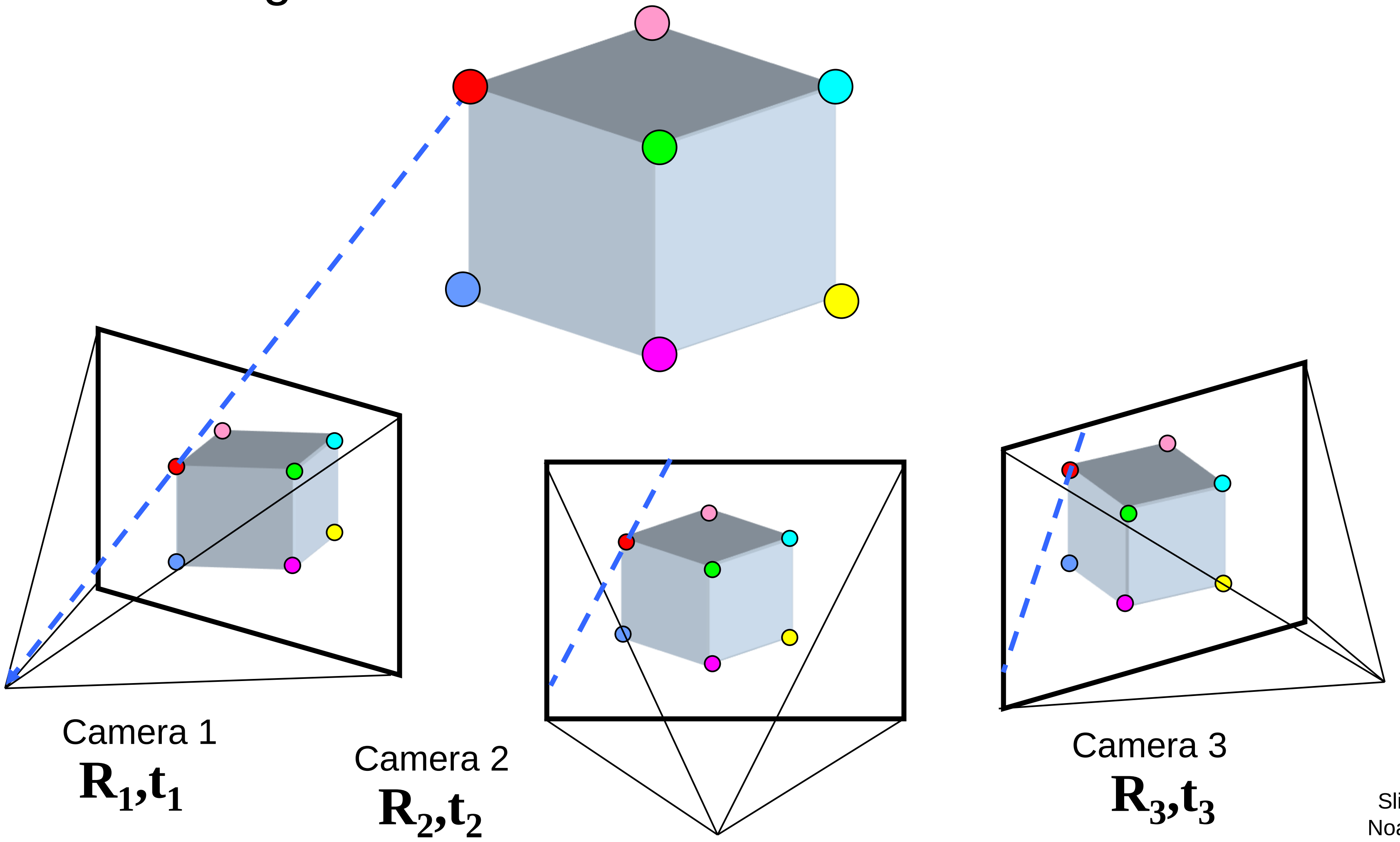

Left stereo image
Ground truth (GT)
GT overlaid on image
msmdnet
all:10 = 2.20 fg:10 = 1.29 bg:10 = 2.20
all:5 = 2.54 fg:5 = 1.61 bg:5 = 2.55
all:3 = 8.22 fg:3 = 3.87 bg:3 = 8.25
4Fun
all:10 = 2.32 fg:10 = 1.29 bg:10 = 2.33
all:5 = 3.03 fg:5 = 1.29 bg:5 = 3.04
all:3 = 7.06 fg:3 = 3.23 bg:3 = 7.09
SMD-stereo
all:10 = 2.55 fg:10 = 1.61 bg:10 = 2.56
all:5 = 3.28 fg:5 = 1.61 bg:5 = 3.30
all:3 = 7.02 fg:3 = 3.23 bg:3 = 7.05
cicero-stereo
all:10 = 2.03 fg:10 = 1.61 bg:10 = 2.04
all:5 = 2.71 fg:5 = 2.58 bg:5 = 2.71
all:3 = 4.00 fg:3 = 3.87 bg:3 = 4.00
NPU-IAP-Stereo
all:10 = 3.19 fg:10 = 1.61 bg:10 = 3.20
all:5 = 4.43 fg:5 = 1.94 bg:5 = 4.45
all:3 = 12.35 fg:3 = 7.10 bg:3 = 12.39


Ground truth
msmdnet
4Fun
SMD-stereo
cicero-stereo
NPU-IAP-Stereo


Ground truth
msmdnet
4Fun
SMD-stereo
cicero-stereo
NPU-IAP-Stereo

Multi-view geometry problems

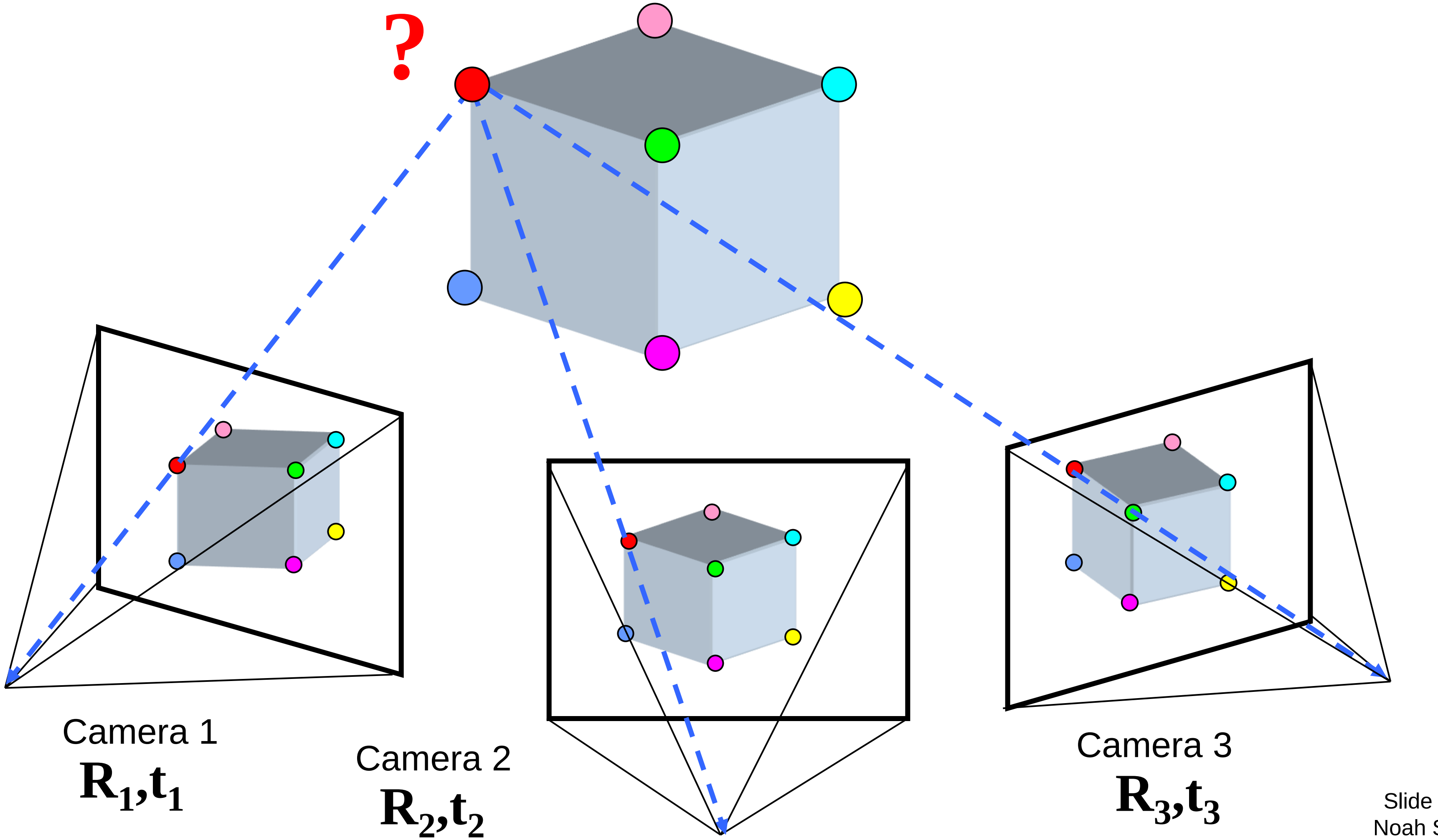
- **Stereo correspondence:** Given a point in one of the images, where could its corresponding points be in the other images?



Slide credit: Noah Snavely

Multi-view geometry problems

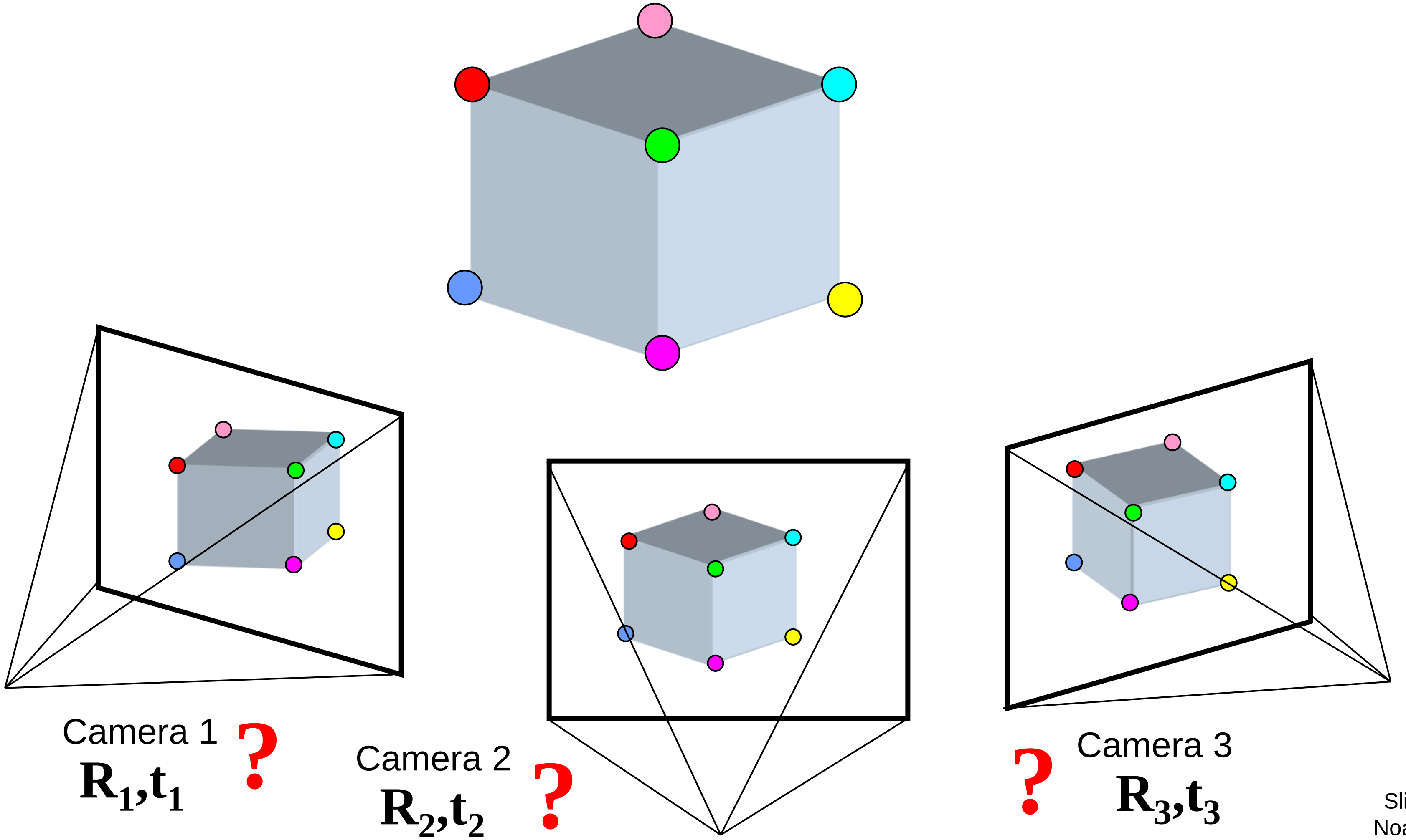
- **Structure:** Given projections of the same 3D point in two or more images, compute the 3D coordinates of that point



Slide credit: Noah Snavely

Multi-view geometry problems

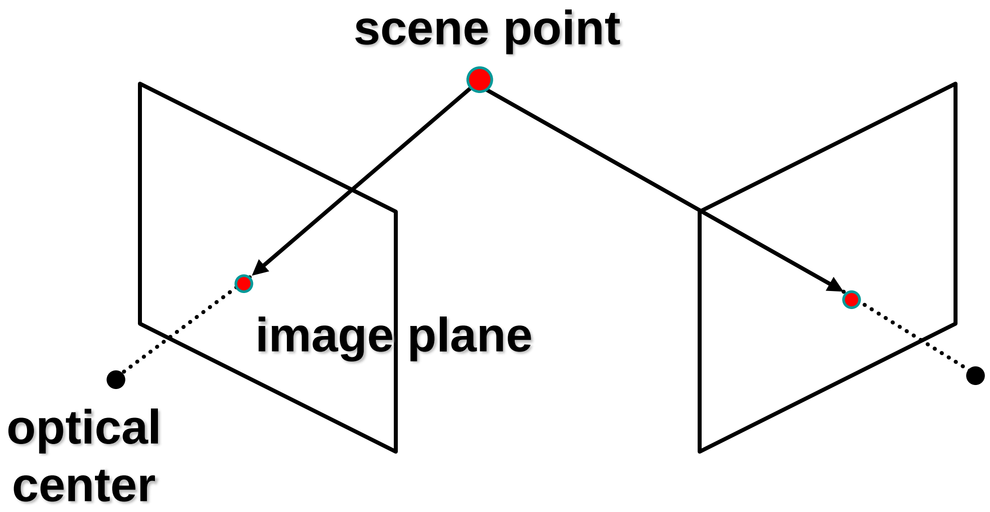
- **Motion:** Given a set of corresponding points in two or more images, compute the camera parameters

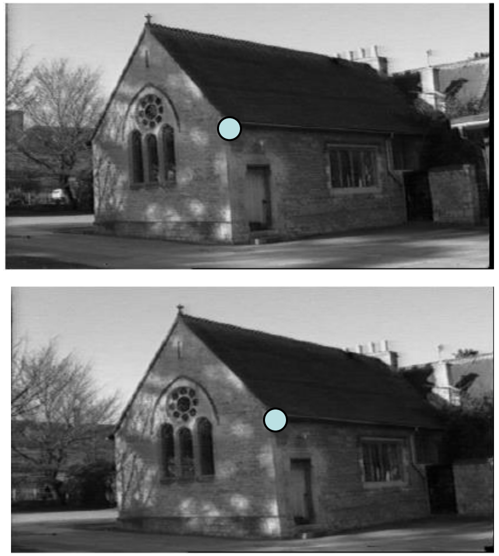


Slide credit: Noah Snavely

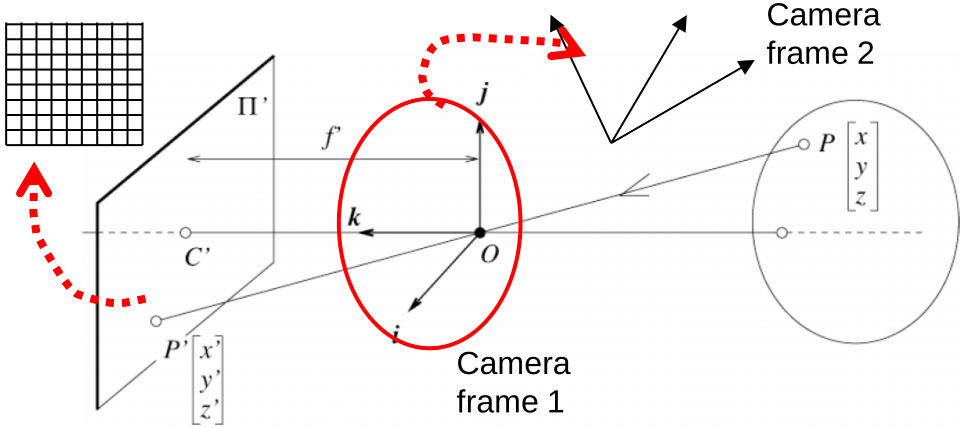
Estimating depth with stereo

- **Stereo**: shape from “motion” between two views
- We’ll need to consider:
  - Info on camera pose (“calibration”)
  - Image point correspondences





Camera parameters



Extrinsic parameters:
Camera frame 1 ←→ Camera frame 2

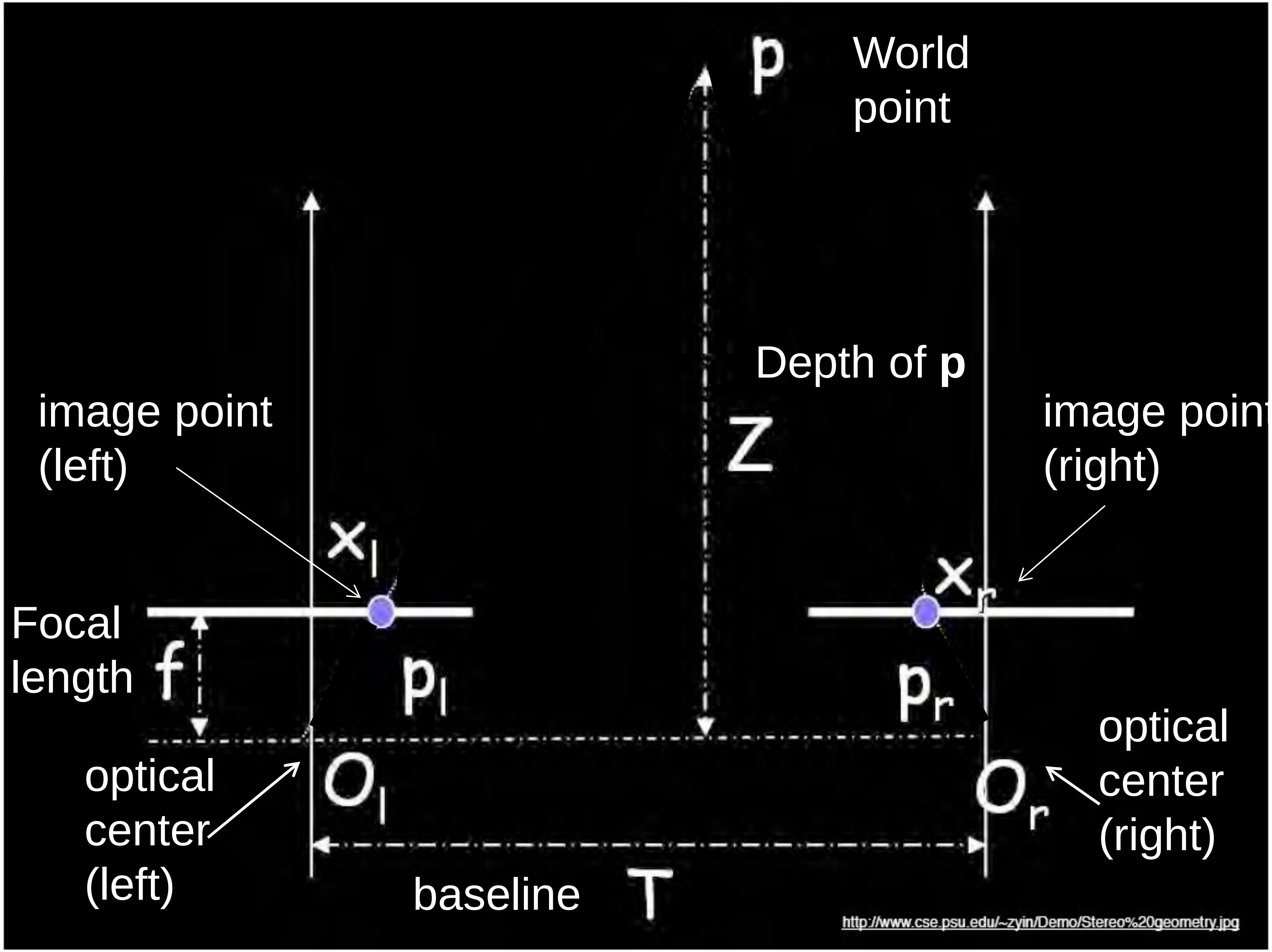
Intrinsic parameters:
Image coordinates relative to camera ←→ Pixel coordinates

- *Extrinsic* params: rotation matrix and translation vector
- *Intrinsic* params: focal length, pixel sizes (mm), image center point, radial distortion parameters

We'll assume for now that these parameters are given and fixed.

Geometry for a simple stereo system

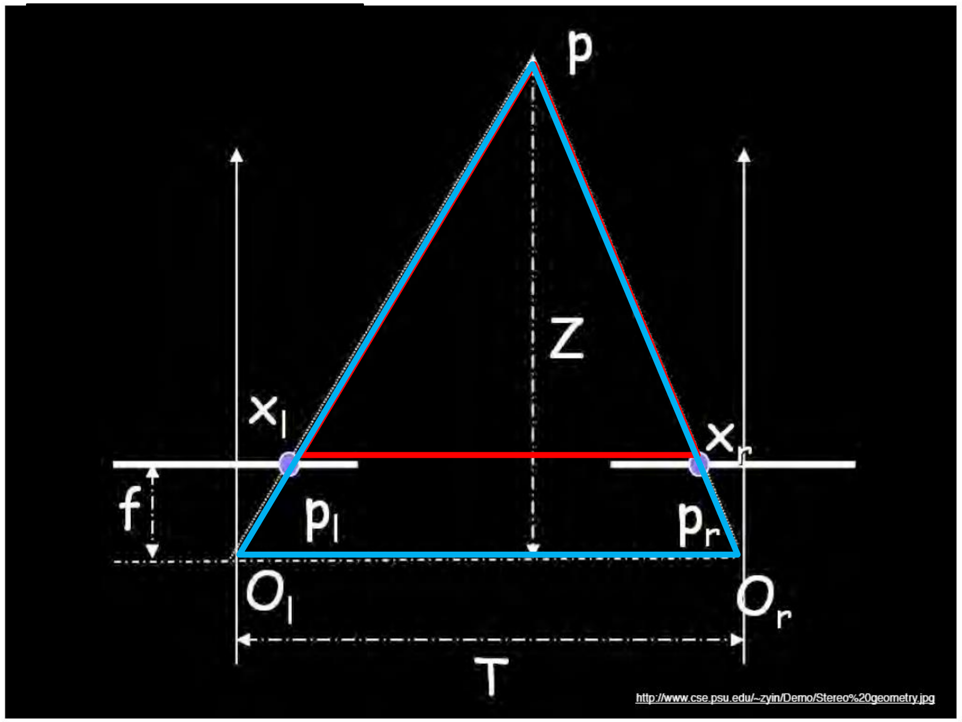
- First, assuming parallel optical axes, known camera parameters (i.e., calibrated cameras):



p
World
point
Depth of p
Z
image point
(left)
image point
(right)
x l
x r
Focal
length
f
p l
p r
optical
center
(left)
O l
O r
optical
center
(right)
baseline
T
http://www.cse.psu.edu/~zyin/Demo/Stereo%20geometry.jpg

Geometry for a simple stereo system

- Assume parallel optical axes, known camera parameters (i.e., calibrated cameras). **What is expression for Z?**



Similar triangles (p_l, P, p_r) and (O_l, P, O_r):

$$\frac{T - x_l + x_r}{Z - f} = \frac{T}{Z}$$

$$Z = f\,\frac{T}{x_l - x_r}$$

disparity

To be continued