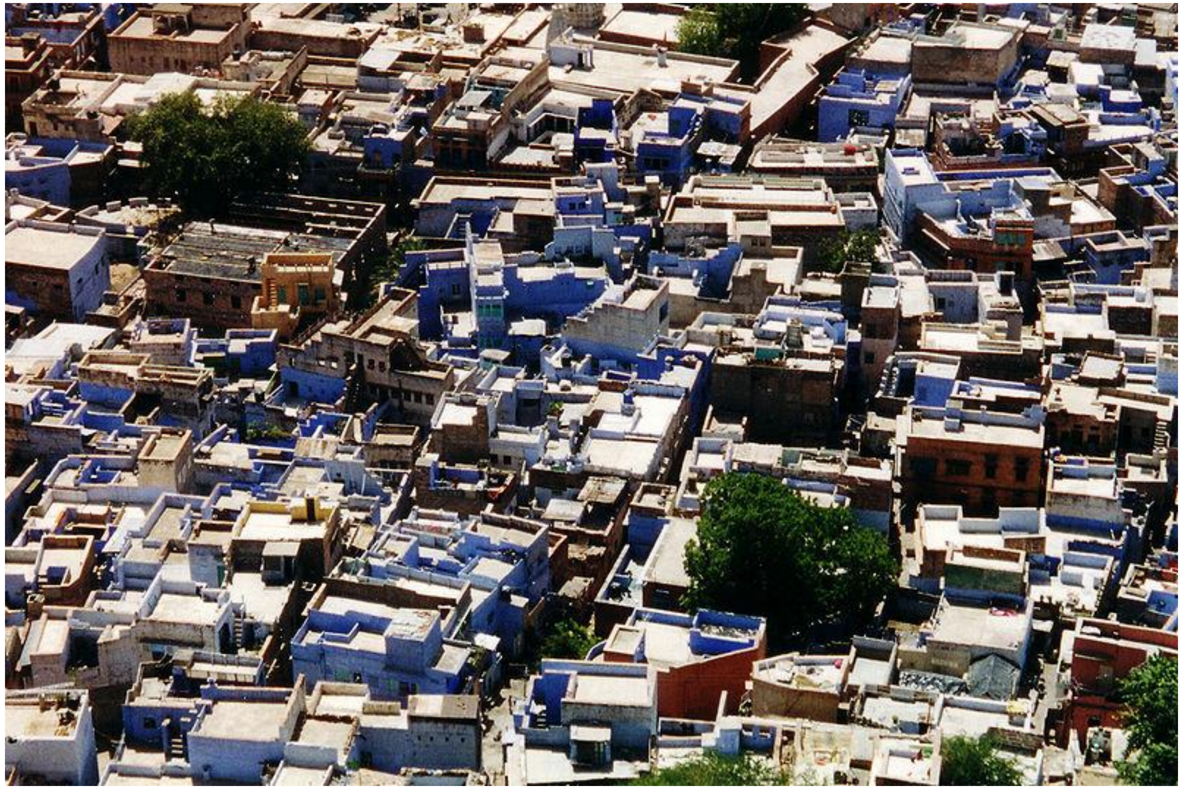
Miniature faking

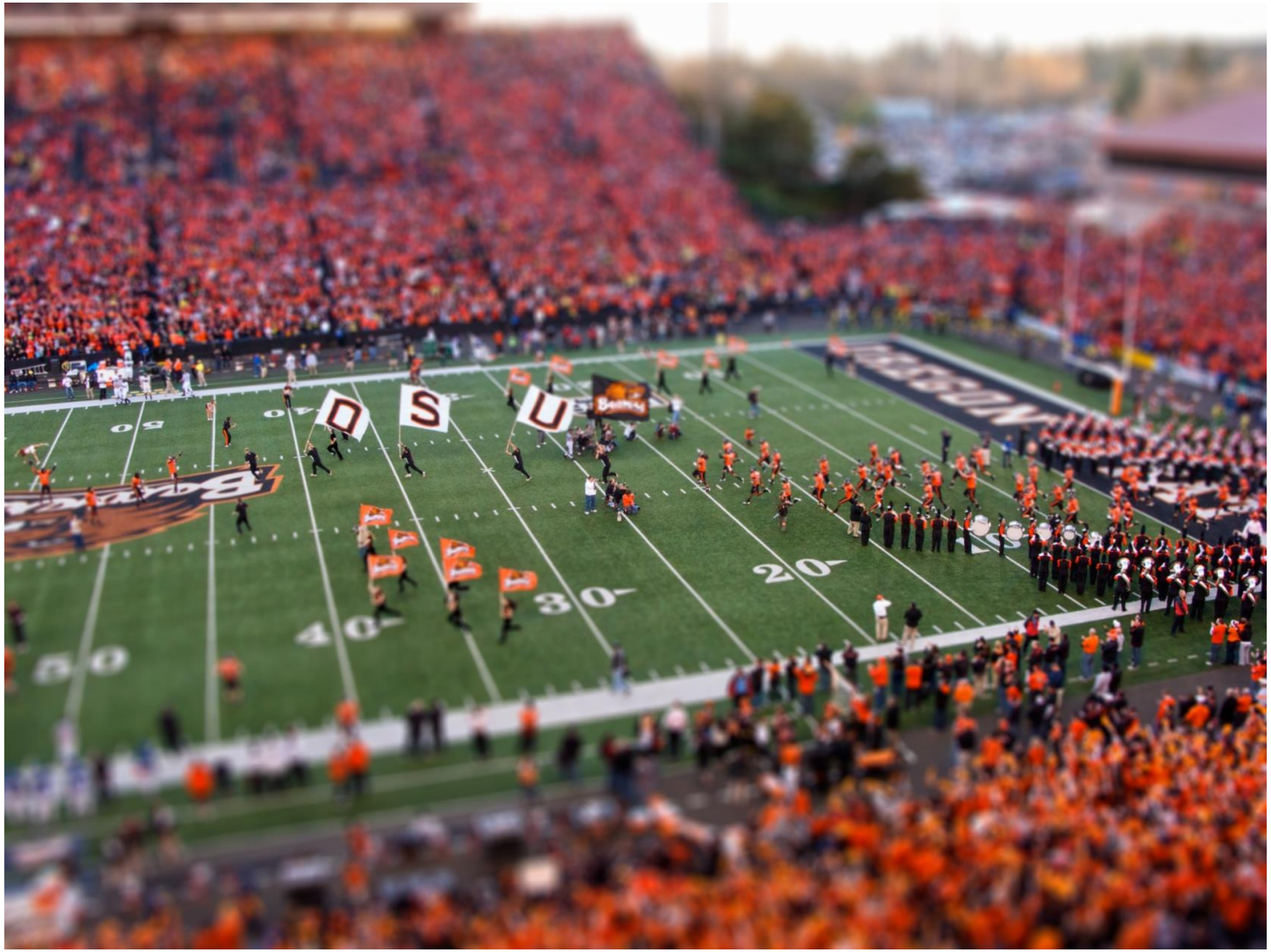
In close-up photo, the depth of field is limited.

http://en.wikipedia.org/wiki/File:Jodhpur_tilt_shift.jpg

Miniature faking



Miniature faking



http://en.wikipedia.org/wiki/File:Oregon_State_Beavers_Tilt-Shift_Miniature_Greg_Keene.jpg

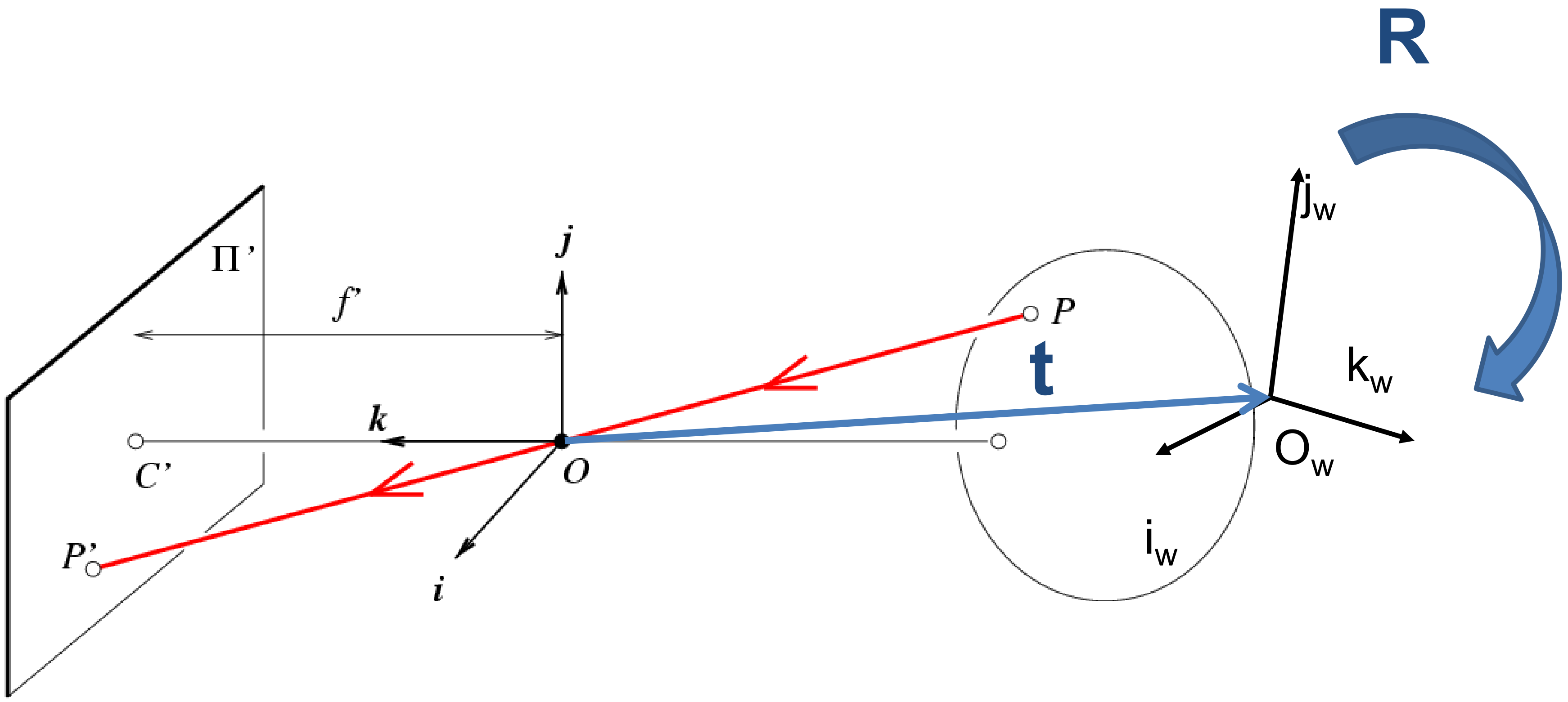
This section – multiple views

- Today – Camera Calibration. Multiple views and Stereo. Epipolar Geometry and Fundamental Matrix.

- Later: Dense Stereo Matching.

- Both topics are extra credit for project 2.

Recap: Oriented and Translated Camera



Recap: Degrees of freedom

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

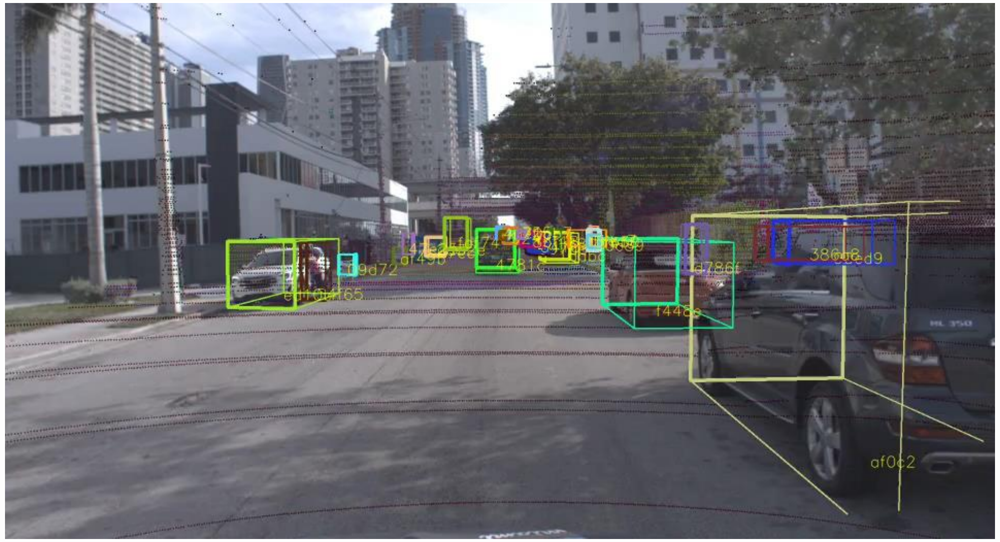
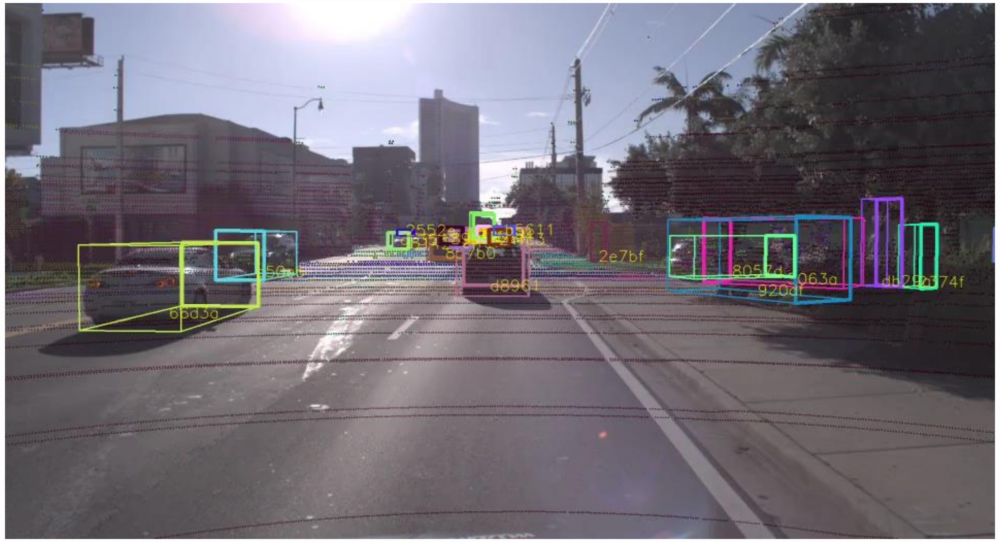
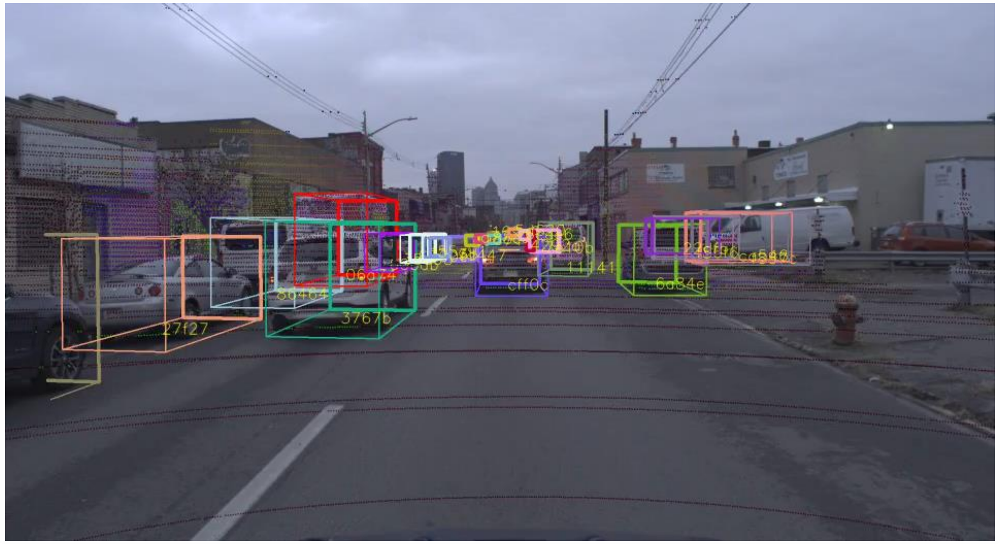
$$w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \overset{5}{\begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}} \overset{6}{\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix}} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

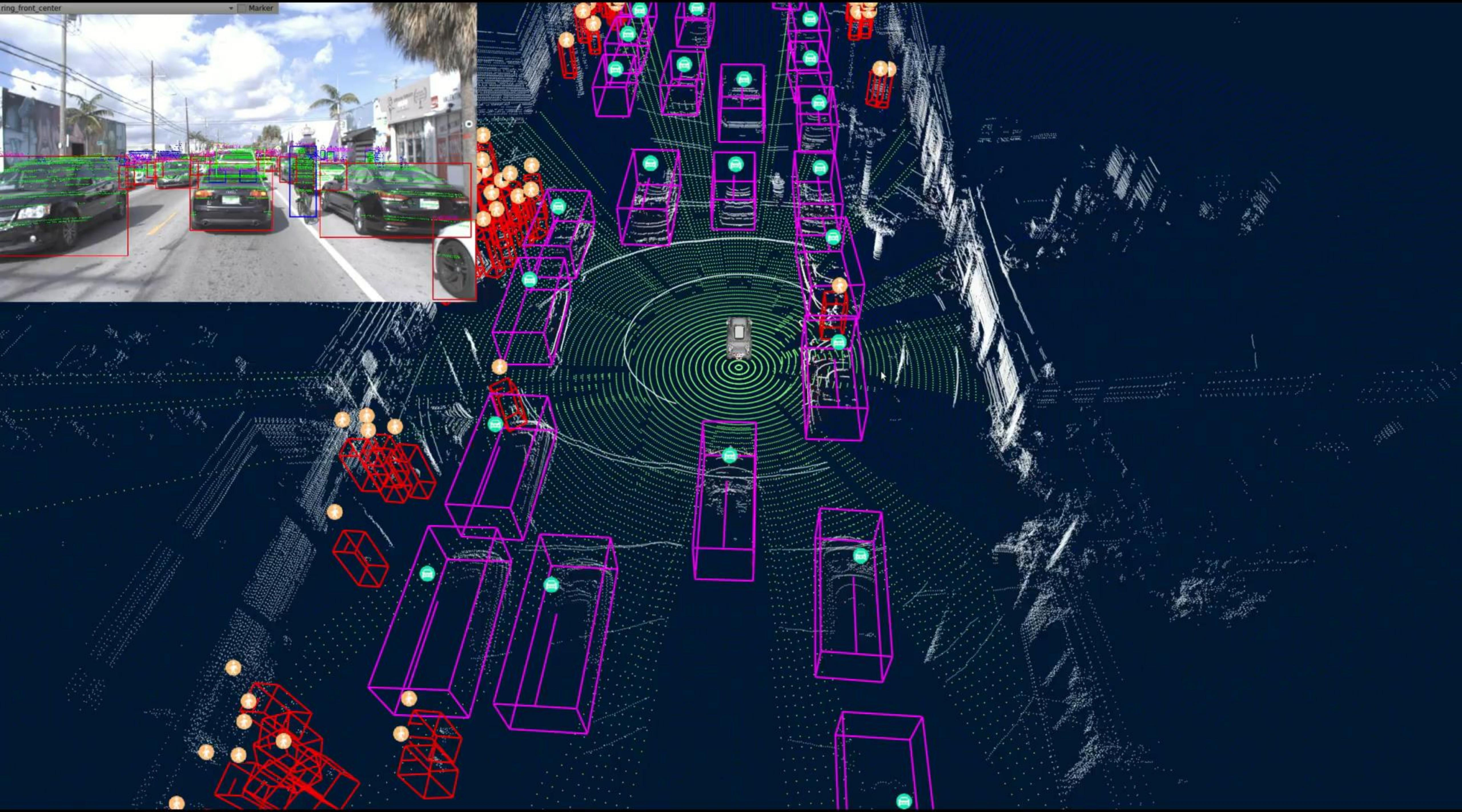
This Lecture: How to calibrate the camera?

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

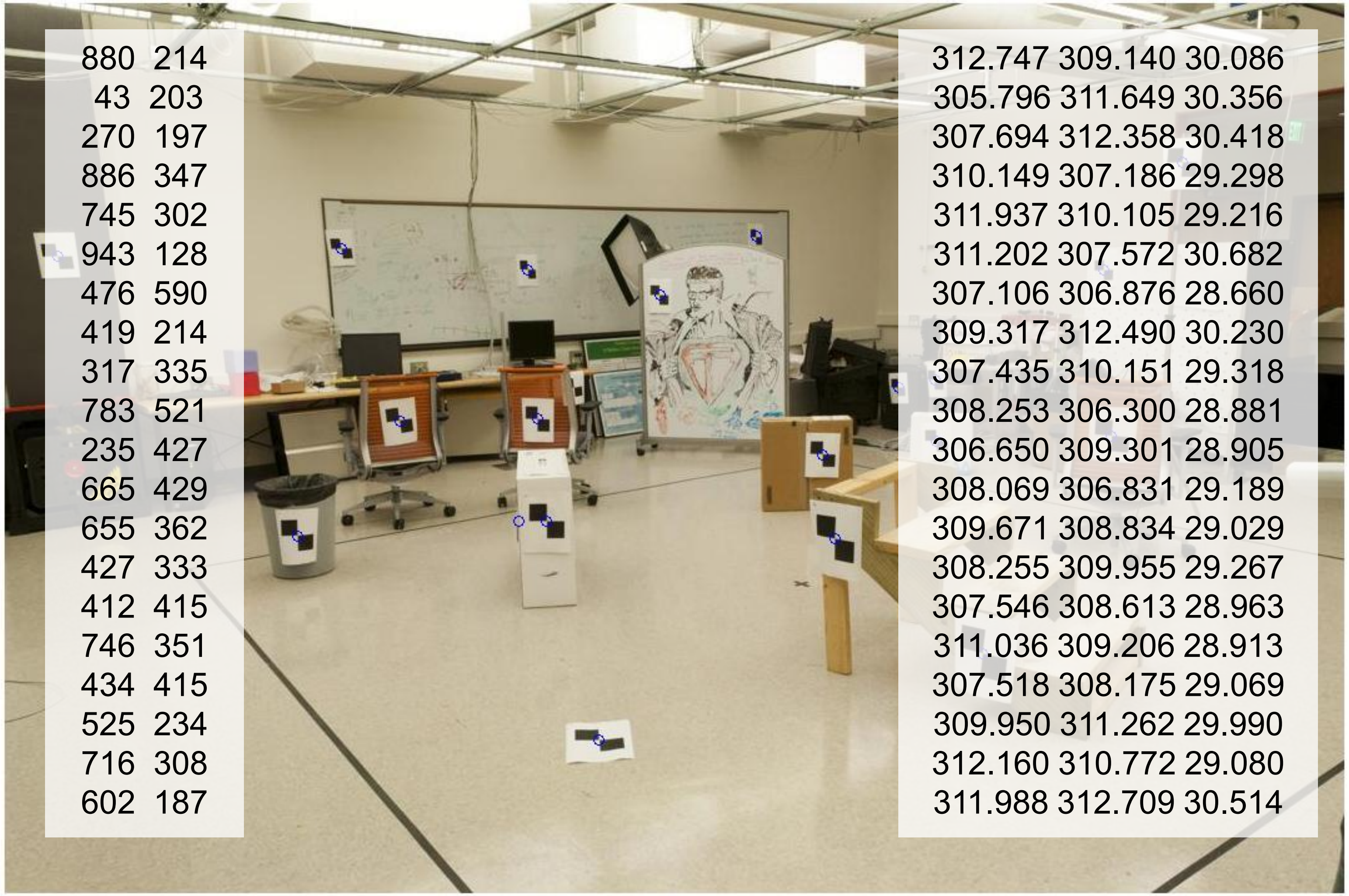
$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

What can we do with camera calibration?

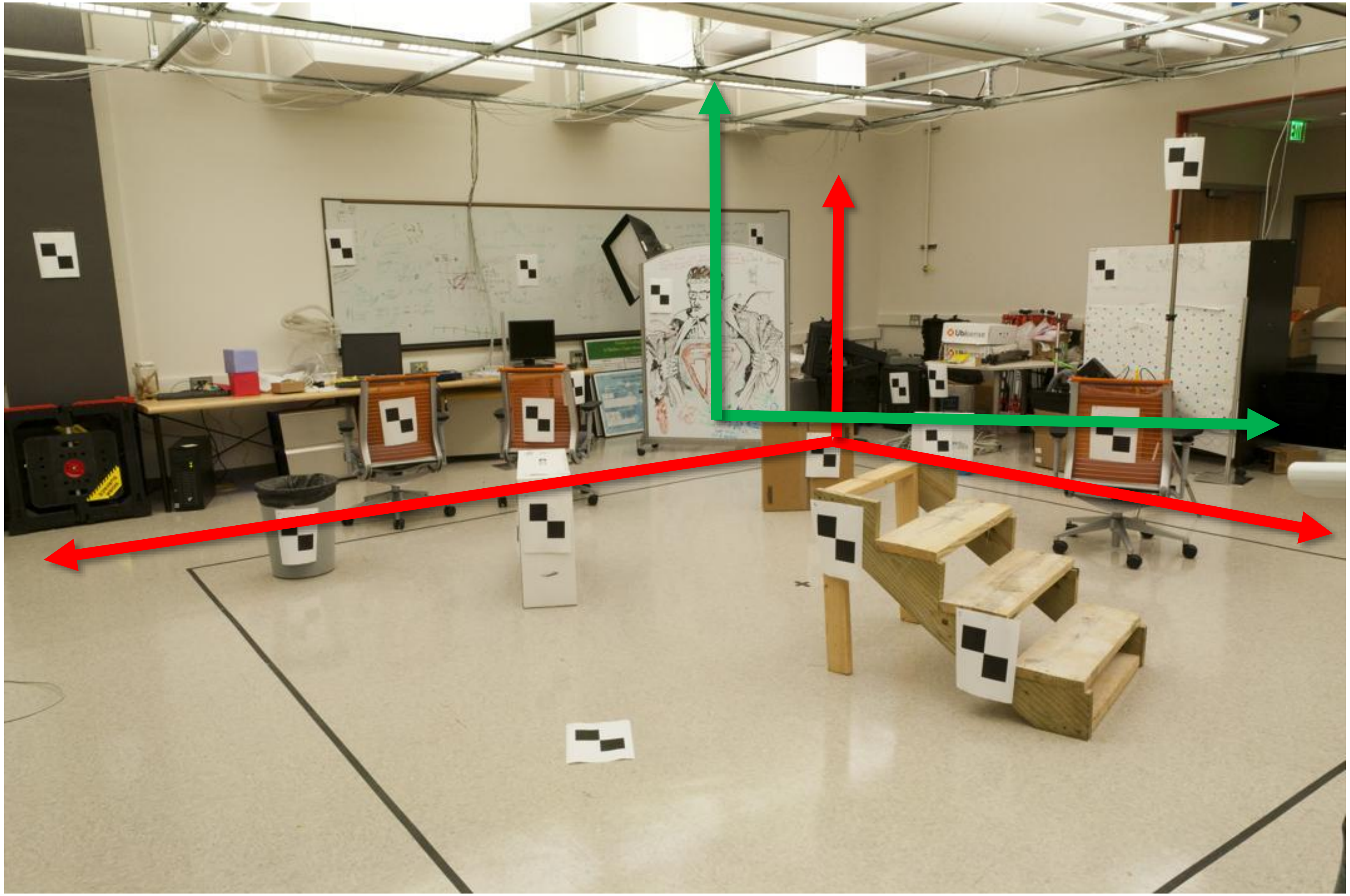






ring_front_center
Marker

How do we calibrate a camera?

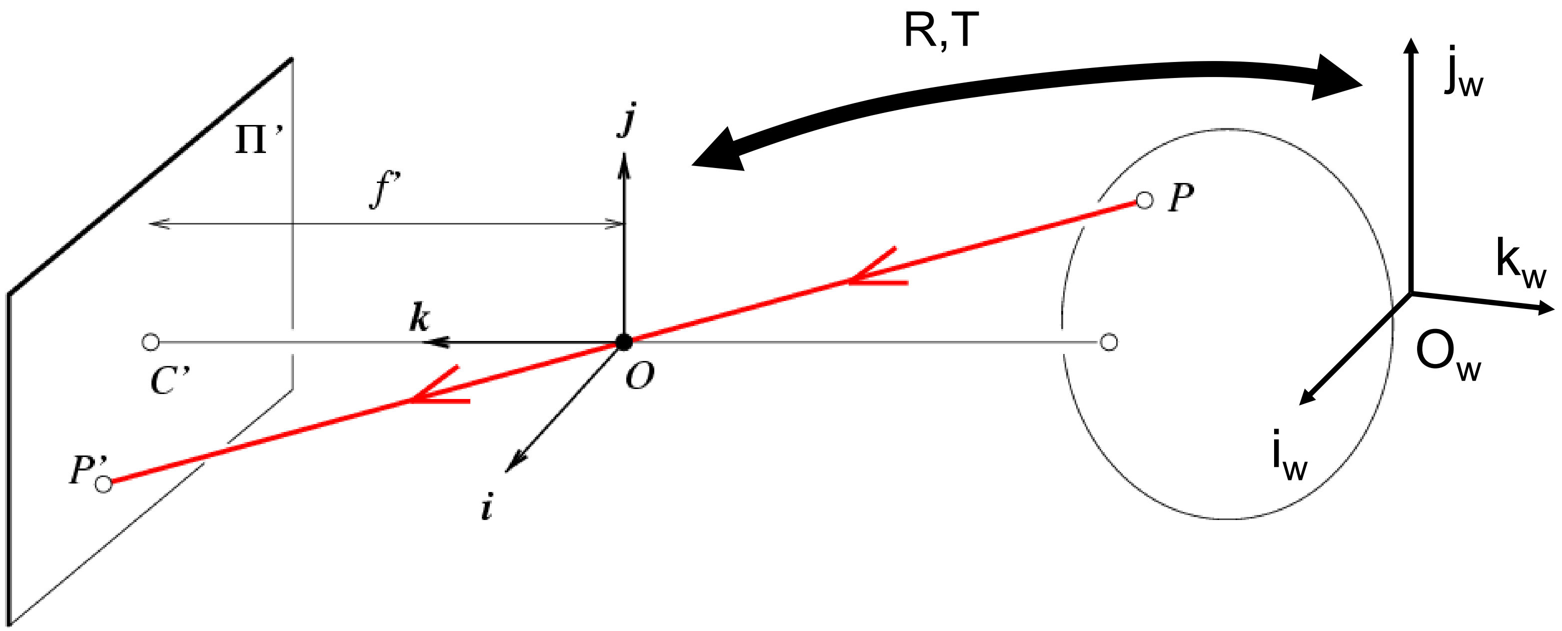


World vs Camera coordinates



Slide Credit: Saverese

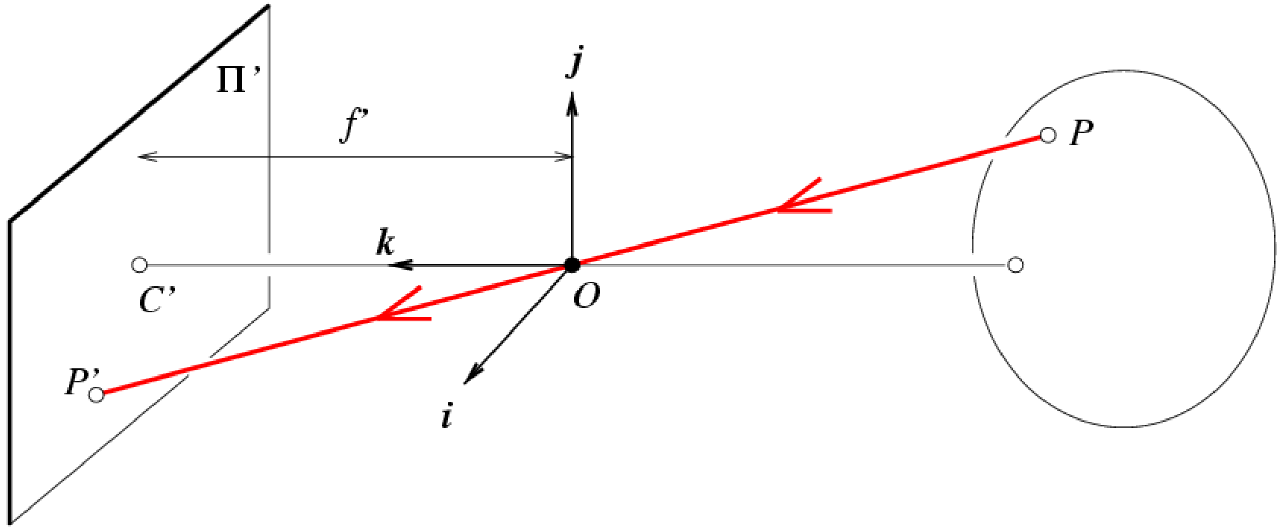
Projection matrix



$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

x: Image Coordinates: (u,v,1)
K: Intrinsic Matrix (3x3)
R: Rotation (3x3)
t: Translation (3x1)
X: World Coordinates: (X,Y,Z,1)

Projection matrix



Intrinsic Assumptions
- Unit aspect ratio
- Optical center at (0,0)
- No skew

Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{X} \Rightarrow w\begin{bmatrix}u\\v\\1\end{bmatrix} = \begin{bmatrix}f & 0 & 0 & 0\\0 & f & 0 & 0\\0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}$$

Slide Credit: Saverese

Remove assumption: known optical center

Intrinsic Assumptions

- Unit aspect ratio
- No skew

Extrinsic Assumptions

- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{X} \quad \Rightarrow \quad w\begin{bmatrix}u\\ v\\ 1\end{bmatrix} = \begin{bmatrix}\boxed{\begin{matrix}f & 0 & u_0\\ 0 & f & v_0\\ 0 & 0 & 1\end{matrix}} & \begin{matrix}0\\0\\0\end{matrix}\end{bmatrix}\begin{bmatrix}x\\ y\\ z\\ 1\end{bmatrix}$$

Remove assumption: square pixels

Intrinsic Assumptions
- No skew

Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{X} \Rightarrow w\begin{bmatrix}u \\ v \\ 1\end{bmatrix} = \begin{bmatrix}\alpha & 0 & u_0 & 0 \\ 0 & \beta & v_0 & 0 \\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x \\ y \\ z \\ 1\end{bmatrix}$$

Remove assumption: non-skewed pixels

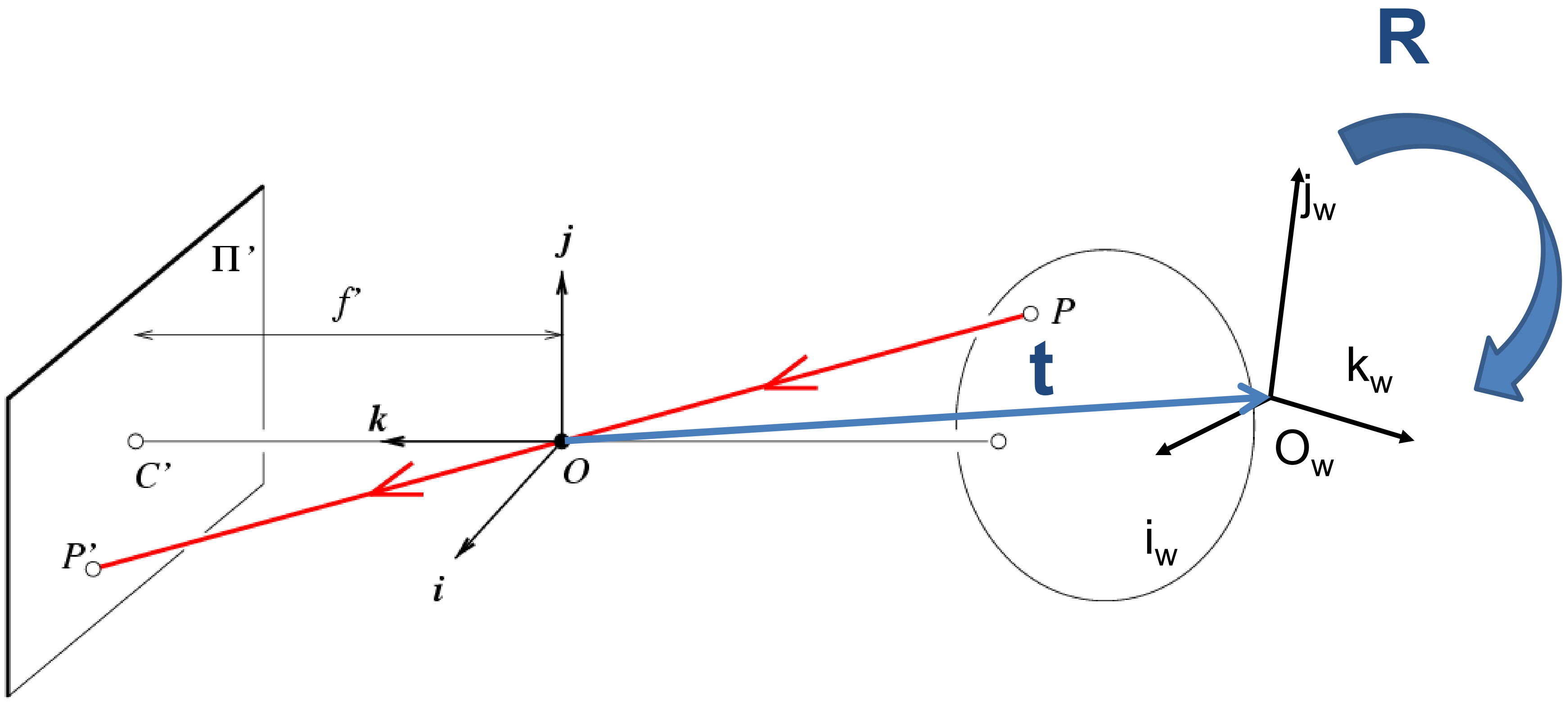
Intrinsic Assumptions

Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{X} \quad \Rightarrow \quad w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 & 0 \\ 0 & \beta & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Note: different books use different notation for parameters

Oriented and Translated Camera



Allow camera translation

Intrinsic Assumptions

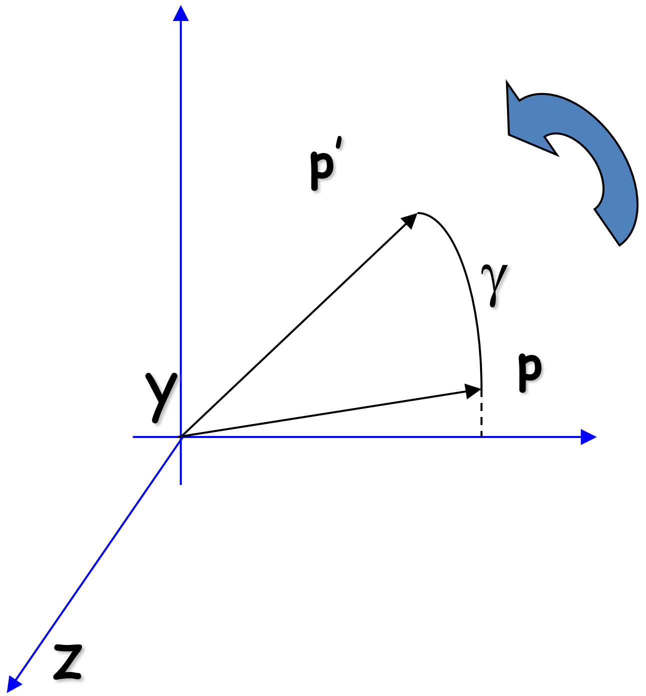
Extrinsic Assumptions
- No rotation

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{t}\end{bmatrix}\mathbf{X} \Rightarrow w\begin{bmatrix}u \\ v \\ 1\end{bmatrix} = \begin{bmatrix}\alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z\end{bmatrix}\begin{bmatrix}x \\ y \\ z \\ 1\end{bmatrix}$$

Slide Credit: Saverese

3D Rotation of Points

Rotation around the coordinate axes, counter-clockwise:



$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

$$R_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

$$R_z(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Allow camera rotation

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

$$w\begin{bmatrix}u\\v\\1\end{bmatrix} = \begin{bmatrix}\alpha & s & u_0\\0 & \beta & v_0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}r_{11} & r_{12} & r_{13} & t_x\\r_{21} & r_{22} & r_{23} & t_y\\r_{31} & r_{32} & r_{33} & t_z\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}$$

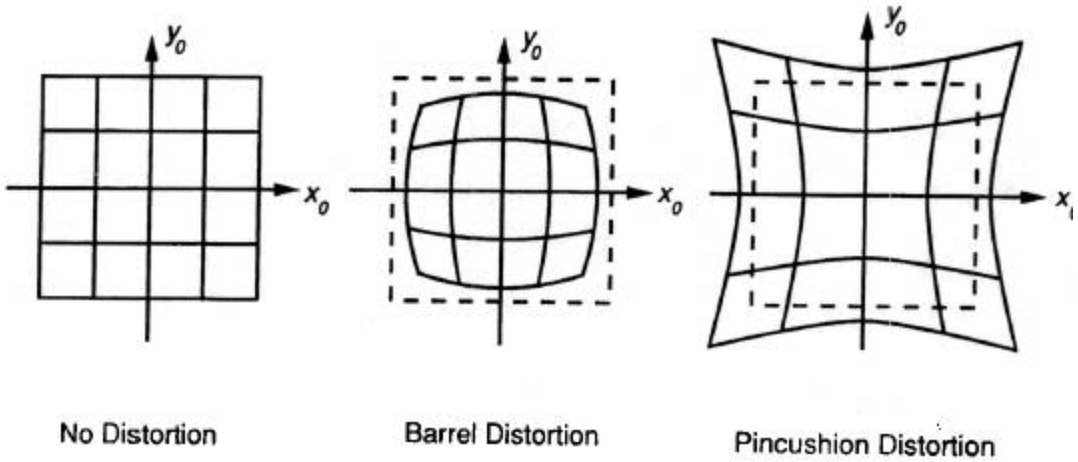
Degrees of freedom

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

$$w\begin{bmatrix}u\\ v\\ 1\end{bmatrix} = \overset{5}{\begin{bmatrix}\alpha & s & u_0\\ 0 & \beta & v_0\\ 0 & 0 & 1\end{bmatrix}}\overset{6}{\begin{bmatrix}r_{11} & r_{12} & r_{13} & t_x\\ r_{21} & r_{22} & r_{23} & t_y\\ r_{31} & r_{32} & r_{33} & t_z\end{bmatrix}}\begin{bmatrix}x\\ y\\ z\\ 1\end{bmatrix}$$

Beyond Pinholes: Radial Distortion

- Common in wide-angle lenses or for special applications (e.g., security)
- Creates non-linear terms in projection
- Usually handled by through solving for non-linear terms and then correcting image



Corrected Barrel Distortion

Image from Martin Habbecke

How to calibrate the camera?

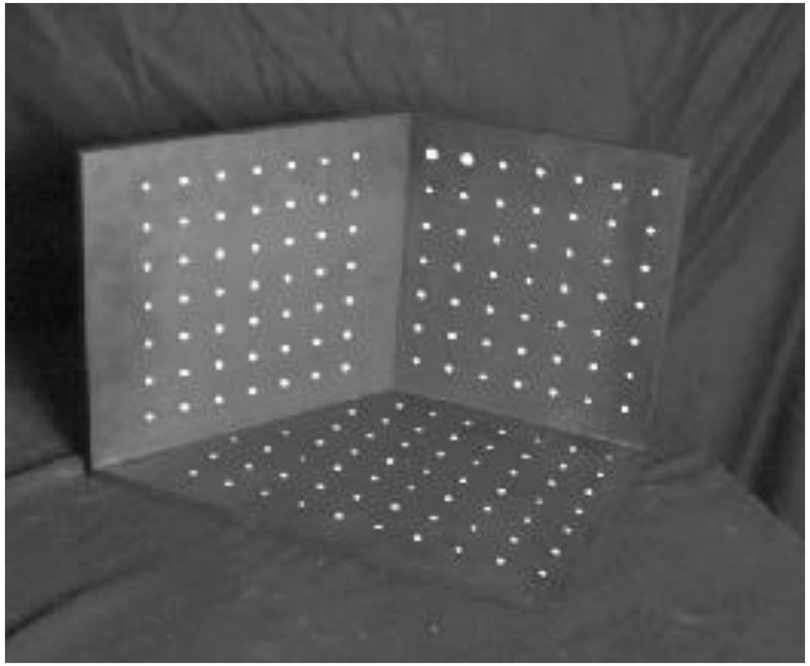
$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Calibrating the Camera

Use a scene with known geometry

- Correspond image points to 3d points
- Get least squares solution (or non-linear solution)



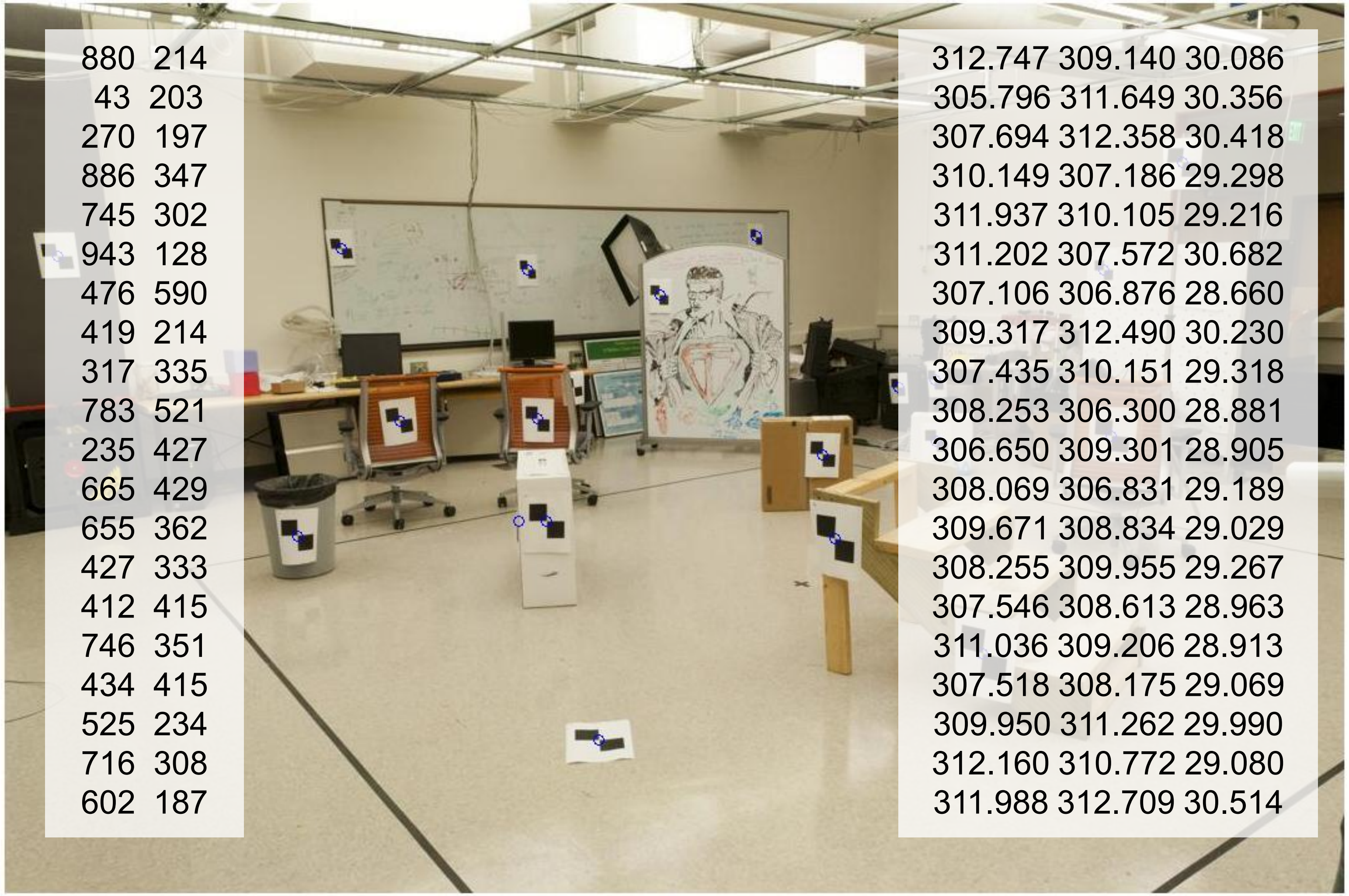
Known 2d image coords

Known 3d locations

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

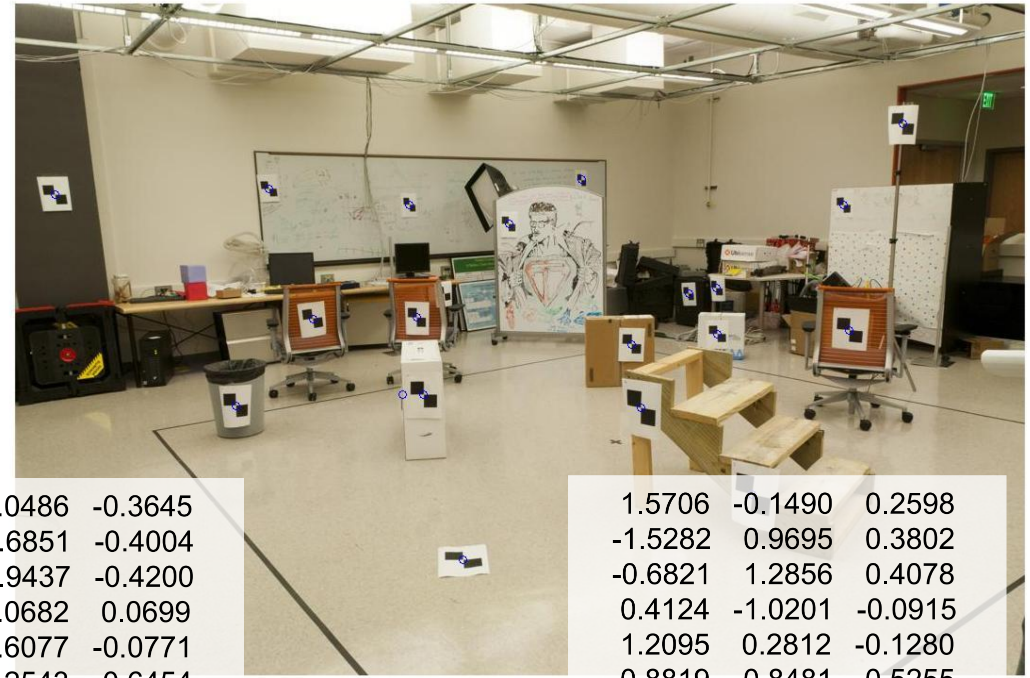
Unknown Camera Parameters

How do we calibrate a camera?



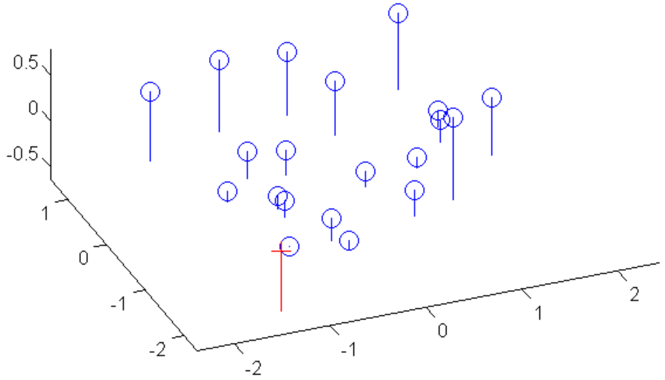
Known 2d image coords		Known 3d locations		
880	214	312.747	309.140	30.086
43	203	305.796	311.649	30.356
270	197	307.694	312.358	30.418
886	347	310.149	307.186	29.298
745	302	311.937	310.105	29.216
943	128	311.202	307.572	30.682
476	590	307.106	306.876	28.660
419	214	309.317	312.490	30.230
317	335	307.435	310.151	29.318
783	521	308.253	306.300	28.881
235	427	306.650	309.301	28.905
665	429	308.069	306.831	29.189
655	362	309.671	308.834	29.029
427	333	308.255	309.955	29.267
412	415	307.546	308.613	28.963
746	351	311.036	309.206	28.913
434	415	307.518	308.175	29.069
525	234	309.950	311.262	29.990
716	308	312.160	310.772	29.080
602	187	311.988	312.709	30.514

Estimate of camera center



1.0486	-0.3645
-1.6851	-0.4004
-0.9437	-0.4200
1.0682	0.0699
0.6077	-0.0771
1.2543	-0.6454
-0.2709	0.8635
-0.4571	-0.3645
-0.7902	0.0307
0.7318	0.6382
-1.0580	0.3312
0.3464	0.3377
0.3137	0.1189
-0.4310	0.0242
-0.4799	0.2920
0.6109	0.0830
-0.4081	0.2920
-0.1109	-0.2992
0.5129	-0.0575
0.1406	-0.4527

1.5706	-0.1490	0.2598
-1.5282	0.9695	0.3802
-0.6821	1.2856	0.4078
0.4124	-1.0201	-0.0915
1.2095	0.2812	-0.1280
0.8819	-0.8481	0.5255
-0.9442	-1.1583	-0.3759
0.0415	1.3445	0.3240
-0.7975	0.3017	-0.0826
-0.4329	-1.4151	-0.2774
-1.1475	-0.0772	-0.2667
-0.5149	-1.1784	-0.1401
0.1993	-0.2854	-0.2114
-0.4320	0.2143	-0.1053
-0.7481	-0.3840	-0.2408
0.8078	-0.1196	-0.2631
-0.7605	-0.5792	-0.1936
0.3237	0.7970	0.2170
1.3089	0.5786	-0.1887
1.2323	1.4421	0.4506



Unknown Camera Parameters

Known 2d image coords

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Known 3d locations

$$su = m_{11}X + m_{12}Y + m_{13}Z + m_{14}$$

$$sv = m_{21}X + m_{22}Y + m_{23}Z + m_{24}$$

$$s = m_{31}X + m_{32}Y + m_{33}Z + m_{34}$$

$$(m_{31}X + m_{32}Y + m_{33}Z + m_{34})u = m_{11}X + m_{12}Y + m_{13}Z + m_{14}$$

$$(m_{31}X + m_{32}Y + m_{33}Z + m_{34})v = m_{21}X + m_{22}Y + m_{23}Z + m_{24}$$

$$m_{31}uX + m_{32}uY + m_{33}uZ + m_{34}u = m_{11}X + m_{12}Y + m_{13}Z + m_{14}$$

$$m_{31}vX + m_{32}vY + m_{33}vZ + m_{34}v = m_{21}X + m_{22}Y + m_{23}Z + m_{24}$$

Unknown Camera Parameters

Known 2d image coords

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Known 3d locations

$$m_{31}uX + m_{32}uY + m_{33}uZ + m_{34}u = m_{11}X + m_{12}Y + m_{13}Z + m_{14}$$
$$m_{31}vX + m_{32}vY + m_{33}vZ + m_{34}v = m_{21}X + m_{22}Y + m_{23}Z + m_{24}$$

$$0 = m_{11}X + m_{12}Y + m_{13}Z + m_{14} - m_{31}uX - m_{32}uY - m_{33}uZ - m_{34}u$$
$$0 = m_{21}X + m_{22}Y + m_{23}Z + m_{24} - m_{31}vX - m_{32}vY - m_{33}vZ - m_{34}v$$

Unknown Camera Parameters

Known 2d image coords

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Known 3d locations

$$0 = m_{11}X + m_{12}Y + m_{13}Z + m_{14} - m_{31}uX - m_{32}uY - m_{33}uZ - m_{34}u$$

$$0 = m_{21}X + m_{22}Y + m_{23}Z + m_{24} - m_{31}vX - m_{32}vY - m_{33}vZ - m_{34}v$$

- Method 1 – homogeneous linear system. Solve for m's entries using linear least squares

$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1X_1 & -u_1Y_1 & -u_1Z_1 & -u_1 \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1X_1 & -v_1Y_1 & -v_1Z_1 & -v_1 \\ & & & & & & \vdots & & & & & \\ X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -u_nX_n & -u_nY_n & -u_nZ_n & -u_n \\ 0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -v_nX_n & -v_nY_n & -v_nZ_n & -v_n \end{bmatrix} \begin{bmatrix} m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{31} \\ m_{32} \\ m_{33} \\ m_{34} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

```
[U, S, V] = svd(A);
M = V(:,end);
M = reshape(M,[],3)';
```

For python, see numpy.linalg.svd

Unknown Camera Parameters

Known 2d image coords

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Known 3d locations

- Method 2 – nonhomogeneous linear system. Solve for m's entries using linear least squares

Ax=b form

$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1X_1 & -u_1Y_1 & -u_1Z_1 \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1X_1 & -v_1Y_1 & -v_1Z_1 \\ & & & & & & \vdots & & & & \\ X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -u_nX_n & -u_nY_n & -u_nZ_n \\ 0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -v_nX_n & -v_nY_n & -v_nZ_n \end{bmatrix} \begin{bmatrix} m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{31} \\ m_{32} \\ m_{33} \end{bmatrix} = \begin{bmatrix} u_1 \\ v_1 \\ \vdots \\ u_n \\ v_n \end{bmatrix}$$

```
M = A\Y;
M = [M;1];
M = reshape(M,[],3)';
```

For python, see numpy.linalg.lstsq

Calibration with linear method

- Advantages
  - Easy to formulate and solve
  - Provides initialization for non-linear methods

- Disadvantages
  - Doesn’t directly give you human-interpretable camera parameters
  - Doesn’t model radial distortion
  - Can’t impose constraints, such as known focal length

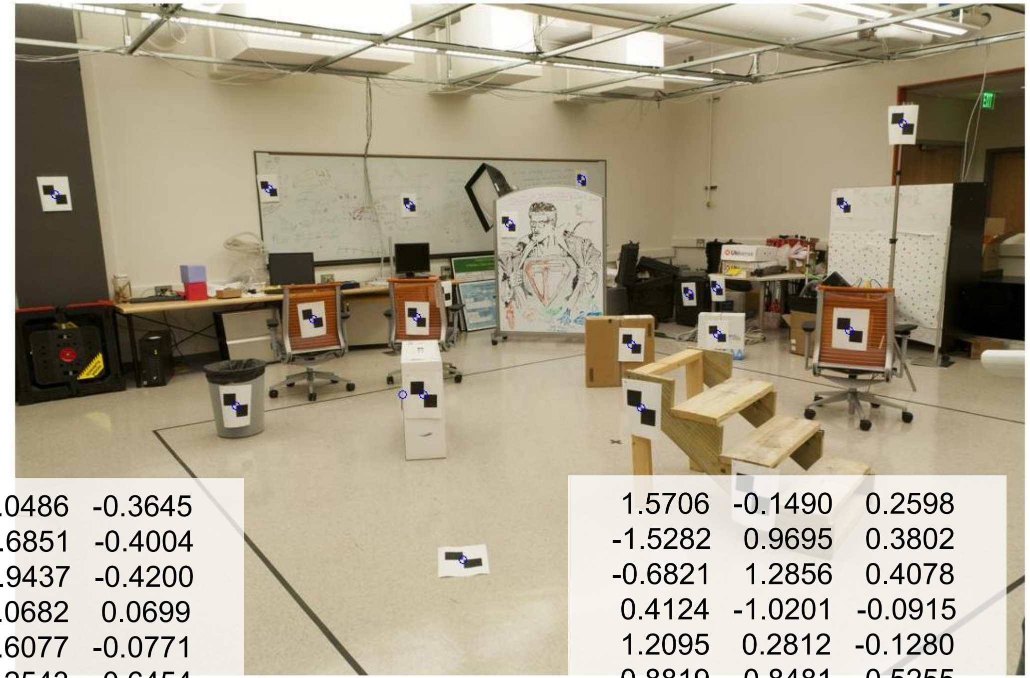
- Non-linear methods are preferred
  - Define error as difference between projected points and measured points
  - Minimize error using Newton’s method or other non-linear optimization

Can we factorize M back to K [R | T]?

- Yes!
- You can use *RQ* factorization (note – not the more familiar *QR* factorization). *R* (right diagonal) is K, and *Q* (orthogonal basis) is R. T, the last column of [R | T], is inv(K) * last column of M.
  - But you need to do a bit of post-processing to make sure that the matrices are valid. See http://ksimek.github.io/2012/08/14/decompose/

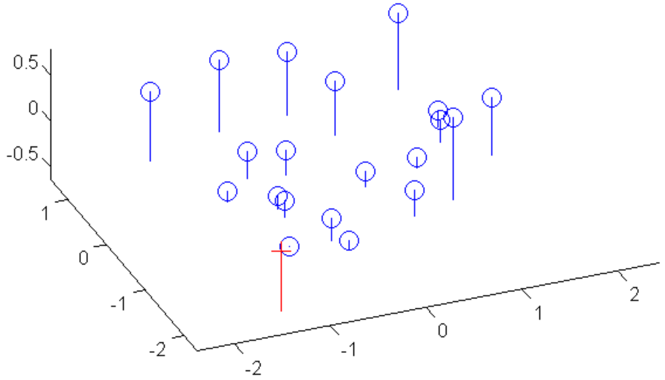
For project 2, we want the camera center

Estimate of camera center

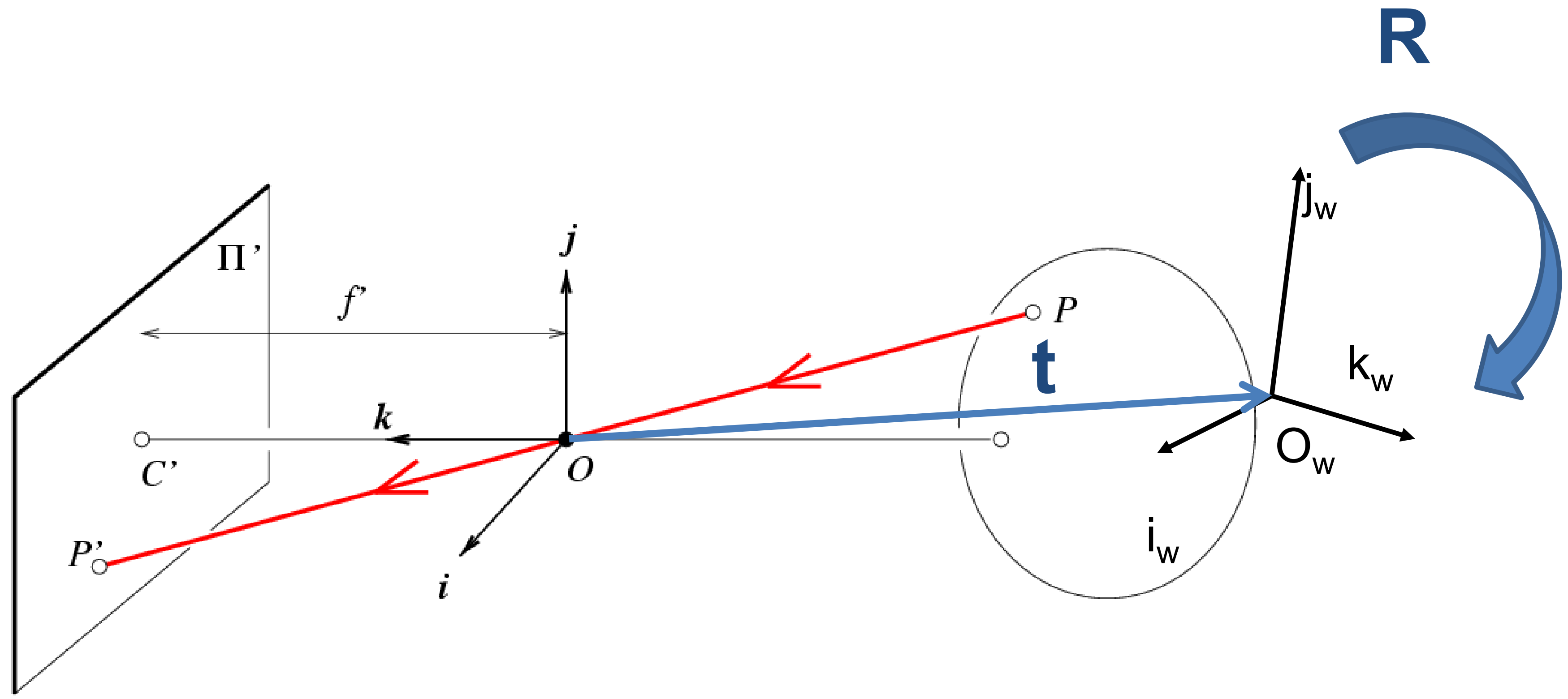


1.0486	-0.3645
-1.6851	-0.4004
-0.9437	-0.4200
1.0682	0.0699
0.6077	-0.0771
1.2543	-0.6454
-0.2709	0.8635
-0.4571	-0.3645
-0.7902	0.0307
0.7318	0.6382
-1.0580	0.3312
0.3464	0.3377
0.3137	0.1189
-0.4310	0.0242
-0.4799	0.2920
0.6109	0.0830
-0.4081	0.2920
-0.1109	-0.2992
0.5129	-0.0575
0.1406	-0.4527

1.5706	-0.1490	0.2598
-1.5282	0.9695	0.3802
-0.6821	1.2856	0.4078
0.4124	-1.0201	-0.0915
1.2095	0.2812	-0.1280
0.8819	-0.8481	0.5255
-0.9442	-1.1583	-0.3759
0.0415	1.3445	0.3240
-0.7975	0.3017	-0.0826
-0.4329	-1.4151	-0.2774
-1.1475	-0.0772	-0.2667
-0.5149	-1.1784	-0.1401
0.1993	-0.2854	-0.2114
-0.4320	0.2143	-0.1053
-0.7481	-0.3840	-0.2408
0.8078	-0.1196	-0.2631
-0.7605	-0.5792	-0.1936
0.3237	0.7970	0.2170
1.3089	0.5786	-0.1887
1.2323	1.4421	0.4506



Oriented and Translated Camera



Recovering the camera center

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

$$w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} r_{11} & r_{12} & r_{13} & \boxed{t_x} \\ r_{21} & r_{22} & r_{23} & \boxed{t_y} \\ r_{31} & r_{32} & r_{33} & \boxed{t_z} \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

This is not the camera center -C. It is –RC (because a point will be rotated before t_x, t_y, and t_z are added)

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Q

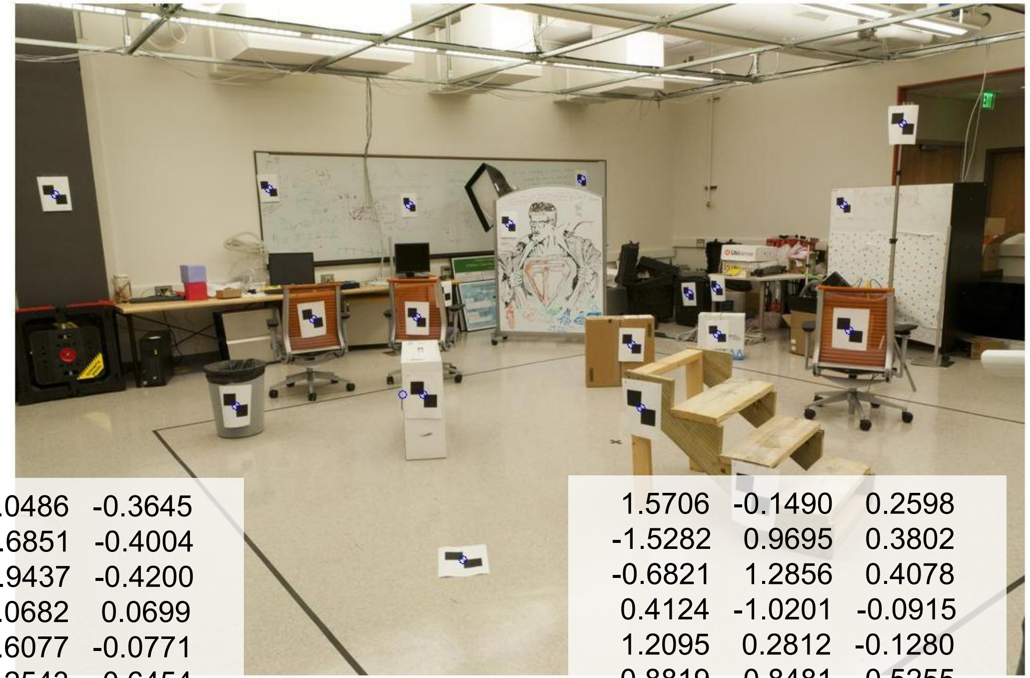
This, m_4, is K * **t**

So $K^{-1} m_4$ is **t**

So we need $-R^{-1} K^{-1} m_4$ to get C

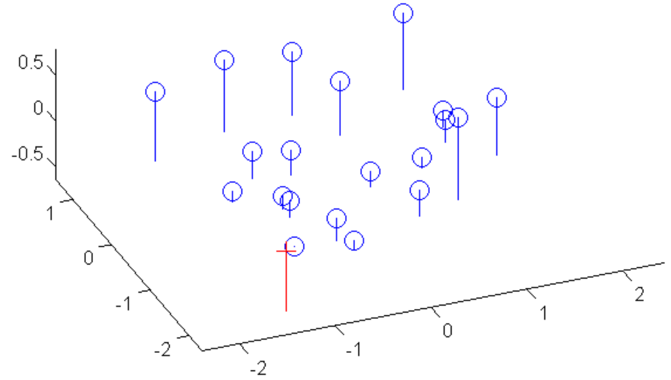
Q is K * R. So we just need $-Q^{-1} m_4$

Estimate of camera center



1.0486	-0.3645
-1.6851	-0.4004
-0.9437	-0.4200
1.0682	0.0699
0.6077	-0.0771
1.2543	-0.6454
-0.2709	0.8635
-0.4571	-0.3645
-0.7902	0.0307
0.7318	0.6382
-1.0580	0.3312
0.3464	0.3377
0.3137	0.1189
-0.4310	0.0242
-0.4799	0.2920
0.6109	0.0830
-0.4081	0.2920
-0.1109	-0.2992
0.5129	-0.0575
0.1406	-0.4527

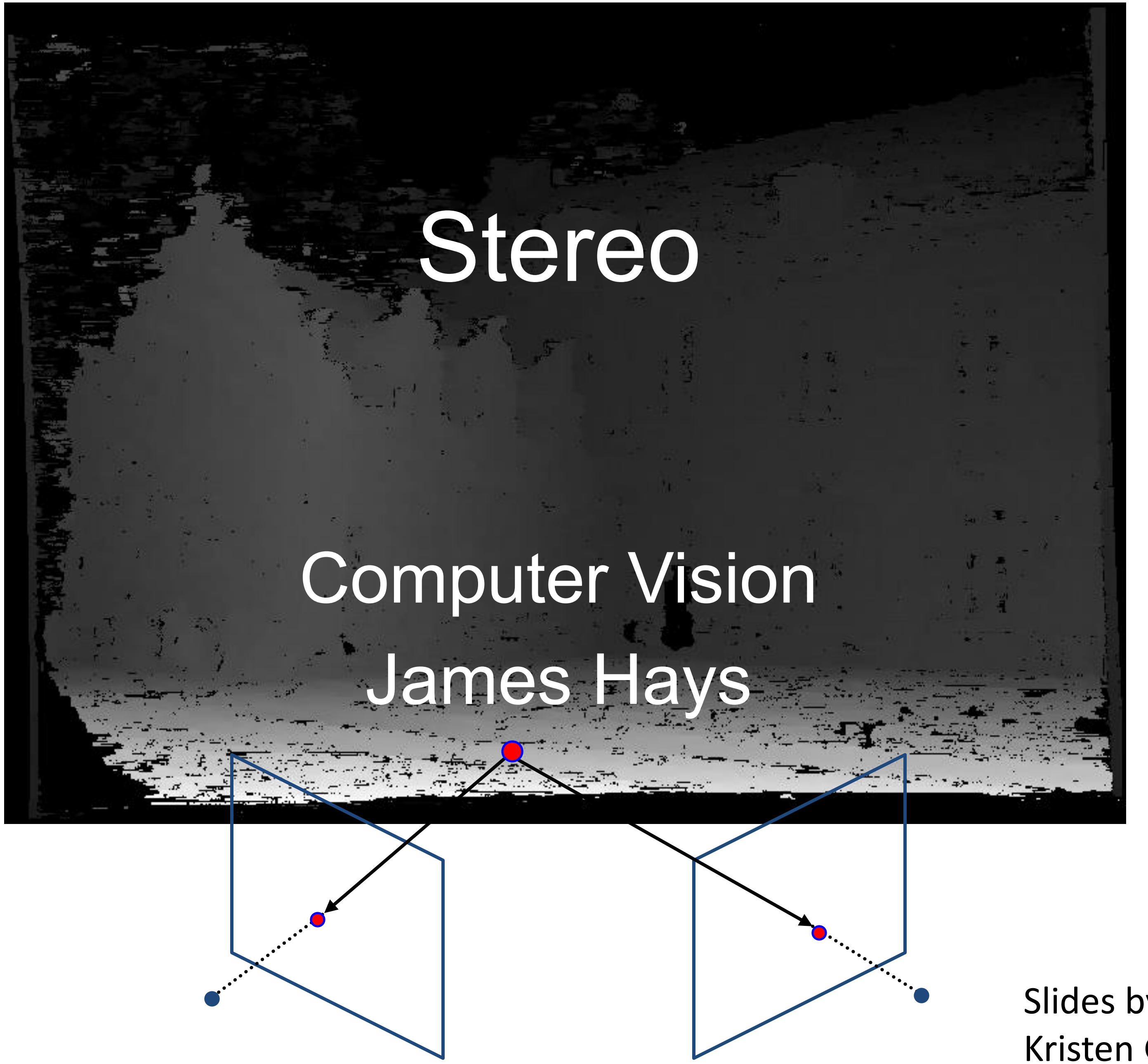
1.5706	-0.1490	0.2598
-1.5282	0.9695	0.3802
-0.6821	1.2856	0.4078
0.4124	-1.0201	-0.0915
1.2095	0.2812	-0.1280
0.8819	-0.8481	0.5255
-0.9442	-1.1583	-0.3759
0.0415	1.3445	0.3240
-0.7975	0.3017	-0.0826
-0.4329	-1.4151	-0.2774
-1.1475	-0.0772	-0.2667
-0.5149	-1.1784	-0.1401
0.1993	-0.2854	-0.2114
-0.4320	0.2143	-0.1053
-0.7481	-0.3840	-0.2408
0.8078	-0.1196	-0.2631
-0.7605	-0.5792	-0.1936
0.3237	0.7970	0.2170
1.3089	0.5786	-0.1887
1.2323	1.4421	0.4506



Stereo

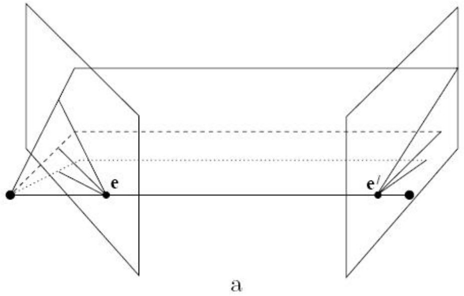
Computer Vision

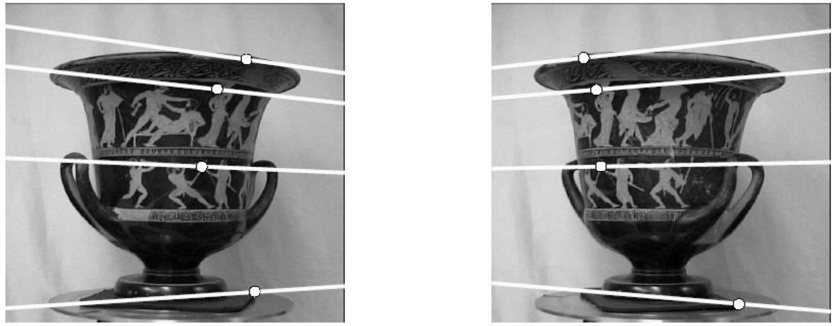
James Hays



Slides by
Kristen Grauman

Multiple views





Hartley and Zisserman

stereo vision
structure from motion
optical flow (later in course)

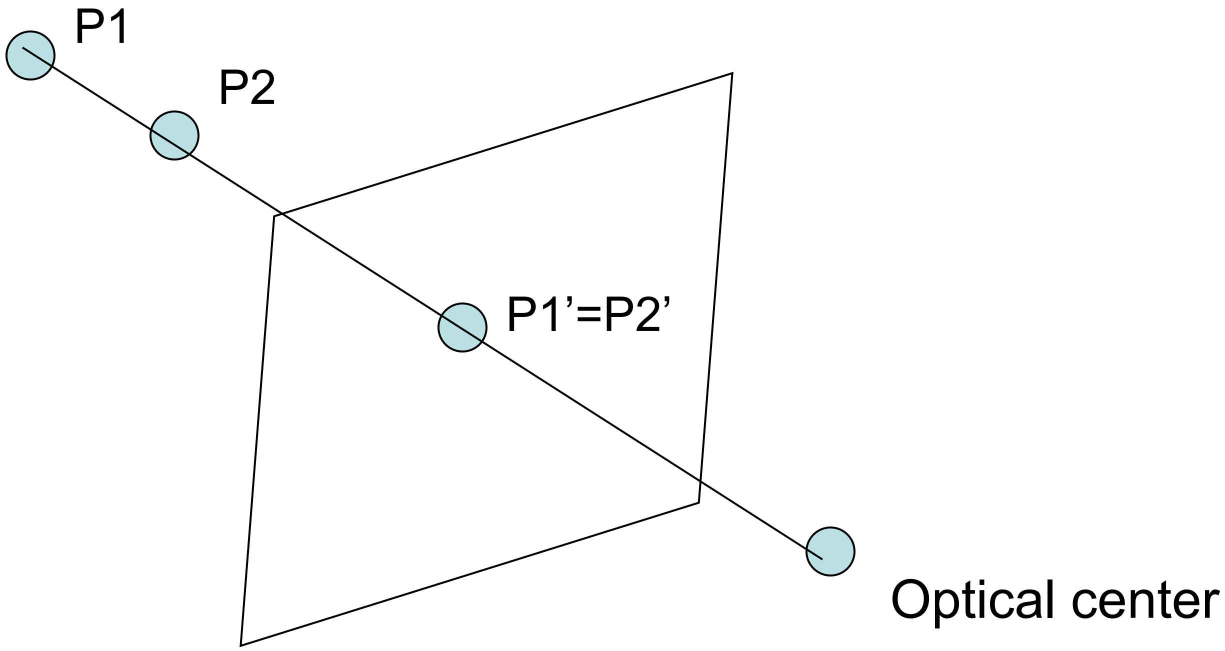
Why multiple views?

- Structure and depth are inherently ambiguous from single views.

Images from Lana Lazebnik

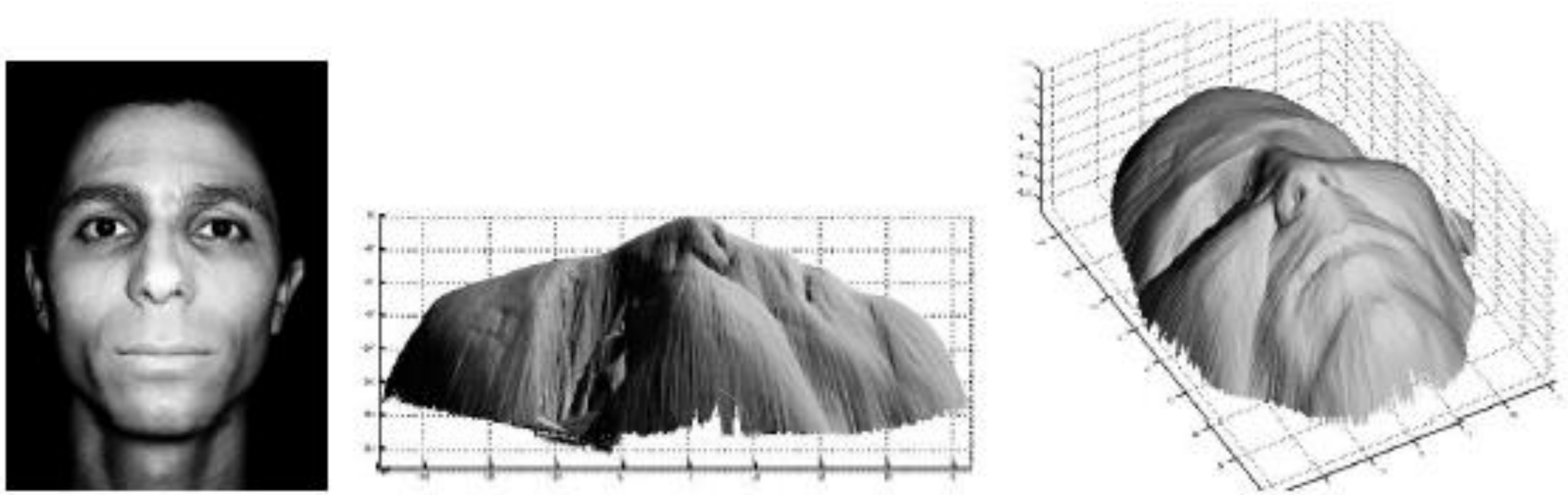
Why multiple views?

- Structure and depth are inherently ambiguous from single views.



- What cues help us to perceive 3d shape and depth?

Shading



a) b) c)

[Figure from Prados & Faugeras 2006]

Shading from multiple light sources: Photometric stereo

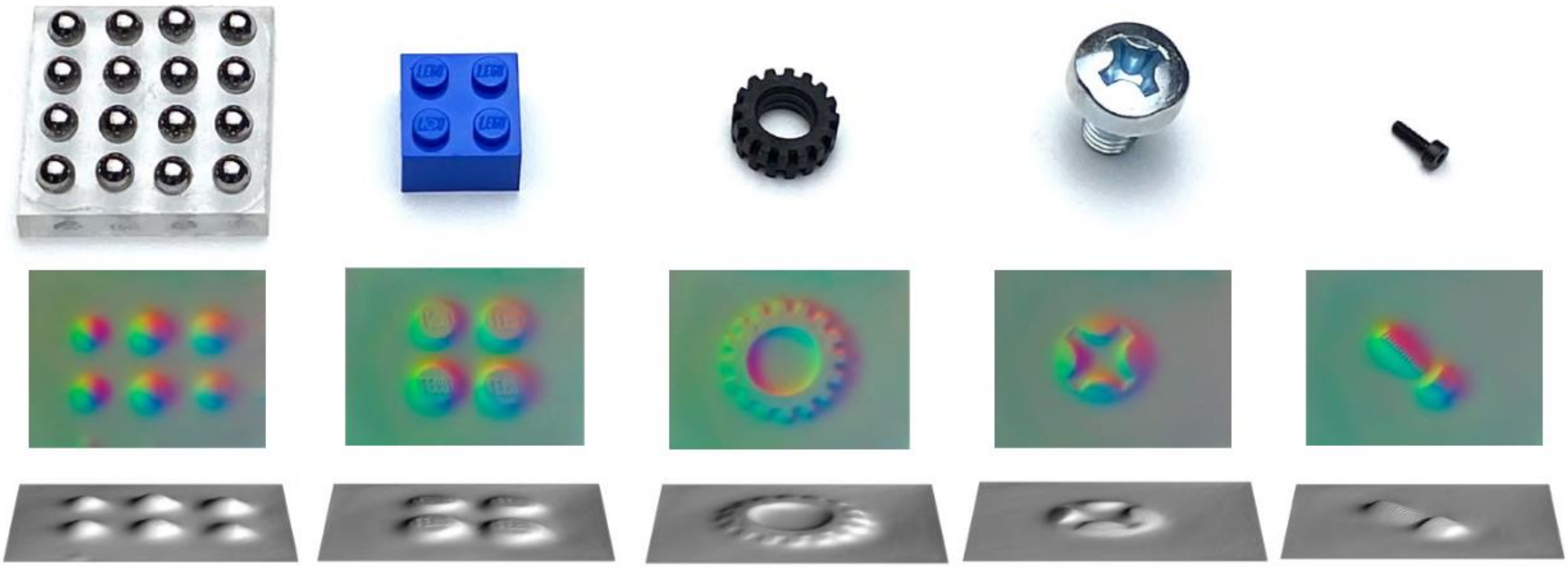
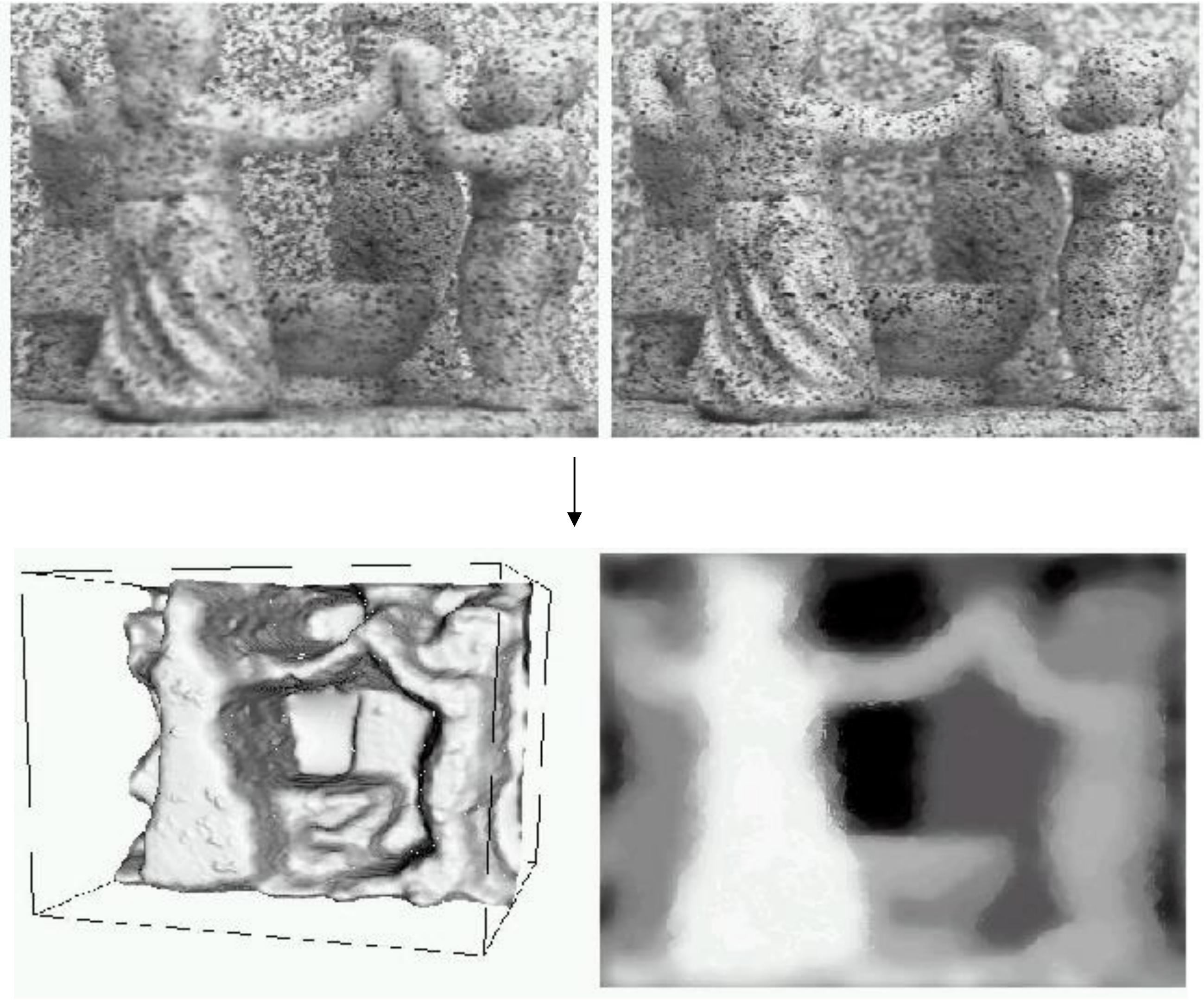


Fig. 7. From top row to bottom: visual images, GelSight imprints, and inferred depth of a ball array, a Lego block, a rubber tyre, a screw cap, a M2 screw.

Focus/defocus

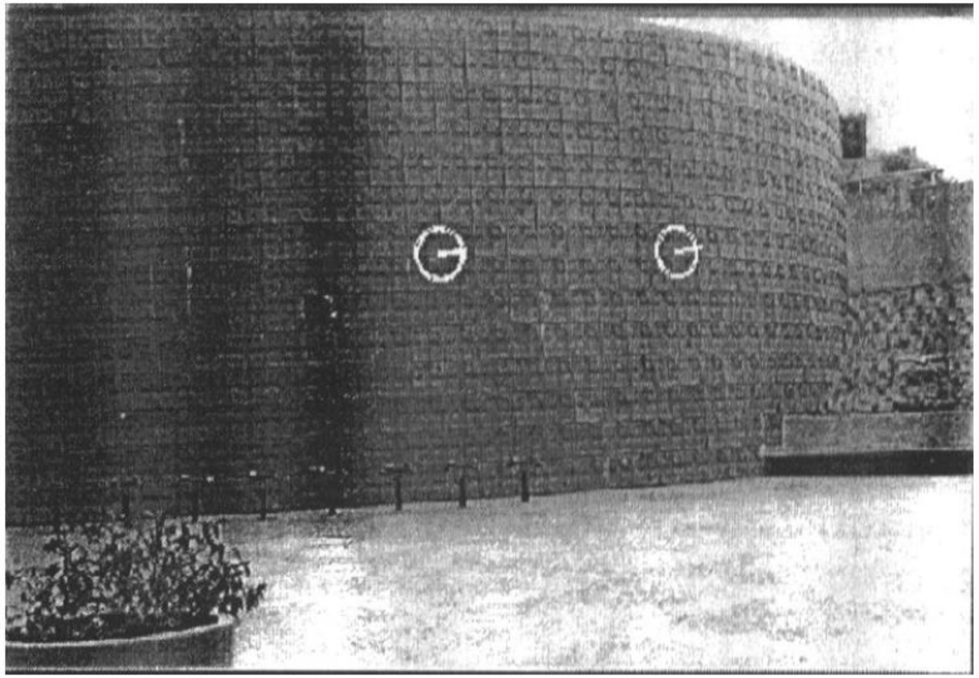


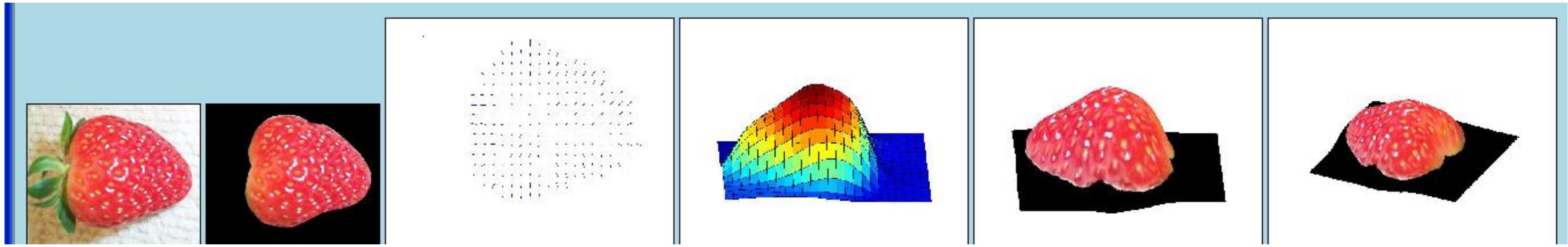
Images from same point of view, different camera parameters

3d shape / depth estimates

[figs from H. Jin and P. Favaro, 2002]

Texture



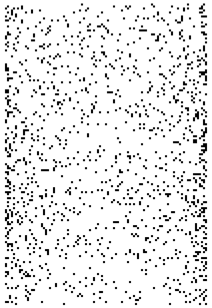


[From A.M. Loh. The recovery of 3-D structure using visual texture patterns. PhD thesis]

Perspective effects

Image credit: S. Seitz

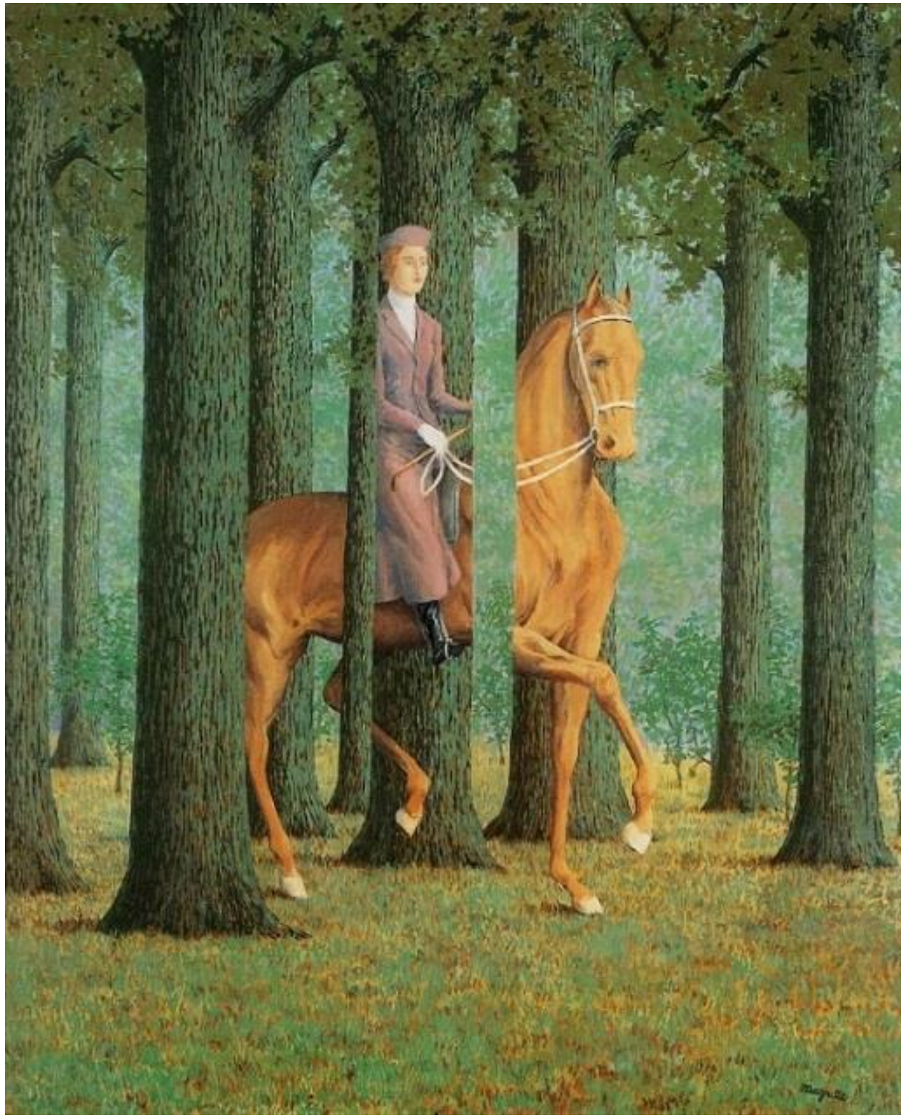
Motion



Figures from L. Zhang

http://www.brainconnection.com/teasers/?main=illusion/motion-shape

Occlusion

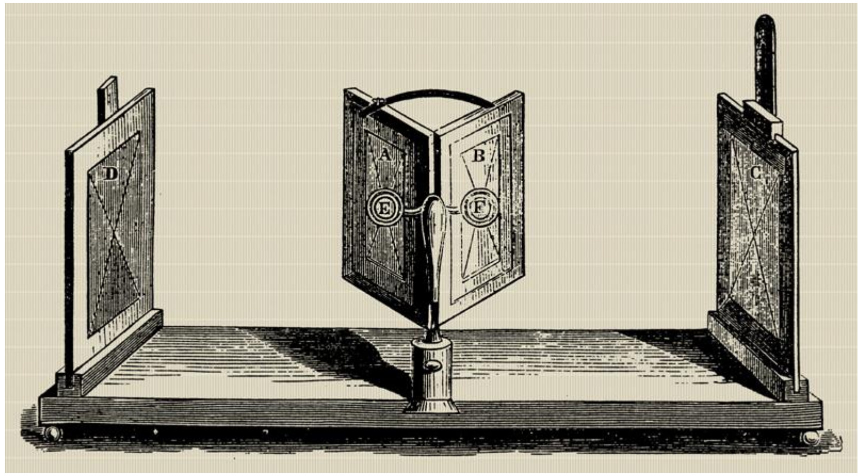


Rene Magritt'e famous painting *Le Blanc-Seing* (literal translation: "The Blank Signature") roughly translates as "free hand“. 1965

If stereo were critical for depth perception, navigation, recognition, etc., then this would be a problem

Stereo photography and stereo viewers

Take two pictures of the same subject from two slightly different viewpoints and display so that each eye sees only one of the images.



Invented by Sir Charles Wheatstone, 1838

Image from fisher-price.com

http://www.johnsonshawmuseum.org

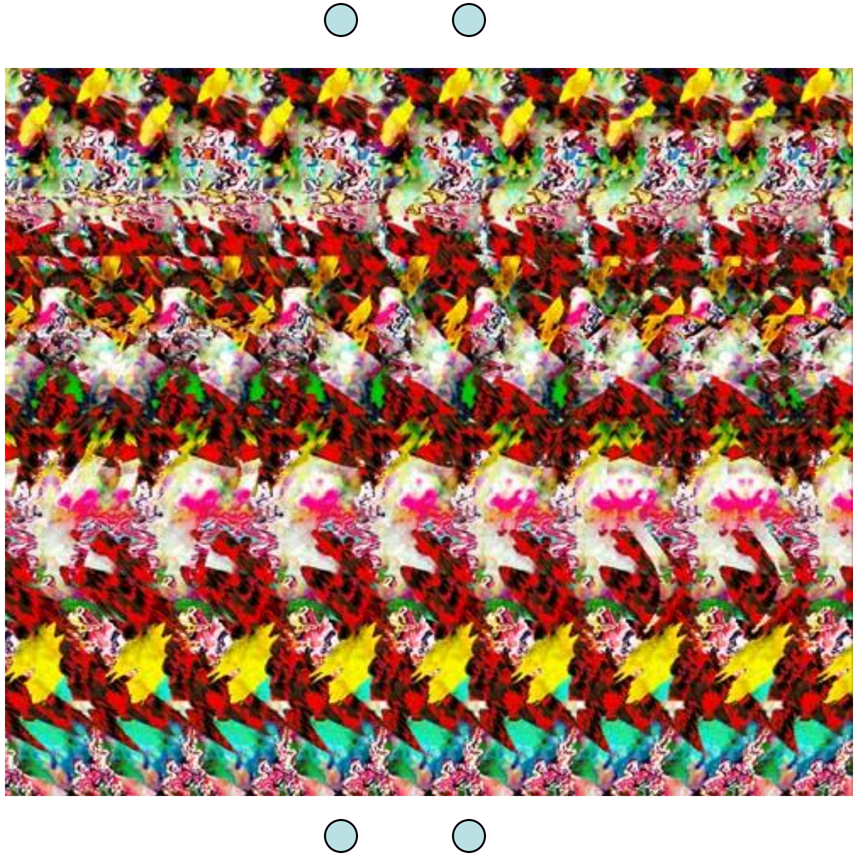
http://www.johnsonshawmuseum.org

Public Library, Stereoscopic Looking Room, Chicago, by Phillips, 1923

http://www.well.com/~jimg/stereo/stereo_list.html

http://www.well.com/~jimg/stereo/stereo_list.html

Autostereograms



Exploit disparity as depth cue using single image.

(Single image random dot stereogram, Single image stereogram)

Images from magiceye.com

Autostereograms

Images from magiceye.com

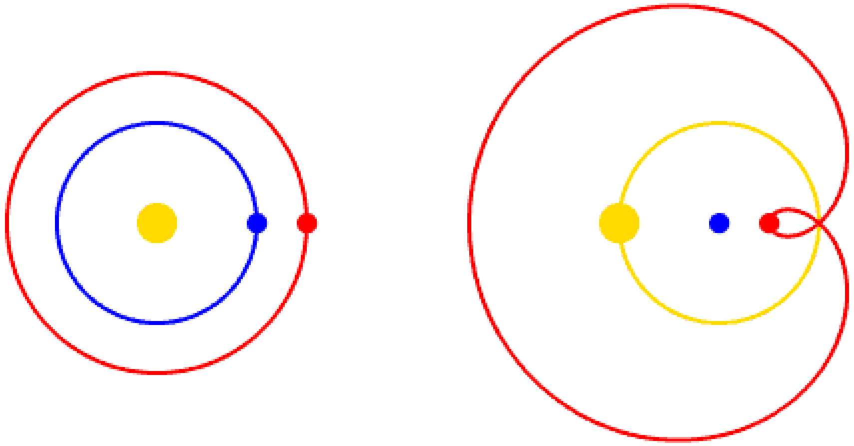
Parallax and our universe

Look again at that dot. That's here. That's home. That's us. On it everyone you love, everyone you know, everyone you ever heard of, every human being who ever was, lived out their lives. The aggregate of our joy and suffering, thousands of confident religions, ideologies, and economic doctrines, every hunter and forager, every hero and coward, every creator and destroyer of civilization, every king and peasant, every young couple in love, every mother and father, hopeful child, inventor and explorer, every teacher of morals, every corrupt politician, every "superstar," every "supreme leader," every saint and sinner in the history of our species lived there--on a mote of dust suspended in a sunbeam.

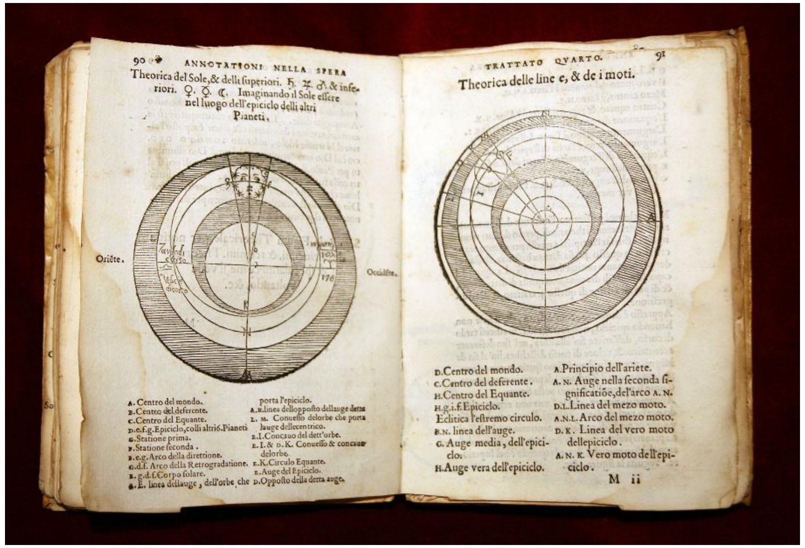
— Carl Sagan

https://en.wikipedia.org/wiki/Pale_Blue_Dot

Nicolaus Copernicus



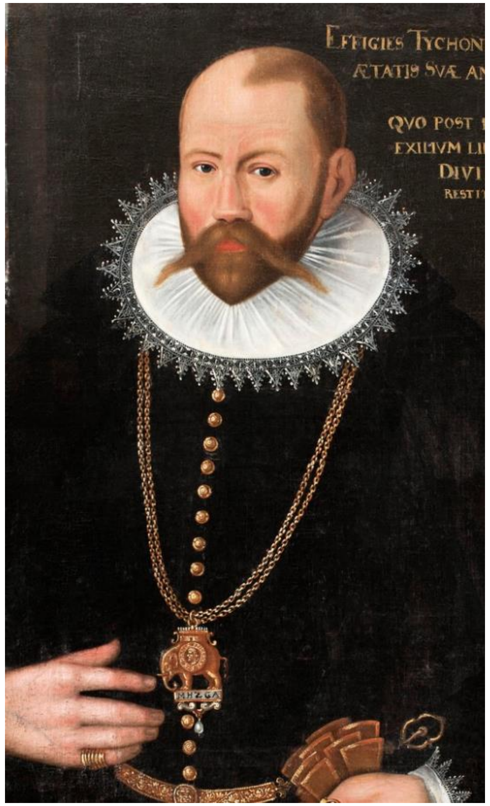
Motion of Sun (yellow), Earth (blue), and Mars (red). At left, Copernicus' heliocentric motion. At right, traditional geocentric motion, including the retrograde motion of Mars.



geocentric model (often exemplified specifically by the **Ptolemaic system**)

https://en.wikipedia.org/wiki/Heliocentrism

If the apparent motion of the planets is caused by parallax, why aren't we seeing parallax for stars?



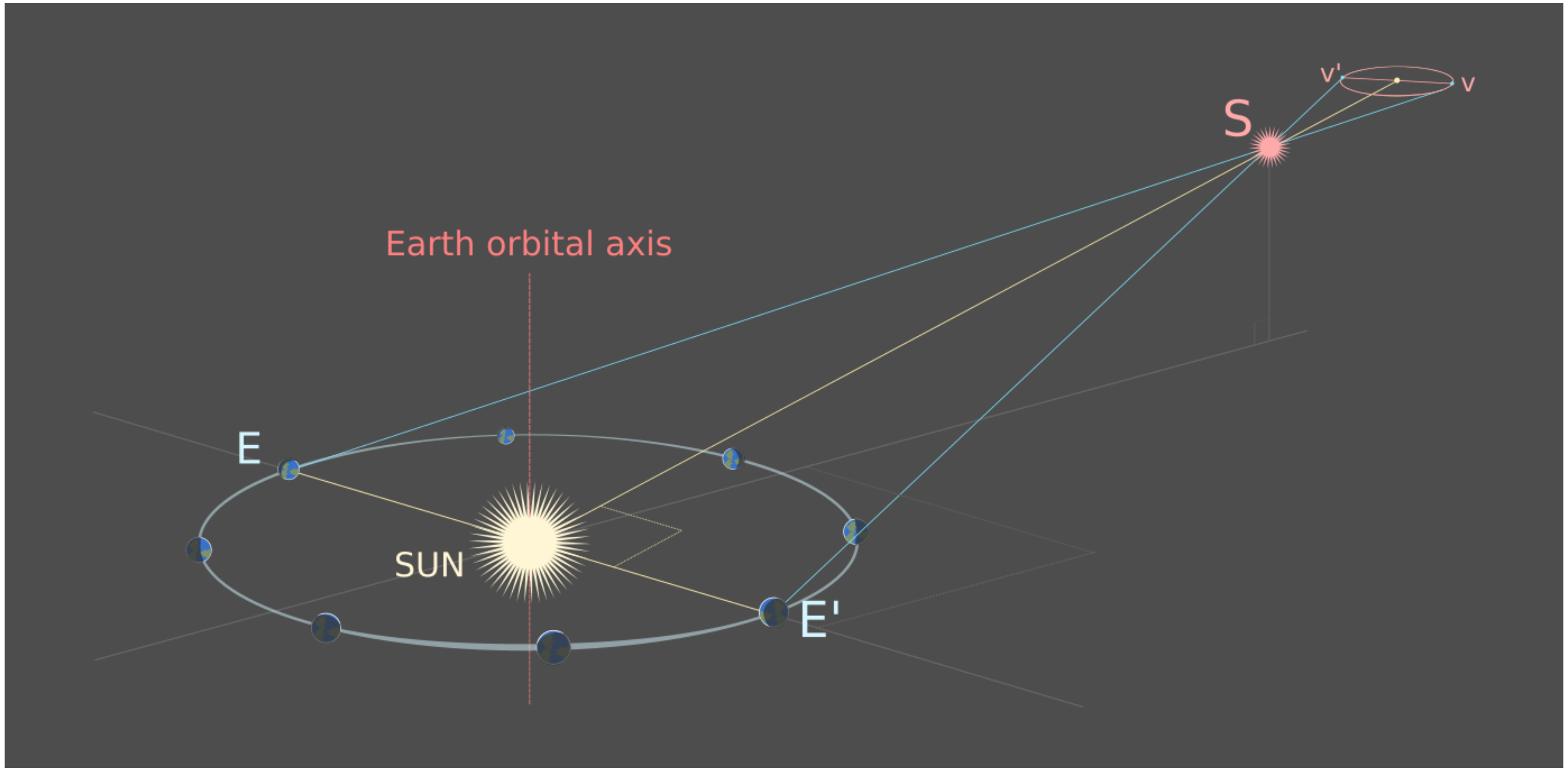
Tycho Brahe

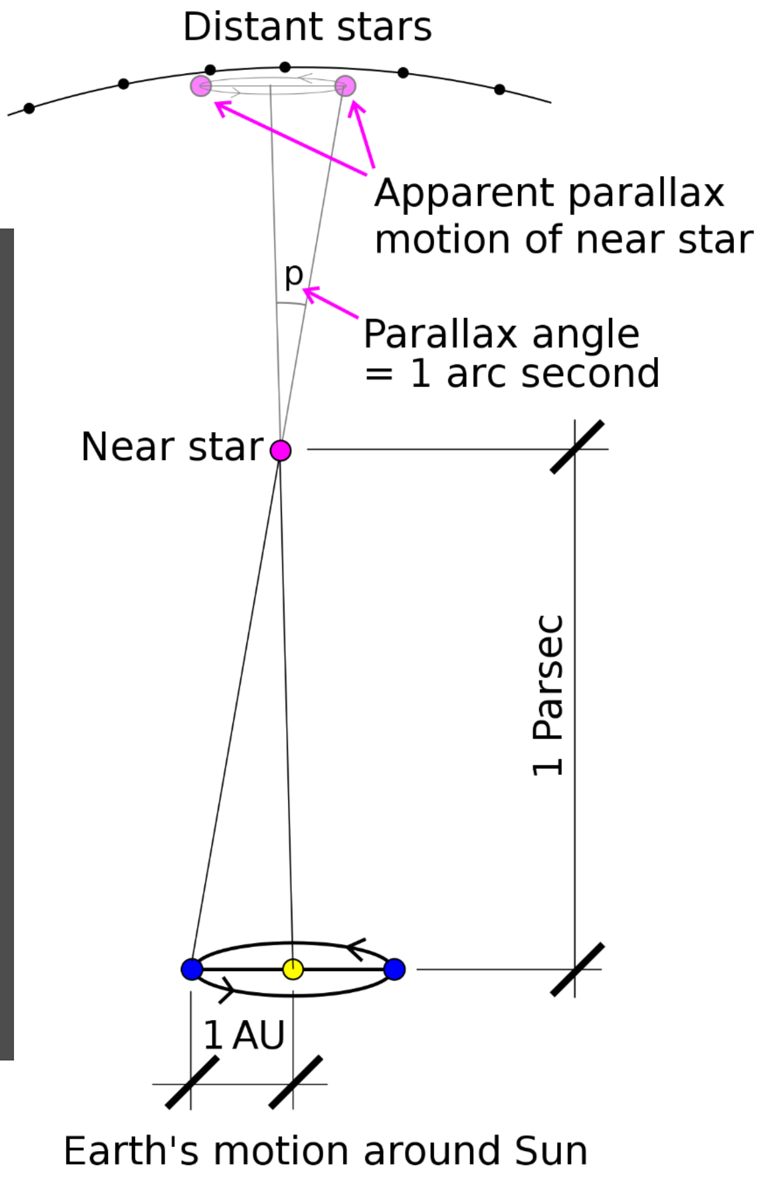
It was one of Tycho Brahe's principal objections to Copernican heliocentrism that for it to be compatible with the lack of observable stellar parallax, there would have to be an enormous and unlikely void between the orbit of Saturn and the eighth sphere (the fixed stars).

Saturn is about 0.000126 light years away. Proxima Centauri is 4.24 light years away.

The angles involved in these calculations are very small and thus difficult to measure. The nearest star to the Sun (and also the star with the largest parallax), Proxima Centauri, has a parallax of 0.7685 ± 0.0002 arcsec.[1] This angle is approximately that subtended by an object 2 centimeters in diameter located 5.3 kilometers away. First reliable measurements of parallax were not made until 1838, by Friedrich Bessel

https://en.wikipedia.org/wiki/Stellar_parallax





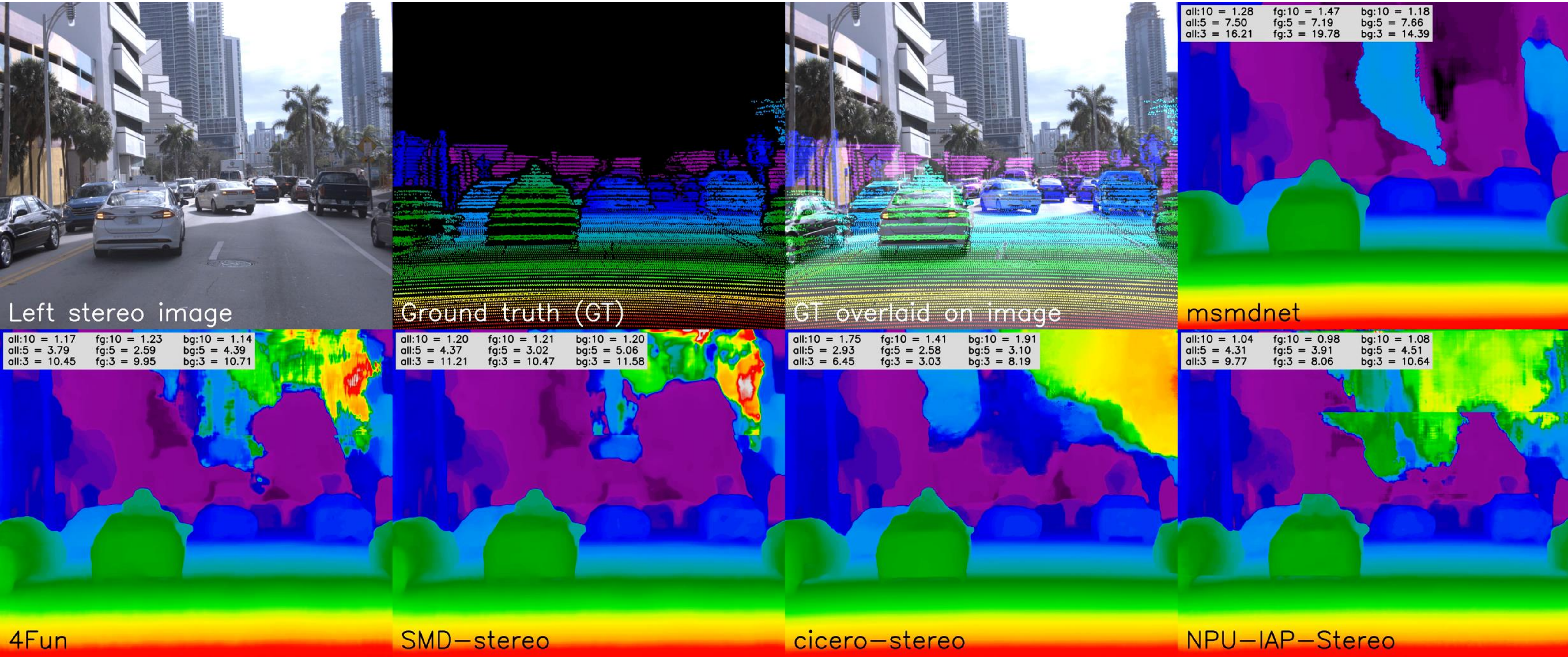
https://en.wikipedia.org/wiki/Stellar_parallax

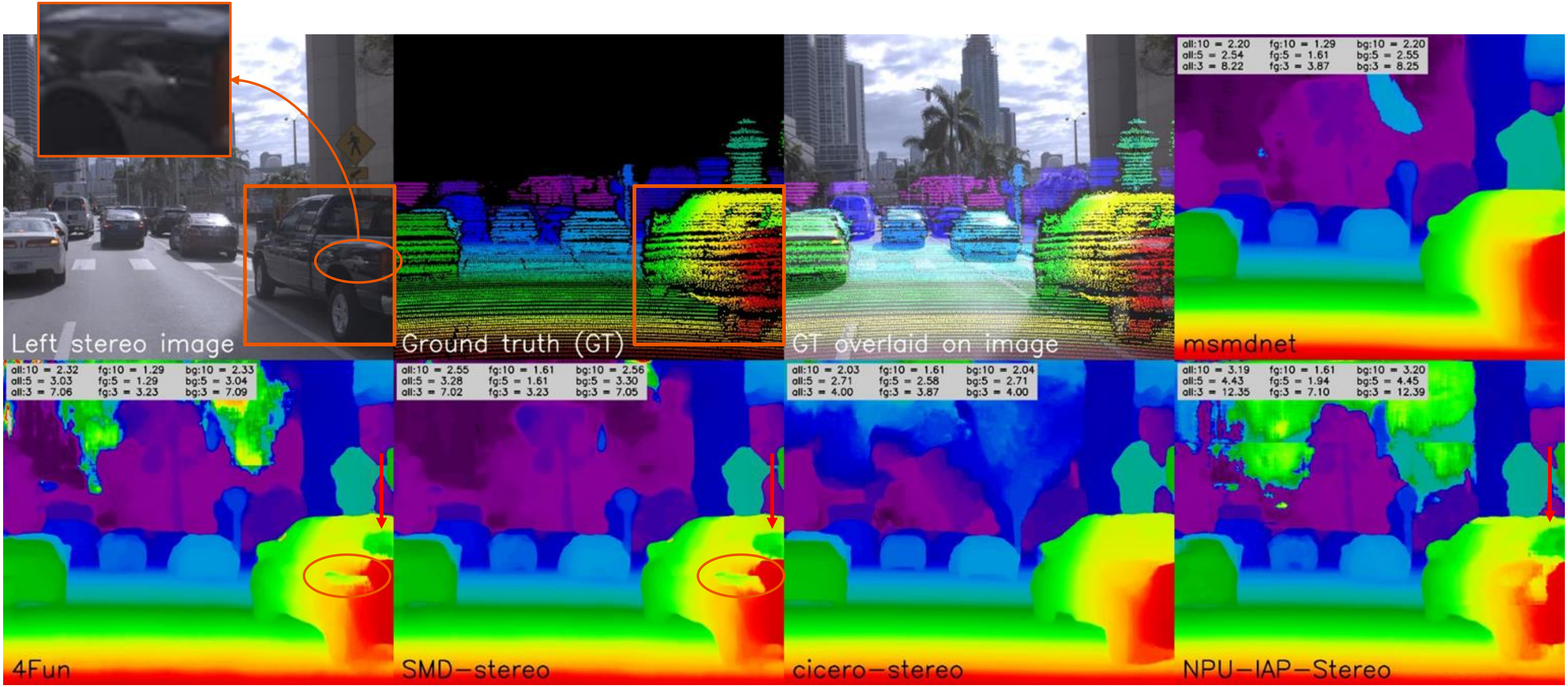
Stereo vision

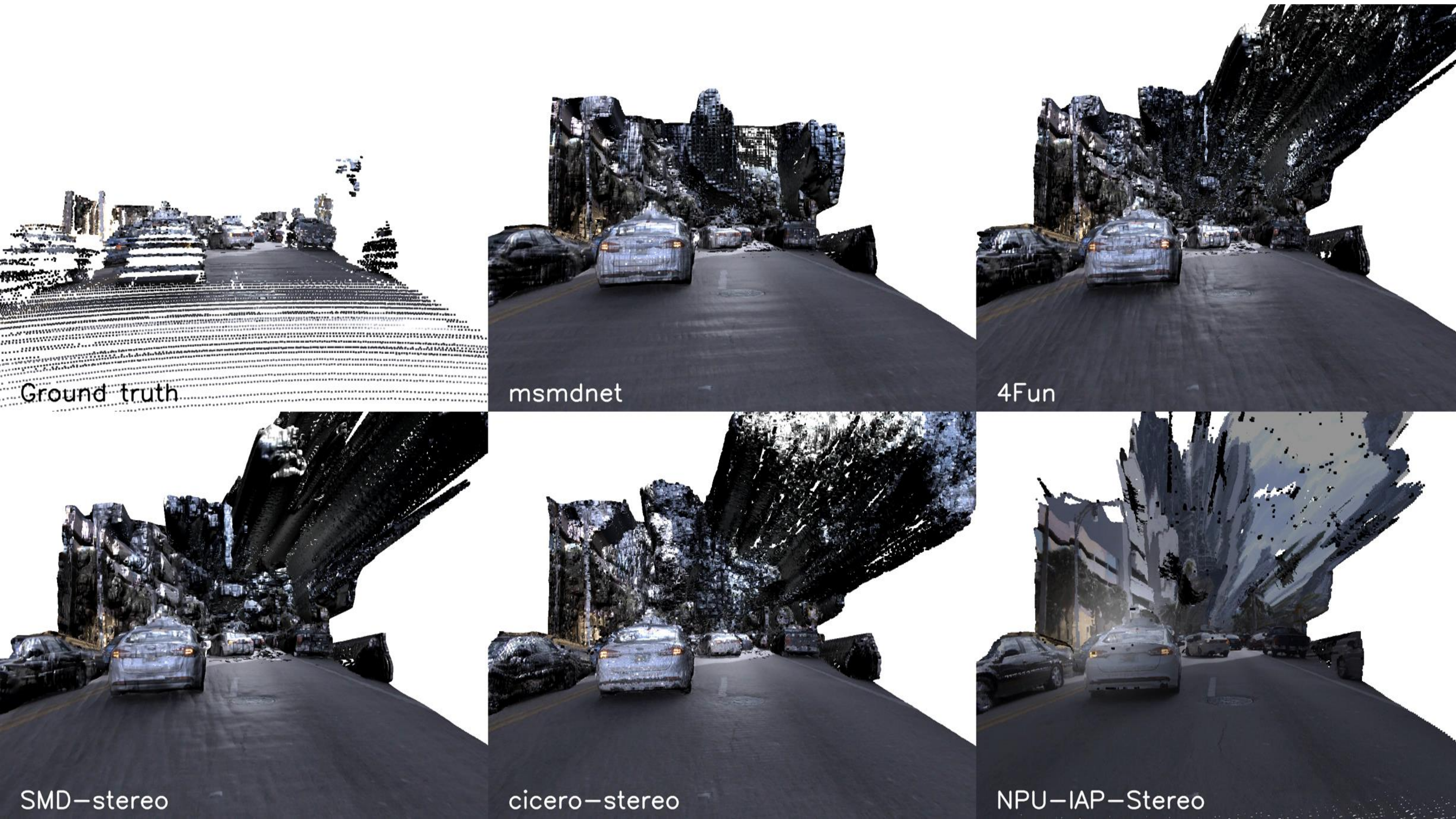
Two cameras, simultaneous views

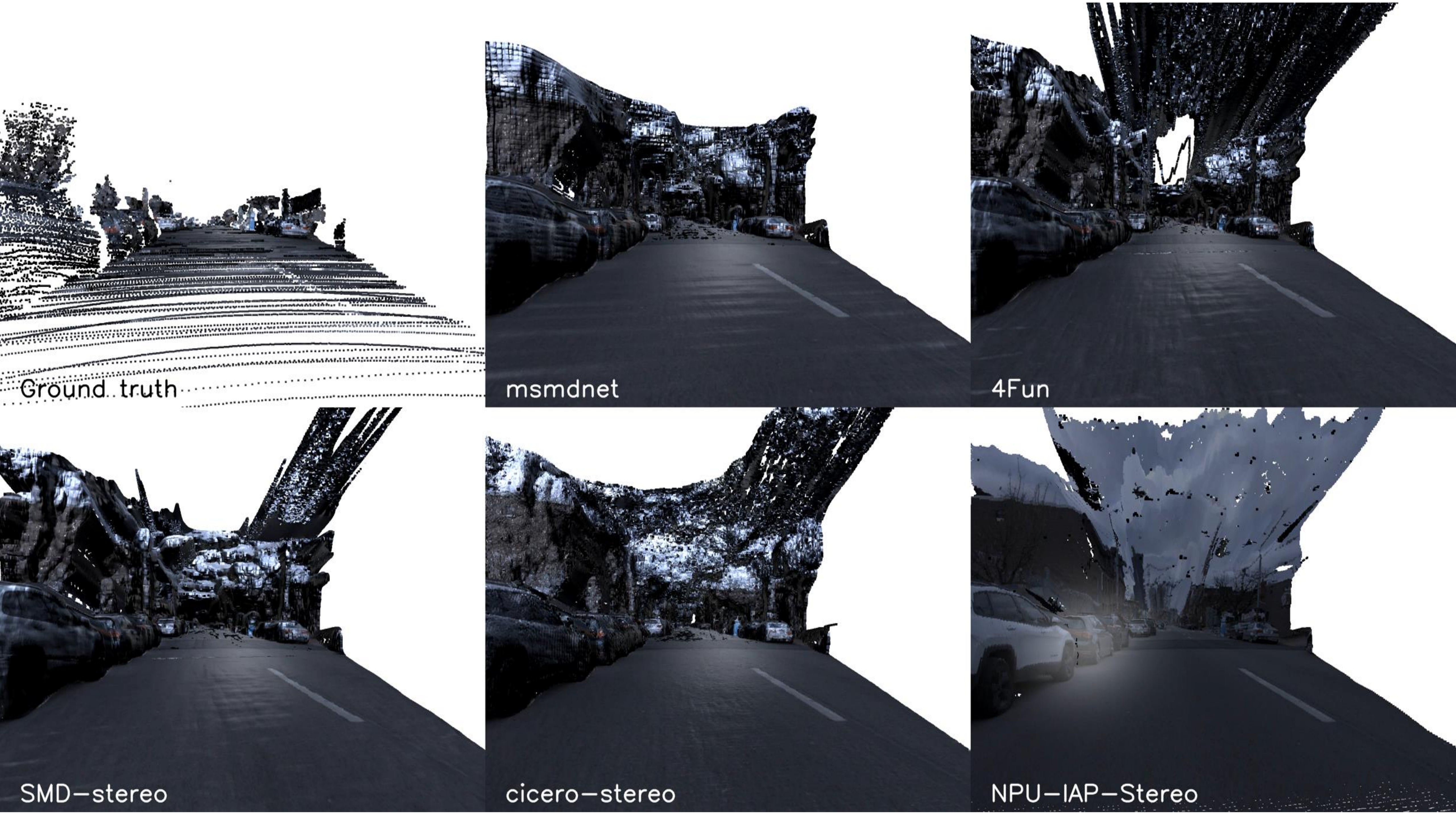
Single moving camera and static scene

Modern stereo depth estimation example



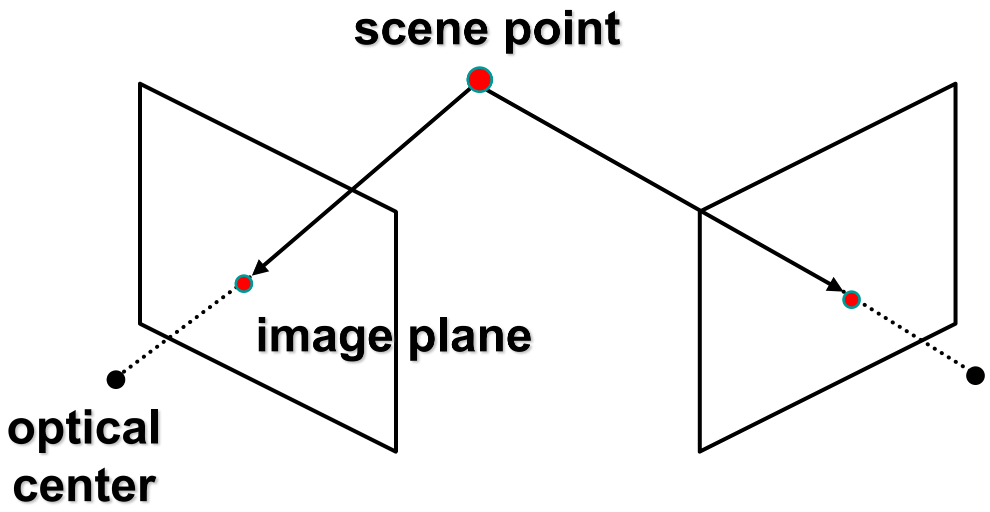

Left stereo image
Ground truth (GT)
GT overlaid on image
all:10 = 2.20 fg:10 = 1.29 bg:10 = 2.20
all:5 = 2.54 fg:5 = 1.61 bg:5 = 2.55
all:3 = 8.22 fg:3 = 3.87 bg:3 = 8.25
msmdnet
all:10 = 2.32 fg:10 = 1.29 bg:10 = 2.33
all:5 = 3.03 fg:5 = 1.29 bg:5 = 3.04
all:3 = 7.06 fg:3 = 3.23 bg:3 = 7.09
4Fun
all:10 = 2.55 fg:10 = 1.61 bg:10 = 2.56
all:5 = 3.28 fg:5 = 1.61 bg:5 = 3.30
all:3 = 7.02 fg:3 = 3.23 bg:3 = 7.05
SMD−stereo
all:10 = 2.03 fg:10 = 1.61 bg:10 = 2.04
all:5 = 2.71 fg:5 = 2.58 bg:5 = 2.71
all:3 = 4.00 fg:3 = 3.87 bg:3 = 4.00
cicero−stereo
all:10 = 3.19 fg:10 = 1.61 bg:10 = 3.20
all:5 = 4.43 fg:5 = 1.94 bg:5 = 4.45
all:3 = 12.35 fg:3 = 7.10 bg:3 = 12.39
NPU−IAP−Stereo

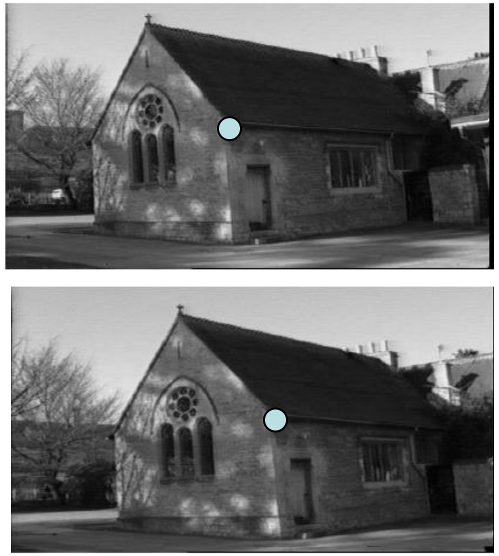

Ground truth
msmdnet
4Fun
SMD-stereo
cicero-stereo
NPU-IAP-Stereo


Ground truth
msmdnet
4Fun
SMD-stereo
cicero-stereo
NPU-IAP-Stereo

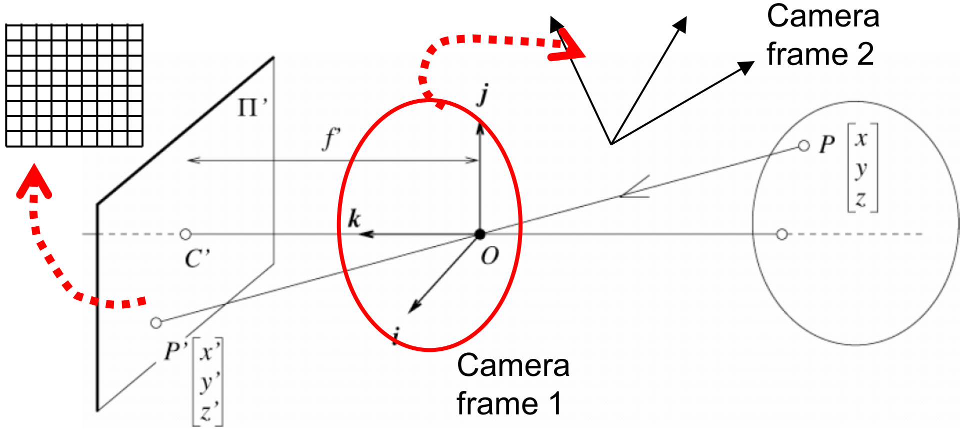
Estimating depth with stereo

- **Stereo**: shape from “motion” between two views
- We’ll need to consider:
  - Info on camera pose (“calibration”)
  - Image point correspondences





Camera parameters



Extrinsic parameters:
Camera frame 1 ←→ Camera frame 2

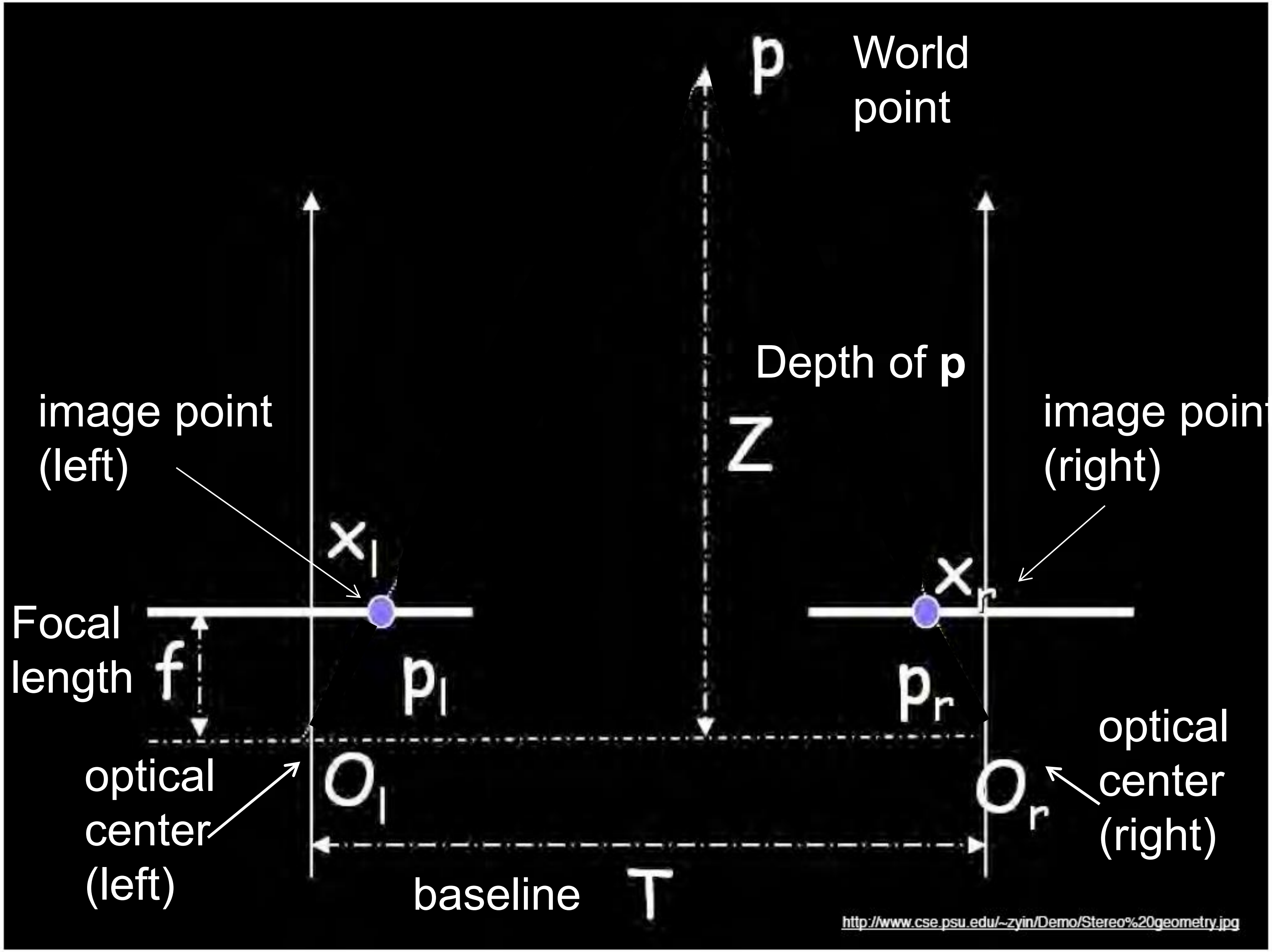
Intrinsic parameters:
Image coordinates relative to camera ←→ Pixel coordinates

- *Extrinsic* params: rotation matrix and translation vector
- *Intrinsic* params: focal length, pixel sizes (mm), image center point, radial distortion parameters

We'll assume for now that these parameters are given and fixed.

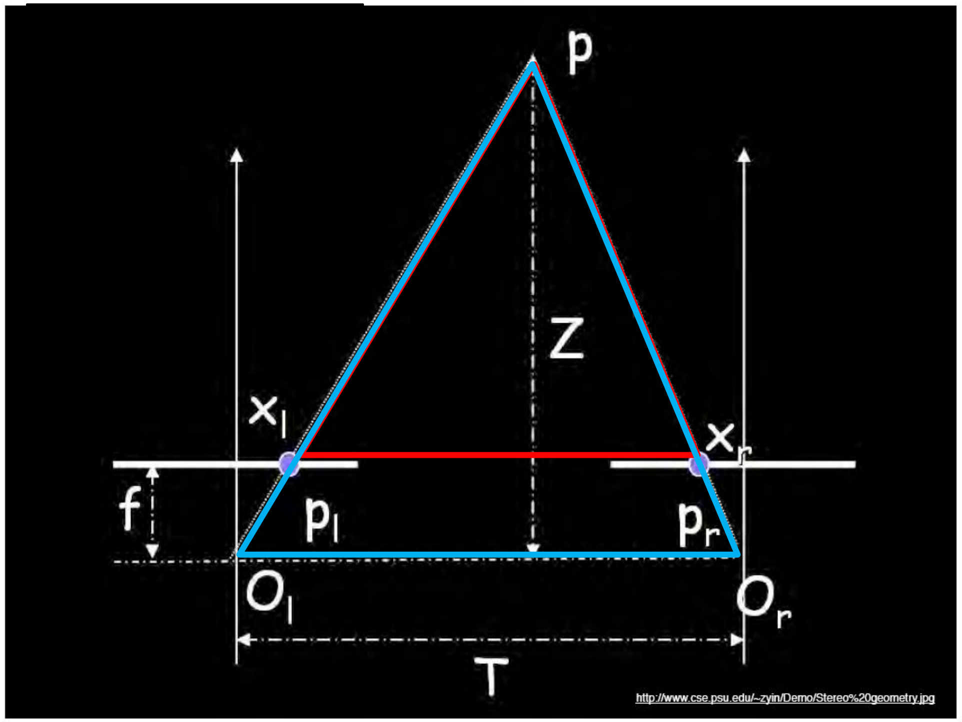
Geometry for a simple stereo system

- First, assuming parallel optical axes, known camera parameters (i.e., calibrated cameras):


p
World point
Depth of **p**
Z
image point (left)
image point (right)
x_l
x_r
Focal length
f
p_l
p_r
optical center (left)
O_l
O_r
optical center (right)
baseline T
http://www.cse.psu.edu/~zyin/Demo/Stereo%20geometry.jpg

Geometry for a simple stereo system

- Assume parallel optical axes, known camera parameters (i.e., calibrated cameras). **What is expression for Z?**



Similar triangles (p_l, P, p_r) and (O_l, P, O_r):

$$\frac{T - x_l + x_r}{Z - f} = \frac{T}{Z}$$

$$Z = f \frac{T}{x_l - x_r}$$

disparity

Depth from disparity

image I(x,y)

Disparity map D(x,y)

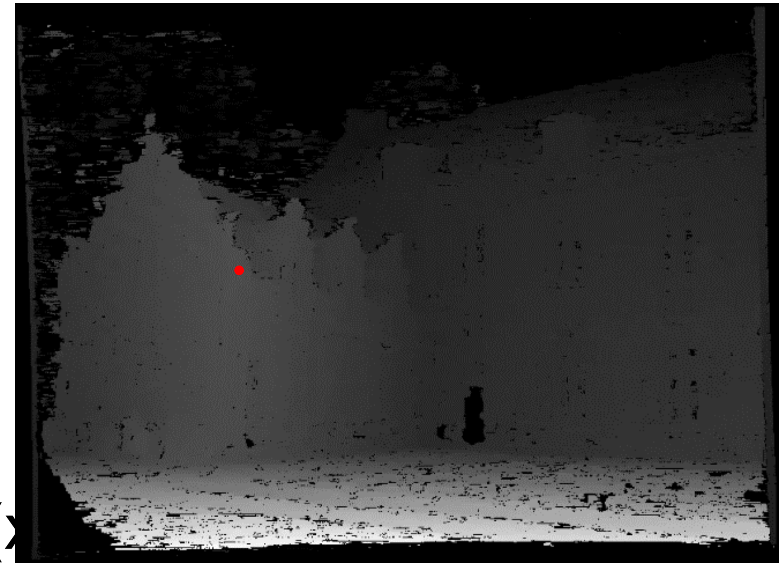
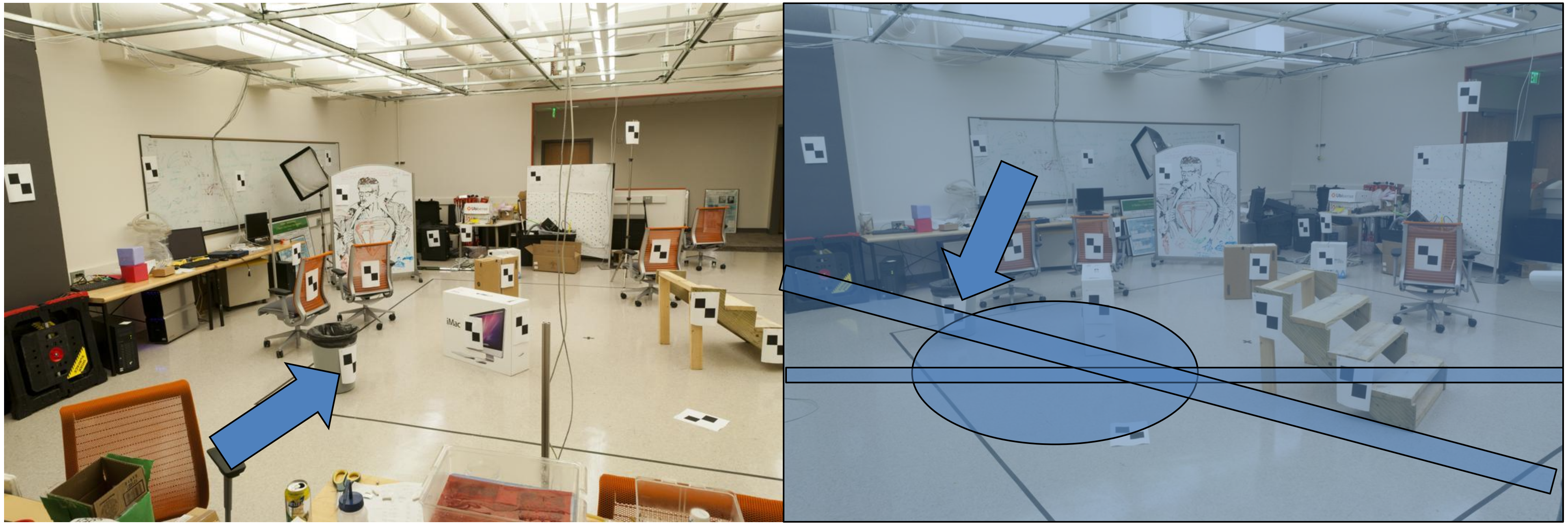


image I´(x´,y´)

So if we could find the **corresponding points** in two images, we could **estimate relative depth**…

If we have a 2D point of interest, where do we need to search for its corresponding point in another view?



- Epipolar Geometry
  - Finding epipolar relationship between two images
  - Using epipolar geometry to rule out outliers
  - Finding dense correspondence along epipolar lines

Epipolar Geometry and Stereo Vision

Chapter 11.3 in Szeliski

Many slides adapted from Derek Hoiem, Lana Lazebnik, Silvio Saverese, Steve Seitz, many figures from Hartley & Zisserman

- Epipolar geometry
  - Relates cameras from two positions

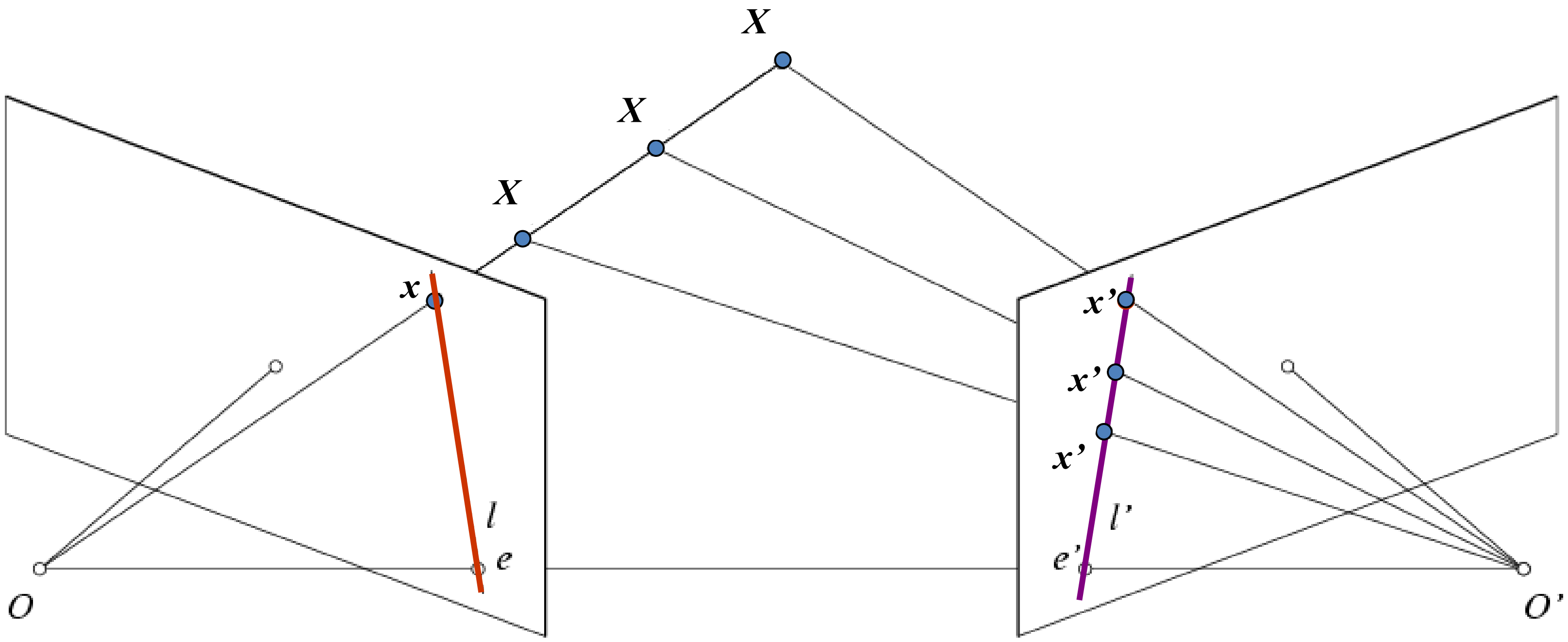
Correspondence Problem

- We have two images taken from cameras with different intrinsic and extrinsic parameters

- How do we match a point in the first image to a point in the second? How can we constrain our search?

Key idea: Epipolar constraint

Key idea: Epipolar constraint

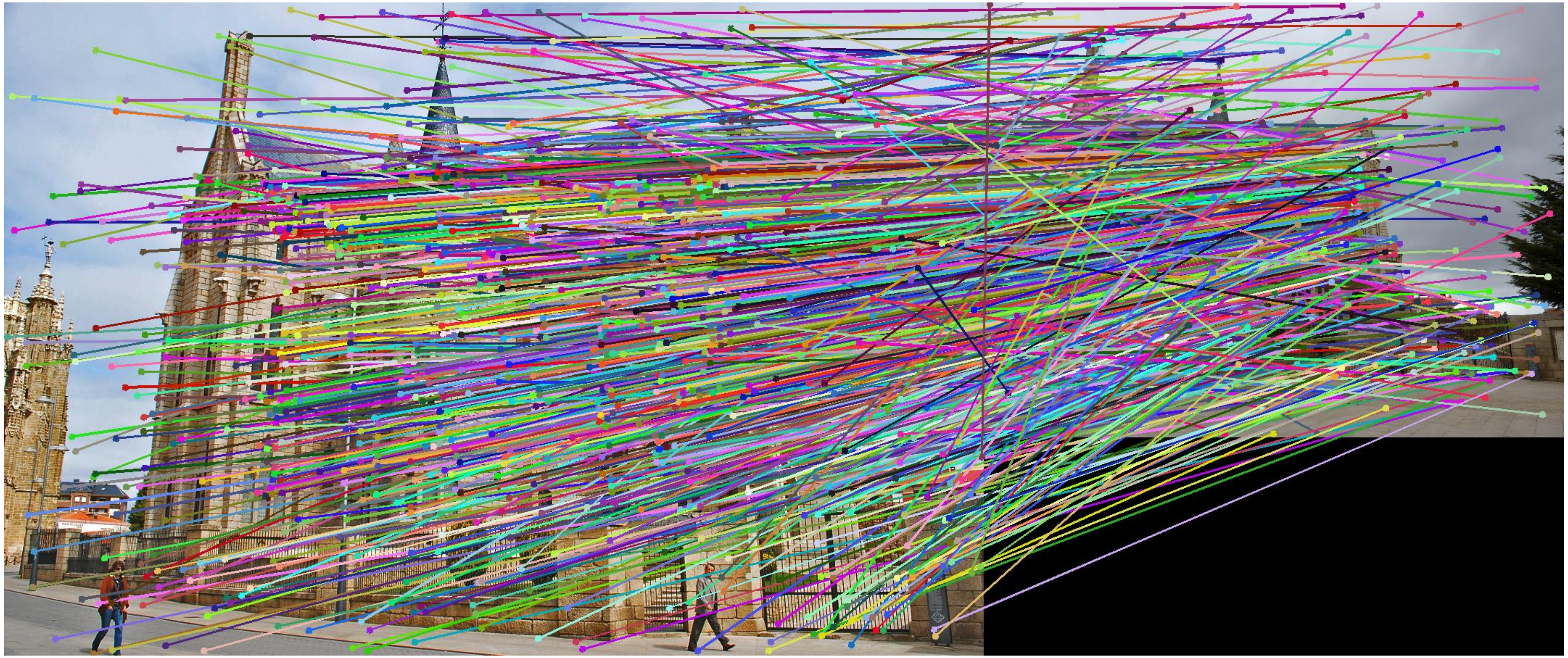


Potential matches for *x* have to lie on the corresponding line *l'*.

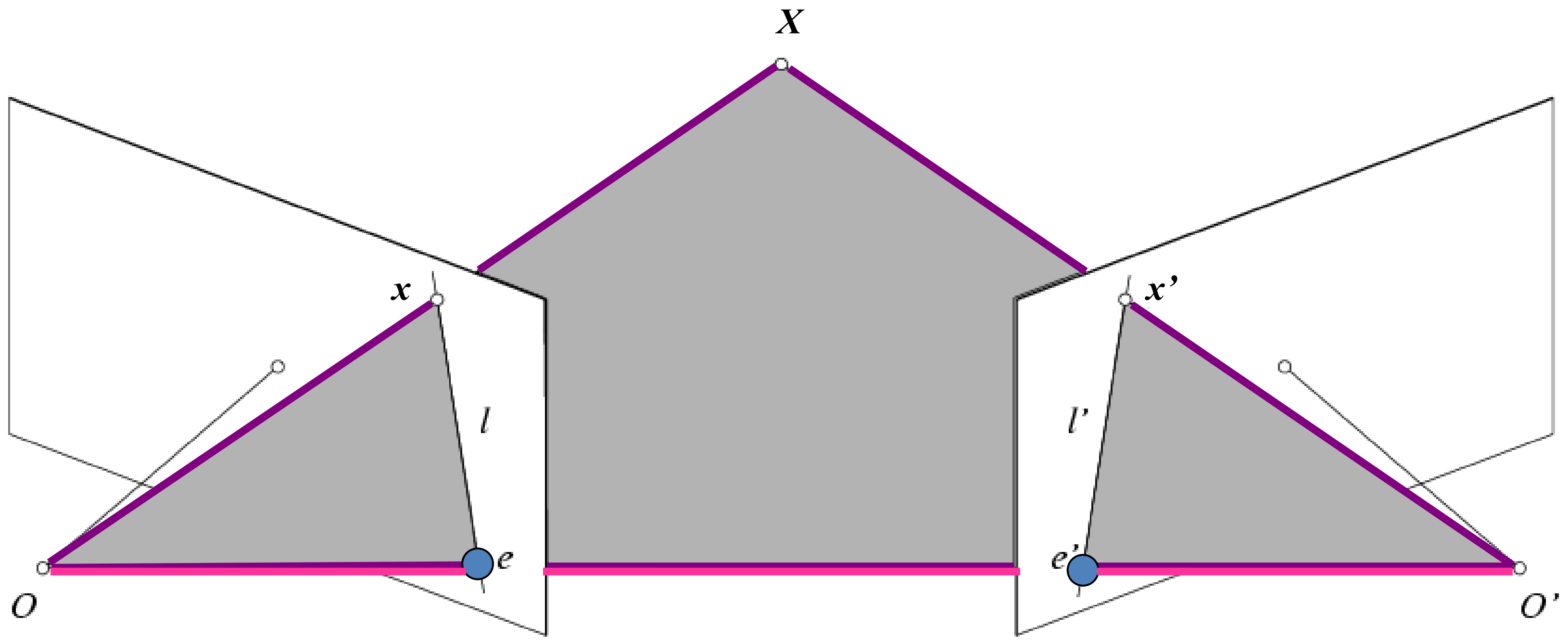
Potential matches for *x'* have to lie on the corresponding line *l*.

Wouldn’t it be nice to know where matches can live? To constrain our 2d search to 1d.

VLFeat's 800 most confident matches among 10,000+ local features.

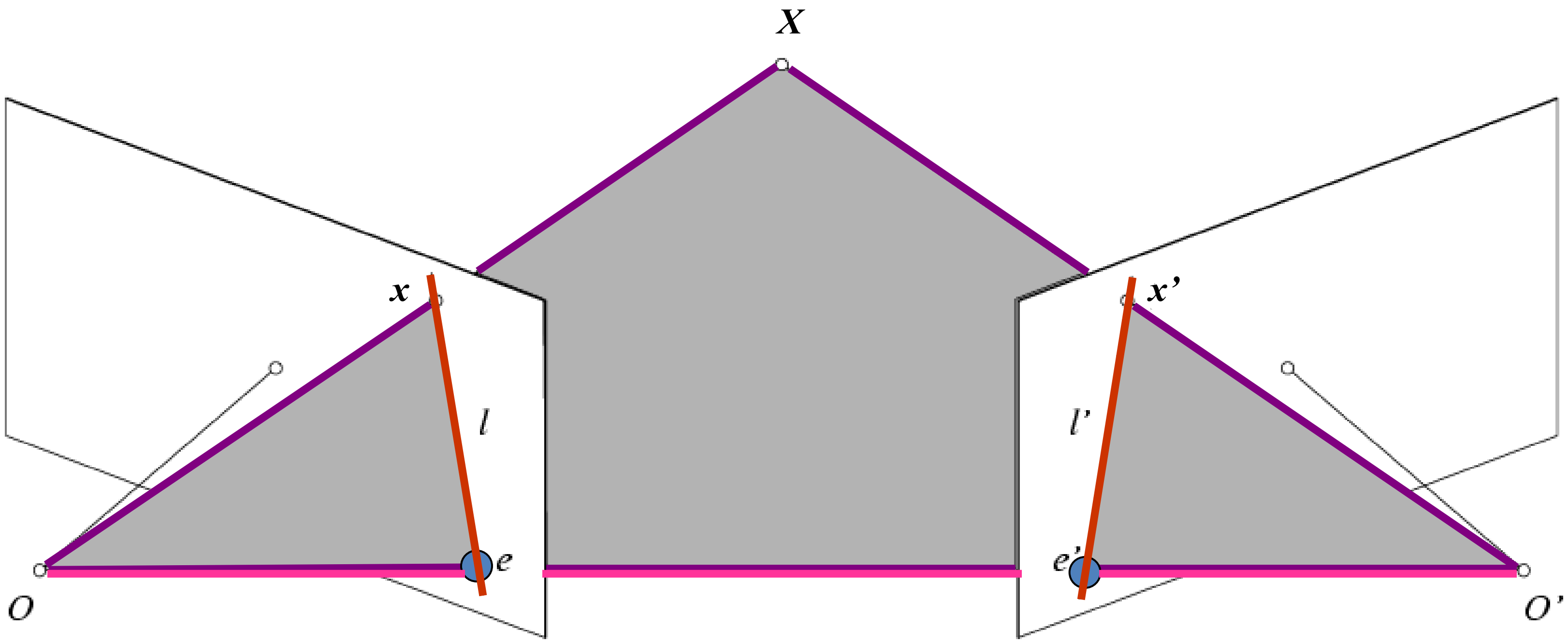


Epipolar geometry: notation



- **Baseline** – line connecting the two camera centers
- **Epipoles**
  = intersections of baseline with image planes
  = projections of the other camera center
- **Epipolar Plane** – plane containing baseline (1D family)

Epipolar geometry: notation

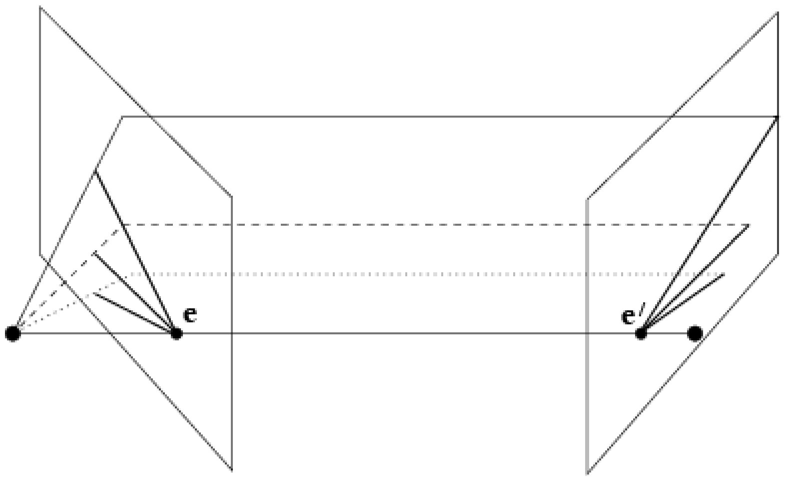


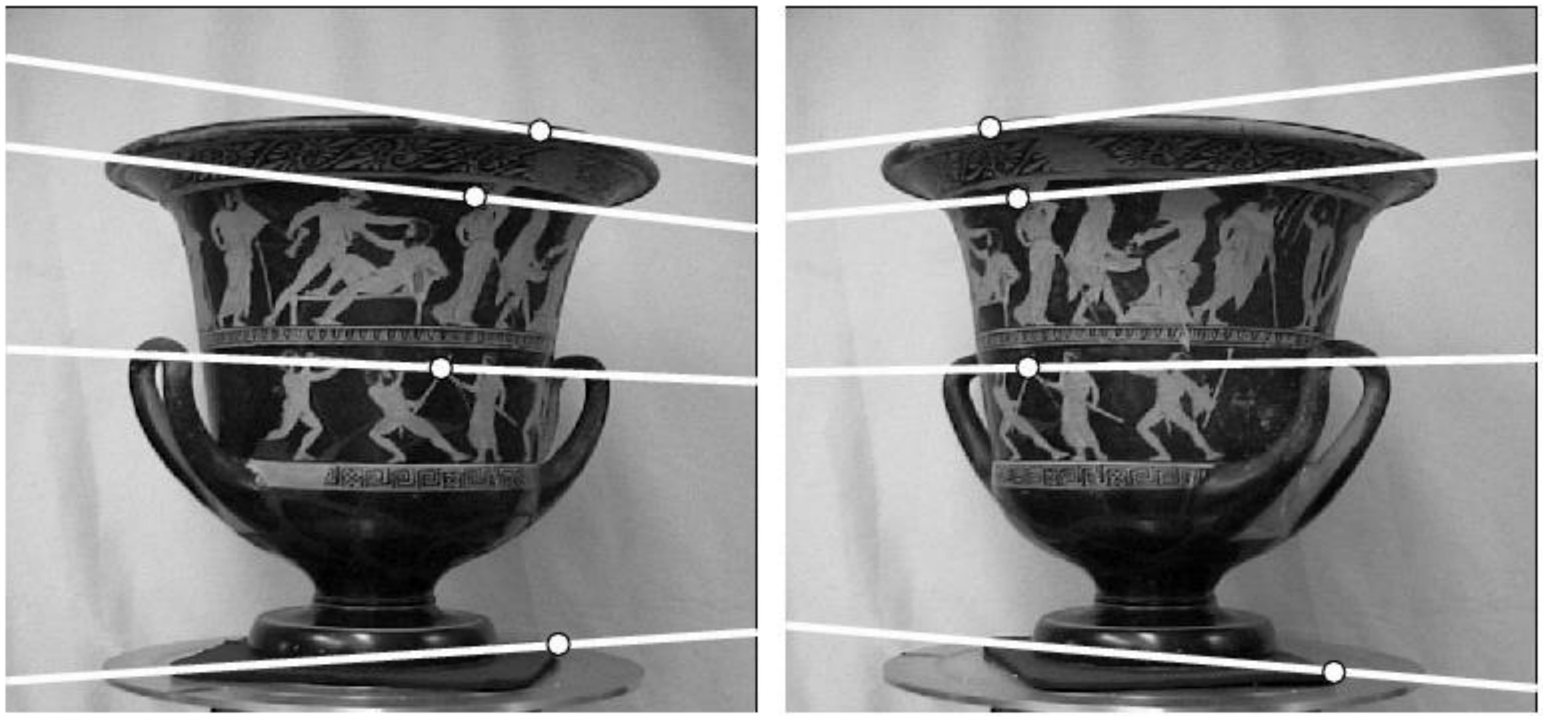
- **Baseline** – line connecting the two camera centers
- **Epipoles**

  = intersections of baseline with image planes

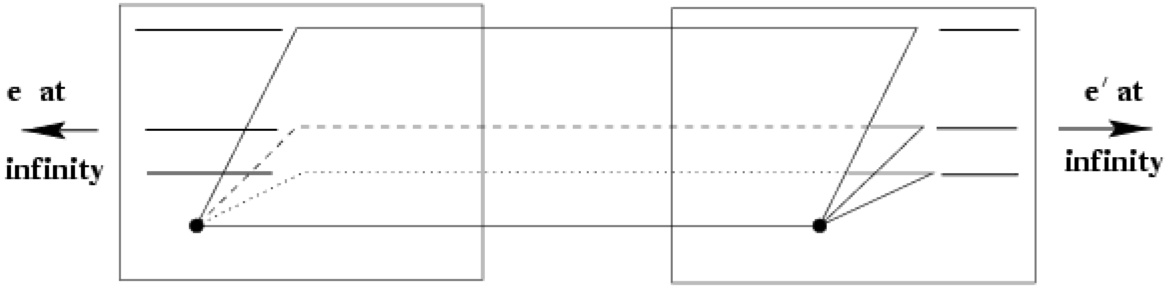
  = projections of the other camera center
- **Epipolar Plane** – plane containing baseline (1D family)
- **Epipolar Lines** - intersections of epipolar plane with image planes (always come in corresponding pairs)

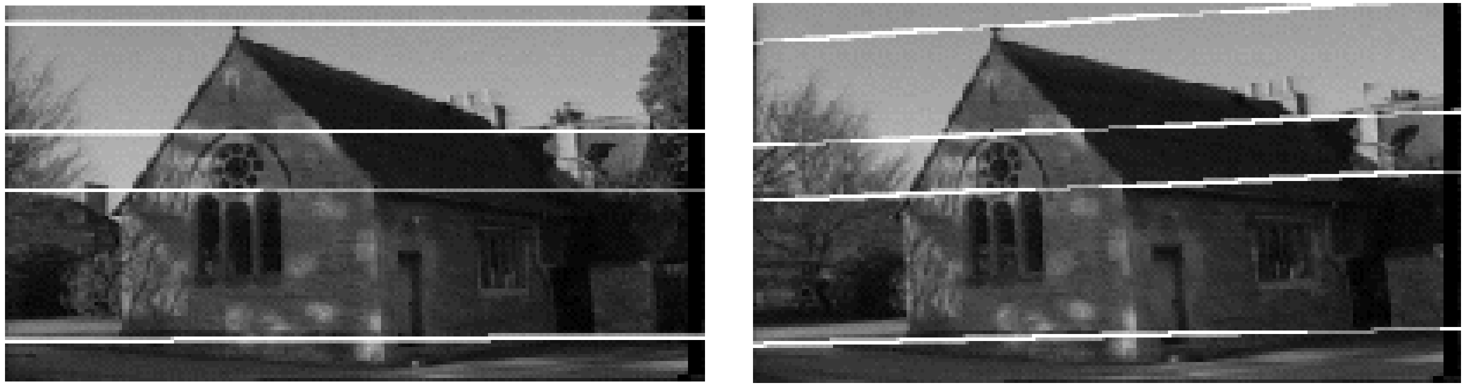
Example: Converging cameras





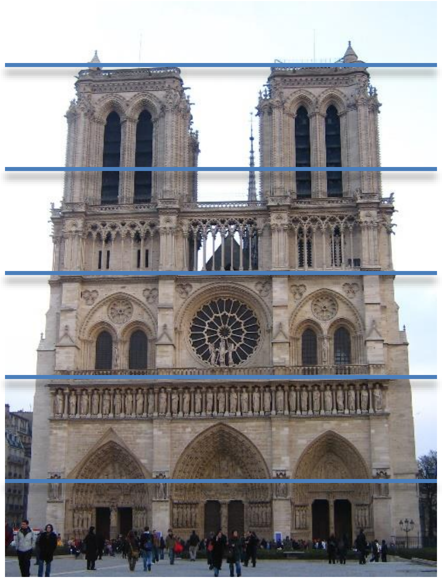
Example: Motion or displacement parallel to image plane



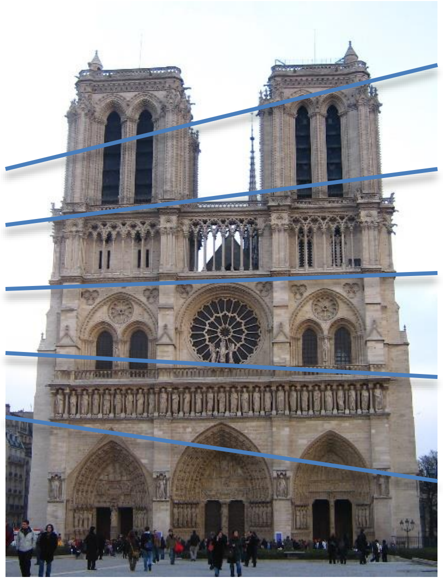


Example: Forward motion

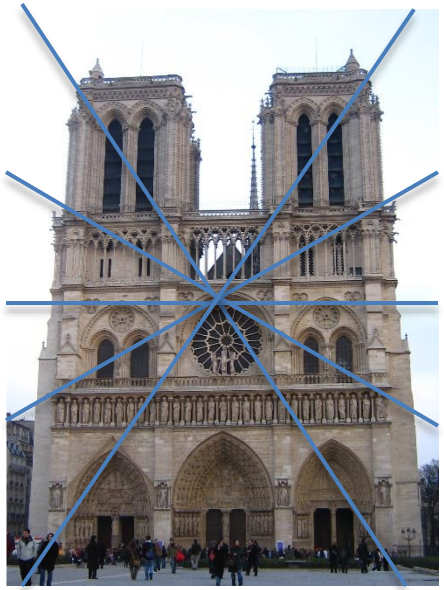
What would the epipolar lines look like if the camera moves forward?



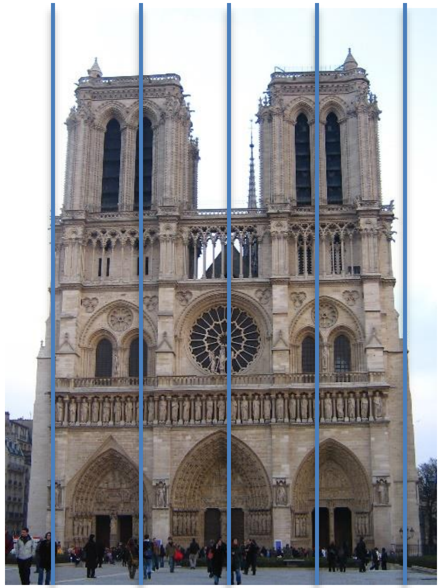
a



b

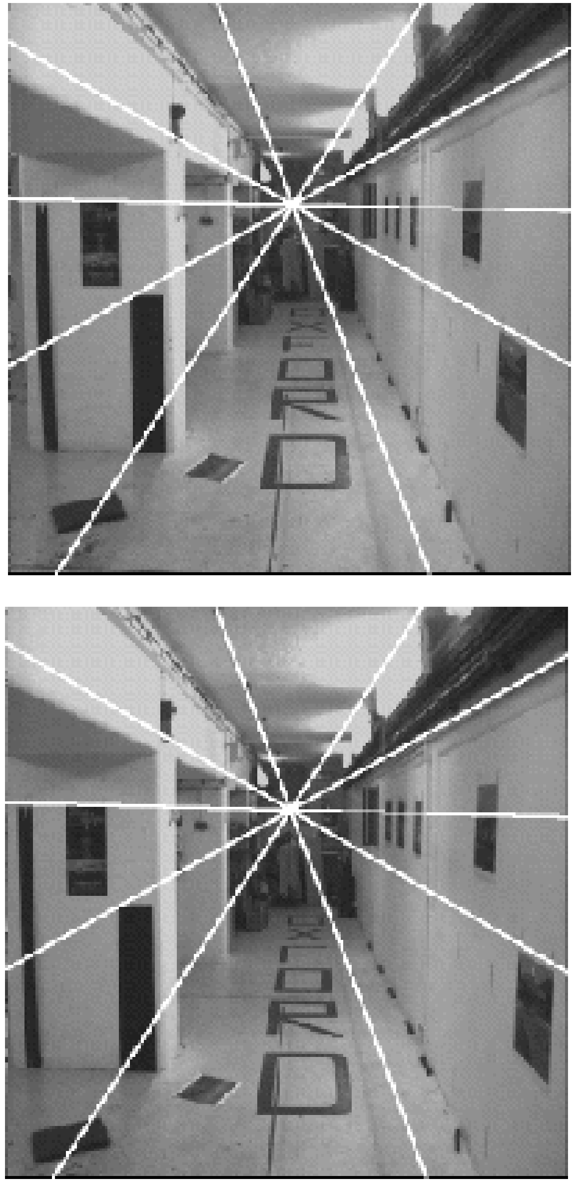


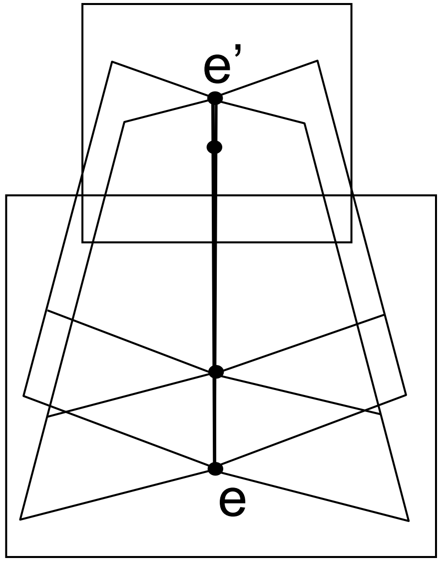
c



d

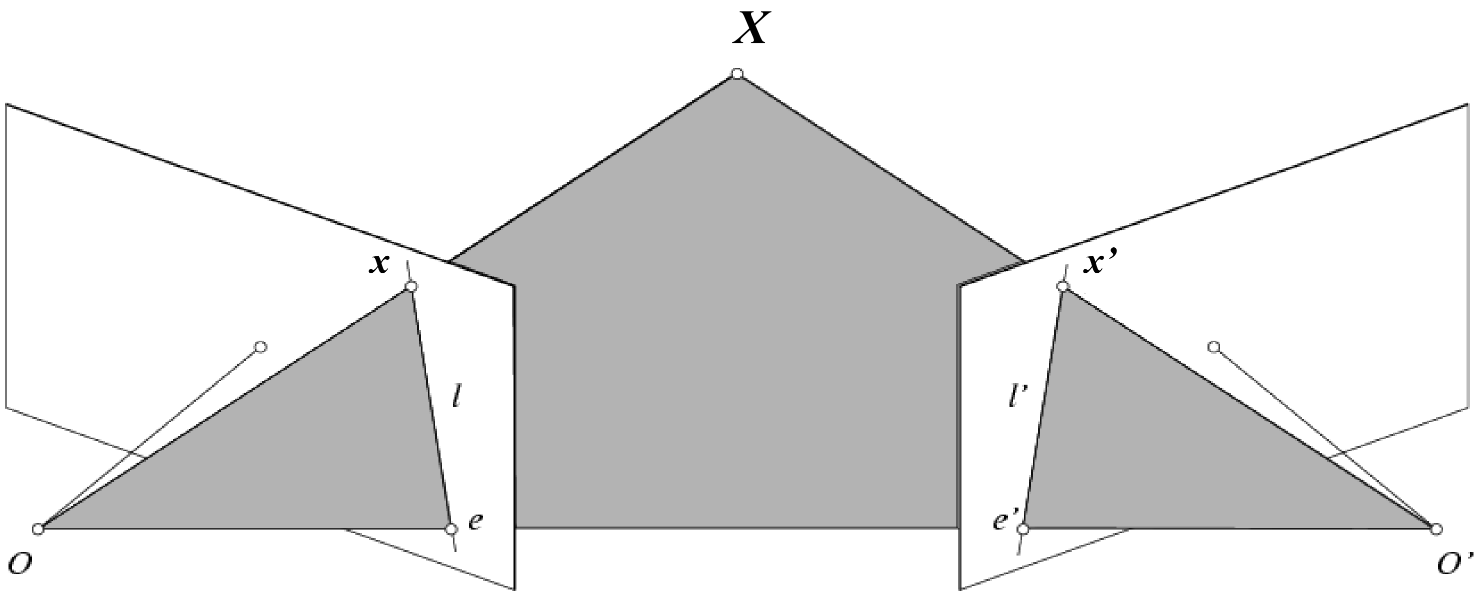
Example: Forward motion





Epipole has same coordinates in both images.
Points move along lines radiating from e: "Focus of expansion"

Epipolar constraint: Calibrated case



Given the intrinsic parameters of the cameras:

1. Convert to normalized coordinates by pre-multiplying all points with the inverse of the calibration matrix; set first camera's coordinate system to world coordinates

$$\hat{x} = K^{-1} x = X \qquad \hat{x}' = K'^{-1} x' = X'$$

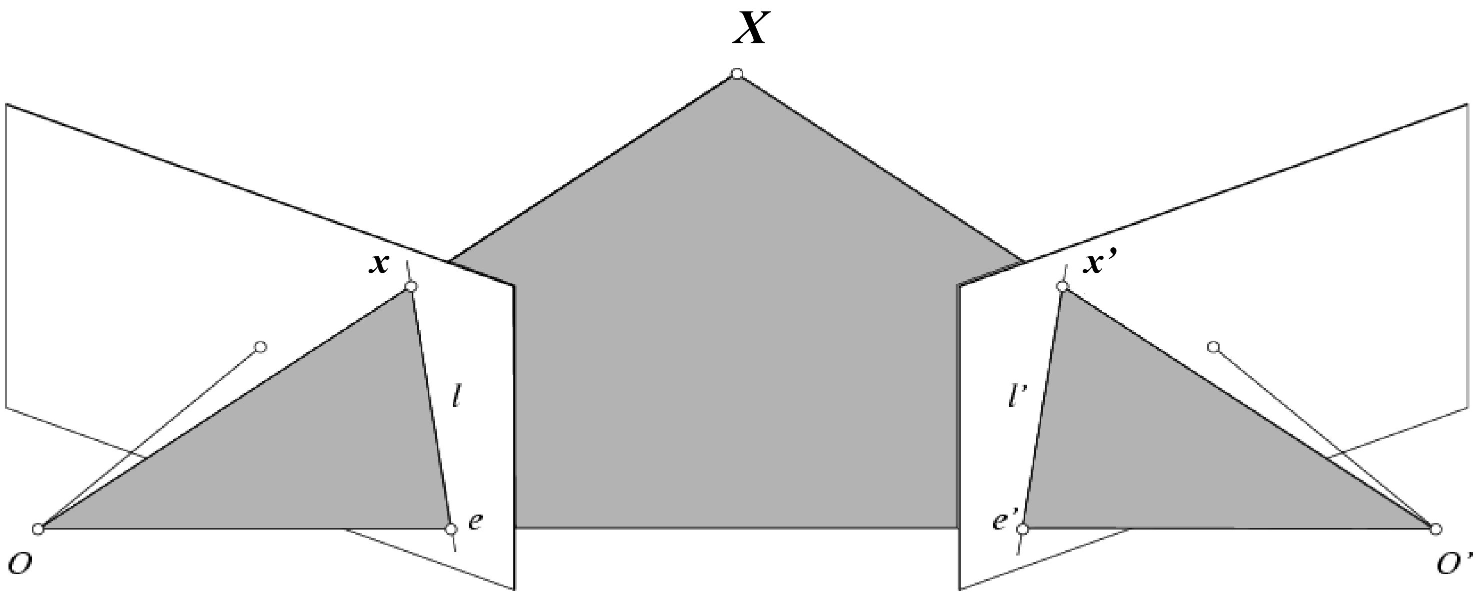
Homogeneous 2d point (3D ray towards X)

2D pixel coordinate (homogeneous)

3D scene point

3D scene point in 2nd camera's 3D coordinates

Epipolar constraint: Calibrated case



Given the intrinsic parameters of the cameras:

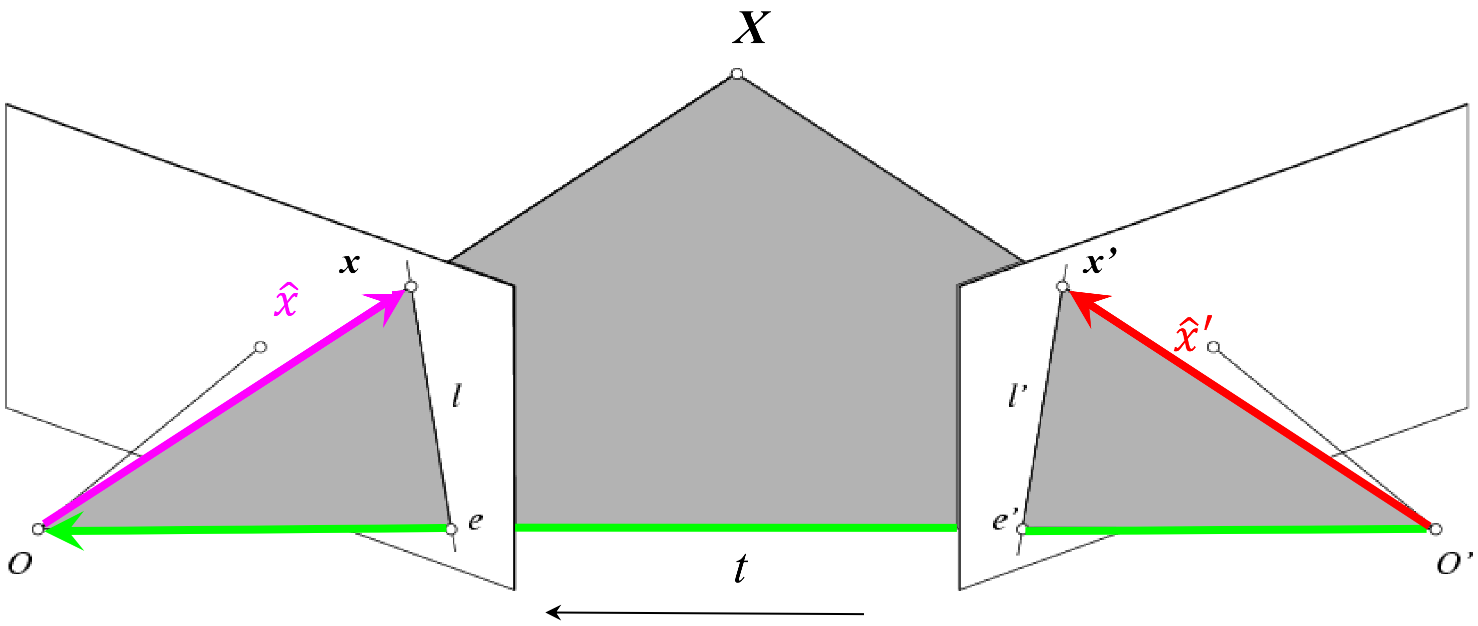
1. Convert to normalized coordinates by pre-multiplying all points with the inverse of the calibration matrix; set first camera's coordinate system to world coordinates
2. Define some *R* and *t* that relate X to X' as below

for some scale factor

$$\hat{x} = K^{-1}x = X \qquad \hat{x}' = K'^{-1}x' = X'$$

$$\hat{x} = R\hat{x}' + t$$

Epipolar constraint: Calibrated case

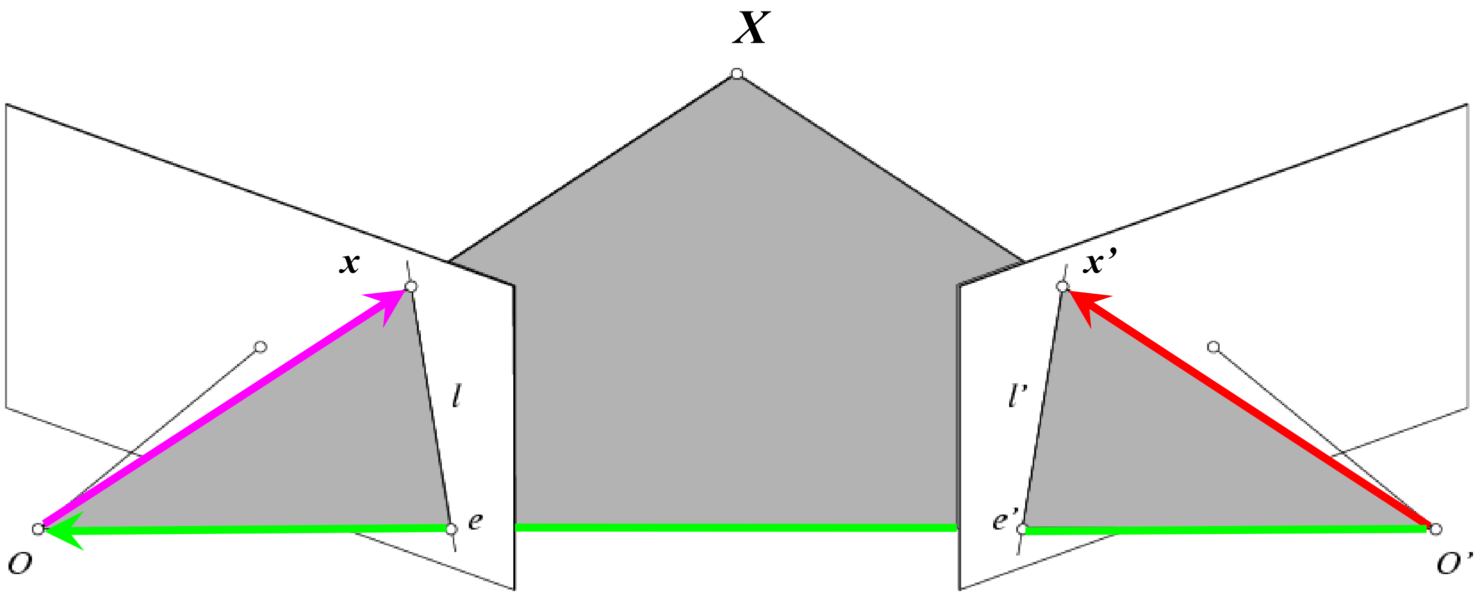


$$\hat{x} = K^{-1}x = X \qquad \hat{x}' = K'^{-1}x' = X'$$

$$\hat{x} = R\hat{x}' + t \quad \Rightarrow \quad \hat{x} \cdot [t \times (R\hat{x}')] = 0$$

(because $\hat{x}, R\hat{x}'$, and t are co-planar)

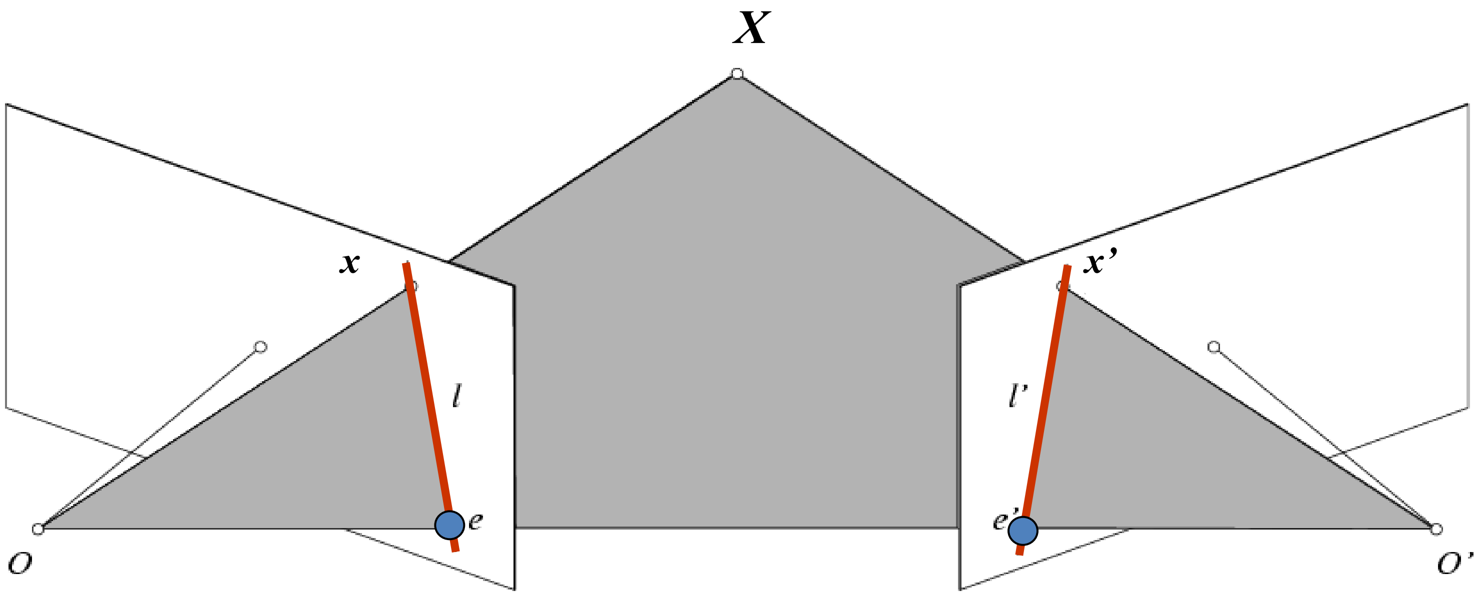
Essential matrix



$\hat{x} \cdot [t \times (R\hat{x}')] = 0 \Rightarrow \hat{x}^T E \hat{x}' = 0$ with $E = [t]_\times R$

Essential Matrix
(Longuet-Higgins, 1981)

Properties of the Essential matrix



$$\hat{x} \cdot [t \times (R\hat{x}')] = 0 \quad \Rightarrow \quad \hat{x}^T E \hat{x}' = 0 \quad \text{with} \quad E = [t]_{\times} R$$

Drop ^ below to simplify notation

Skew-symmetric matrix

- *E x'* is the epipolar line associated with *x'* (*l = E x'*)
- $E^T x$ is the epipolar line associated with *x* ($l' = E^T x$)
- *E e'* = 0 and $E^T e = 0$
- *E* is singular (rank two)
- *E* has five degrees of freedom
  - (3 for R, 2 for t because it's up to a scale)

The Fundamental Matrix

Without knowing K and K', we can define a similar relation using *unknown* normalized coordinates

$$\hat{x}^T E \hat{x}' = 0$$

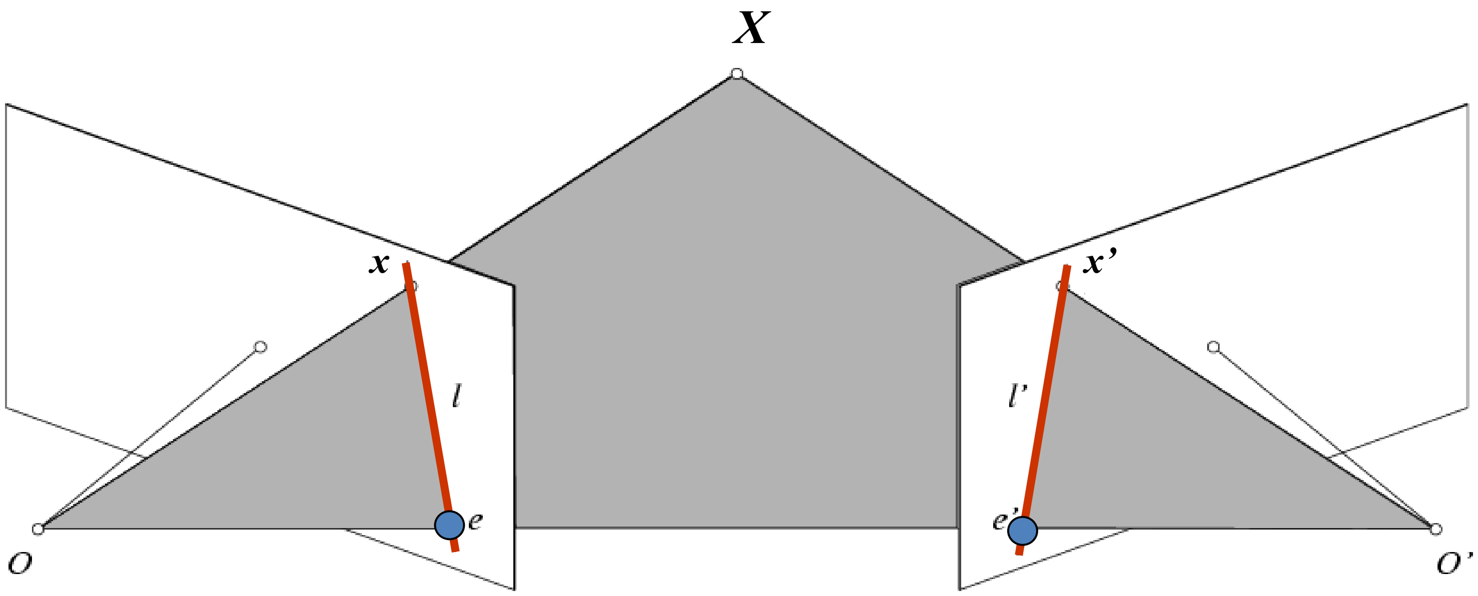
$$\hat{x} = K^{-1} x$$

$$\hat{x}' = K'^{-1} x'$$

$$x^T F x' = 0 \quad \text{with} \quad F = K^{-T} E K'^{-1}$$

Fundamental Matrix
(Faugeras and Luong, 1992)

Properties of the Fundamental matrix



$$x^T F x' = 0 \quad \text{with} \quad F = K^{-T} E K'^{-1}$$

- $F x' = 0$ is the epipolar line associated with x'
- $F^T x = 0$ is the epipolar line associated with x
- $F e' = 0$ and $F^T e = 0$
- F is singular (rank two): det(F)=0
- F has seven degrees of freedom: 9 entries but defined up to scale, det(F)=0

Estimating the Fundamental Matrix

- 8-point algorithm
  - Least squares solution using SVD on equations from 8 pairs of correspondences
  - Enforce det(F)=0 constraint using SVD on F

- 7-point algorithm
  - Use least squares to solve for null space (two vectors) using SVD and 7 pairs of correspondences
  - Solve for linear combination of null space vectors that satisfies det(F)=0

- Minimize reprojection error
  - Non-linear least squares

Note: estimation of F (or E) is degenerate for a planar scene.

8-point algorithm

1. Solve a system of homogeneous linear equations
   a. Write down the system of equations

$$\mathbf{x}^T F \mathbf{x}' = 0$$

$$uu'f_{11} + uv'f_{12} + uf_{13} + vu'f_{21} + vv'f_{22} + vf_{23} + u'f_{31} + v'f_{32} + f_{33} = 0$$

$$\mathrm{A}\boldsymbol{f} = \begin{bmatrix} u_1u_1{}' & u_1v_1{}' & u_1 & v_1u_1{}' & v_1v_1{}' & v_1 & u_1{}' & v_1{}' & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_nu_v' & u_nv_n{}' & u_n & v_nu_n{}' & v_nv_n{}' & v_n & u_n{}' & v_n{}' & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ \vdots \\ f_{33} \end{bmatrix} = \mathbf{0}$$

8-point algorithm

1. Solve a system of homogeneous linear equations
   a. Write down the system of equations
   b. Solve **f** from A**f**=**0** using SVD

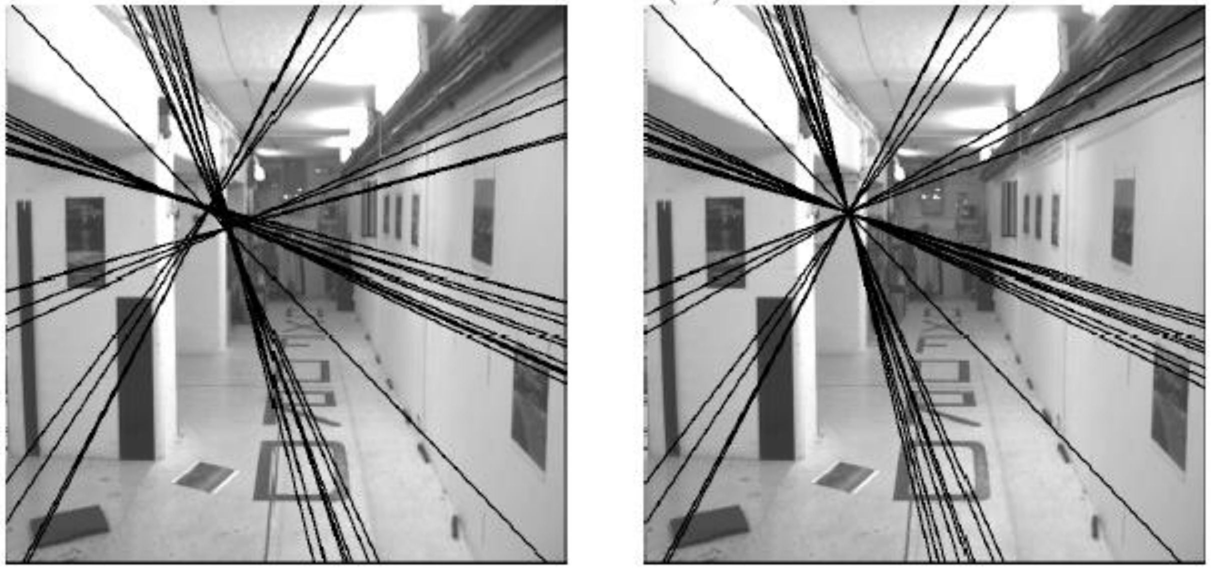
Matlab:
```
[U, S, V] = svd(A);
f = V(:, end);
F = reshape(f, [3 3])';
```

For python, see

numpy.linalg.svd

Need to enforce singularity constraint

Fundamental matrix has rank 2 : $\det(\mathbf{F}) = 0$.



Left : Uncorrected $\mathtt{F}$ – epipolar lines are not coincident.

Right : Epipolar lines from corrected $\mathtt{F}$.

8-point algorithm

1. Solve a system of homogeneous linear equations
   a. Write down the system of equations
   b. Solve **f** from A**f**=**0** using SVD

Matlab:
```
[U, S, V] = svd(A);
f = V(:, end);
F = reshape(f, [3 3])’;
```

2. Resolve det(F) = 0 constraint using SVD

Matlab:
```
[U, S, V] = svd(F);
S(3,3) = 0;
F = U*S*V’;
```

For python, see numpy.linalg.svd

Problem with eight-point algorithm

$$\begin{bmatrix} u'u & u'v & u' & v'u & v'v & v' & u & v \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \end{bmatrix} = -1$$

Problem with eight-point algorithm

$$\begin{bmatrix} 250906.36 & 183269.57 & 921.81 & 200931.10 & 146766.13 & 738.21 & 272.19 & 198.81 \\ 2692.28 & 131633.03 & 176.27 & 6196.73 & 302975.59 & 405.71 & 15.27 & 746.79 \\ 416374.23 & 871684.30 & 935.47 & 408110.89 & 854384.92 & 916.90 & 445.10 & 931.81 \\ 191183.60 & 171759.40 & 410.27 & 416435.62 & 374125.90 & 893.65 & 465.99 & 418.65 \\ 48988.86 & 30401.76 & 57.89 & 298604.57 & 185309.58 & 352.87 & 846.22 & 525.15 \\ 164786.04 & 546559.67 & 813.17 & 1998.37 & 6628.15 & 9.86 & 202.65 & 672.14 \\ 116407.01 & 2727.75 & 138.89 & 169941.27 & 3982.21 & 202.77 & 838.12 & 19.64 \\ 135384.58 & 75411.13 & 198.72 & 411350.03 & 229127.78 & 603.79 & 681.28 & 379.48 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \end{bmatrix} = -1$$

Poor numerical conditioning

Can be fixed by rescaling the data

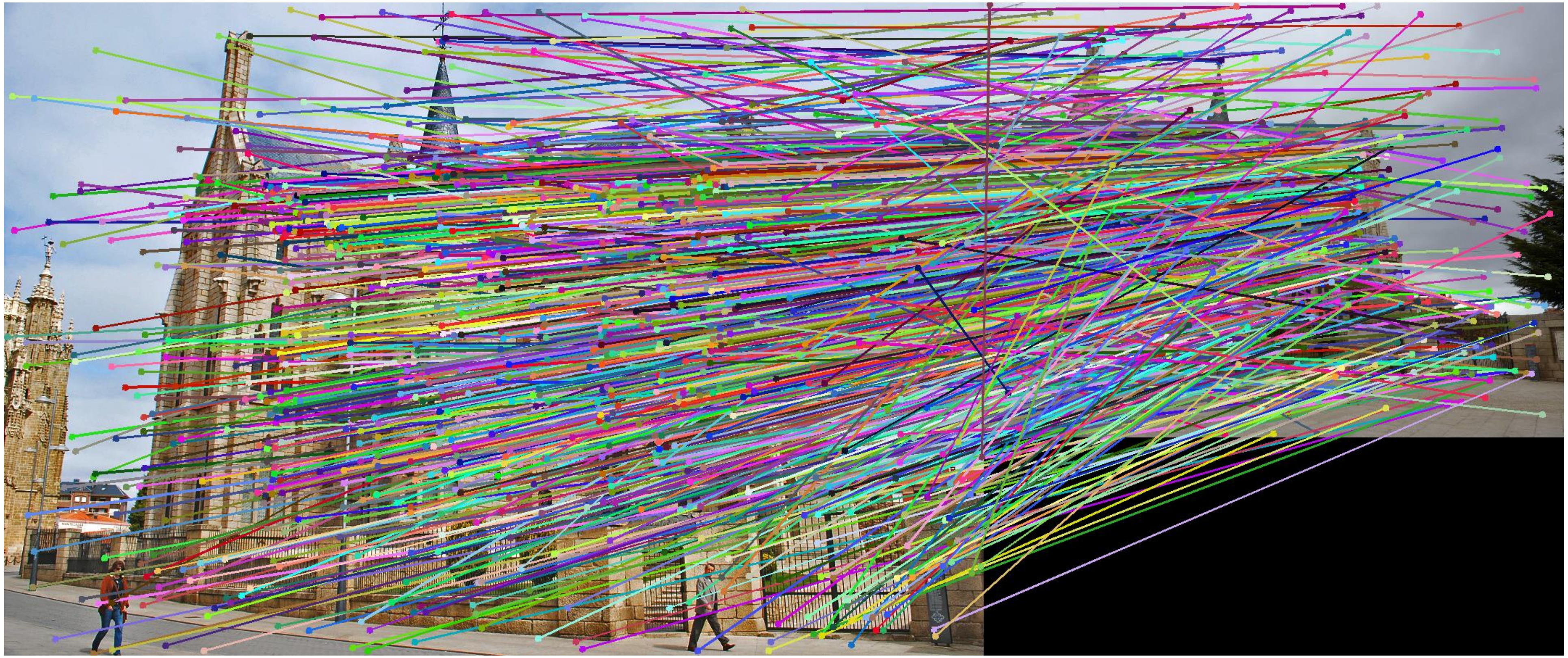
The normalized eight-point algorithm

(Hartley, 1995)

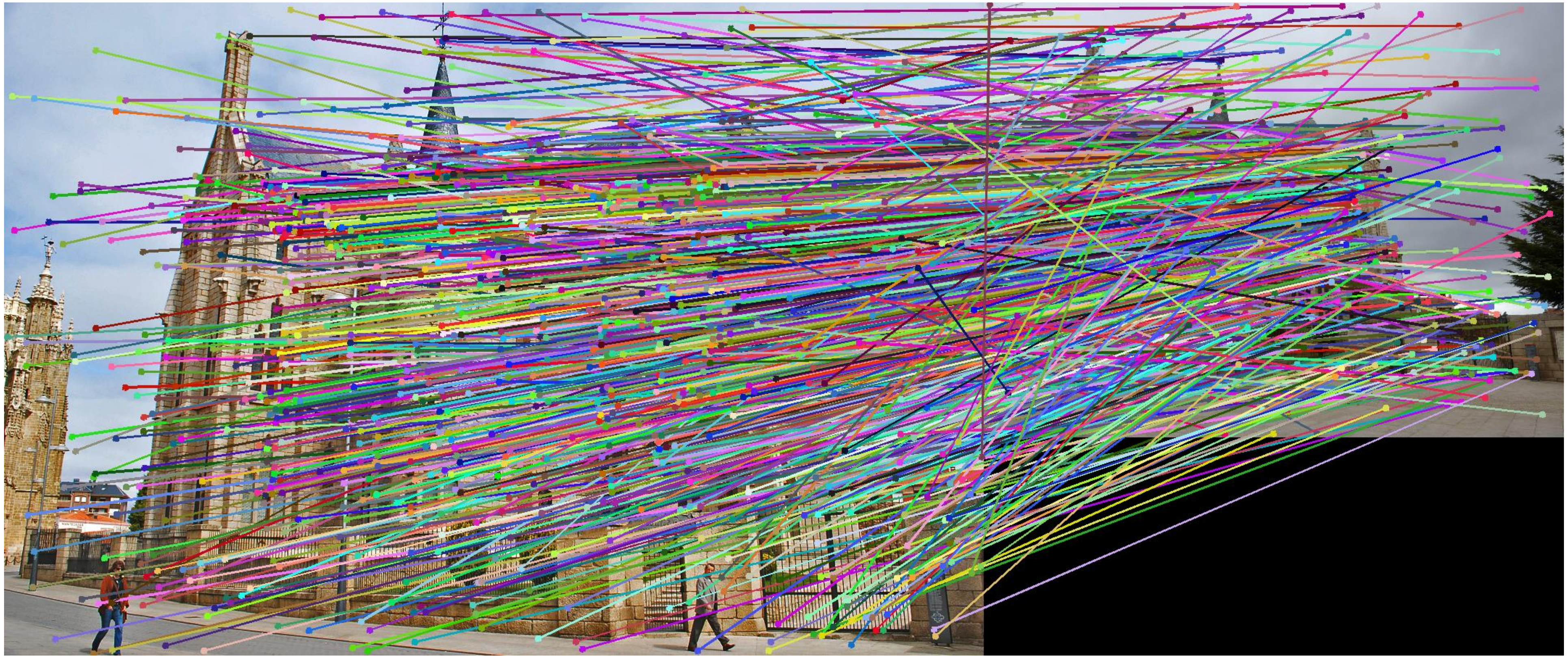
- Center the image data at the origin, and scale it so the mean squared distance between the origin and the data points is 2 pixels
- Use the eight-point algorithm to compute $\boldsymbol{F}$ from the normalized points
- Enforce the rank-2 constraint (for example, take SVD of $\boldsymbol{F}$ and throw out the smallest singular value)
- Transform fundamental matrix back to original units: if $\boldsymbol{T}$ and $\boldsymbol{T'}$ are the normalizing transformations in the two images, than the fundamental matrix in original coordinates is $\boldsymbol{T'}^{T}\,\boldsymbol{F}\,\boldsymbol{T}$

But which 8 points do we choose?

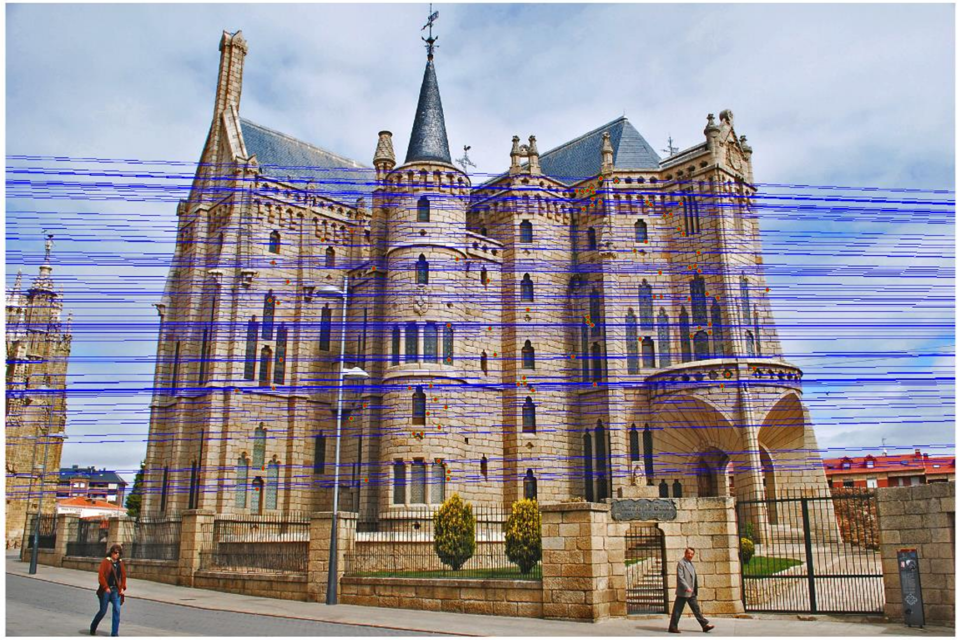
VLFeat's 800 most confident matches among 10,000+ local features.

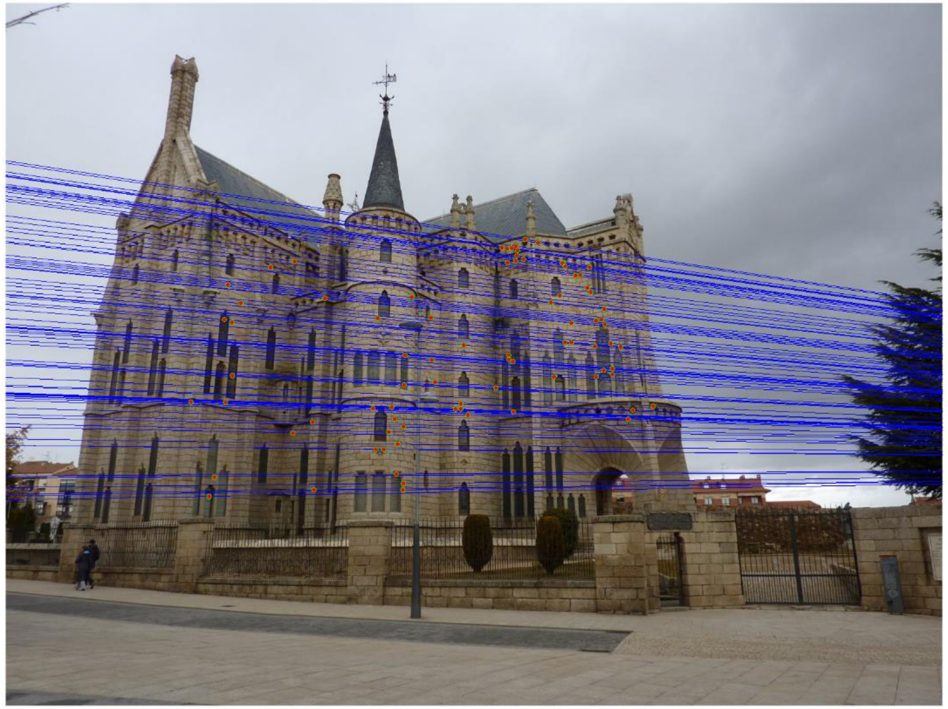


How to test for outliers?

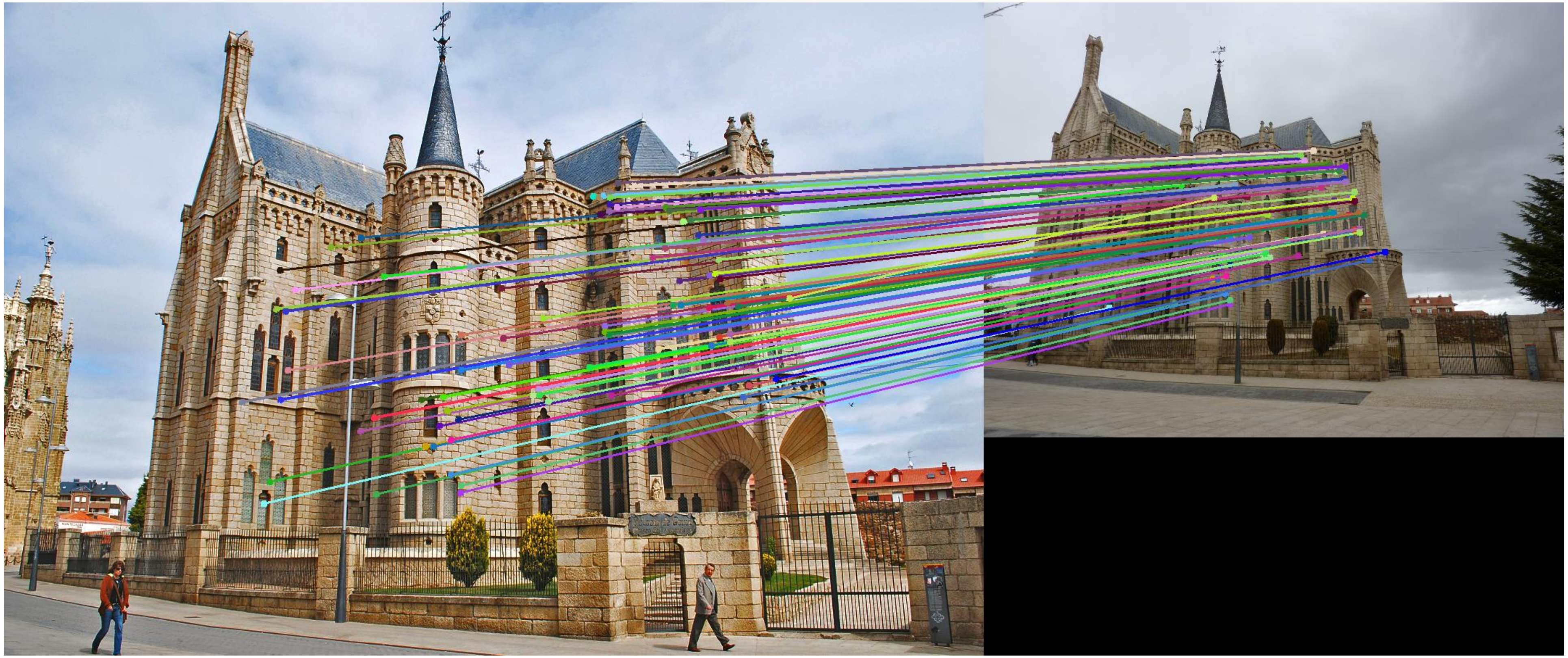


Epipolar lines



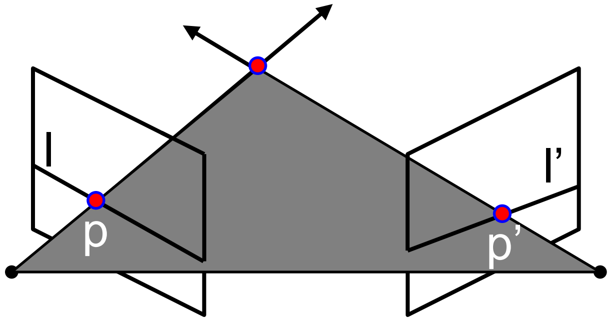


Keep only the matches at are “inliers” with respect to the “best” fundamental matrix



Fundamental matrix

Let *p* be a point in left image, *p’* in right image



Epipolar relation

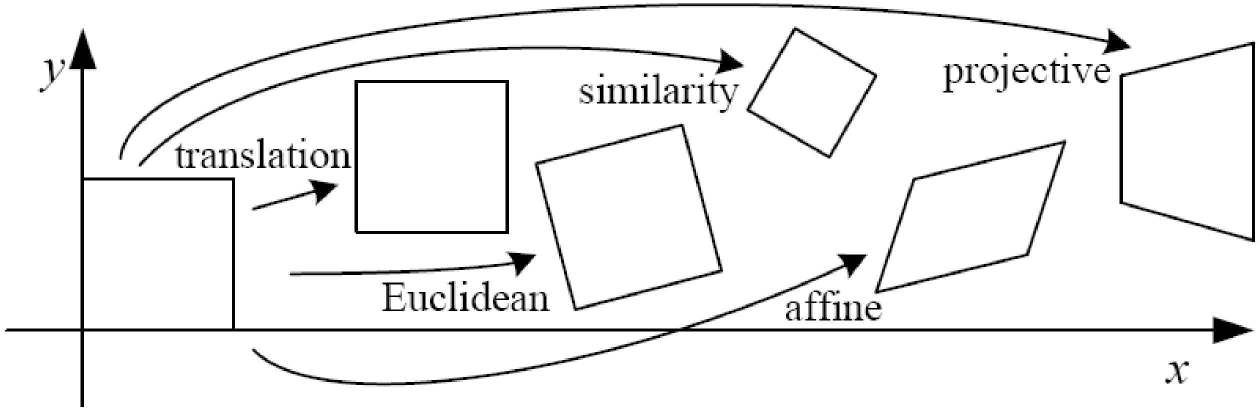
- *p* maps to epipolar line *l’*
- *p’* maps to epipolar line *l*

Epipolar mapping described by a 3x3 matrix *F*

$$p'^T F p = 0$$

Homography vs Fundamental Matrix

2D image transformations (reference table)



Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$\begin{bmatrix} I \mid t \end{bmatrix}_{2\times 3}$	2	orientation $+\cdots$	
rigid (Euclidean)	$\begin{bmatrix} R \mid t \end{bmatrix}_{2\times 3}$	3	lengths $+\cdots$	
similarity	$\begin{bmatrix} sR \mid t \end{bmatrix}_{2\times 3}$	4	angles $+\cdots$	
affine	$\begin{bmatrix} A \end{bmatrix}_{2\times 3}$	6	parallelism $+\cdots$	
projective	$\begin{bmatrix} \tilde{H} \end{bmatrix}_{3\times 3}$	8	straight lines	

Szeliski 2.1

Textbook 2.1.1

Projective. This transformation, also known as a *perspective transform* or *homography*, operates on homogeneous coordinates,

$$\tilde{\mathbf{x}}' = \tilde{\mathbf{H}}\tilde{\mathbf{x}}, \tag{2.20}$$

where $\tilde{\mathbf{H}}$ is an arbitrary 3×3 matrix. Note that $\tilde{\mathbf{H}}$ is homogeneous, i.e., it is only defined up to a scale, and that two $\tilde{\mathbf{H}}$ matrices that differ only by scale are equivalent. The resulting homogeneous coordinate $\tilde{\mathbf{x}}'$ must be normalized in order to obtain an inhomogeneous result $\mathbf{x}$, i.e.,

$$x' = \frac{h_{00}x + h_{01}y + h_{02}}{h_{20}x + h_{21}y + h_{22}} \quad \text{and} \quad y' = \frac{h_{10}x + h_{11}y + h_{12}}{h_{20}x + h_{21}y + h_{22}}. \tag{2.21}$$

Perspective transformations preserve straight lines (i.e., they remain straight after the transformation).

Homography vs Fundamental Matrix

Epipolar mapping described by a 3x3 matrix *F*

$$p'^T F p = 0$$

Projective. This transformation, also known as a *perspective transform* or *homography*, operates on homogeneous coordinates,

$$\tilde{\mathbf{x}}' = \tilde{\mathbf{H}}\tilde{\mathbf{x}}, \tag{2.20}$$