Reminder: last lecture on stereo matching

Textureless regions are non-distinct; high ambiguity for matches.
Computer Vision

Motion and Optical Flow

Many slides adapted from S. Seitz, R. Szeliski, M. Pollefeys, K. Grauman and others…
Video

- A video is a sequence of frames captured over time
- Now our image data is a function of space \((x, y)\) and time \((t)\)
Motion and perceptual organization

Gestalt psychology
(Max Wertheimer, 1880-1943)
Motion and perceptual organization

- Sometimes, motion is the only cue

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- Even “impoverished” motion data can evoke a strong percept

Motion and perceptual organization

• Even “impoverished” motion data can evoke a strong percept

Motion and perceptual organization

Animation from:
An experimental study of apparent behavior.

Courtesy of:
Department of Psychology
University of Kansas, Lawrence.

Experimental study of apparent behavior.
Fritz Heider & Marianne Simmel. 1944
Motion estimation: Optical flow

*Optical flow* is the **apparent** motion of objects or surfaces

Will start by estimating motion of each pixel separately
Then will consider motion of entire image

The term “scene flow” is used to describe 3d motion estimation
Motion field + camera motion

Zoom out  Zoom in  Pan right to left
Problem definition: optical flow

How to estimate pixel motion from image $I(x,y,t)$ to $I(x,y,t+1)$?

- Solve pixel correspondence problem
  - given a pixel in $I(x,y,t)$, look for nearby pixels of the same color in $I(x,y,t+1)$

Key assumptions

- **color constancy**: a point in $I(x,y,t)$ looks the same in $I(x,y,t+1)$
  - For grayscale images, this is brightness constancy
- **small motion**: points do not move very far

This is called the optical flow problem
Motion and Optic Flow

CS 4495 Computer Vision

– A. Bobick

Optical flow constraints (grayscale images)

Let's look at these constraints more closely

- brightness constancy constraint (equation)
  \[ I(x, y, t) = I(x + u, y + v, t + 1) \]

- small motion: \((u \text{ and } v \text{ are less than 1 pixel, or smooth})\)
  Taylor series expansion of the spatial changes of \(I\):
  \[ I(x + u, y + v) = I(x, y) + \frac{\partial I}{\partial x} u + \frac{\partial I}{\partial y} v + [\text{higher order terms}] \]
  
  \approx I(x, y) + \frac{\partial I}{\partial x} u + \frac{\partial I}{\partial y} v \]
Optical flow equation

• Combining these two equations

\[ 0 = I(x + u, y + v, t + 1) - I(x, y, t) \]
\[ \approx I(x, y, t + 1) + I_x u + I_y v - I(x, y, t) \]

(Short hand: \( I_x = \frac{\partial I}{\partial x} \)

for \( t \) or \( t+1 \))
Optical flow equation

- Combining these two equations

\[ 0 = I(x+u, y+v, t+1) - I(x, y, t) \]

\[ \approx I(x, y, t+1) + I_x u + I_y v - I(x, y, t) \]

\[ \approx [I(x, y, t+1) - I(x, y, t)] + I_x u + I_y v \]

\[ \approx I_t + I_x u + I_y v \]

\[ \approx I_t + \nabla I \cdot <u, v> \]

(Short hand: \( I_x = \frac{\partial I}{\partial x} \)
for \( t \) or \( t+1 \))
Optical flow equation

• Combining these two equations

\[ 0 = I(x + u, y + v, t + 1) - I(x, y, t) \]
\[ \approx I(x, y, t + 1) + I_x u + I_y v - I(x, y, t) \]
\[ \approx [I(x, y, t + 1) - I(x, y, t)] + I_x u + I_y v \]
\[ \approx I_t + I_x u + I_y v \]
\[ \approx I_t + \nabla I \cdot <u, v> \]

In the limit as u and v go to zero, this becomes exact

\[ 0 = I_t + \nabla I \cdot <u, v> \]

*Brightness constancy constraint equation*

\[ I_x u + I_y v + I_t = 0 \]
How does this make sense?

**Brightness constancy constraint equation**

\[ I_x u + I_y v + I_t = 0 \]

- What do the static image gradients have to do with motion estimation?

If I told you
- \( I_t \) is -5
- \( I_x \) is 2.5
- \( I_y \) is 0

What was the pixel shift \((u,v)\)?
The brightness constancy constraint

Can we use this equation to recover image motion \((u,v)\) at each pixel?

\[ 0 = I_t + \nabla I \cdot <u, v> \quad \text{or} \quad I_x u + I_y v + I_t = 0 \]

• How many equations and unknowns per pixel?

  • One equation (this is a scalar equation!), two unknowns \((u,v)\)

The component of the motion perpendicular to the gradient (i.e., parallel to the edge) cannot be measured

If \((u, v)\) satisfies the equation, so does \((u+u', v+v')\) if

\[ \nabla I \cdot [u' \ v']^T = 0 \]
Aperture problem
Aperture problem
Aperture problem
The barber pole illusion

http://en.wikipedia.org/wiki/Barberpole_illusion
The barber pole illusion

http://en.wikipedia.org/wiki/Barberpole_illusion
Solving the ambiguity...


- How to get more equations for a pixel?
- **Spatial coherence constraint**
  - Assume the pixel’s neighbors have the same \((u,v)\)
    - If we use a 5x5 window, that gives us 25 equations per pixel
      \[
      0 = I_t(p_i) + \nabla I(p_i) \cdot [u \ v]
      \]

\[
\begin{bmatrix}
  I_x(p_1) & I_y(p_1) \\
  I_x(p_2) & I_y(p_2) \\
  \vdots & \vdots \\
  I_x(p_{25}) & I_y(p_{25})
\end{bmatrix}
\begin{bmatrix}
  u \\
  v
\end{bmatrix}
= -
\begin{bmatrix}
  I_t(p_1) \\
  I_t(p_2) \\
  \vdots \\
  I_t(p_{25})
\end{bmatrix}
\]
Solving the ambiguity...

• Least squares problem:

\[
\begin{bmatrix}
I_x(p_1) & I_y(p_1) \\
I_x(p_2) & I_y(p_2) \\
\vdots & \vdots \\
I_x(p_{25}) & I_y(p_{25})
\end{bmatrix}
\begin{bmatrix}
u \\
v
\end{bmatrix}
= -
\begin{bmatrix}
I_t(p_1) \\
I_t(p_2) \\
\vdots \\
I_t(p_{25})
\end{bmatrix}
\]

\[A \cdot d = b\]

25x2 2x1 25x1
Matching patches across images

- Overconstrained linear system

\[
\begin{bmatrix}
I_x(p_1) & I_y(p_1) \\
I_x(p_2) & I_y(p_2) \\
\vdots & \vdots \\
I_x(p_{25}) & I_y(p_{25})
\end{bmatrix}
\begin{bmatrix}
u \\
v
\end{bmatrix} = -
\begin{bmatrix}
I_t(p_1) \\
I_t(p_2) \\
\vdots \\
I_t(p_{25})
\end{bmatrix}
\]

Least squares solution for \(d\) given by \((A^T A) d = A^T b\)

\[
\begin{bmatrix}
\sum I_x I_x & \sum I_x I_y \\
\sum I_x I_y & \sum I_y I_y
\end{bmatrix}
\begin{bmatrix}
u \\
v
\end{bmatrix} = -
\begin{bmatrix}
\sum I_x I_t \\
\sum I_y I_t
\end{bmatrix}
\]

\[A^T A \quad A^T b\]

The summations are over all pixels in the K x K window.
Conditions for solvability

Optimal $(u, v)$ satisfies Lucas-Kanade equation

$$
\begin{bmatrix}
\sum I_x I_x & \sum I_x I_y \\
\sum I_x I_y & \sum I_y I_y
\end{bmatrix}
\begin{bmatrix}
u \\
v
\end{bmatrix}
= -
\begin{bmatrix}
\sum I_x I_t \\
\sum I_y I_t
\end{bmatrix}

A^T A

A^T b
$$

When is this solvable? I.e., what are good points to track?

- $A^T A$ should be invertible
- $A^T A$ should not be too small due to noise
  - eigenvalues $\lambda_1$ and $\lambda_2$ of $A^T A$ should not be too small
- $A^T A$ should be well-conditioned
  - $\lambda_1 / \lambda_2$ should not be too large ($\lambda_1$ = larger eigenvalue)

Does this remind you of anything?

Criteria for Harris corner detector
Low texture region

\[ \sum \nabla I (\nabla I)^T \]
- gradients have small magnitude
- small \( \lambda_1 \), small \( \lambda_2 \)
\[ \sum \nabla I (\nabla I)^T \]

- large gradients, all the same
- large \( \lambda_1 \), small \( \lambda_2 \)
High textured region

\[ \sum \nabla I(\nabla I)^T \]

- gradients are different, large magnitudes
- large \( \lambda_1 \), large \( \lambda_2 \)
The aperture problem resolved
The aperture problem resolved

Perceived motion
Revisiting the small motion assumption

- Is this motion small enough?
  - Probably not—it’s much larger than one pixel
  - How might we solve this problem?
Optical Flow: Aliasing

Temporal aliasing causes ambiguities in optical flow because images can have many pixels with the same intensity. I.e., how do we know which ‘correspondence’ is correct?

To overcome aliasing: coarse-to-fine estimation.
Reduce the resolution!
Coarse-to-fine optical flow estimation

Gaussian pyramid of image 1

Gaussian pyramid of image 2

- u = 1.25 pixels
- u = 2.5 pixels
- u = 5 pixels
- u = 10 pixels
Coarse-to-fine optical flow estimation

1. Gaussian pyramid of image 1
2. Gaussian pyramid of image 2
3. Run iterative L-K
4. Warp & upsample
5. Run iterative L-K
6. Repeat steps 3-5

Coarse-to-fine optical flow estimation
Optical Flow Results

Lucas-Kanade without pyramids

Fails in areas of large motion

* From Khurram Hassan-Shafique CAP5415 Computer Vision 2003
Optical Flow Results

Lucas-Kanade with Pyramids

* From Khurram Hassan-Shafique CAP5415 Computer Vision 2003
State-of-the-art optical flow in 2009

Start with something similar to Lucas-Kanade
+ gradient constancy
+ energy minimization with smoothing term
+ region matching
+ keypoint matching (long-range)

Large displacement optical flow, Brox et al., CVPR 2009
State-of-the-art optical flow in 2015

Deep convolutional network which accepts a pair of input frames and upsamples the estimated flow back to input resolution. Very fast because of deep network, near the state-of-the-art in terms of end-point-error.

Deep optical flow, 2015

Synthetic Training data

Deep optical flow, 2015

Results on Sintel

Optical flow

• Definition: optical flow is the *apparent* motion of brightness patterns in the image

• Ideally, optical flow would be the same as the motion field

• Have to be careful: apparent motion can be caused by lighting changes without any actual motion
  – Think of a uniform rotating sphere under fixed lighting vs. a stationary sphere under moving illumination