

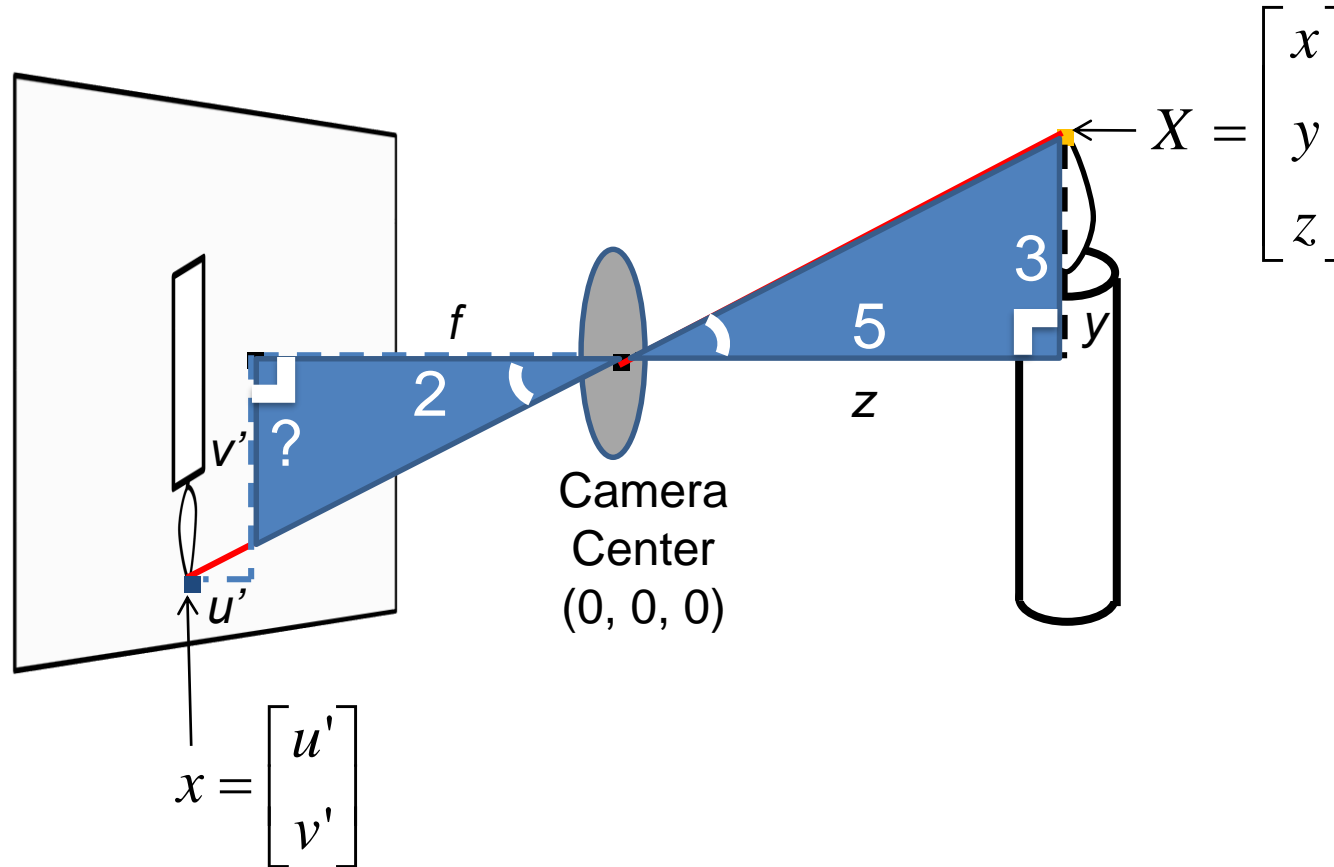
Welcome back!

- Project 0 will be out today
- Project 1 will be out soon
- Read Szeliski 2.1, especially 2.1.4
- Today
 - Image projection
 - Filtering

Image formation

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Projection: world coordinates \rightarrow image coordinates



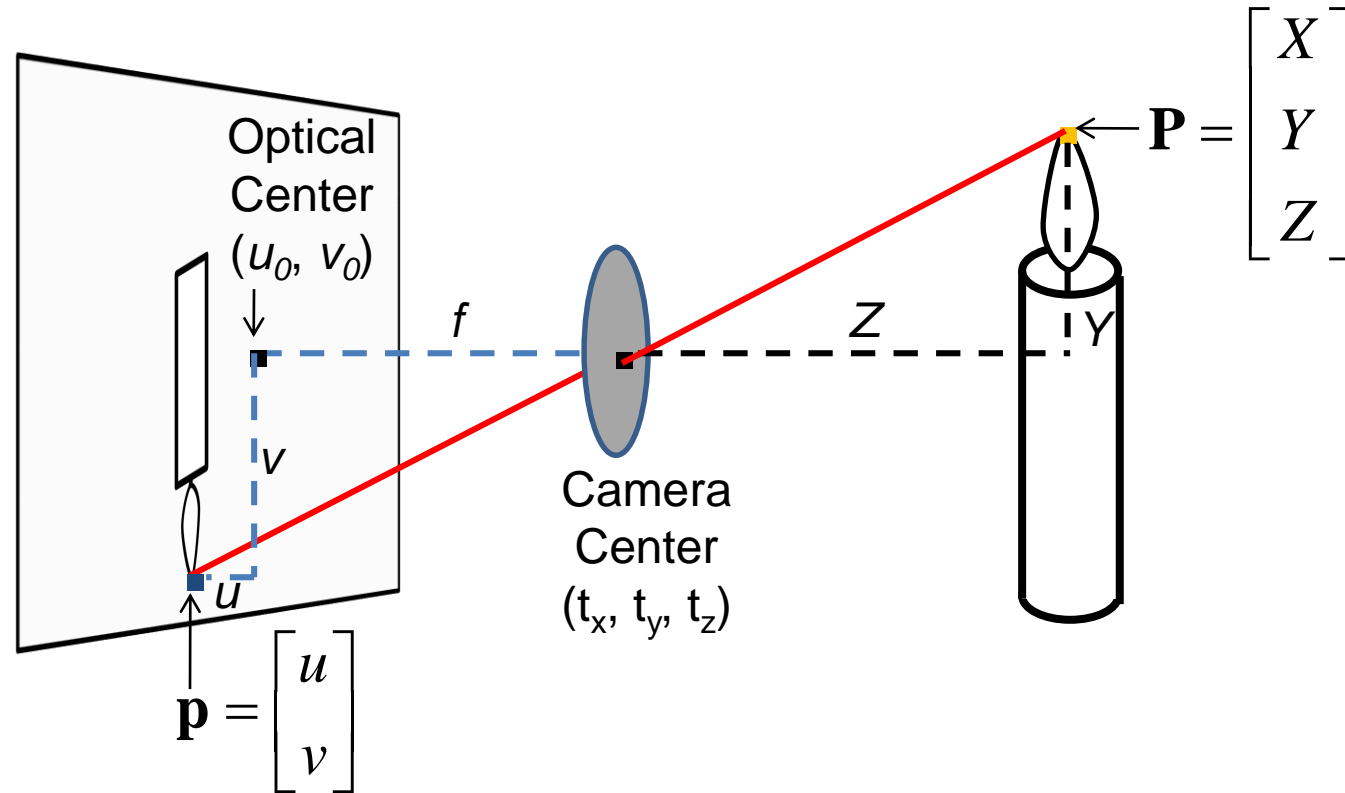
If $x = 2$, $y = 3$,
 $z = 5$, and $f = 2$
What are u' and v' ?

$$\frac{v'}{-f} = \frac{y}{z}$$

$$u' = -x * \frac{f}{z}$$
$$v' = -y * \frac{f}{z}$$

$$u' = -2 * \frac{2}{5}$$
$$v' = -3 * \frac{2}{5}$$

Projection: world coordinates \rightarrow image coordinates



How do we handle the general case?

Interlude: why does this matter?

Relating multiple views



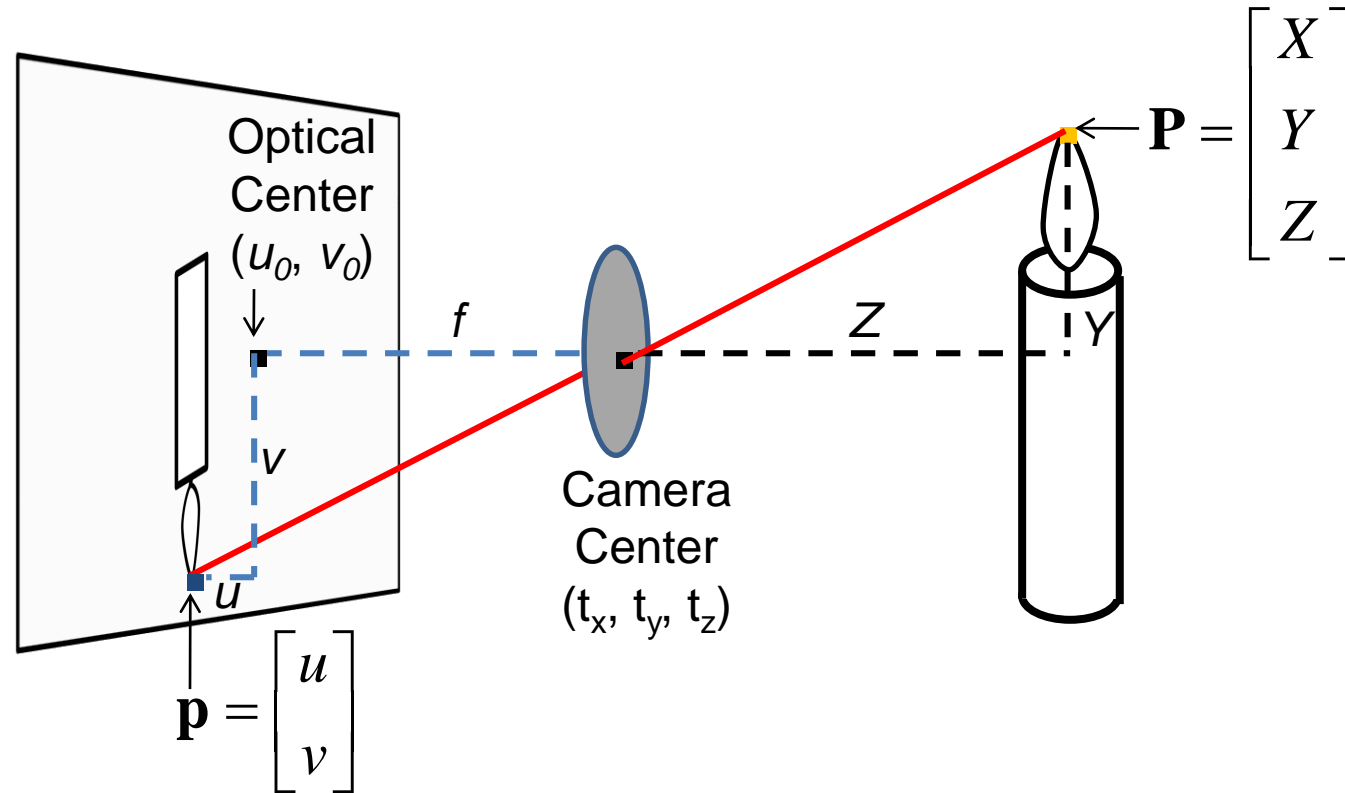
Photo Tourism

Exploring photo collections in 3D

Noah Snavely Steven M. Seitz Richard Szeliski
University of Washington *Microsoft Research*

SIGGRAPH 2006

Projection: world coordinates \rightarrow image coordinates



How do we handle the general case?

Homogeneous coordinates

Conversion

Converting to *homogeneous* coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image
coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene
coordinates

Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Homogeneous coordinates

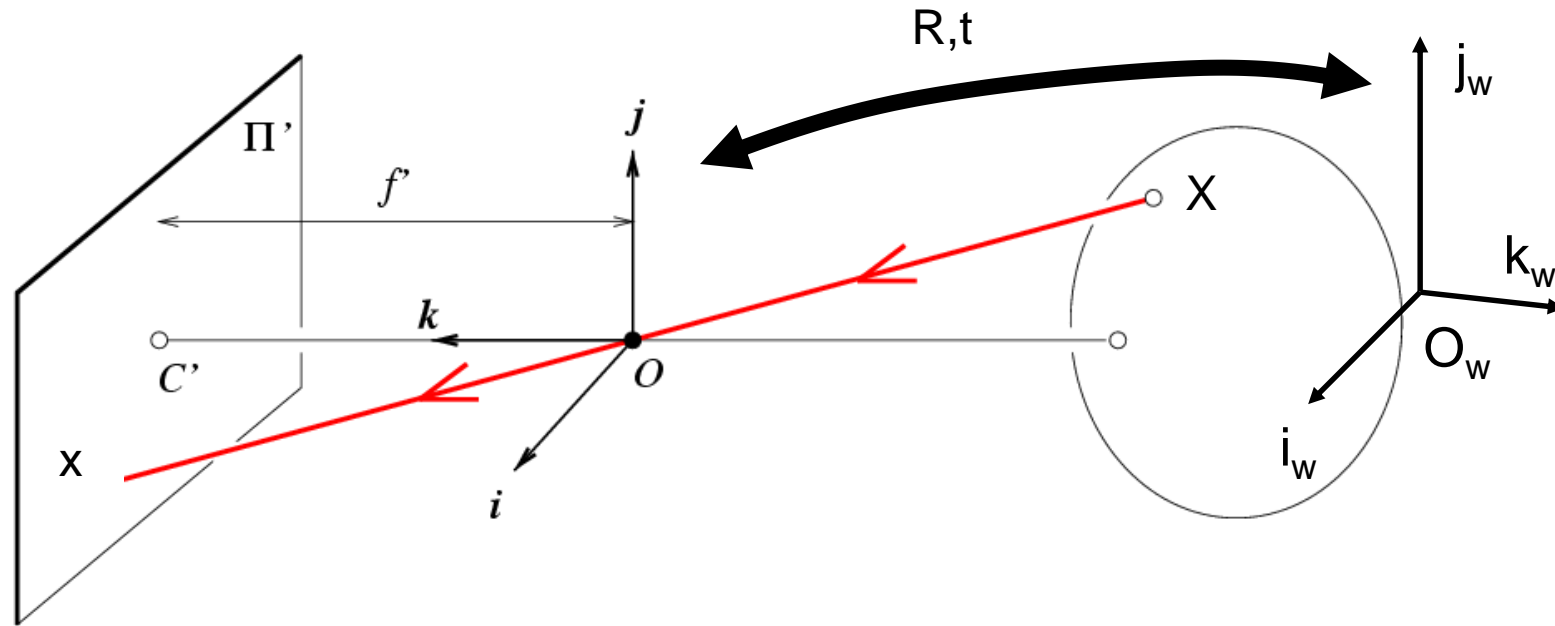
Invariant to scaling

$$k \begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} kx \\ ky \\ kw \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{kx}{kw} \\ \frac{ky}{kw} \\ \frac{kw}{kw} \end{bmatrix} = \begin{bmatrix} \frac{x}{w} \\ \frac{y}{w} \\ 1 \end{bmatrix}$$

Homogeneous Coordinates Cartesian Coordinates

Point in Cartesian is ray in Homogeneous

Projection matrix



$$\mathbf{x} = \mathbf{K} \begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix} \mathbf{X}$$

\mathbf{x} : Image Coordinates: $(u, v, 1)$

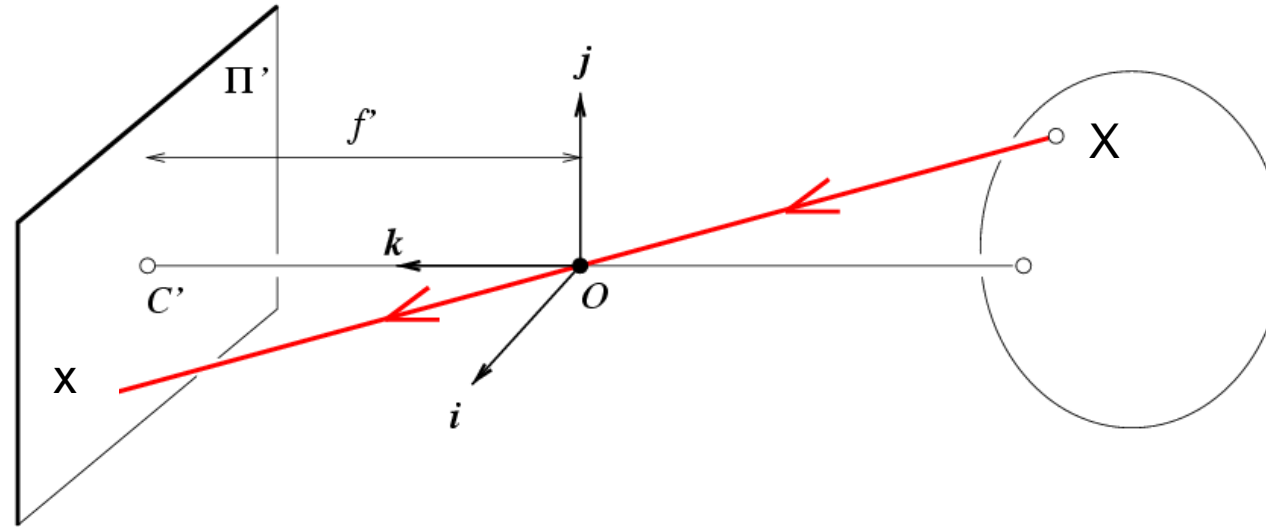
\mathbf{K} : Intrinsic Matrix (3×3)

\mathbf{R} : Rotation (3×3)

\mathbf{t} : Translation (3×1)

\mathbf{X} : World Coordinates: $(X, Y, Z, 1)$

Projection matrix



Intrinsic Assumptions

- Unit aspect ratio
- Optical center at (0,0)
- No skew

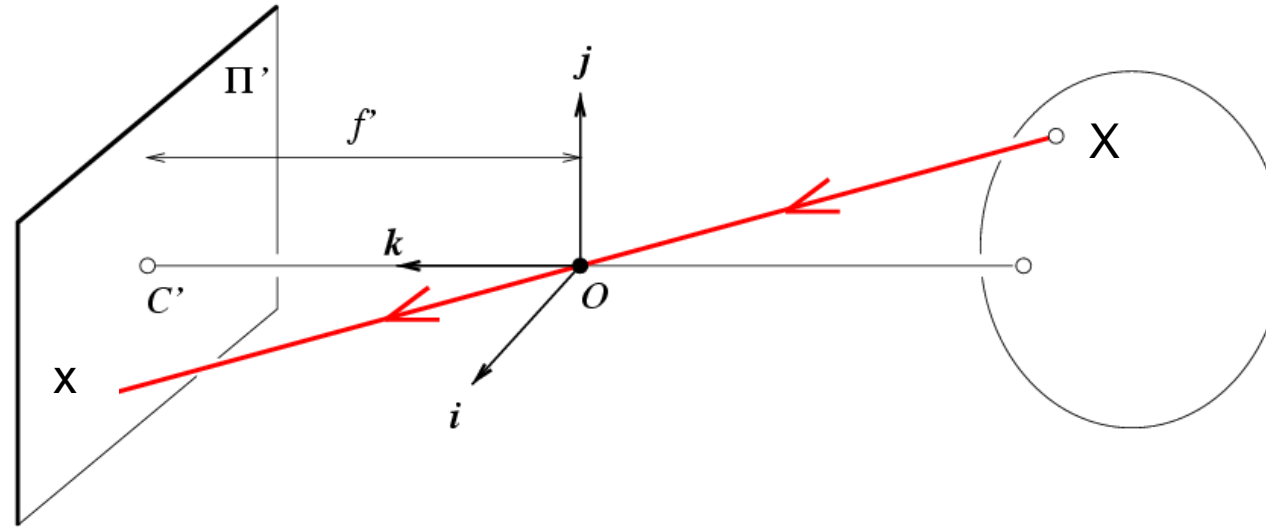
Extrinsic Assumptions

- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \mathbf{X} \Rightarrow w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

\mathbf{K}

Projection matrix



Intrinsic Assumptions

- Unit aspect ratio
- Optical center at $(0,0)$
- No skew

Extrinsic Assumptions

- No rotation
- Camera at $(0,0,0)$

$$\mathbf{x} = \mathbf{K} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \mathbf{X} \Rightarrow w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Remove assumption: known optical center

Intrinsic Assumptions Extrinsic Assumptions

- Unit aspect ratio
- No skew

- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \mathbf{X} \quad \rightarrow \quad w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Remove assumption: square pixels

Intrinsic Assumptions

- No skew

Extrinsic Assumptions

- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \mathbf{X} \Rightarrow w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & 0 & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Remove assumption: non-skewed pixels

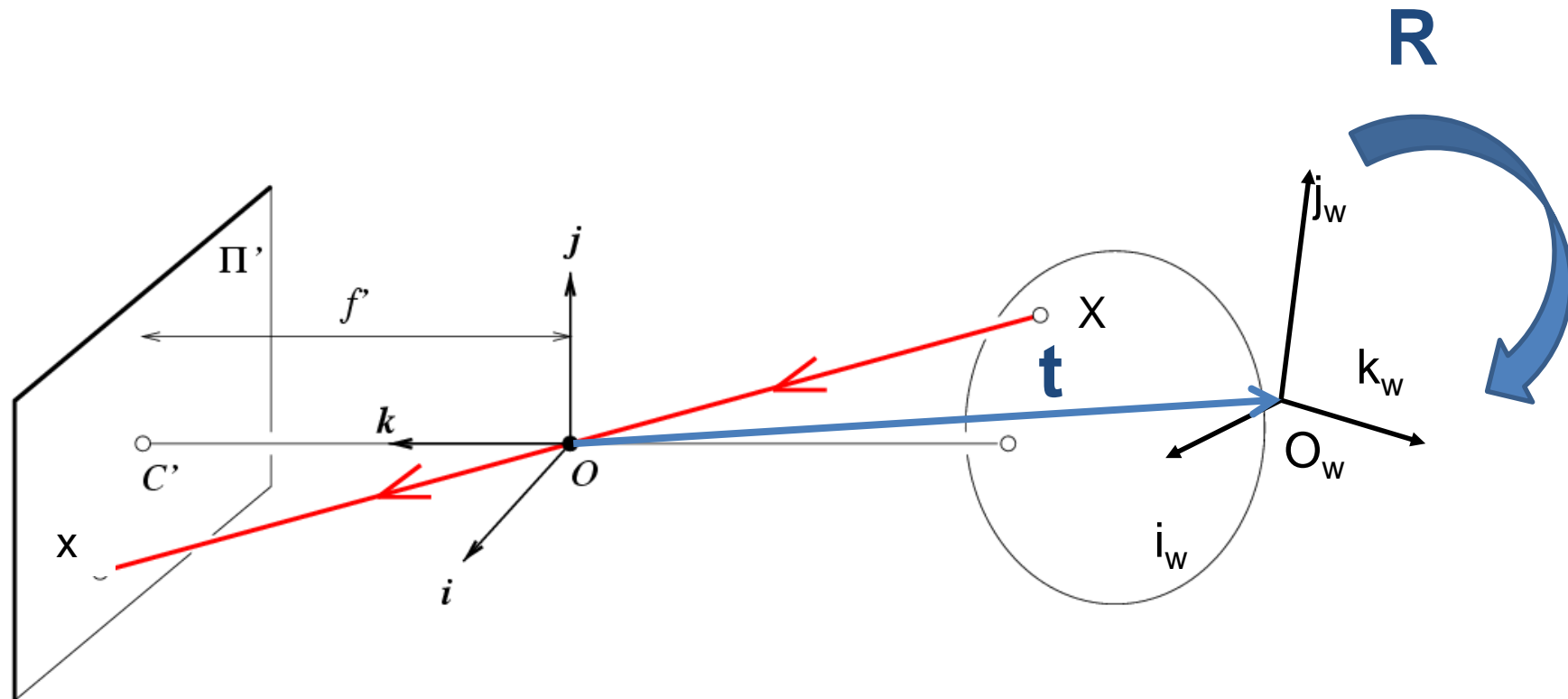
Intrinsic Assumptions Extrinsic Assumptions

- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \mathbf{X} \Rightarrow w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Note: different books use different notation for parameters

Oriented and Translated Camera



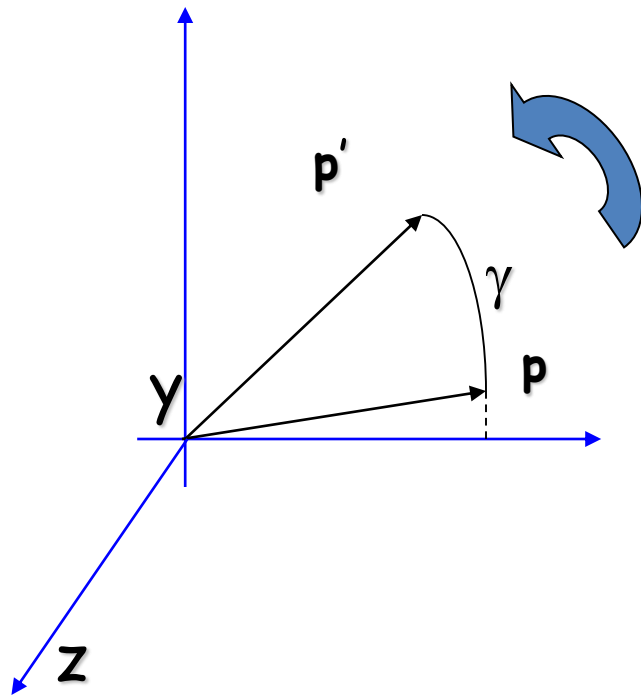
Allow camera translation

Intrinsic Assumptions Extrinsic Assumptions
• No rotation

$$\mathbf{x} = \mathbf{K} \begin{bmatrix} \mathbf{I} & \mathbf{t} \end{bmatrix} \mathbf{X} \Rightarrow w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & 0 & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3D Rotation of Points

Rotation around the coordinate axes, **counter-clockwise**:



$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix}$$

$$R_y(\beta) = \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix}$$

$$R_z(\gamma) = \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

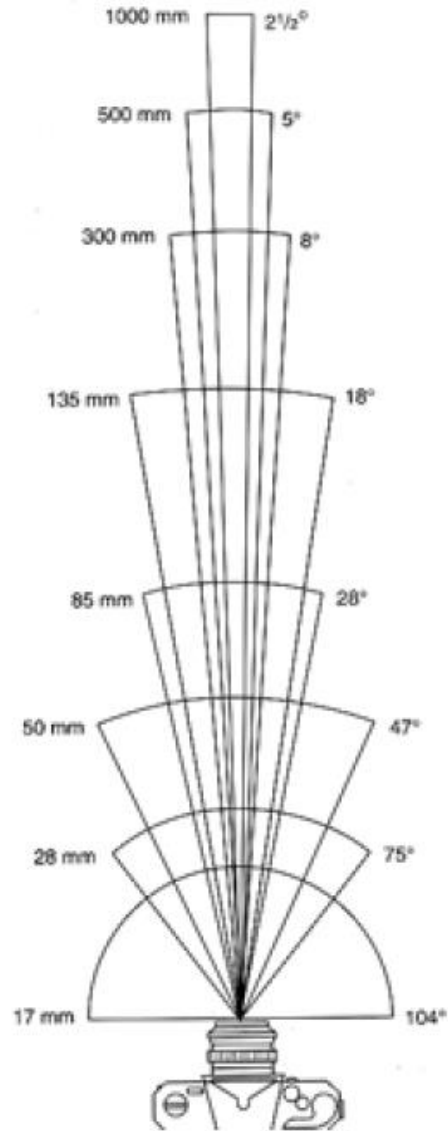
Allow camera rotation

$$\mathbf{x} = \mathbf{K}[\mathbf{R} \quad \mathbf{t}] \mathbf{X}$$



$$w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Field of View (Zoom, focal length)



17mm



28mm



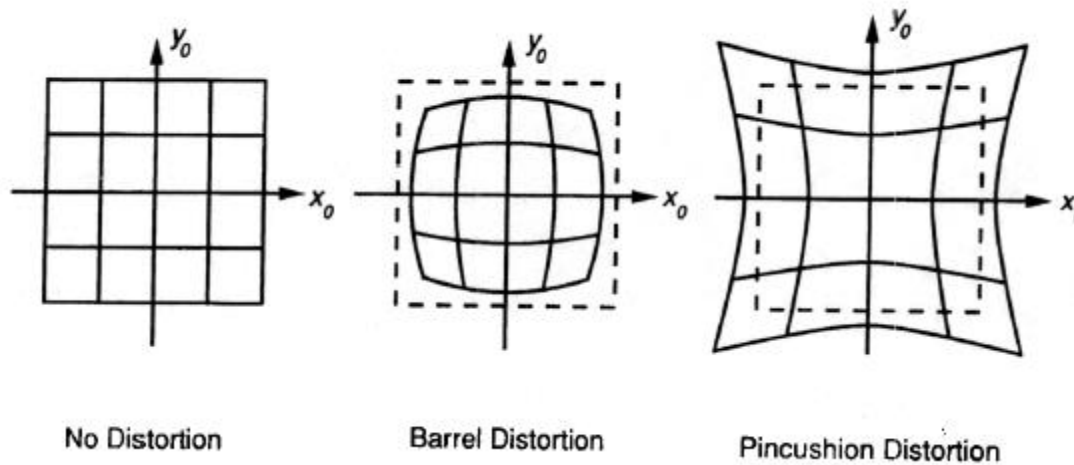
50mm



85mm

From London and Upton

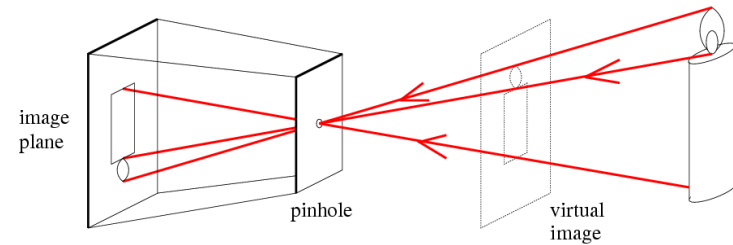
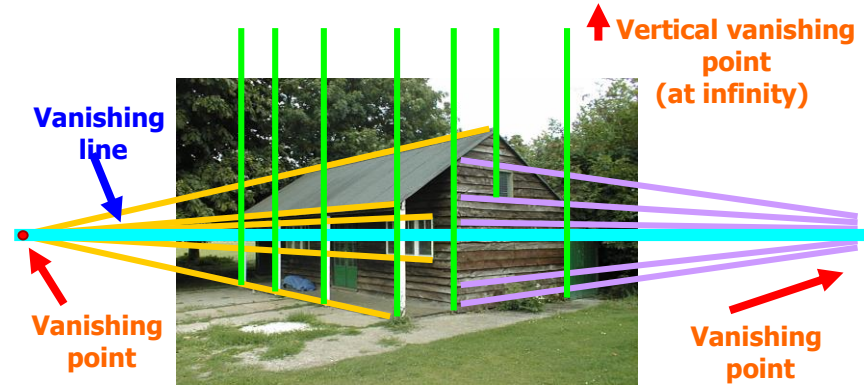
Beyond Pinholes: Radial Distortion



Corrected Barrel Distortion

Things to remember

- Vanishing points and vanishing lines
- Pinhole camera model and camera projection matrix
- Homogeneous coordinates



$$\mathbf{x} = \mathbf{K}[\mathbf{R} \quad \mathbf{t}] \mathbf{X}$$

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Reminder: read your book

- Lectures have assigned readings
- Szeliski 2.1 and especially 2.1.4 cover the geometry of image formation

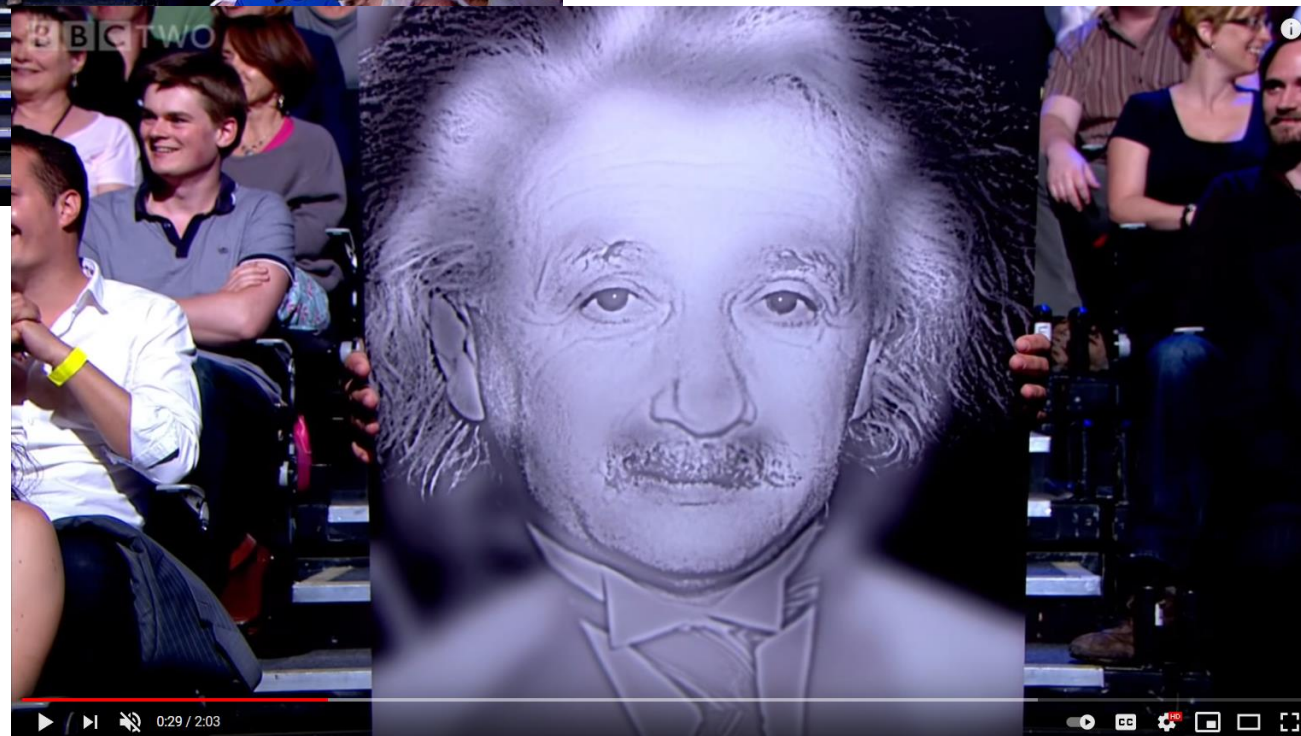
Image Filtering



Computer Vision

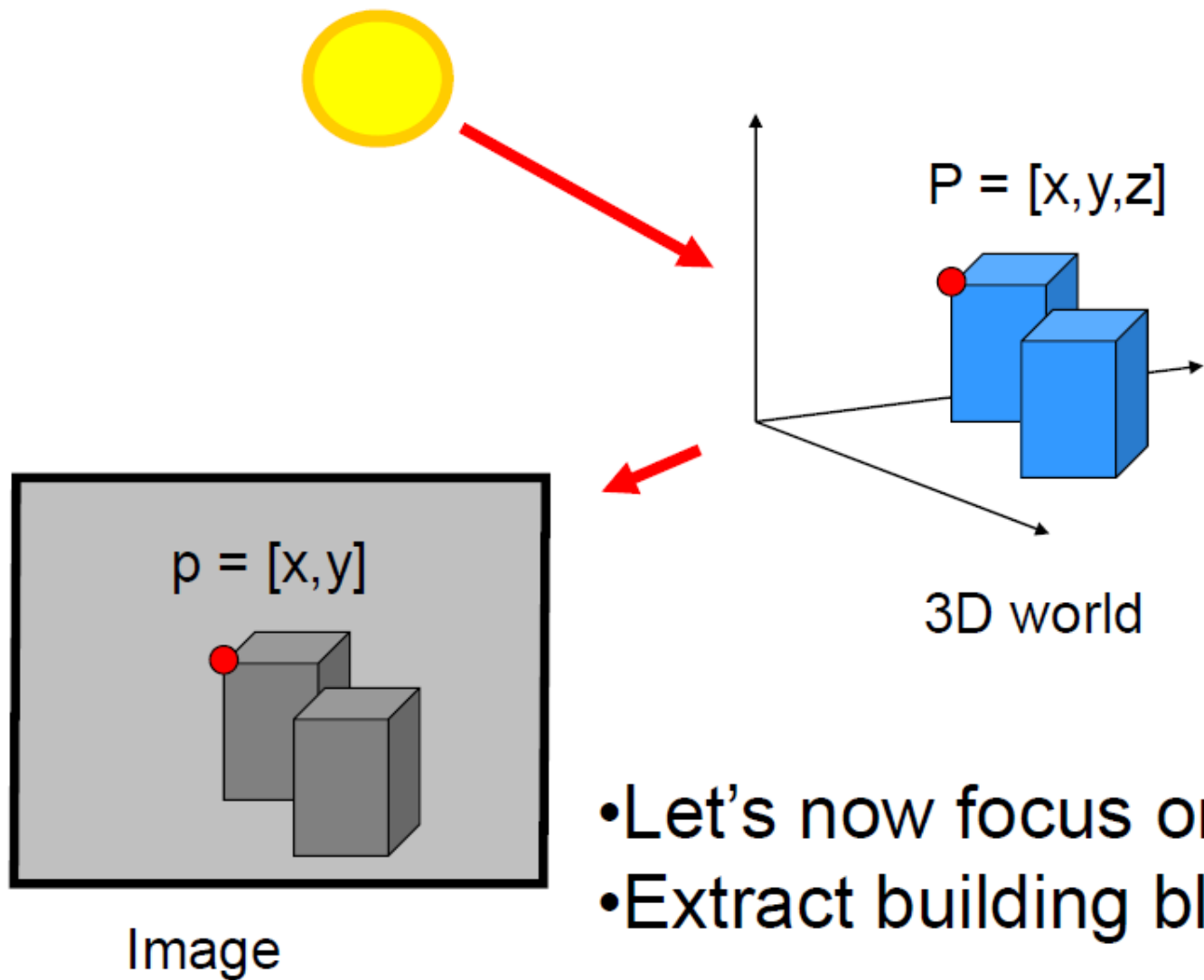
James Hays

Many slides by Derek Hoiem



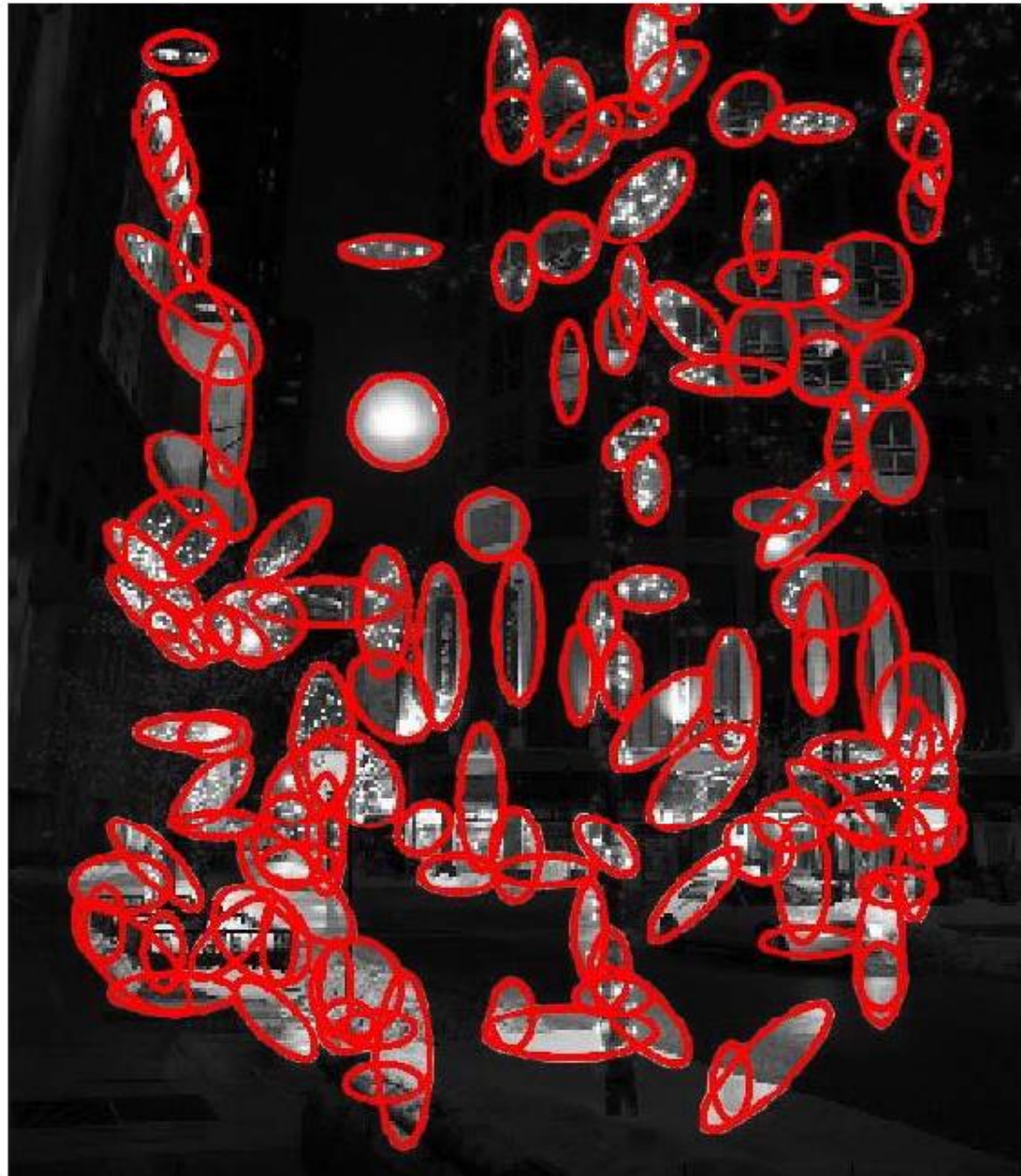
BBC Clip: <https://www.youtube.com/watch/OlumoQ05gS8>

From the 3D to 2D

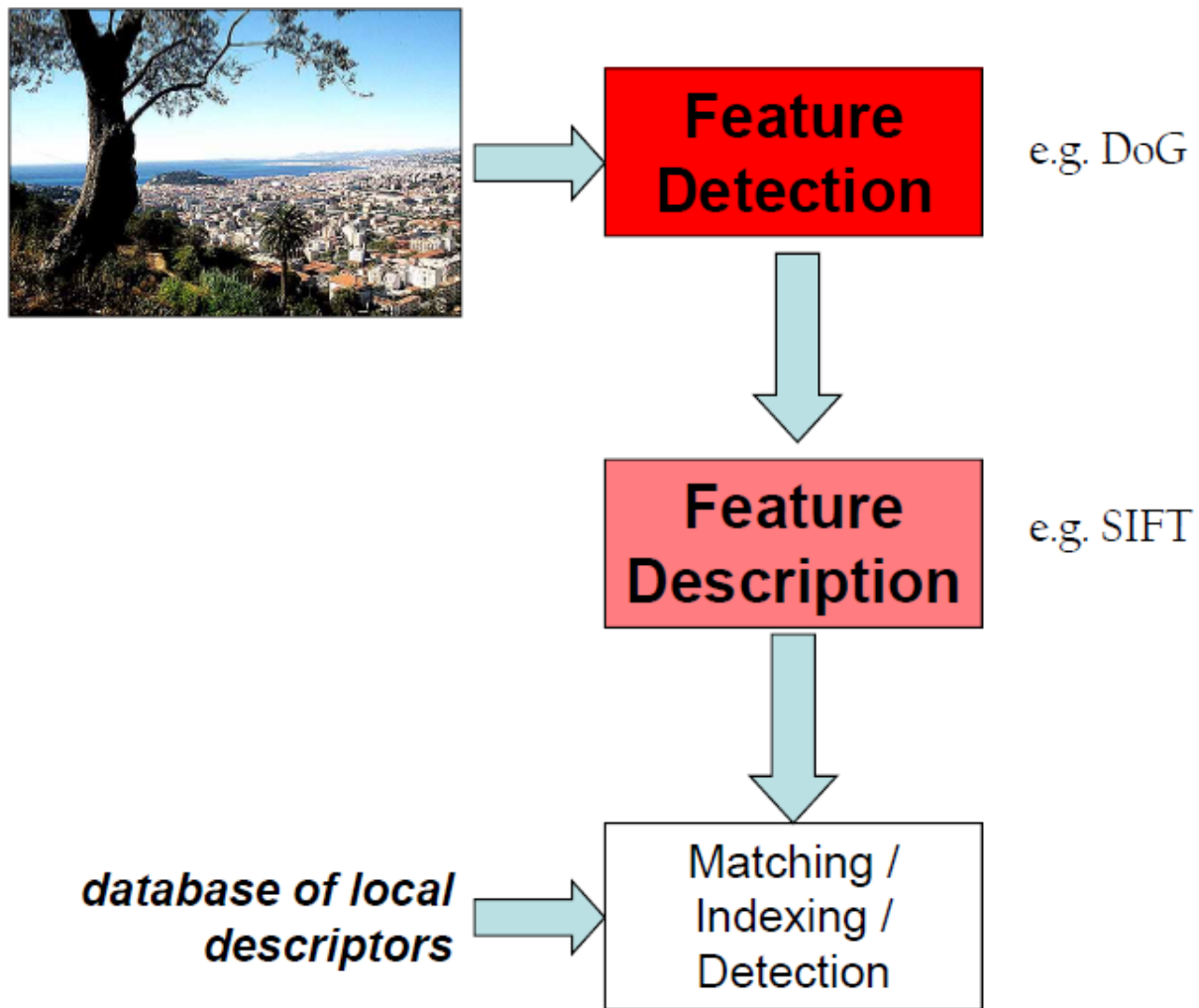


- Let's now focus on 2D
- Extract building blocks

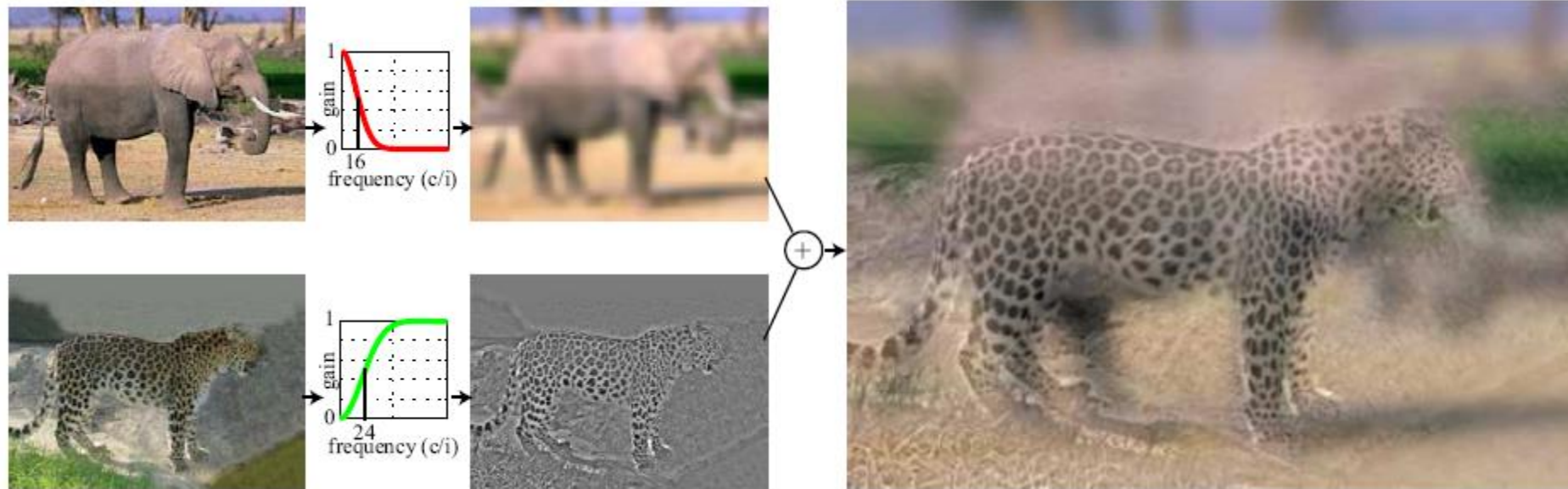
Extract useful building blocks



The big picture...



Hybrid Images



- A. Oliva, A. Torralba, P.G. Schyns, [“Hybrid Images,”](#) SIGGRAPH 2006

Upcoming classes: two views of filtering

- Image filters in spatial domain
 - Filter is a mathematical operation of a grid of numbers
 - Smoothing, sharpening, measuring texture
- Image filters in the frequency domain
 - Filtering is a way to modify the frequencies of images
 - Denoising, sampling, image compression

Image filtering

- Image filtering: compute function of local neighborhood at each position
- Really important!
 - Enhance images
 - Denoise, resize, increase contrast, etc.
 - Extract information from images
 - Texture, edges, distinctive points, etc.
 - Detect patterns
 - Template matching
 - Deep Convolutional Networks

Example: box filter

$g[\cdot, \cdot]$

	1	1	1
1	1	1	1
9	1	1	1

Image filtering

$$g[\cdot, \cdot] \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$f[\cdot, \cdot]$

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	0	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

$h[\cdot, \cdot]$

$$h[m, n] = \sum_{k, l} g[k, l] f[m + k, n + l]$$

Image filtering

$$g[\cdot, \cdot] \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$f[\cdot, \cdot]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[\cdot, \cdot]$

	0	10							

$$h[m, n] = \sum_{k, l} g[k, l] f[m + k, n + l]$$

Image filtering

$$g[\cdot, \cdot] \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$f[\cdot, \cdot]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[\cdot, \cdot]$

	0	10	20						

$$h[m, n] = \sum_{k, l} g[k, l] f[m + k, n + l]$$

Image filtering

$$g[\cdot, \cdot] \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$f[\cdot, \cdot]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[\cdot, \cdot]$

	0	10	20	30					

$$h[m, n] = \sum_{k, l} g[k, l] f[m + k, n + l]$$

Image filtering

$$g[\cdot, \cdot] \frac{1}{9}$$

1	1	1
1	1	1
1	1	1

$f[\cdot, \cdot]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[\cdot, \cdot]$

	0	10	20	30	30				

$$h[m, n] = \sum_{k, l} g[k, l] f[m + k, n + l]$$

Image filtering

$$g[\cdot, \cdot] \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$f[\cdot, \cdot]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[\cdot, \cdot]$

	0	10	20	30	30				
							?		
					50				

$$h[m, n] = \sum_{k, l} g[k, l] f[m + k, n + l]$$

Image filtering

$$g[\cdot, \cdot] = \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$f[\cdot, \cdot]$$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$h[\cdot, \cdot]$$

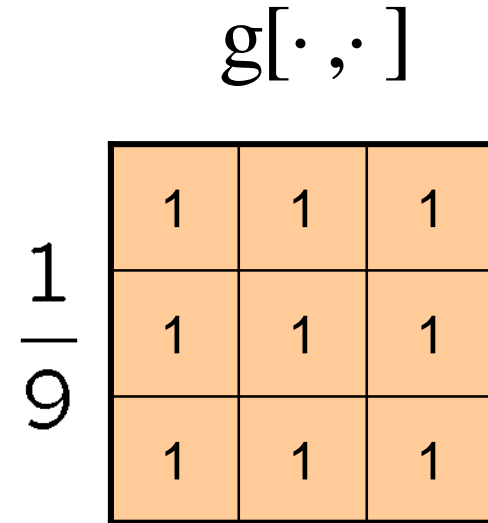
	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

$$h[m, n] = \sum_{k, l} g[k, l] f[m + k, n + l]$$

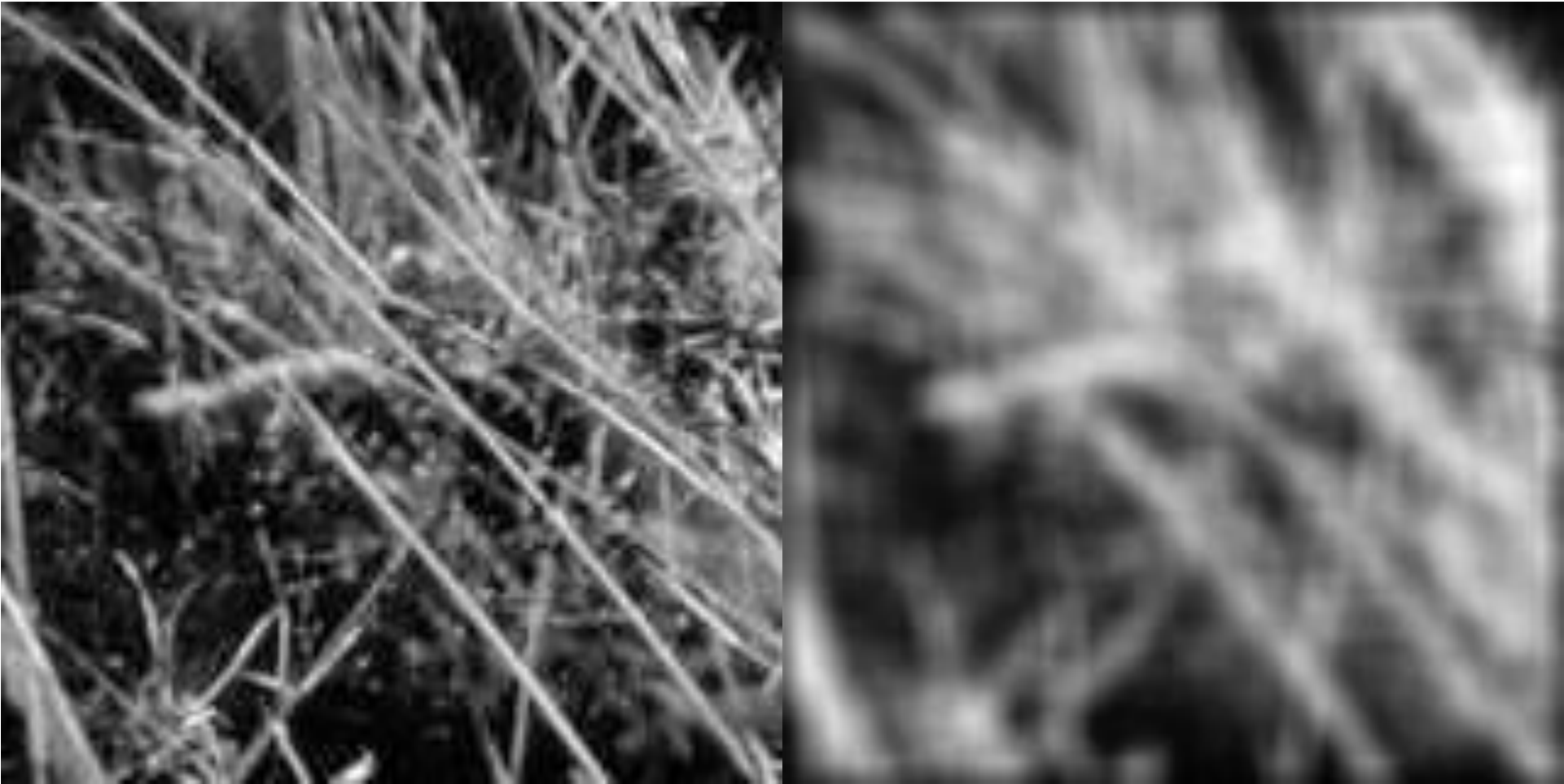
Box Filter

What does it do?

- Replaces each pixel with an average of its neighborhood
- Achieve smoothing effect (remove sharp features)



Smoothing with box filter



Practice with linear filters



Original

0	0	0
0	1	0
0	0	0

?

Practice with linear filters



Original

0	0	0
0	1	0
0	0	0



Filtered
(no change)

Practice with linear filters



Original

0	0	0
0	0	1
0	0	0

?

Practice with linear filters



Original

0	0	0
0	0	1
0	0	0



Shifted left
By 1 pixel

Practice with linear filters



Original

0	0	0
0	2	0
0	0	0

-

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

?

(Note that filter sums to 1)

Practice with linear filters



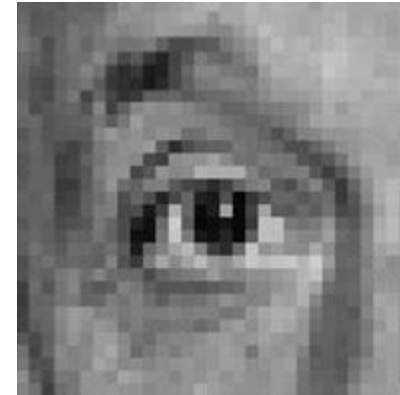
Original

0	0	0
0	2	0
0	0	0

-

$\frac{1}{9}$

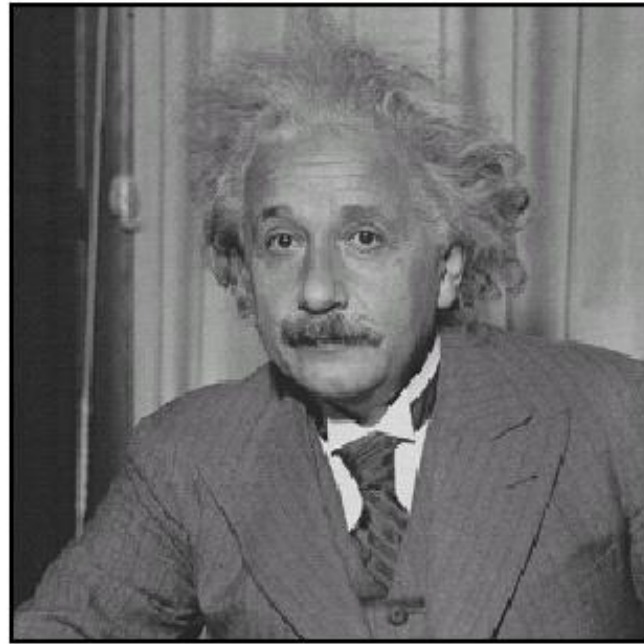
1	1	1
1	1	1
1	1	1



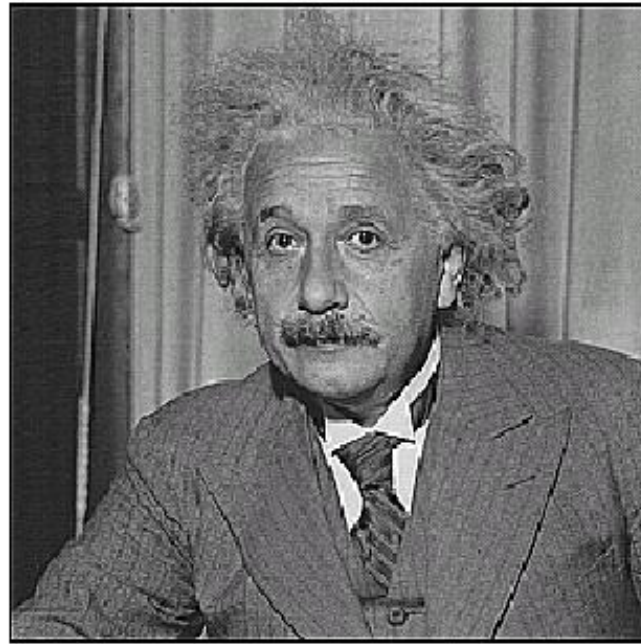
Sharpening filter

- Accentuates differences with local average

Sharpening

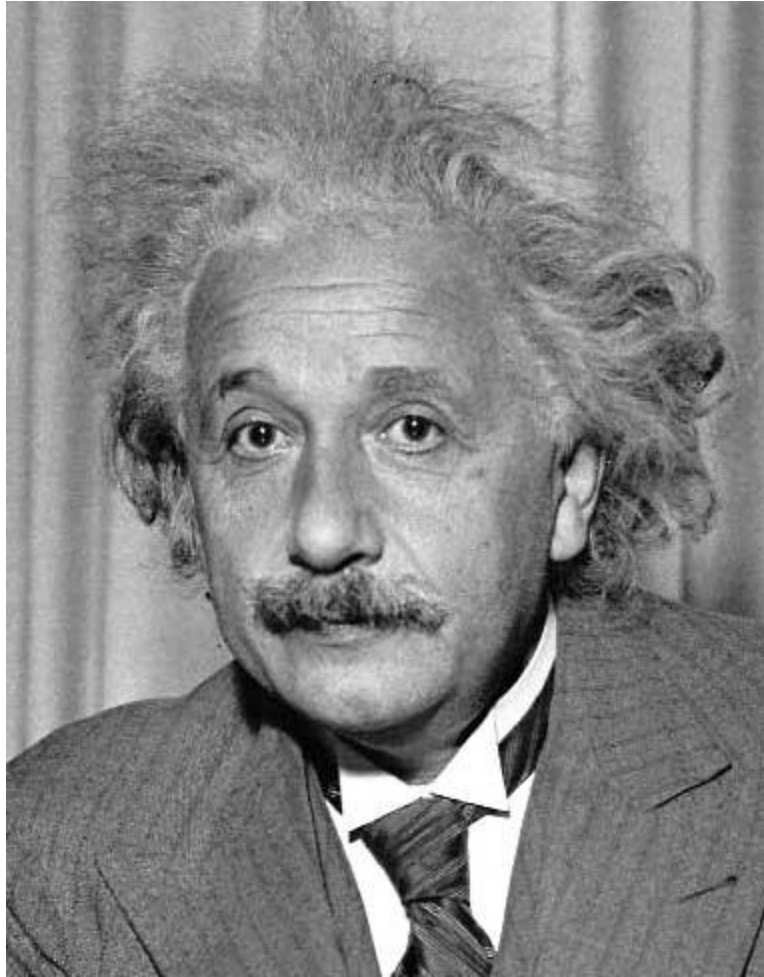


before



after

Other filters



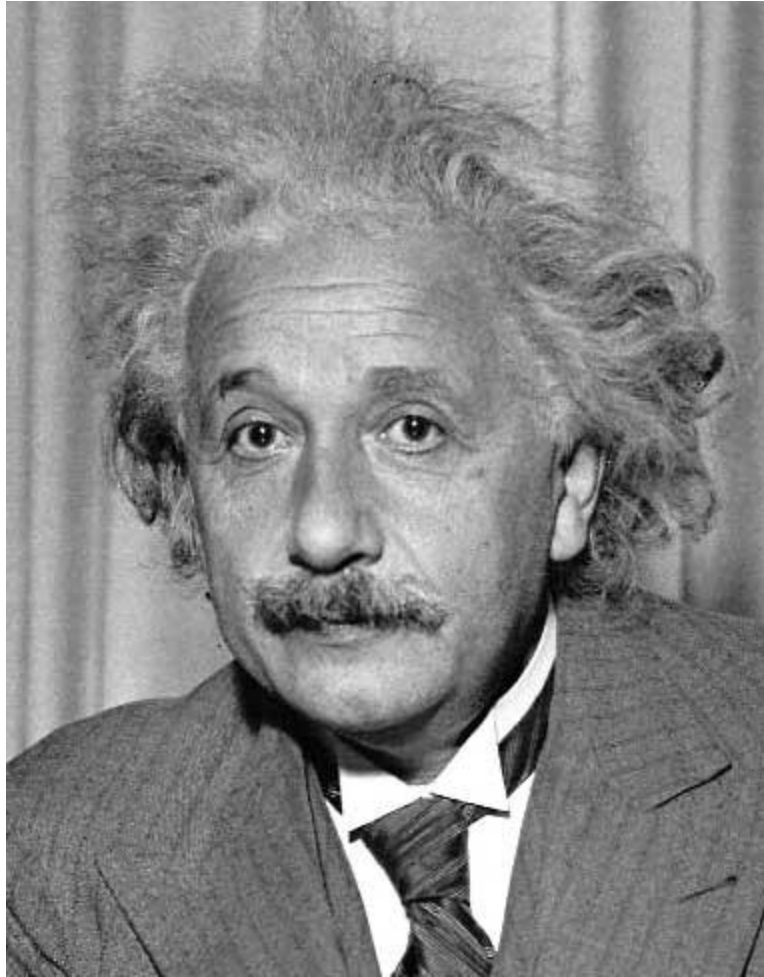
1	0	-1
2	0	-2
1	0	-1

Sobel



Vertical Edge
(absolute value)

Other filters



1	2	1
0	0	0
-1	-2	-1

Sobel



Horizontal Edge
(absolute value)

Filtering vs. Convolution

f=filter, size k x l

I=image, size m x n

- 2d filtering

$$h[m,n] = \sum_{k,l} f[k,l] I[m+k,n+l]$$

- 2d convolution

$$h[m,n] = \sum_{k,l} f[k,l] I[m-k,n-l]$$

Key properties of linear filters

Linearity:

$$\text{imfilter}(I, f_1 + f_2) = \text{imfilter}(I, f_1) + \text{imfilter}(I, f_2)$$

Shift invariance: same behavior regardless of pixel location

$$\text{imfilter}(I, \text{shift}(f)) = \text{shift}(\text{imfilter}(I, f))$$

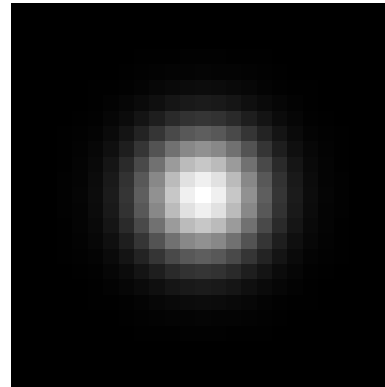
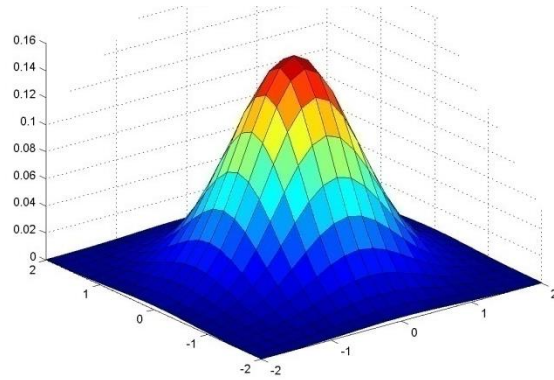
Any linear, shift-invariant operator can be represented as a convolution

More properties

- Commutative: $a * b = b * a$
 - Conceptually no difference between filter and signal
 - But particular filtering implementations might break this equality
- Associative: $a * (b * c) = (a * b) * c$
 - Often apply several filters one after another: $((a * b_1) * b_2) * b_3$
 - This is equivalent to applying one filter: $a * (b_1 * b_2 * b_3)$
- Distributes over addition: $a * (b + c) = (a * b) + (a * c)$
- Scalars factor out: $ka * b = a * kb = k(a * b)$
- Identity: unit impulse $e = [0, 0, 1, 0, 0]$,
 $a * e = a$

Important filter: Gaussian

- Weight contributions of neighboring pixels by nearness

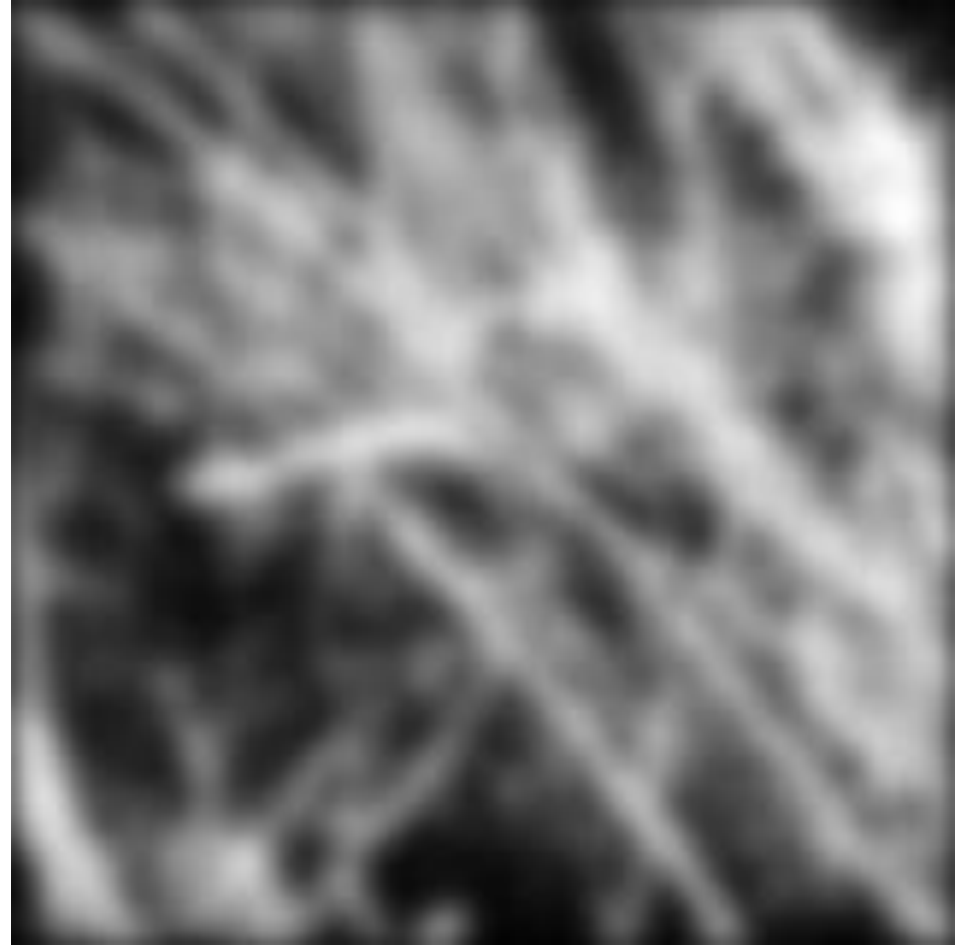


0.003	0.013	0.022	0.013	0.003
0.013	0.059	0.097	0.059	0.013
0.022	0.097	0.159	0.097	0.022
0.013	0.059	0.097	0.059	0.013
0.003	0.013	0.022	0.013	0.003

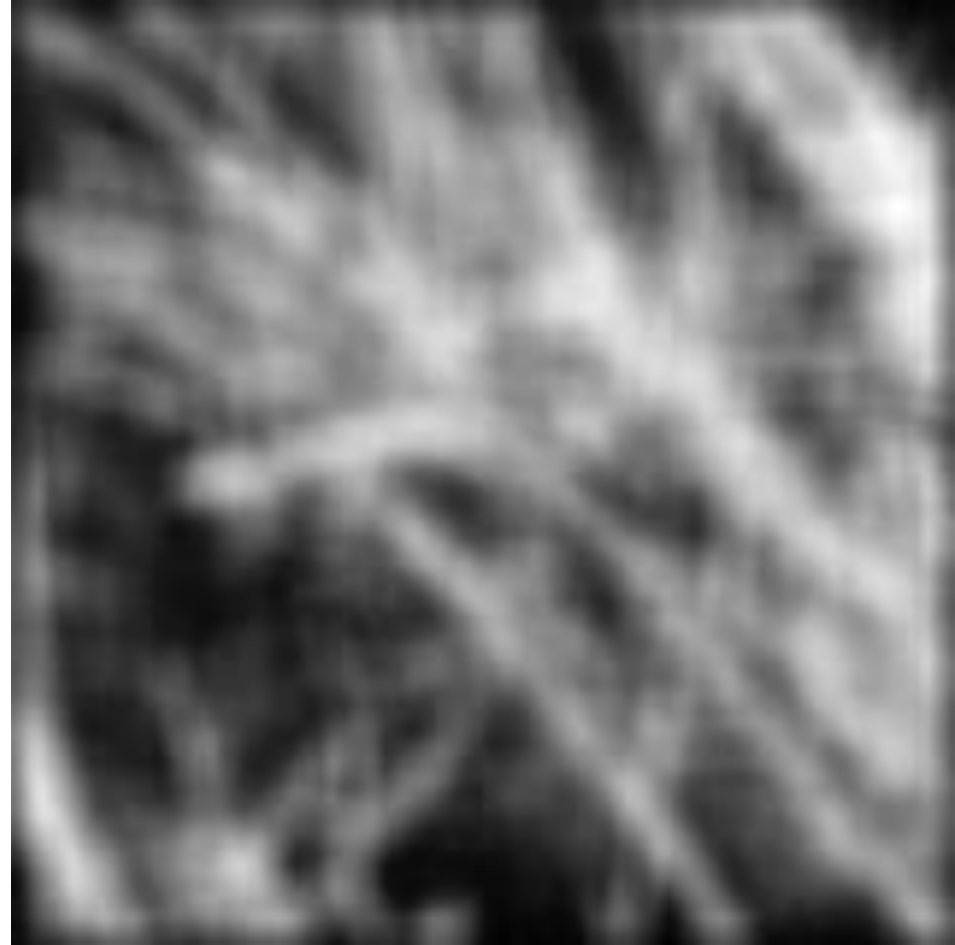
5 x 5, $\sigma = 1$

$$G_{\sigma} = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

Smoothing with Gaussian filter



Smoothing with box filter



Gaussian filters

- Remove “high-frequency” components from the image (low-pass filter)
 - Images become more smooth
- Convolution with self is another Gaussian
 - So can smooth with small-width kernel, repeat, and get same result as larger-width kernel would have
 - Convoluting two times with Gaussian kernel of width σ is same as convoluting once with kernel of width $\sigma\sqrt{2}$
- *Separable* kernel
 - Factors into product of two 1D Gaussians

Separability of the Gaussian filter

$$\begin{aligned} G_{\sigma}(x, y) &= \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right) \\ &= \left(\frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right)\right) \left(\frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{y^2}{2\sigma^2}\right)\right) \end{aligned}$$

The 2D Gaussian can be expressed as the product of two functions, one a function of x and the other a function of y

In this case, the two functions are the (identical) 1D Gaussian

Separability example

2D convolution
(center location only)

$$\begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array} * \begin{array}{|c|c|c|} \hline 2 & 3 & 3 \\ \hline 3 & 5 & 5 \\ \hline 4 & 4 & 6 \\ \hline \end{array}$$

The filter factors
into a product of 1D
filters:

$$\begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array} = \begin{array}{|c|} \hline 1 \\ \hline 2 \\ \hline 1 \\ \hline \end{array} \times \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline \end{array}$$

Perform convolution
along rows:

$$\begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline \end{array} * \begin{array}{|c|c|c|} \hline 2 & 3 & 3 \\ \hline 3 & 5 & 5 \\ \hline 4 & 4 & 6 \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline & 11 & \\ \hline & 18 & \\ \hline & 18 & \\ \hline \end{array}$$

Followed by convolution
along the remaining column:

Separability

- Why is separability useful in practice?

Some practical matters

Practical matters

How big should the filter be?

- Values at edges should be near zero
- Rule of thumb for Gaussian: set filter half-width to about 3σ

Practical matters

- What about near the edge?
 - the filter window falls off the edge of the image
 - need to extrapolate
 - methods:
 - clip filter (black)
 - wrap around
 - copy edge
 - reflect across edge



To be continued...

Next class: Light and Color and Thinking in Frequency

