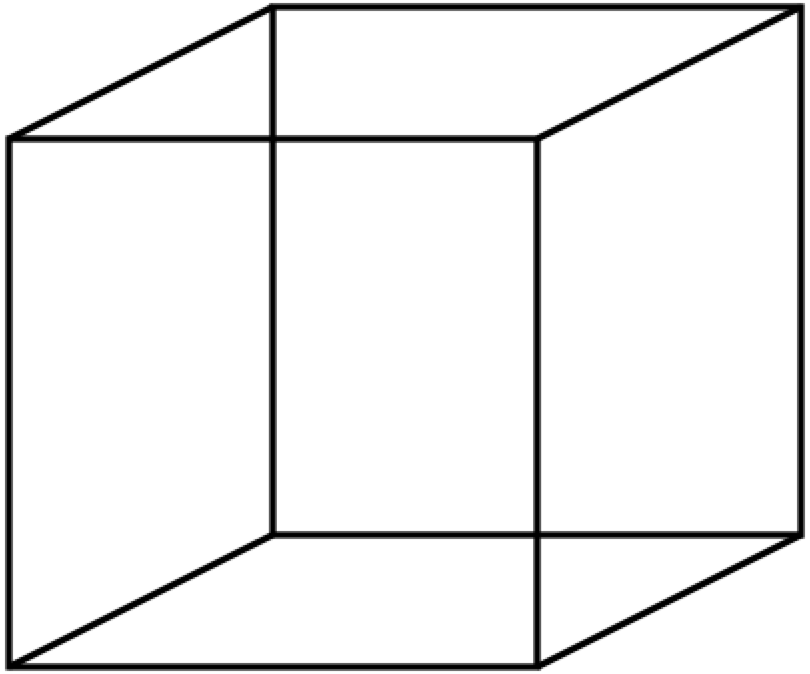
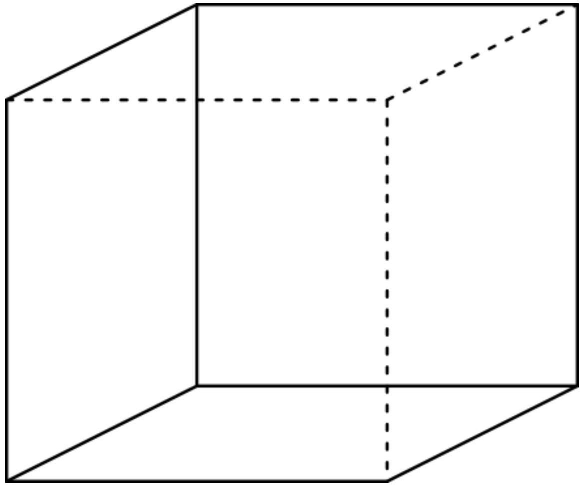
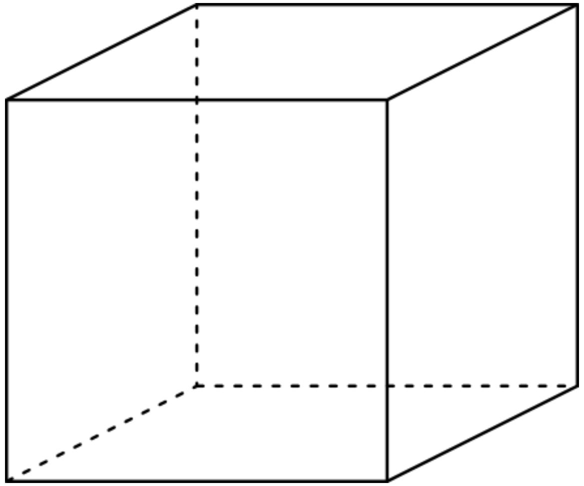
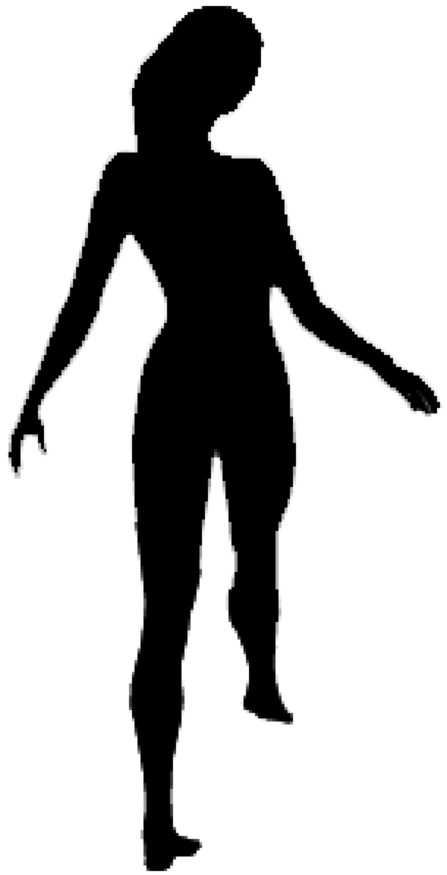
Multi-stable Perception



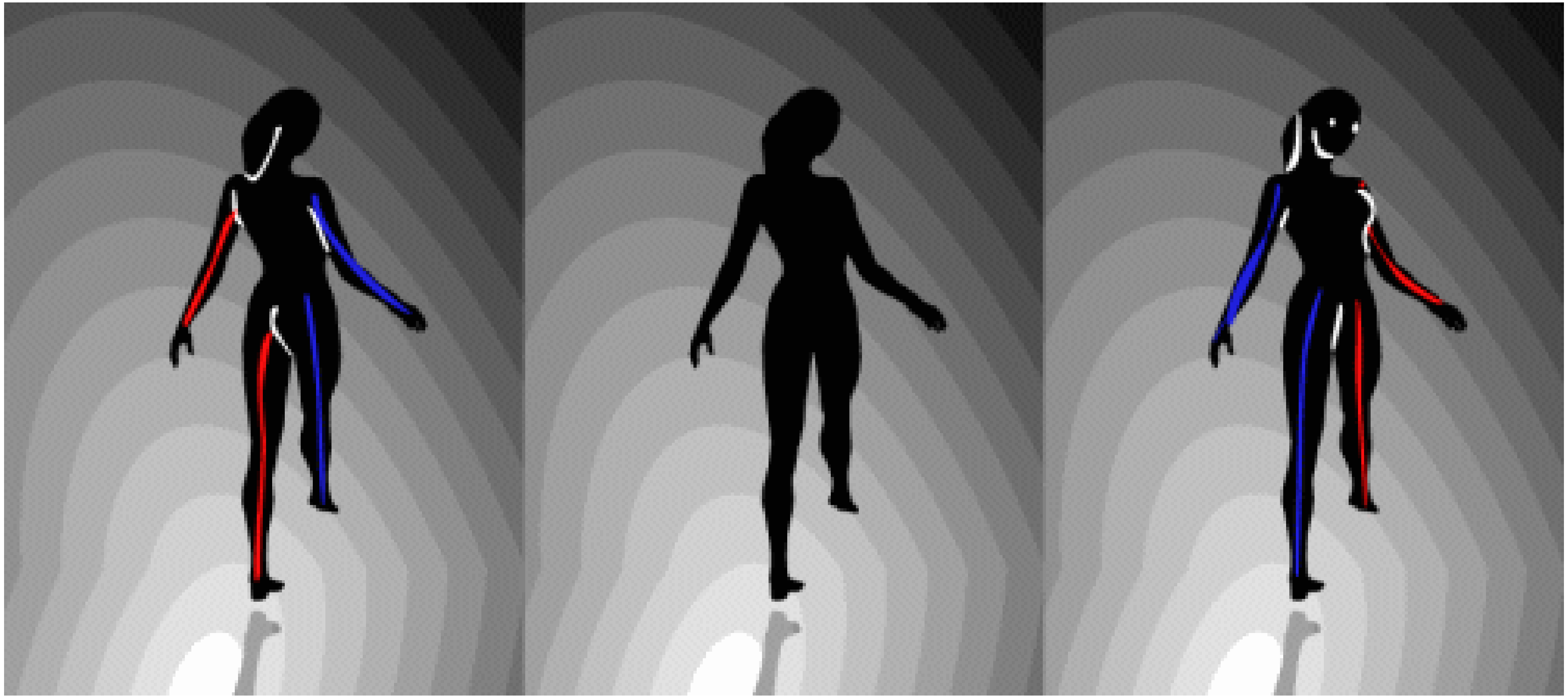
Necker Cube







Spinning dancer illusion, Nobuyuki Kayahara



Feature Matching and Robust Fitting

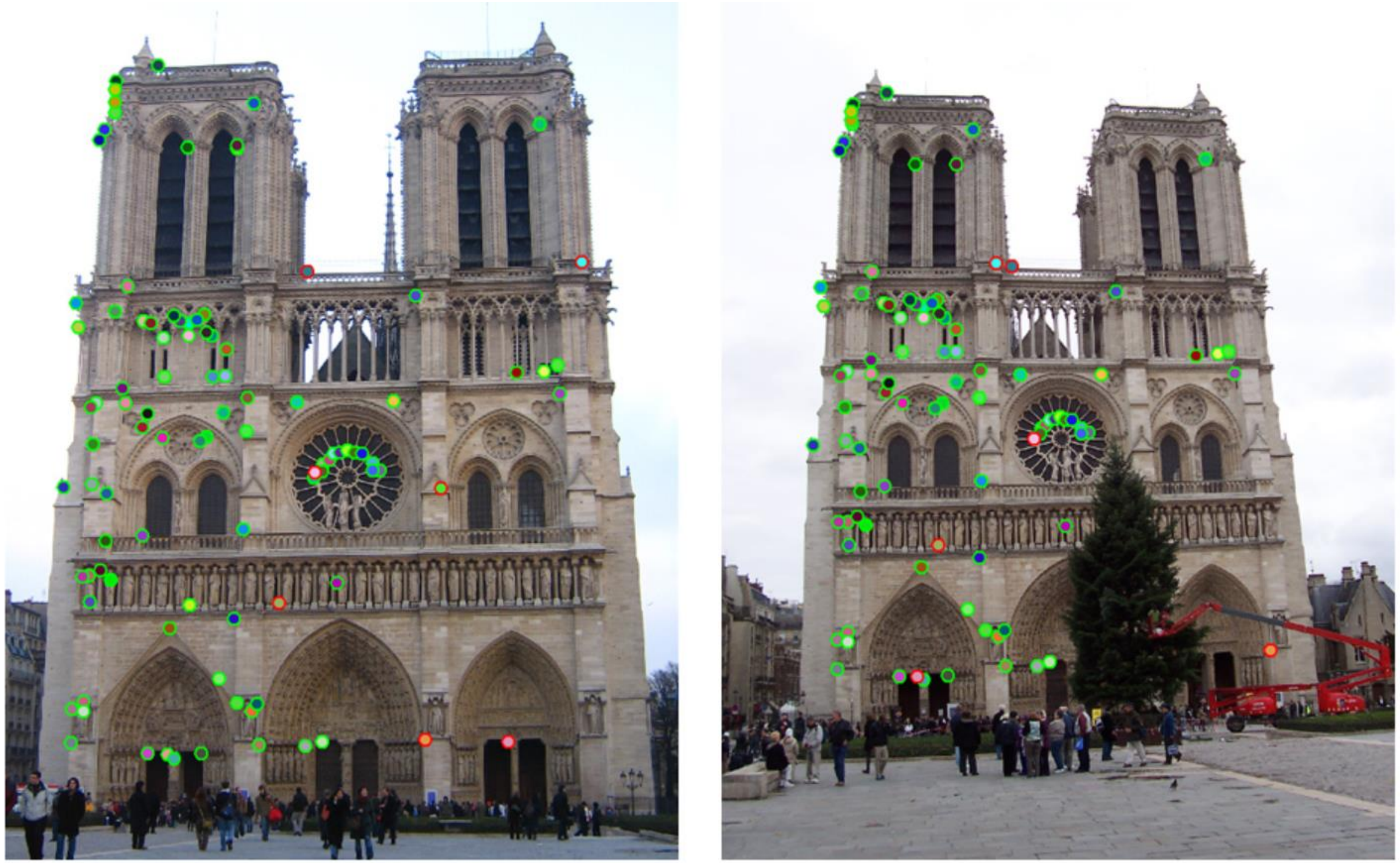
Read Szeliski 7.4.2 and 2.1

Computer Vision

James Hays

Acknowledgment: Many slides from Derek Hoiem and Grauman&Leibe 2008 AAAI Tutorial

Project 2

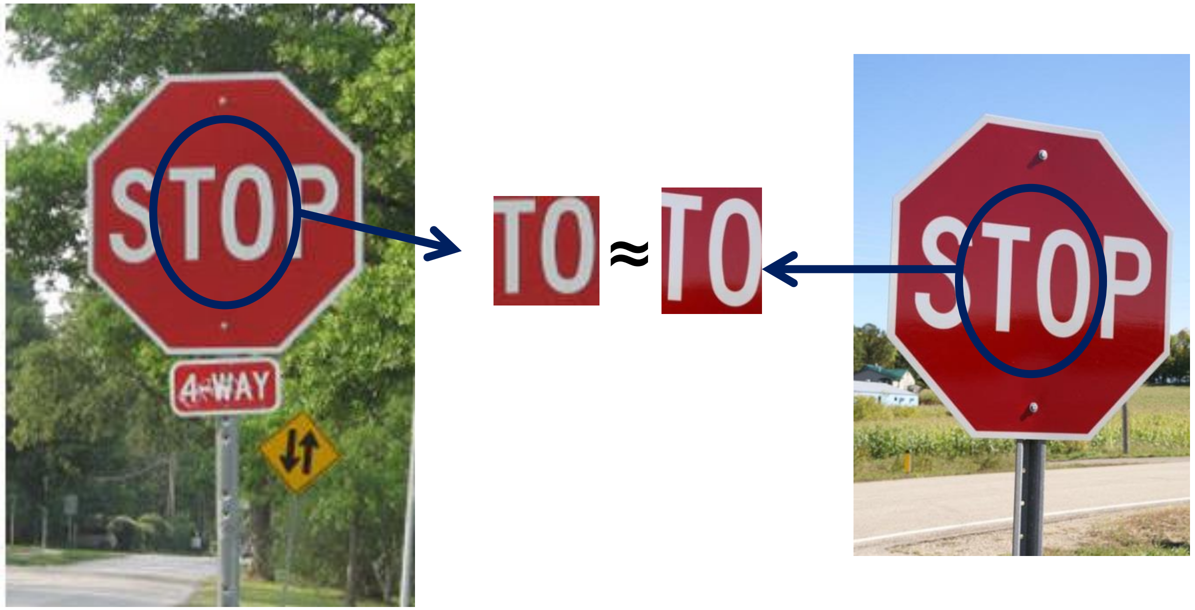


The top 100 most confident local feature matches from a baseline implementation of project 2. In this case, 93 were correct (highlighted in green) and 7 were incorrect (highlighted in red).

Project 2: Local Feature Matching

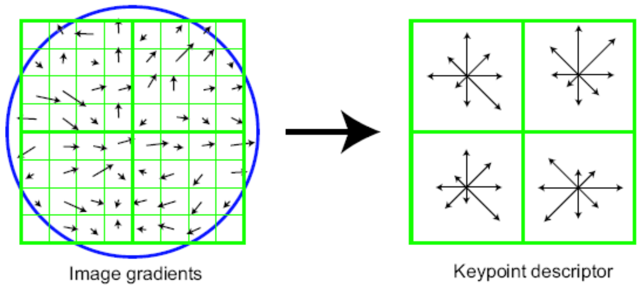
This section: correspondence and alignment

- Correspondence: matching points, patches, edges, or regions across images



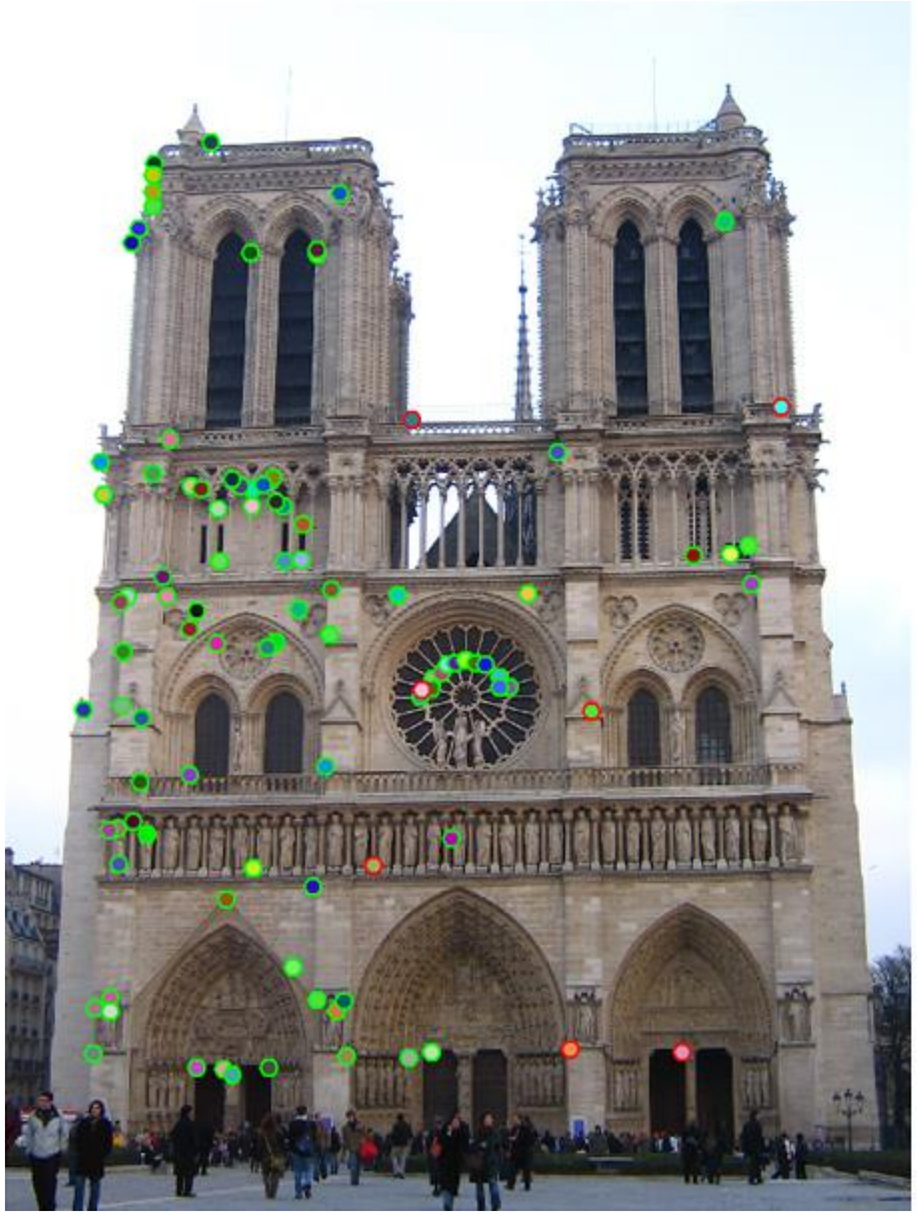
Review: Local Descriptors

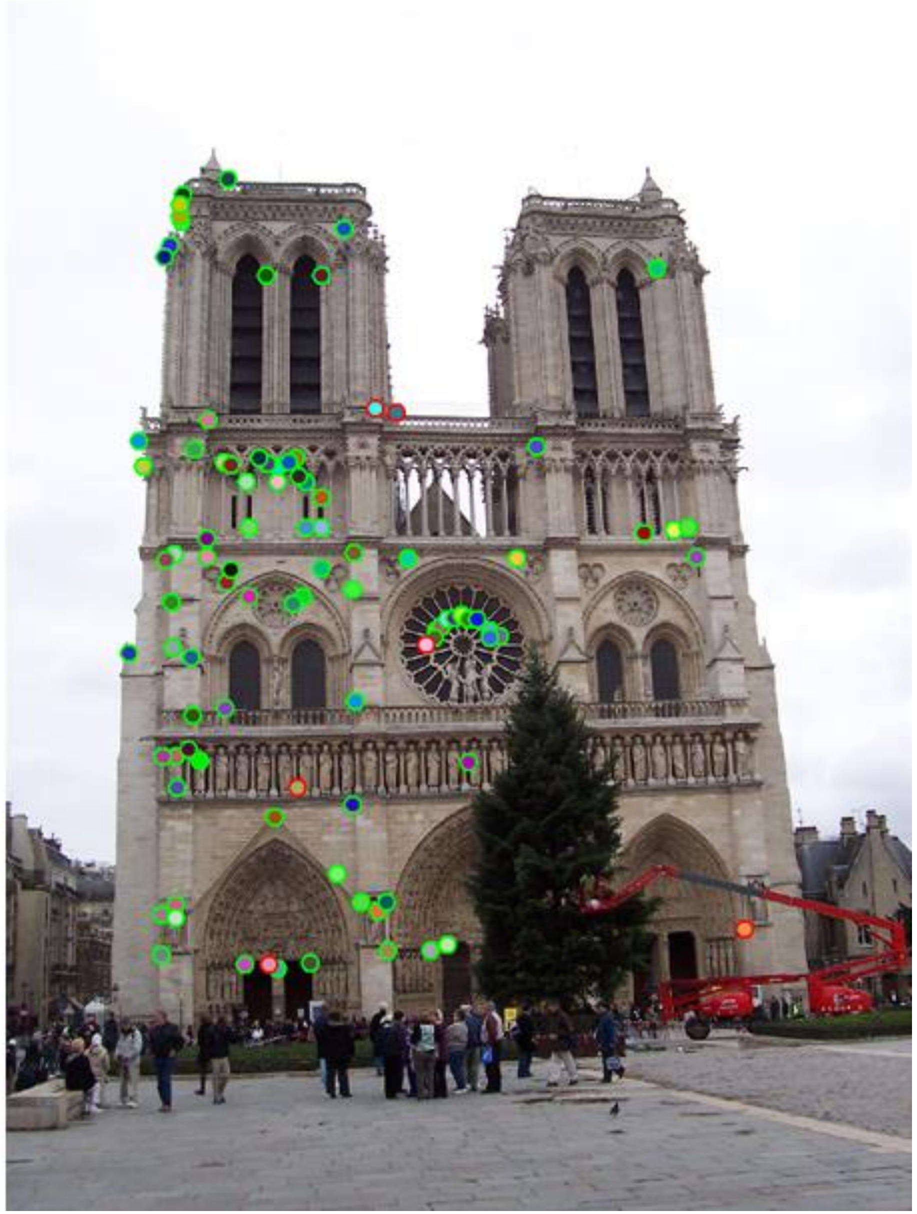
- Most features can be thought of as templates, histograms (counts), or combinations
- The ideal descriptor should be
  – Robust and Distinctive
  – Compact and Efficient



- Most available descriptors focus on edge/gradient information
  – Capture texture information
  – Color rarely used

Can we refine this further?





Fitting: find the parameters of a model that best fit the data

Alignment: find the parameters of the transformation that best align matched points

Fitting and Alignment

- Design challenges
  - Design a suitable **goodness of fit** measure
    - Similarity should reflect application goals
    - Encode robustness to outliers and noise
  - Design an **optimization** method
    - Avoid local optima
    - Find best parameters quickly

Fitting and Alignment: Methods

- Global optimization / Search for parameters
  - Least squares fit
  - Robust least squares
  - Other parameter search methods
- Hypothesize and test
  - Generalized Hough transform
  - RANSAC

Fitting and Alignment: Methods

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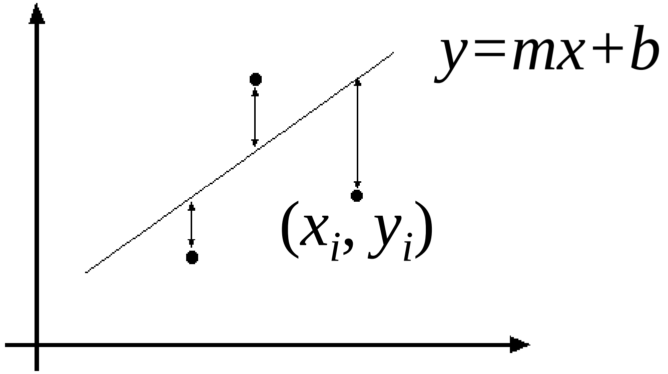
Simple example: Fitting a line

Least squares line fitting

- Data: $(x_1, y_1), \ldots, (x_n, y_n)$
- Line equation: $y_i = m x_i + b$
- Find (m, b) to minimize



$$\boxed{E = \sum_{i=1}^{n} (y_i - m x_i - b)^2}$$

$$E = \sum_{i=1}^{n} \left(\begin{bmatrix} x_i & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} - y_i \right)^2 = \left\| \begin{bmatrix} x_1 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} - \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \right\|^2 = \|\mathbf{Ap} - \mathbf{y}\|^2$$

$$= \mathbf{y}^T \mathbf{y} - 2(\mathbf{Ap})^T \mathbf{y} + (\mathbf{Ap})^T (\mathbf{Ap})$$

$$\frac{dE}{dp} = 2\mathbf{A}^T \mathbf{Ap} - 2\mathbf{A}^T \mathbf{y} = 0$$

$$\mathbf{A}^T \mathbf{Ap} = \mathbf{A}^T \mathbf{y} \Rightarrow \mathbf{p} = \left(\mathbf{A}^T \mathbf{A}\right)^{-1} \mathbf{A}^T \mathbf{y}$$

Matlab: `p = A \ y;`

Python: `p = numpy.linalg.lstsq(A, y)`

Modified from S. Lazebnik

Least squares (global) optimization

Good

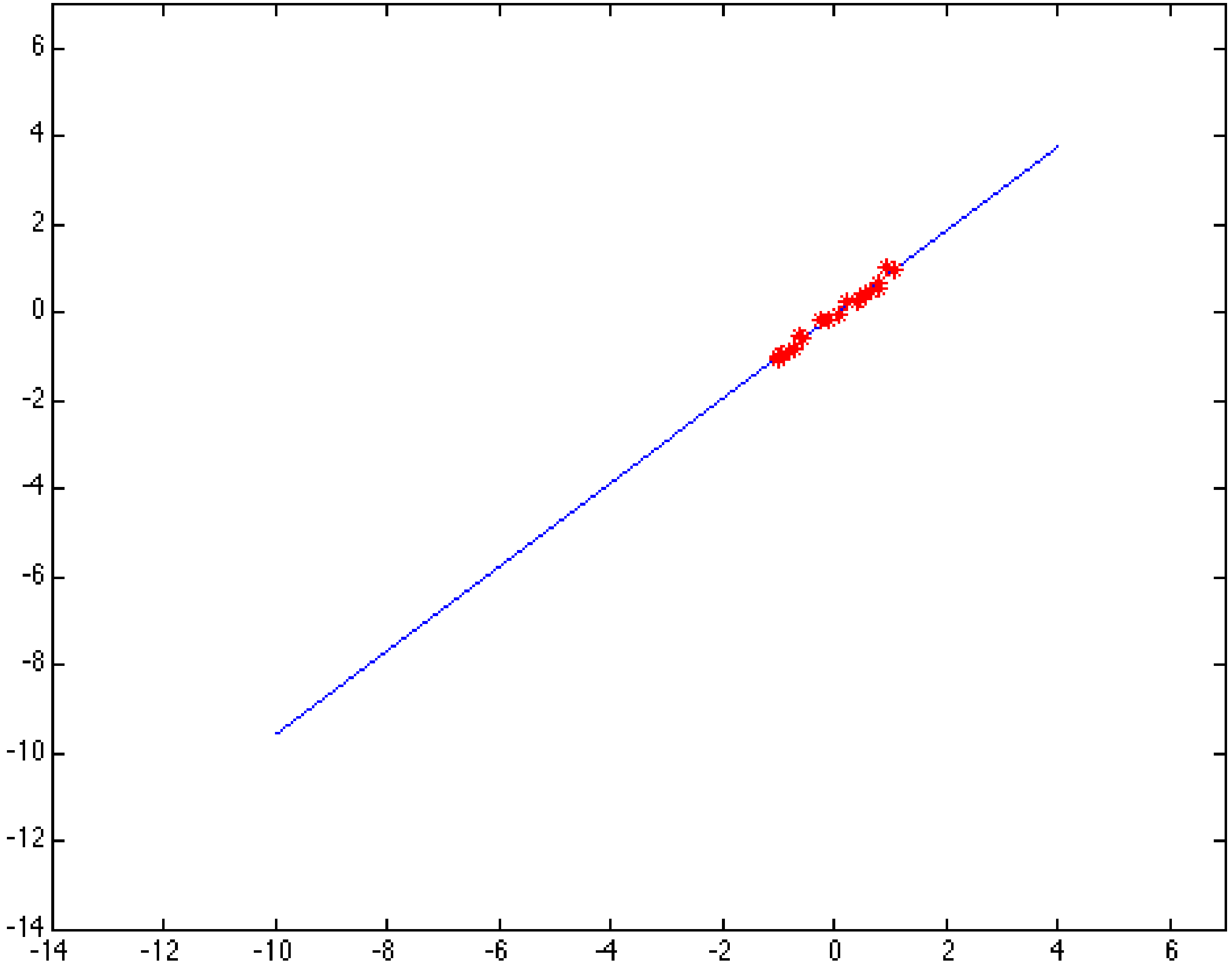
- Clearly specified objective
- Optimization is easy

Bad

- May not be what you want to optimize
- Sensitive to outliers
  - Bad matches, extra points
- Doesn’t allow you to get multiple good fits
  - Detecting multiple objects, lines, etc.

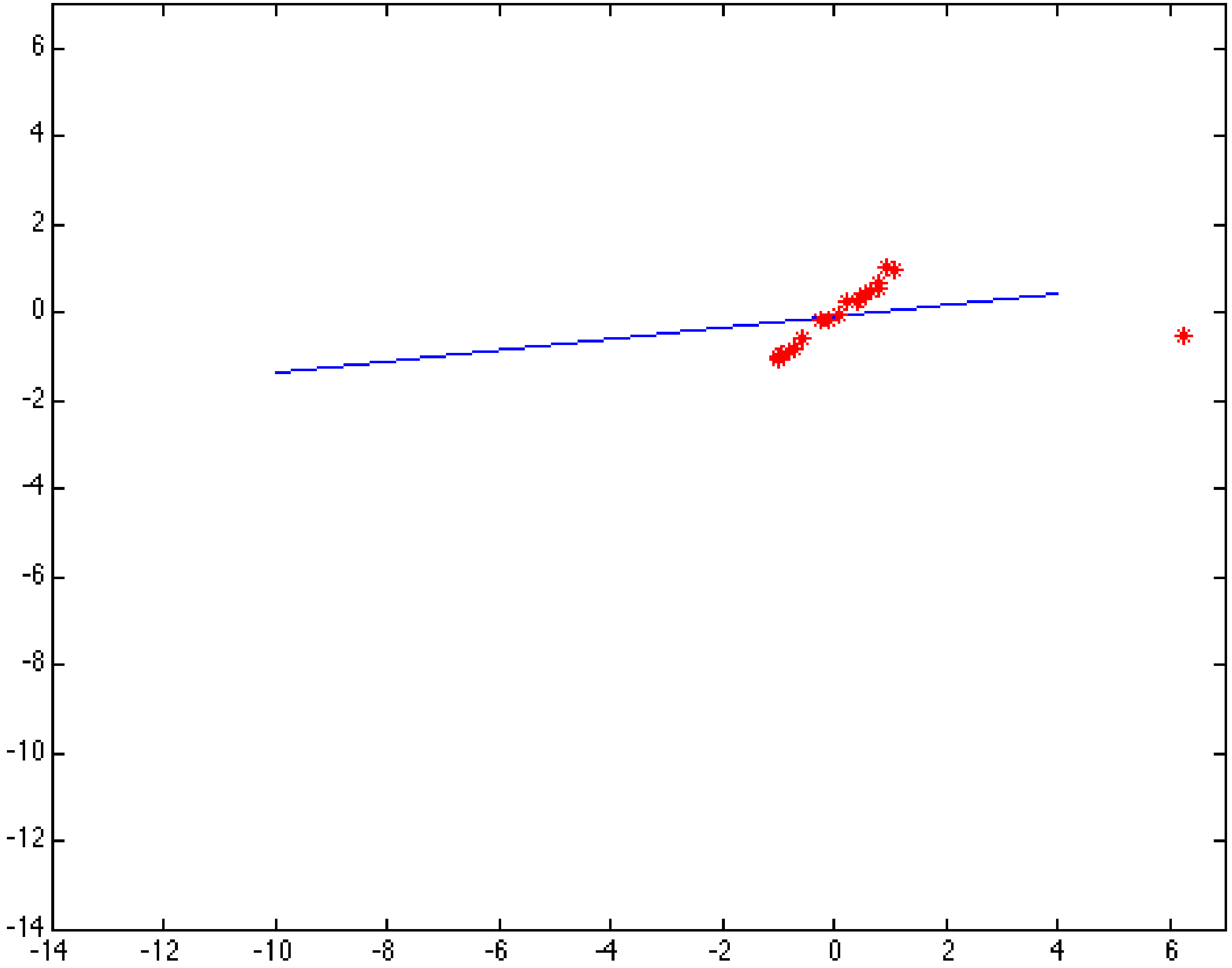
Least squares: Robustness to noise

- Least squares fit to the red points:



Least squares: Robustness to noise

- Least squares fit with an outlier:



Problem: squared error heavily penalizes outliers

Fitting and Alignment: Methods

- Global optimization / Search for parameters
  - Least squares fit
  - Robust least squares
  - Other parameter search methods
- Hypothesize and test
  - Generalized Hough transform
  - RANSAC

Robust least squares (to deal with outliers)

General approach:

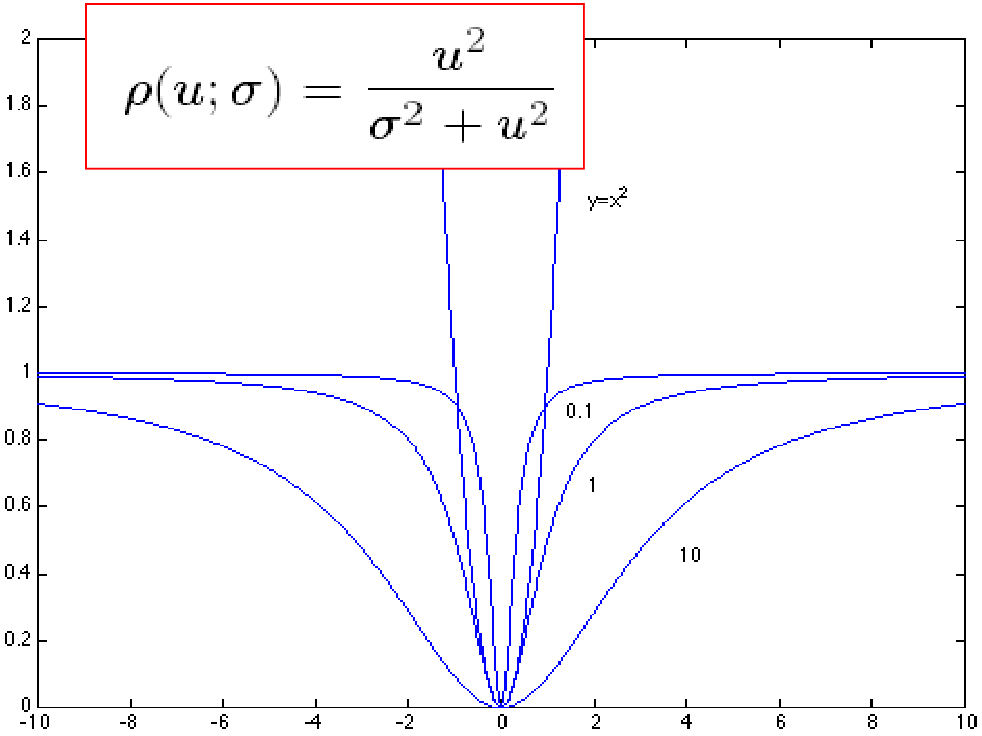
minimize

$$\sum_{\mathrm{i}} \rho\left(\mathrm{u_i}\left(\mathrm{x_i}, \theta\right); \sigma\right) \qquad u^2 = \sum_{i=1}^{n} (y_i - m x_i - b)^2$$

$u_i\,(x_i, \theta)$ – residual of i^th point w.r.t. model parameters ϑ

ρ – robust function with scale parameter σ

$$\rho(u;\sigma) = \frac{u^2}{\sigma^2 + u^2}$$

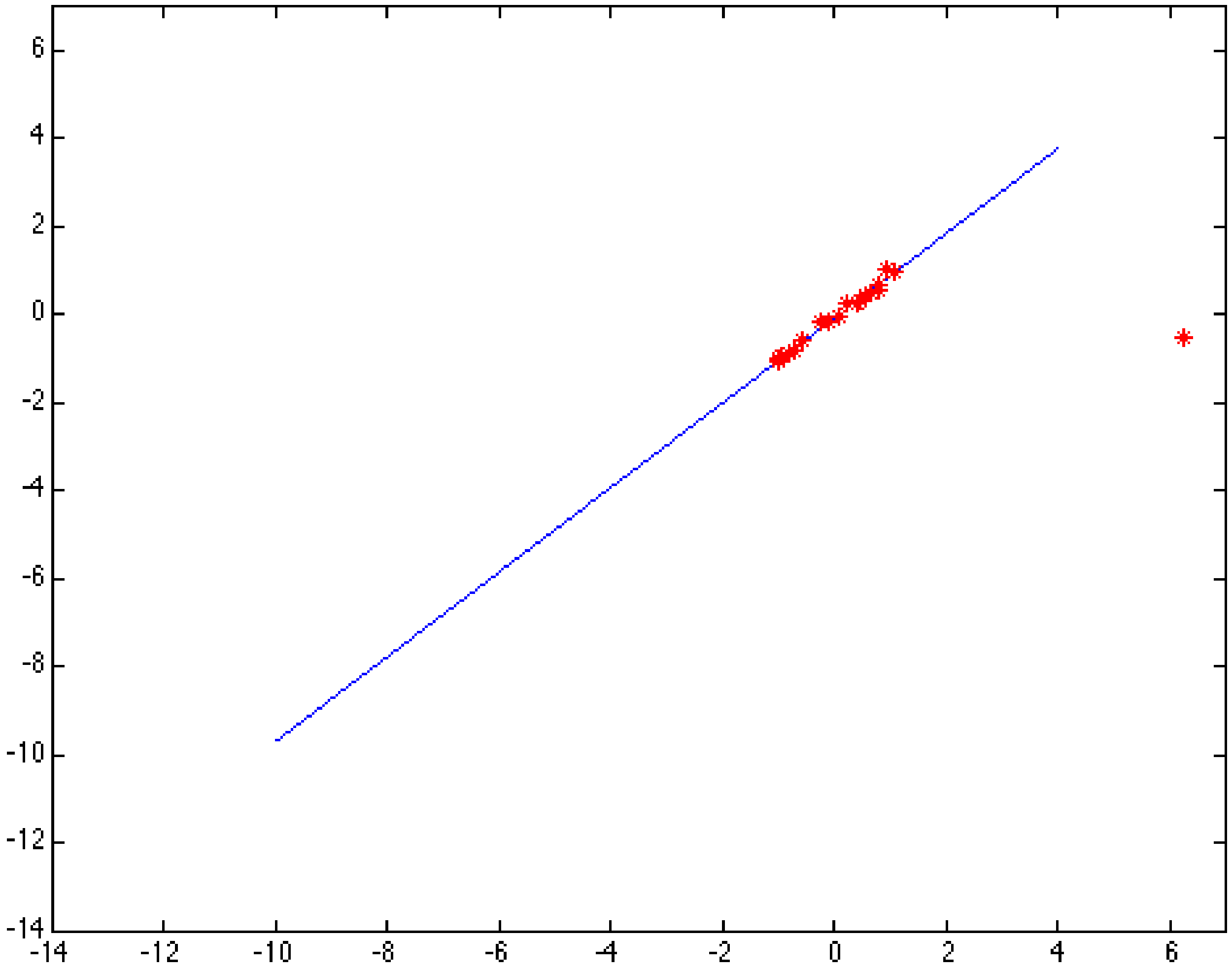


The robust function ρ

- Favors a configuration with small residuals
- Constant penalty for large residuals

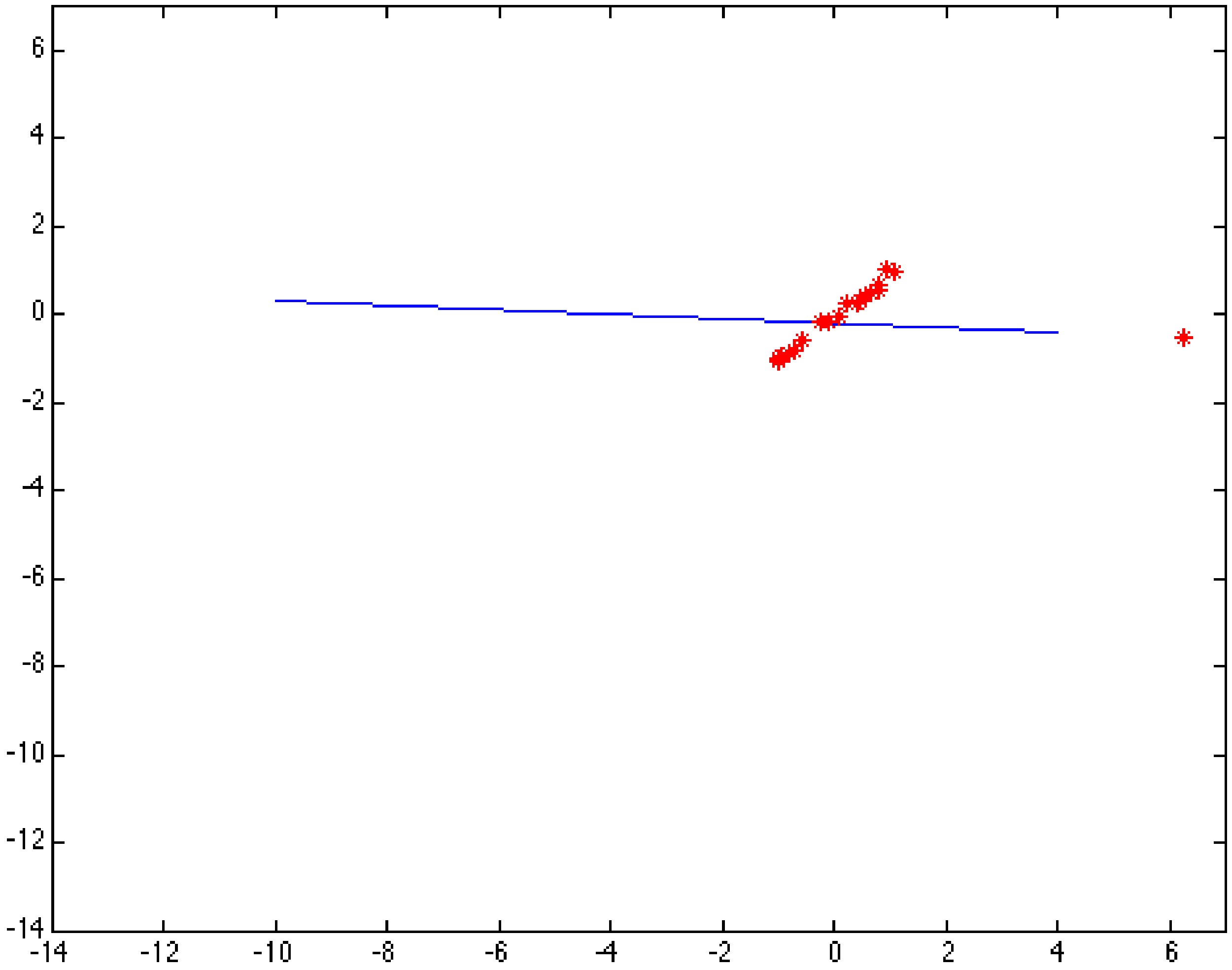
Slide from S. Savarese

Choosing the scale: Just right



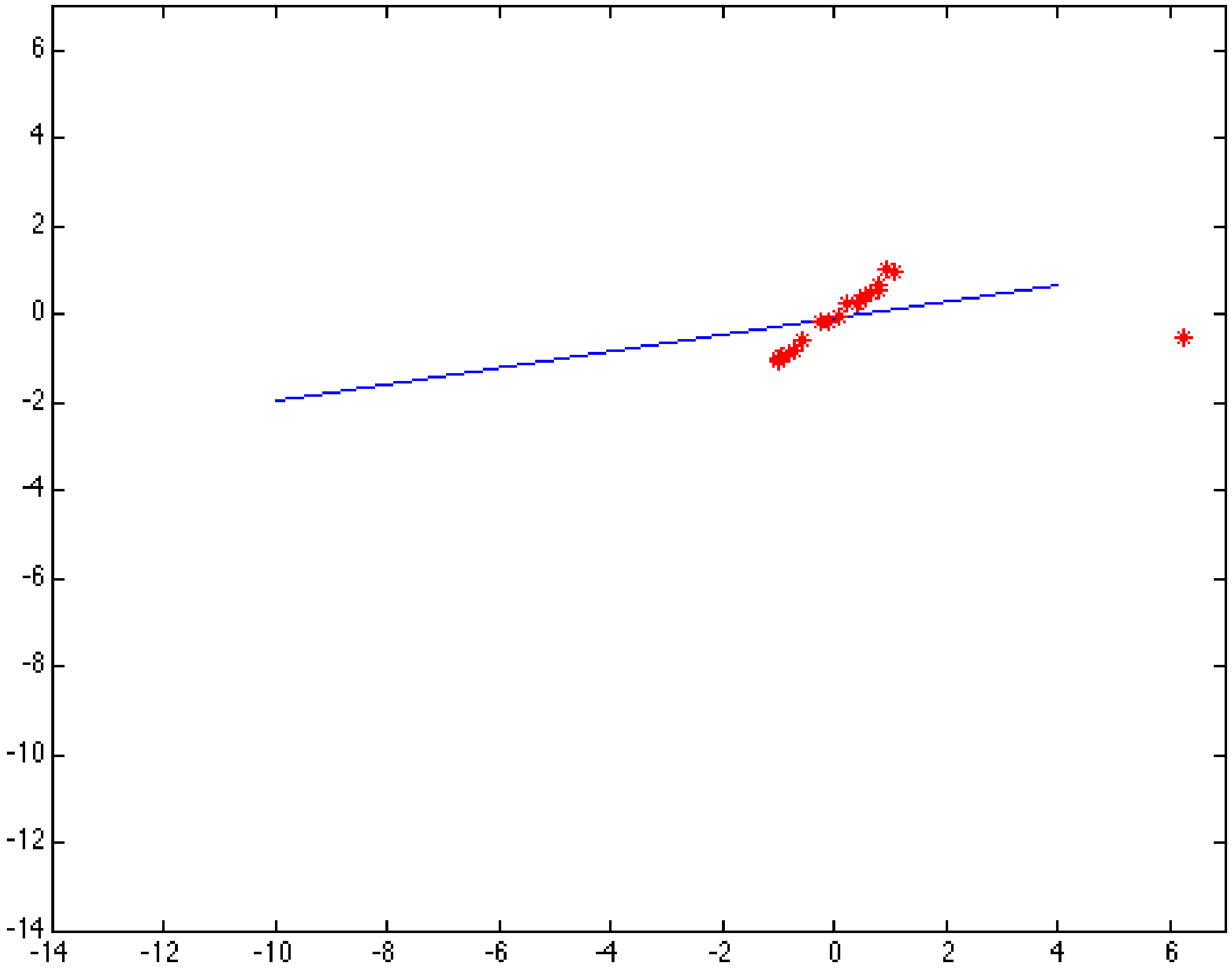
The effect of the outlier is minimized

Choosing the scale: Too small



The error value is almost the same for every point and the fit is very poor

Choosing the scale: Too large



Behaves much the same as least squares

Robust estimation: Details

- Robust fitting is a nonlinear optimization problem that must be solved iteratively
- Least squares solution can be used for initialization
- Scale of robust function should be chosen adaptively based on median residual

Fitting and Alignment: Methods

- Global optimization / Search for parameters
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  - RANSAC

Other ways to search for parameters (for when no closed form solution exists)

- Line search
  1. For each parameter, step through values and choose value that gives best fit
  2. Repeat (1) until no parameter changes

- Grid search
  1. Propose several sets of parameters, evenly sampled in the joint set
  2. Choose best (or top few) and sample joint parameters around the current best; repeat

- Gradient descent
  1. Provide initial position (e.g., random)
  2. Locally search for better parameters by following gradient

Fitting and Alignment: Methods

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Fitting and Alignment: Methods

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Hough Transform: Outline

1. Create a grid of parameter values

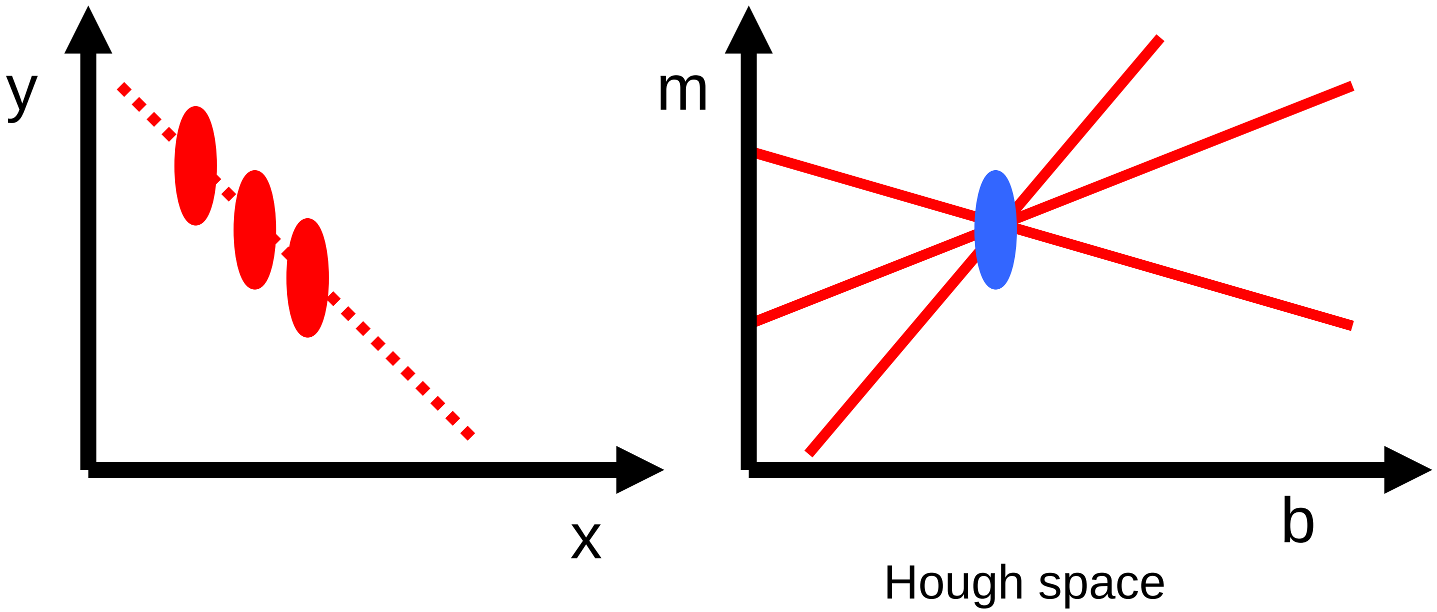
2. Each point votes for a set of parameters, incrementing those values in grid

3. Find maximum or local maxima in grid

Hough transform

P.V.C. Hough, *Machine Analysis of Bubble Chamber Pictures,* Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959

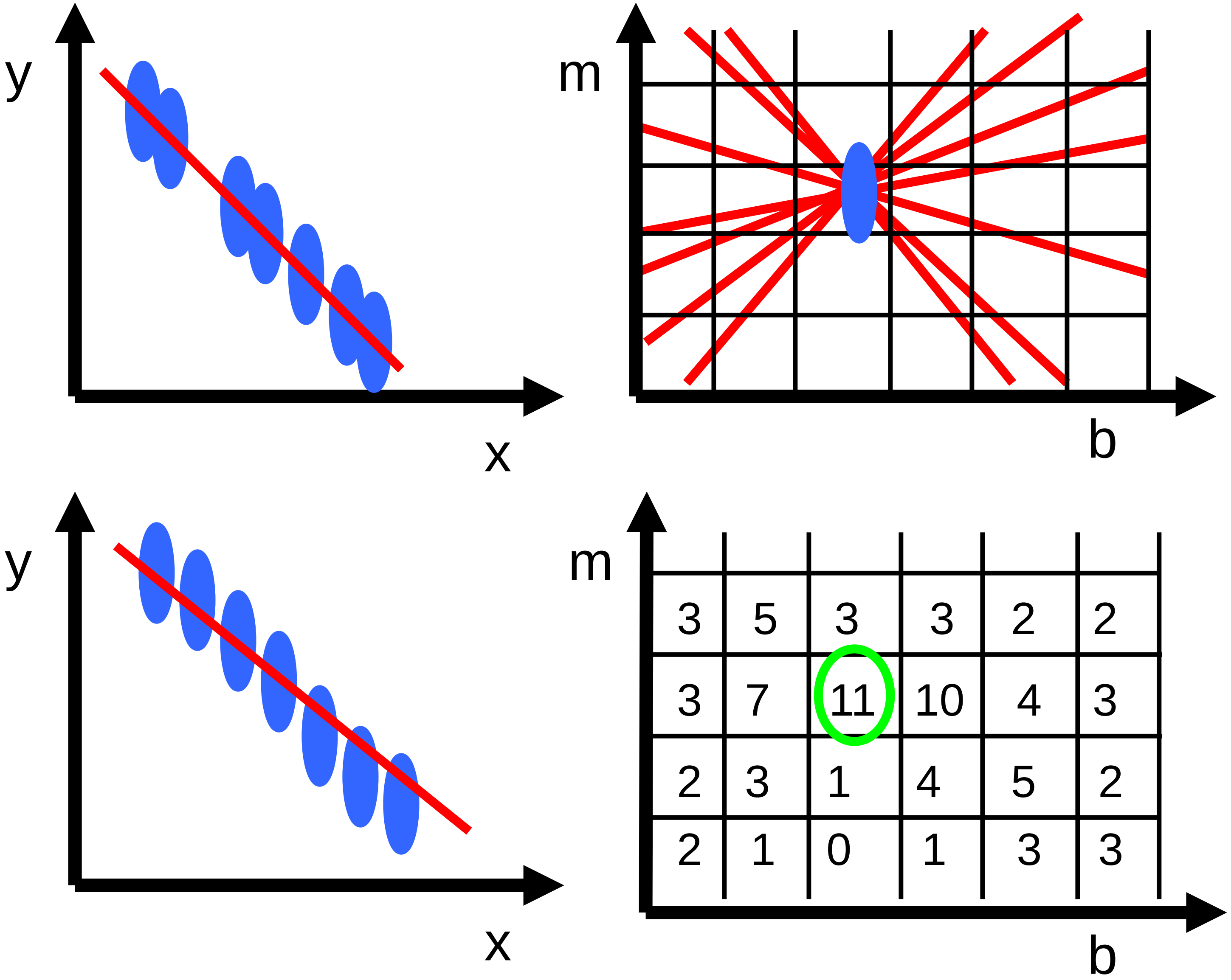
Given a set of points, find the curve or line that explains the data points best



$$y = m\,x + b$$

Slide from S. Savarese

Hough transform



Hough transform

P.V.C. Hough, *Machine Analysis of Bubble Chamber Pictures,* Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959

Issue : parameter space [m,b] is unbounded…

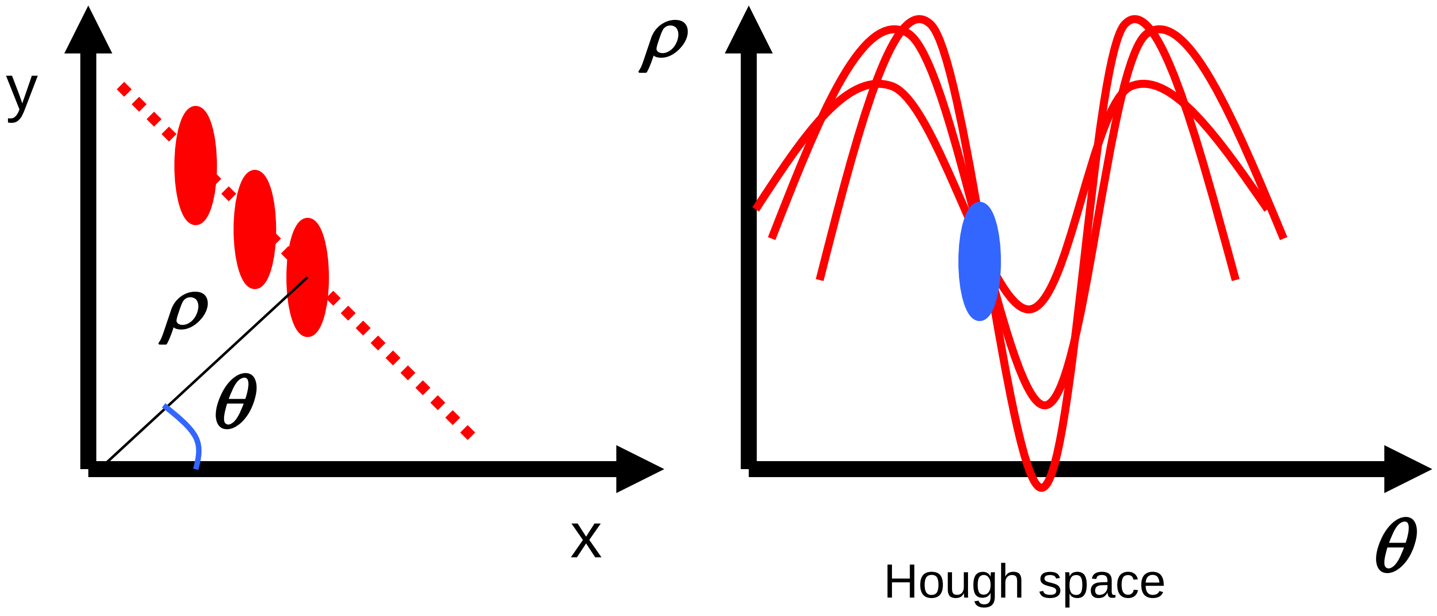
Slide from S. Savarese

Hough transform

P.V.C. Hough, *Machine Analysis of Bubble Chamber Pictures,* Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959

Issue : parameter space [m,b] is unbounded…

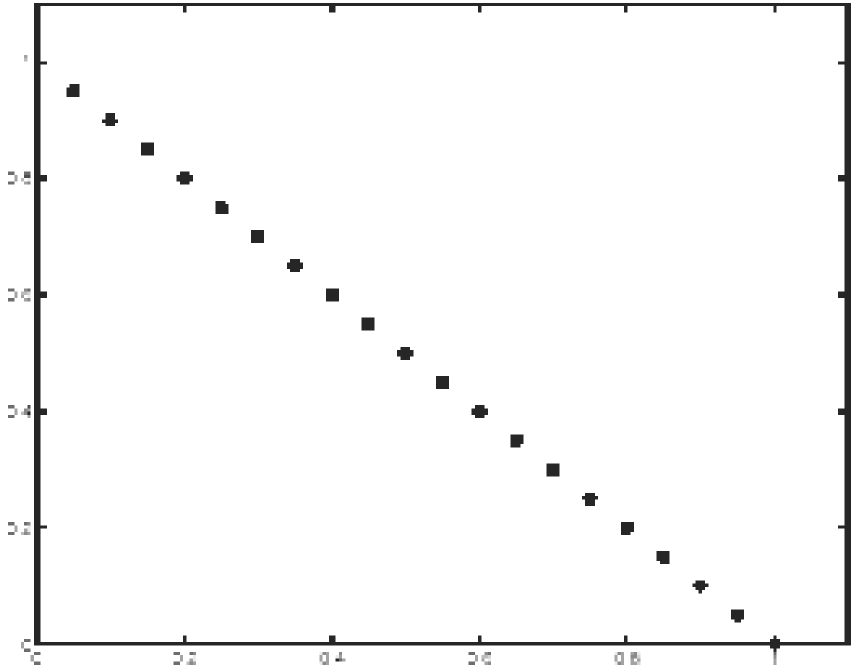
Use a polar representation for the parameter space



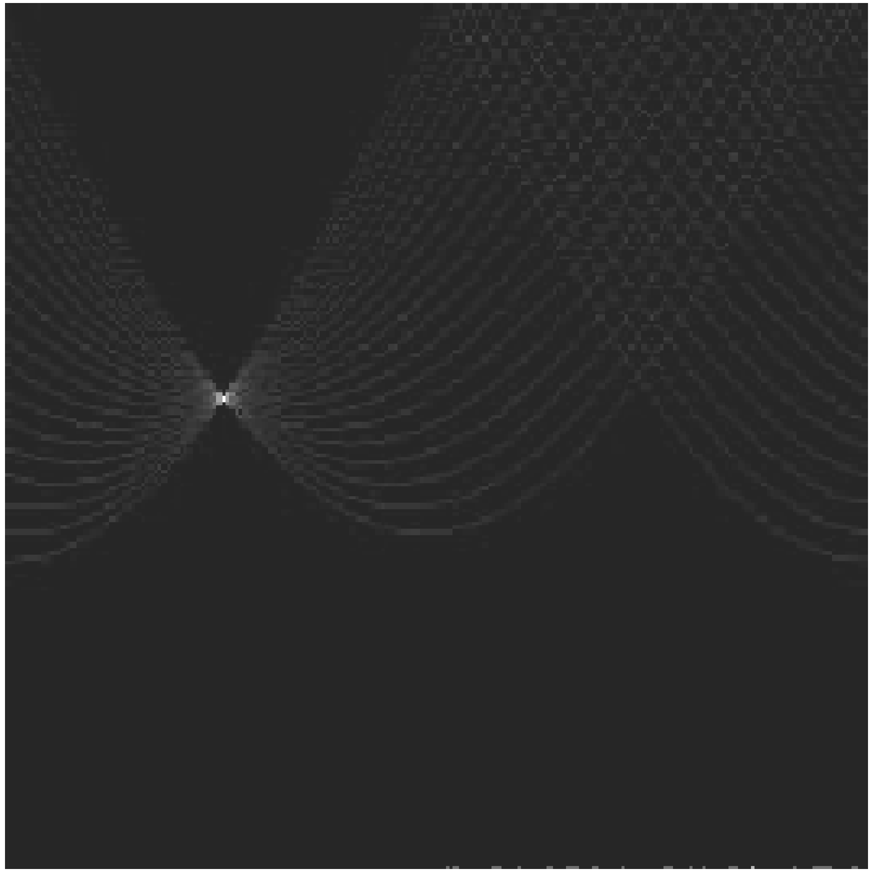
$$x\cos\theta + y\sin\theta = \rho$$

Slide from S. Savarese

Hough transform - experiments



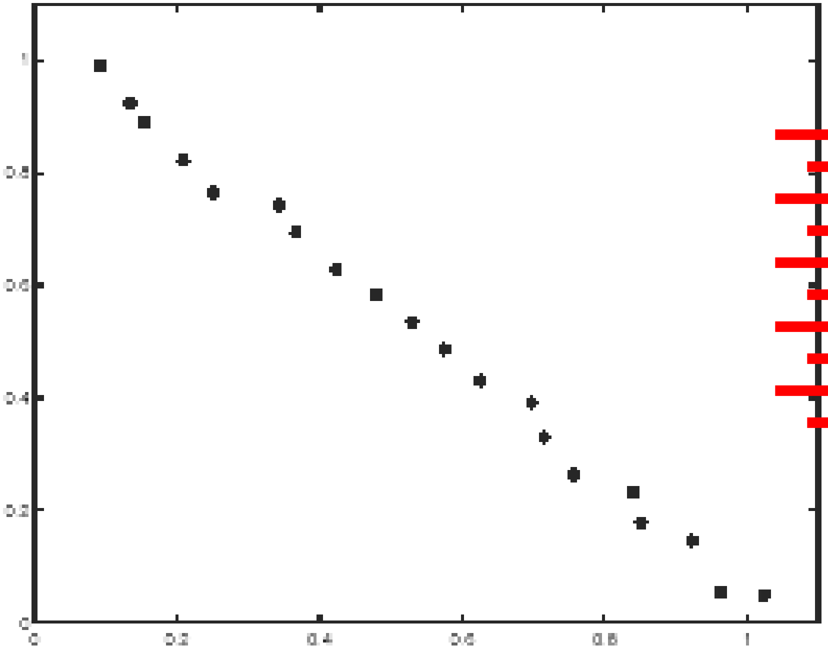
features



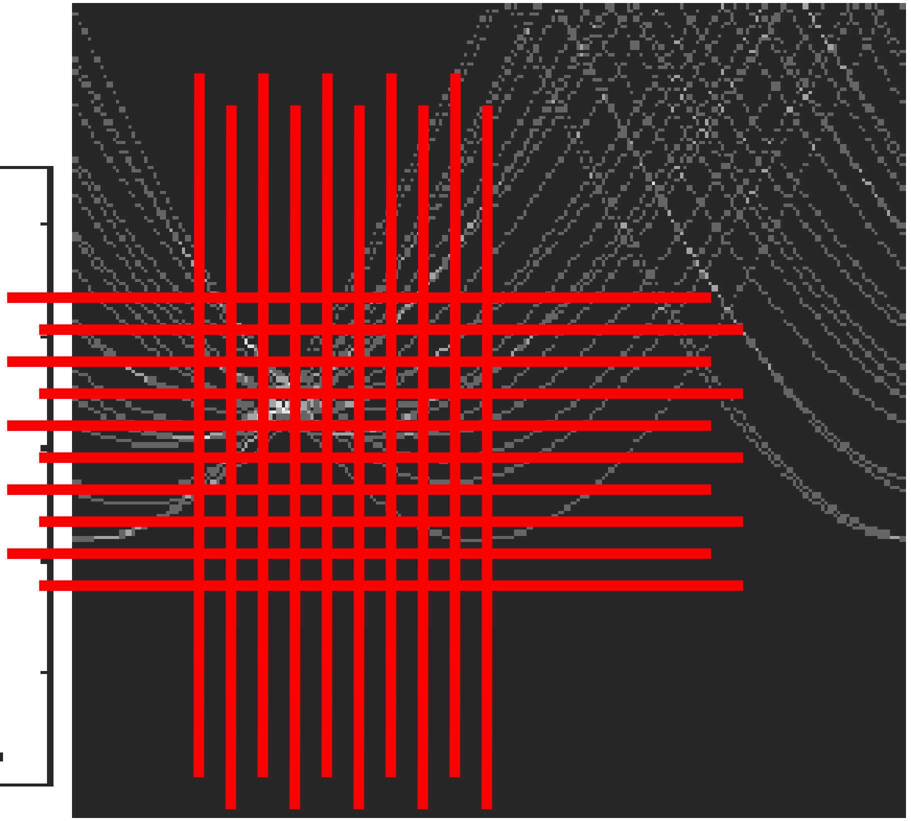
votes

Hough transform - experiments

Noisy data



features

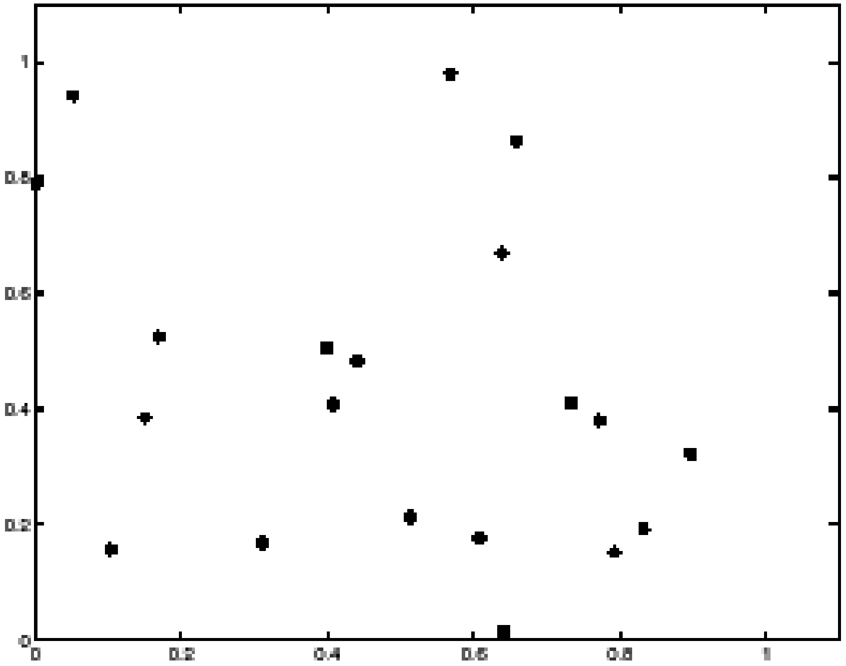


votes

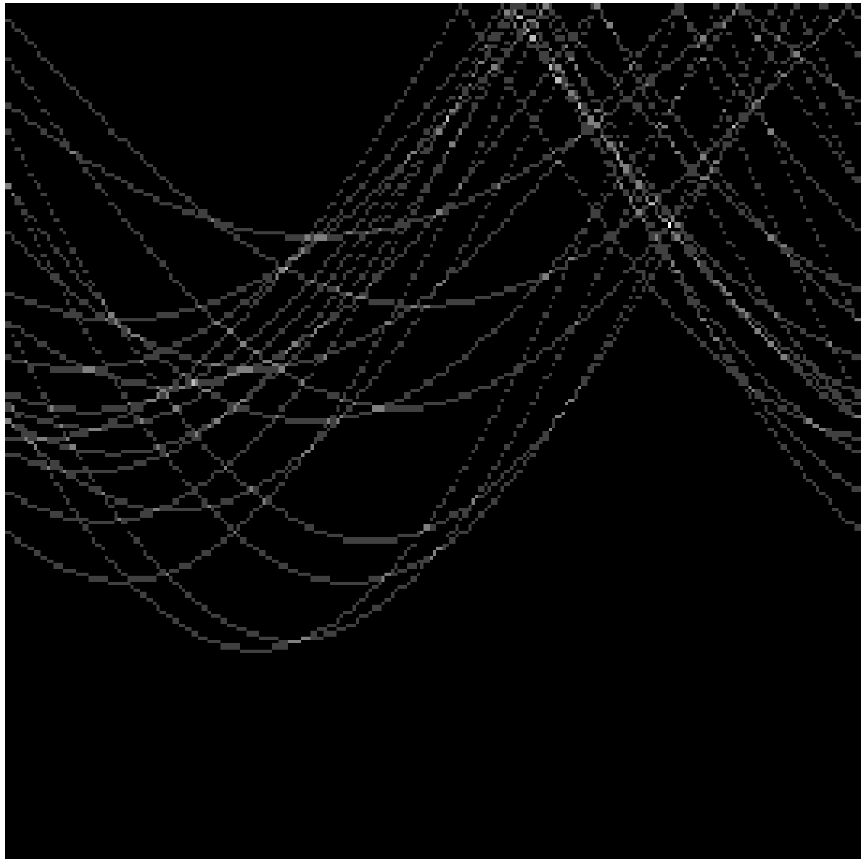
Need to adjust grid size or smooth

Slide from S. Savarese

Hough transform - experiments



features



votes

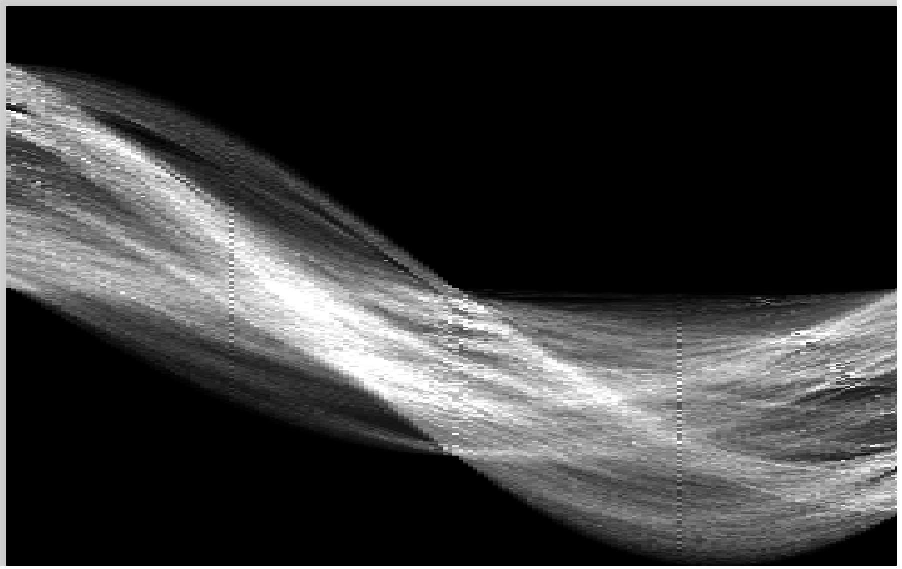
Issue: spurious peaks due to uniform noise

1. Image → Canny Edge Detection

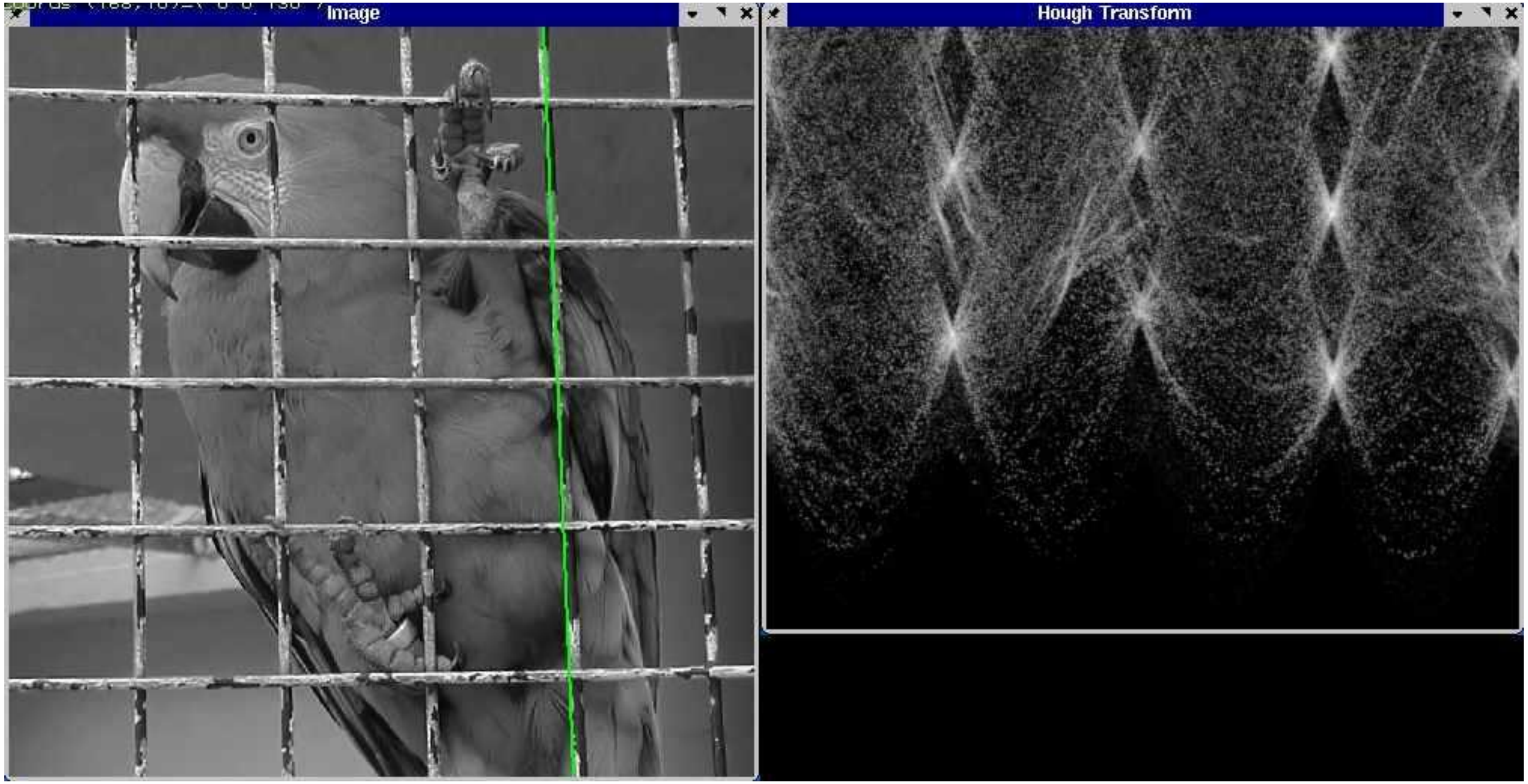
2. Canny → Hough votes

3. Hough votes → Edges

Find peaks and post-process



Hough transform example



http://ostatic.com/files/images/ss_hough.jpg

Finding lines using Hough transform

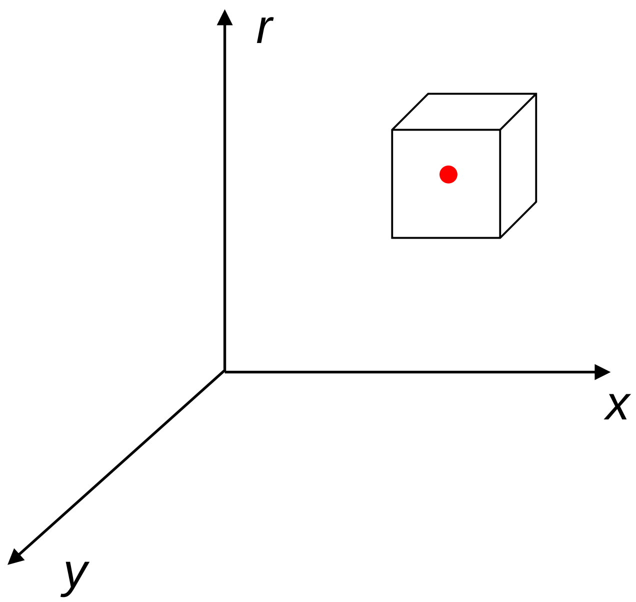
- Using m,b parameterization
- Using r, theta parameterization
  - Using oriented gradients
- Practical considerations
  - Bin size
  - Smoothing
  - Finding multiple lines
  - Finding line segments

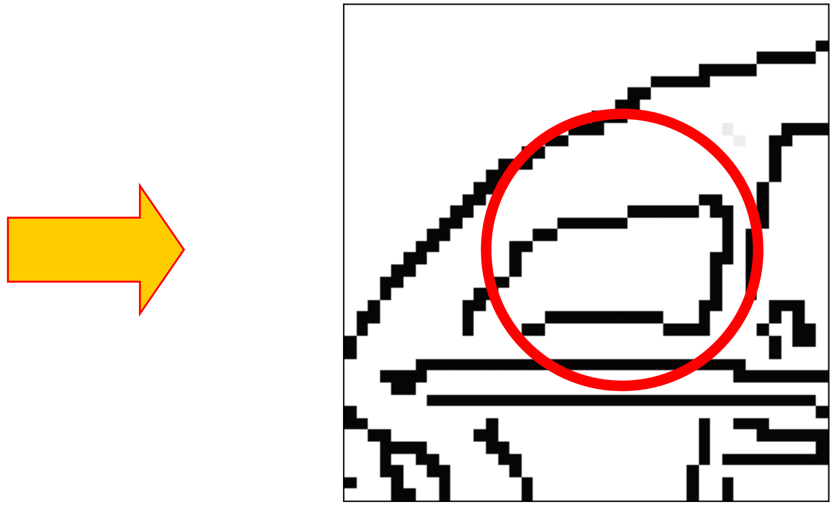
Hough Transform

- How would we find circles?
  - Of fixed radius
  - Of unknown radius
  - Of unknown radius but with known edge orientation

Hough transform for circles

- Conceptually equivalent procedure: for each (x,y,r), draw the corresponding circle in the image and compute its "support"



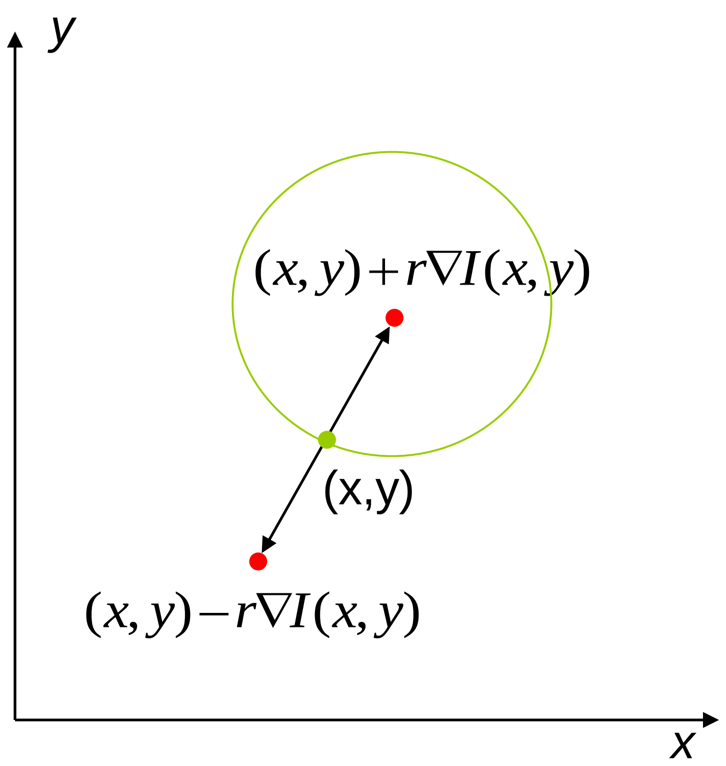


Hough Transform

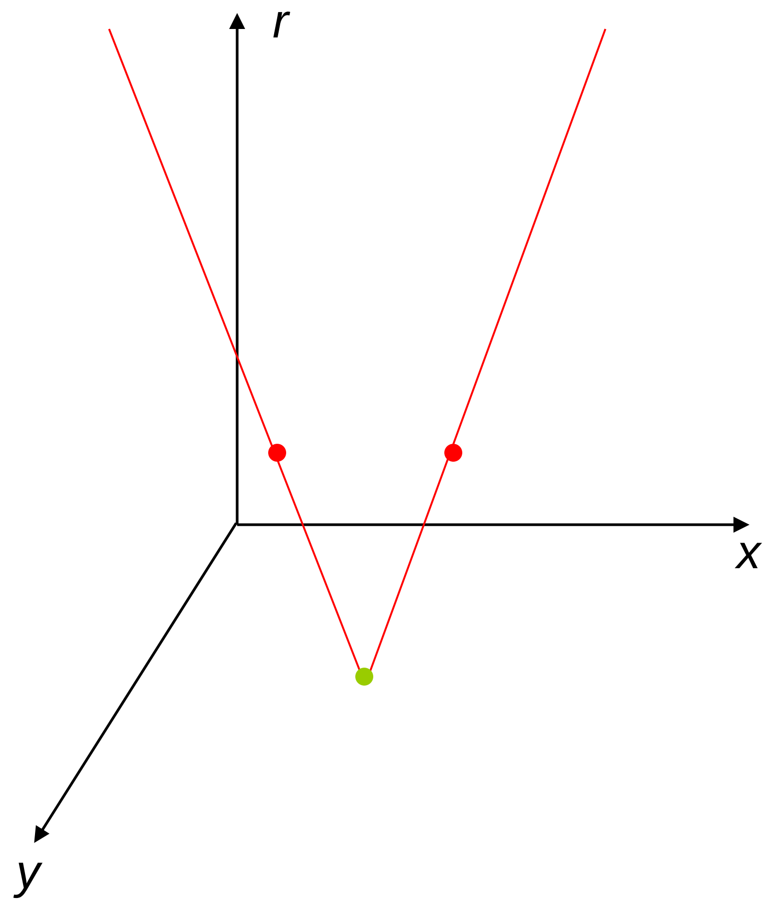
- How would we find circles?
  - Of fixed radius
  - Of unknown radius
  - Of unknown radius but with known edge orientation

Hough transform for circles

image space



Hough parameter space



Hough transform conclusions

Good
- Robust to outliers: each point votes separately
- Fairly efficient (much faster than trying all sets of parameters)
- Provides multiple good fits

Bad
- Some sensitivity to noise
- Bin size trades off between noise tolerance, precision, and speed/memory
  - Can be hard to find sweet spot
- Not suitable for more than a few parameters
  - grid size grows exponentially

Common applications
- Line fitting (also circles, ellipses, etc.)
- Object instance recognition (parameters are affine transform)
- Object category recognition (parameters are position/scale)