



Deep Learning Neural Net Basics

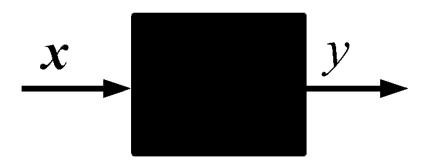
Computer Vision
James Hays

Outline

- Neural Networks
- Convolutional Neural Networks
- Variants
 - Detection
 - Segmentation
 - Siamese Networks
- Visualization of Deep Networks

Supervised Learning

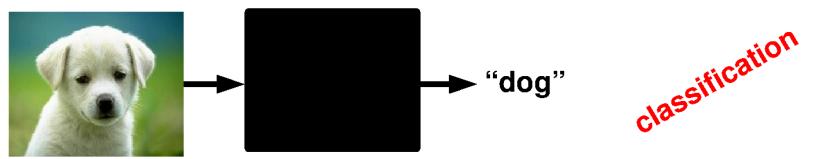
 $\{(x^i, y^i), i=1...P\}$ training dataset x^i i-th input training example y^i i-th target label P number of training examples



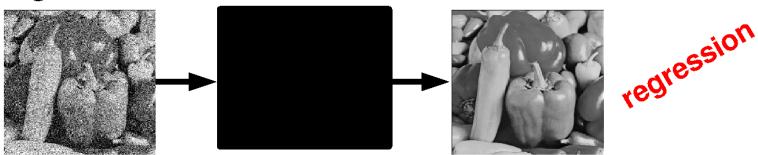
Goal: predict the target label of unseen inputs.

Supervised Learning: Examples

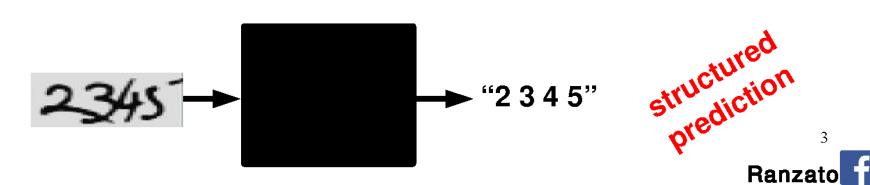
Classification



Denoising

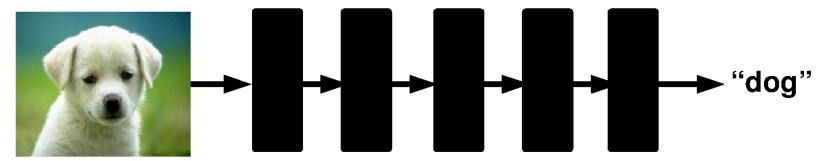


OCR

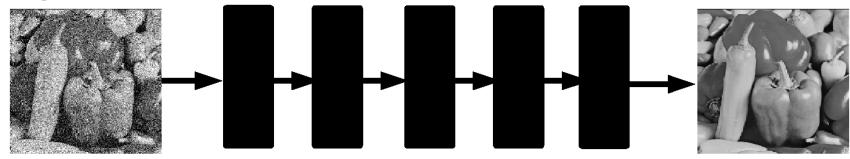


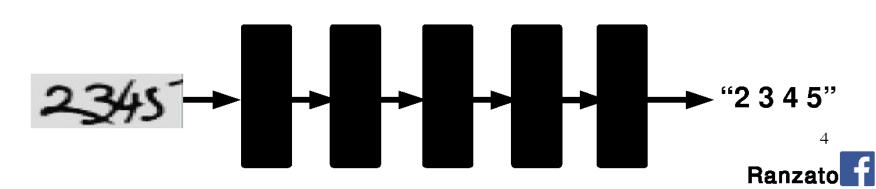
Supervised Deep Learning

Classification



Denoising





Outline

- Supervised Neural Networks
- Convolutional Neural Networks
- Examples
- Tips

Neural Networks

Assumptions (for the next few slides):

- The input image is vectorized (disregard the spatial layout of pixels)
- The target label is discrete (classification)

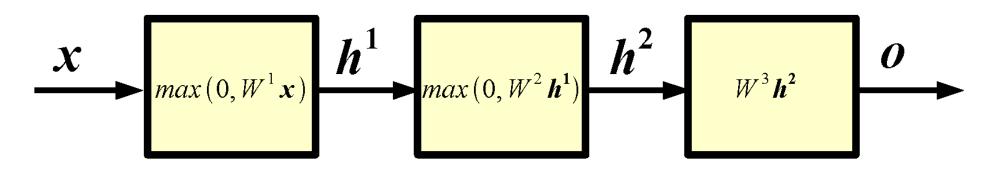
Question: what class of functions shall we consider to map the input into the output?

Answer: composition of simpler functions.

Follow-up questions: Why not a linear combination? What are the "simpler" functions? What is the interpretation?

Answer: later...

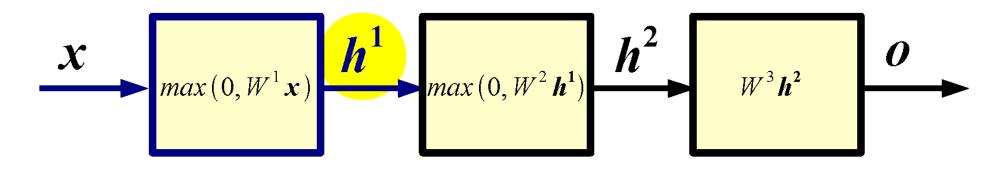
Neural Networks: example



- \boldsymbol{x} input
- h^1 1-st layer hidden units
- h^2 2-nd layer hidden units
- output

Example of a 2 hidden layer neural network (or 4 layer network, counting also input and output).

Def.: Forward propagation is the process of computing the output of the network given its input.



$$\boldsymbol{x} \in R^D \quad W^1 \in R^{N_1 \times D} \quad \boldsymbol{b}^1 \in R^{N_1} \quad \boldsymbol{h}^1 \in R^{N_1}$$

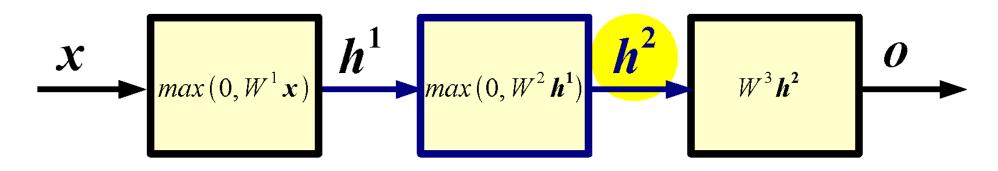
$$\boldsymbol{h}^1 = max(0, W^1 \boldsymbol{x} + \boldsymbol{b}^1)$$

 W^1 1-st layer weight matrix or weights

 \boldsymbol{b}^{1} 1-st layer biases

The non-linearity u = max(0, v) is called **ReLU** in the DL literature. Each output hidden unit takes as input all the units at the previous layer: each such layer is called "**fully connected**".

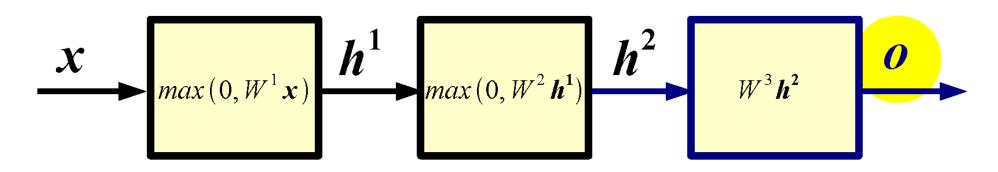
Ranzato



$$h^1 \in R^{N_1} \quad W^2 \in R^{N_2 \times N_1} \quad b^2 \in R^{N_2} \quad h^2 \in R^{N_2}$$

$$\boldsymbol{h^2} = max(0, W^2 \boldsymbol{h^1} + \boldsymbol{b^2})$$

 W^2 2-nd layer weight matrix or weights b^2 2-nd layer biases

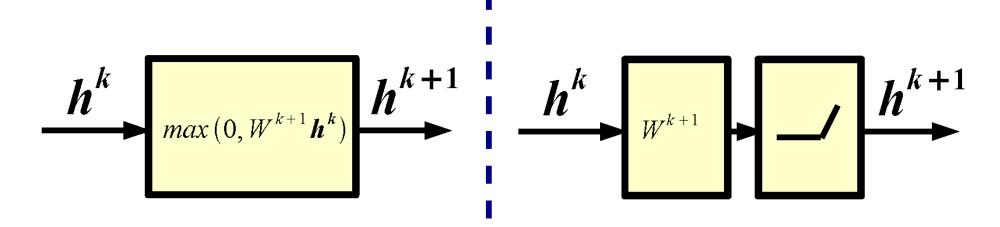


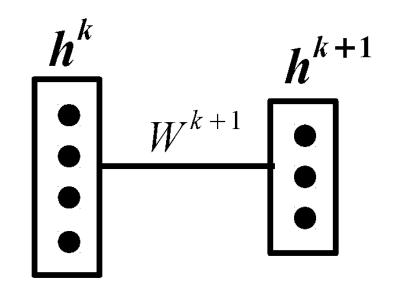
$$h^2 \in R^{N_2} \ W^3 \in R^{N_3 \times N_2} \ b^3 \in R^{N_3} \ o \in R^{N_3}$$

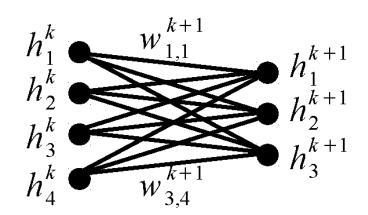
$$\boldsymbol{o} = max(0, W^3 \boldsymbol{h}^2 + \boldsymbol{b}^3)$$

 W^3 3-rd layer weight matrix or weights b^3 3-rd layer biases

Alternative Graphical Representation



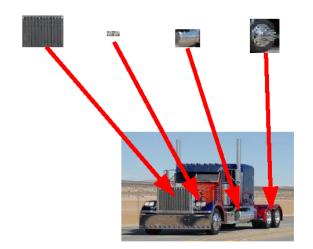




Question: Why do we need many layers?

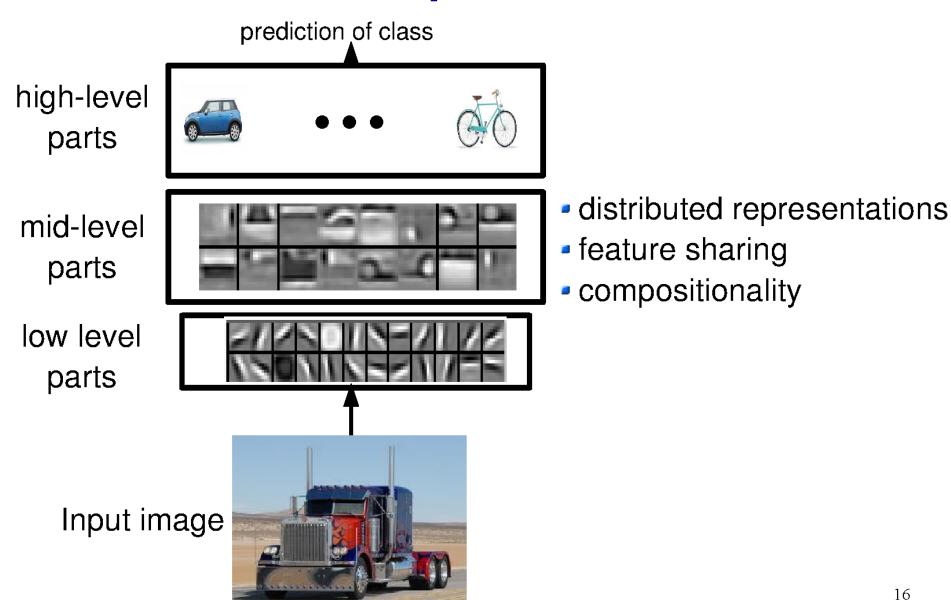
Answer: When input has hierarchical structure, the use of a hierarchical architecture is potentially more efficient because intermediate computations can be re-used. DL architectures are efficient also because they use **distributed representations** which are shared across classes.

[0 0 1 0 0 0 0 1 0 0 1 1 0 0 1 0 ...] truck feature



Exponentially more efficient than a 1-of-N representation (a la k-means)

[1 1 0 0 0 1 0 1 0 0 0 0 1 1 0 1...] motorbike
[0 0 1 0 0 0 1 1 0 0 1 0 0 1 0 ...] truck



Question: What does a hidden unit do?

Answer: It can be thought of as a classifier or feature detector.

Question: How many layers? How many hidden units?

Answer: Cross-validation or hyper-parameter search methods are the answer. In general, the wider and the deeper the network the more complicated the mapping.

Question: How do I set the weight matrices?

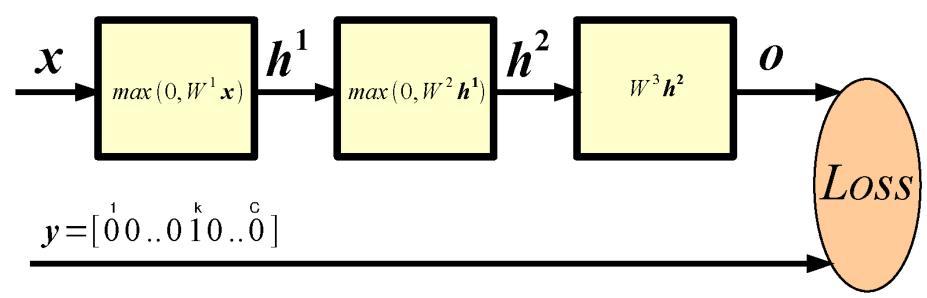
Answer: Weight matrices and biases are learned.

First, we need to define a measure of quality of the current mapping.

Then, we need to define a procedure to adjust the parameters.



How Good is a Network?



Probability of class k given input (softmax):

$$p(c_k=1|\mathbf{x}) = \frac{e^{o_k}}{\sum_{j=1}^{C} e^{o_j}}$$

(Per-sample) **Loss**; e.g., negative log-likelihood (good for classification of small number of classes):

$$L(\boldsymbol{x}, y; \boldsymbol{\theta}) = -\sum_{i} y_{i} \log p(c_{i}|\boldsymbol{x})$$



Training

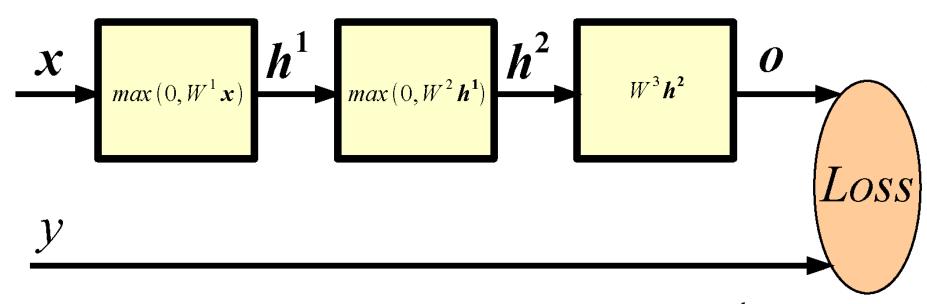
Learning consists of minimizing the loss (plus some regularization term) w.r.t. parameters over the whole training set.

$$\boldsymbol{\theta}^* = arg min_{\boldsymbol{\theta}} \sum_{n=1}^{P} L(\boldsymbol{x}^n, y^n; \boldsymbol{\theta})$$

Question: How to minimize a complicated function of the parameters?

Answer: Chain rule, a.k.a. **Backpropagation!** That is the procedure to compute gradients of the loss w.r.t. parameters in a multi-layer neural network.

Key Idea: Wiggle To Decrease Loss



Let's say we want to decrease the loss by adjusting $W_{i,j}^1$. We could consider a very small $\epsilon = 1\text{e-}6$ and compute:

$$L(\boldsymbol{x}, y; \boldsymbol{\theta})$$

$$L(\boldsymbol{x}, y; \boldsymbol{\theta} \setminus W_{i,j}^1, W_{i,j}^1 + \epsilon)$$

Then, update:

$$W_{i,j}^{1} \leftarrow W_{i,j}^{1} + \epsilon \, sgn(L(\boldsymbol{x}, y; \boldsymbol{\theta}) - L(\boldsymbol{x}, y; \boldsymbol{\theta} \setminus W_{i,j}^{1}, W_{i,j}^{1} + \epsilon))$$
Banzato

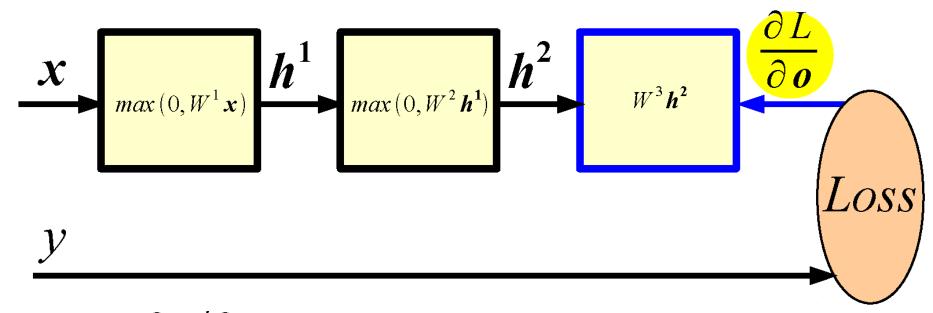
Derivative w.r.t. Input of Softmax

$$p(c_k=1|\mathbf{x}) = \frac{e^{o_k}}{\sum_{j} e^{o_j}}$$

$$L(x, y; \theta) = -\sum_{j} y_{j} \log p(c_{j}|x)$$
 $y = [0.0.010.0]$

By substituting the fist formula in the second, and taking the derivative w.r.t. *o* we get:

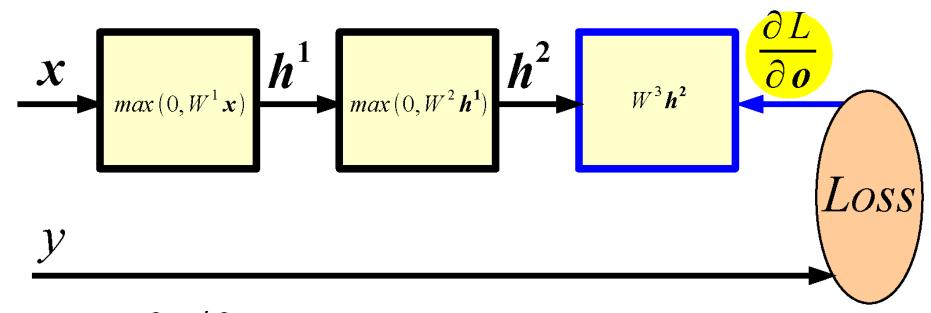
$$\frac{\partial L}{\partial \rho} = p(c|\mathbf{x}) - \mathbf{y}$$



Given $\partial L/\partial \mathbf{o}$ and assuming we can easily compute the Jacobian of each module, we have:

$$\frac{\partial L}{\partial W^3} = \frac{\partial L}{\partial \boldsymbol{o}} \frac{\partial \boldsymbol{o}}{\partial W^3}$$

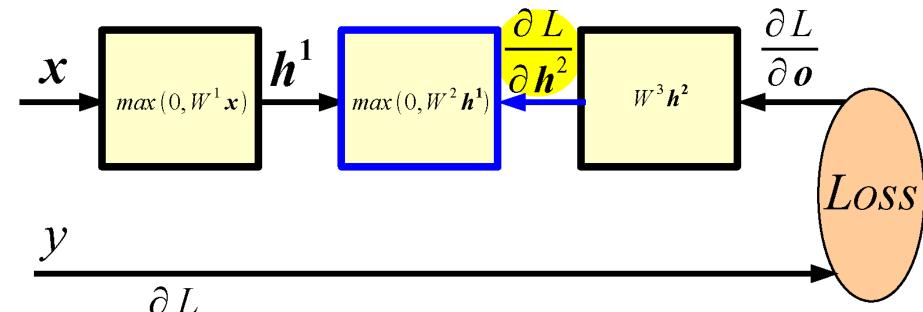
$$\frac{\partial L}{\partial \boldsymbol{h}^2} = \frac{\partial L}{\partial \boldsymbol{o}} \frac{\partial \boldsymbol{o}}{\partial \boldsymbol{h}^2}$$



Given $\partial L/\partial \mathbf{o}$ and assuming we can easily compute the Jacobian of each module, we have:

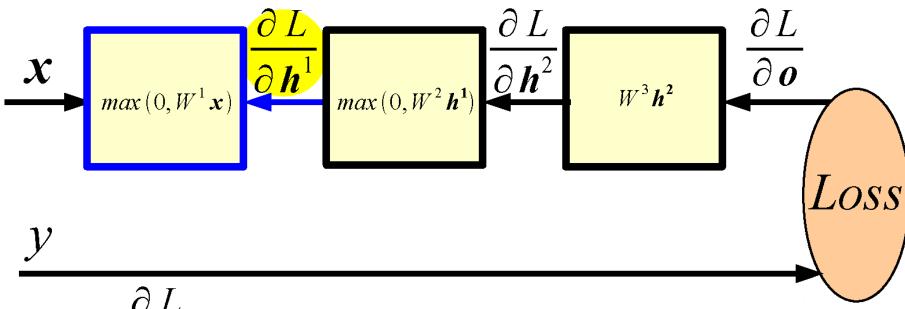
$$\frac{\partial L}{\partial W^3} = \frac{\partial L}{\partial o} \frac{\partial o}{\partial W^3} \qquad \frac{\partial L}{\partial h^2} = \frac{\partial L}{\partial o} \frac{\partial o}{\partial h^2}$$

$$\frac{\partial L}{\partial W^3} = (p(c|\mathbf{x}) - \mathbf{y}) h^{2T} \qquad \frac{\partial L}{\partial h^2} = W^{3T} (p(c|\mathbf{x}) - \mathbf{y})_{23}$$



Given $\frac{\partial L}{\partial \mathbf{h}^2}$ we can compute now:

$$\frac{\partial L}{\partial W^2} = \frac{\partial L}{\partial \boldsymbol{h}^2} \frac{\partial \boldsymbol{h}^2}{\partial W^2} \qquad \frac{\partial L}{\partial \boldsymbol{h}^1} = \frac{\partial L}{\partial \boldsymbol{h}^2} \frac{\partial \boldsymbol{h}^2}{\partial \boldsymbol{h}^1}$$



Given $\frac{\partial L}{\partial \mathbf{h}^1}$ we can compute now:

$$\frac{\partial L}{\partial W^1} = \frac{\partial L}{\partial \boldsymbol{h}^1} \frac{\partial \boldsymbol{h}^1}{\partial W^1}$$

Question: Does BPROP work with ReLU layers only?

Answer: Nope, any a.e. differentiable transformation works.

Question: What's the computational cost of BPROP?

Answer: About twice FPROP (need to compute gradients w.r.t. input and parameters at every layer).

Optimization

Stochastic Gradient Descent (on mini-batches):

$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \eta \frac{\partial L}{\partial \boldsymbol{\theta}}, \eta \in (0, 1)$$

Stochastic Gradient Descent with Momentum:

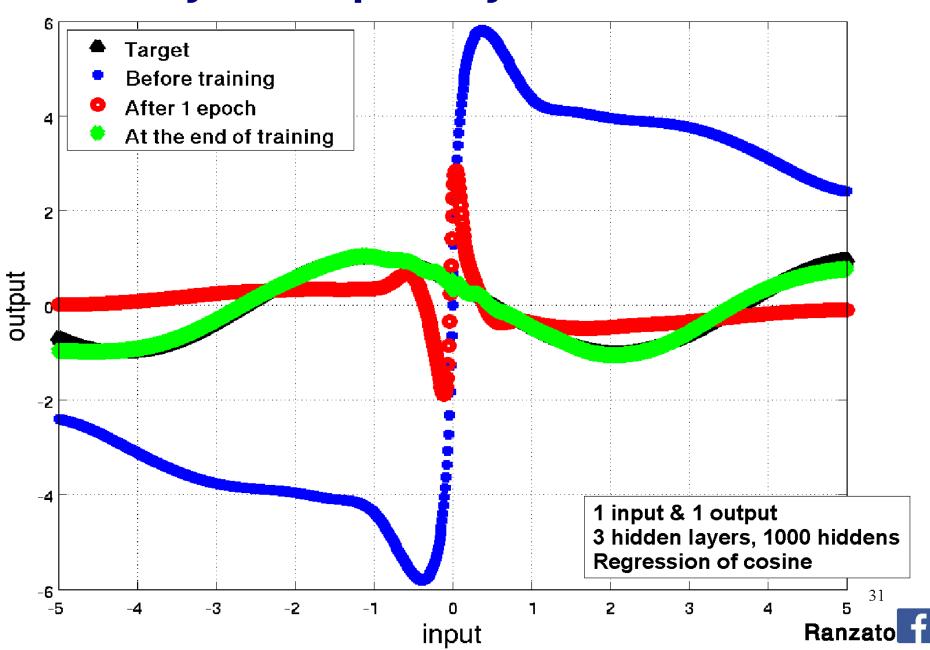
$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \eta \boldsymbol{\Delta}$$

$$\Delta \leftarrow 0.9 \Delta + \frac{\partial L}{\partial \theta}$$

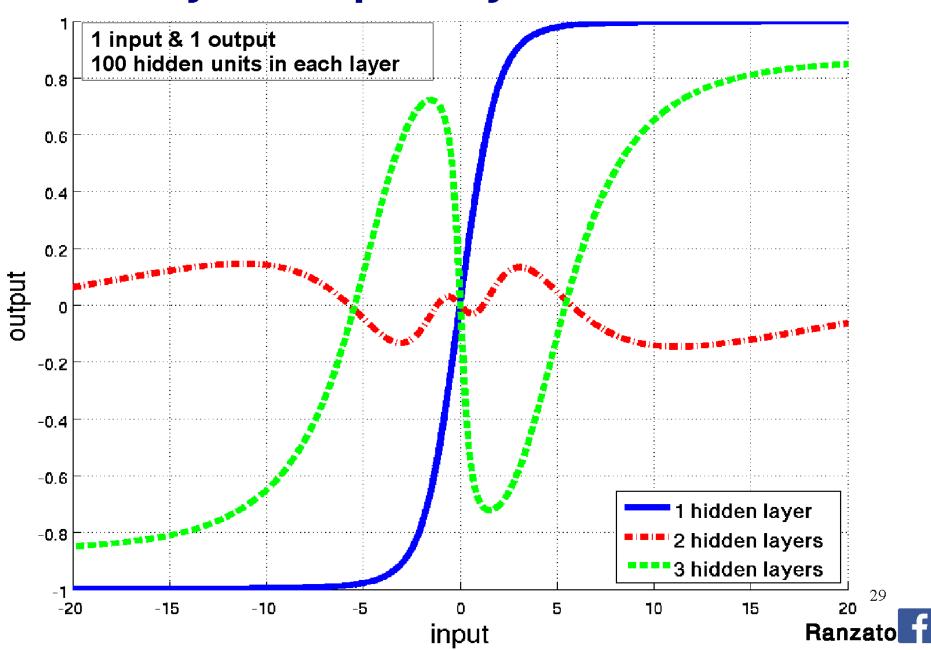
Note: there are many other variants...



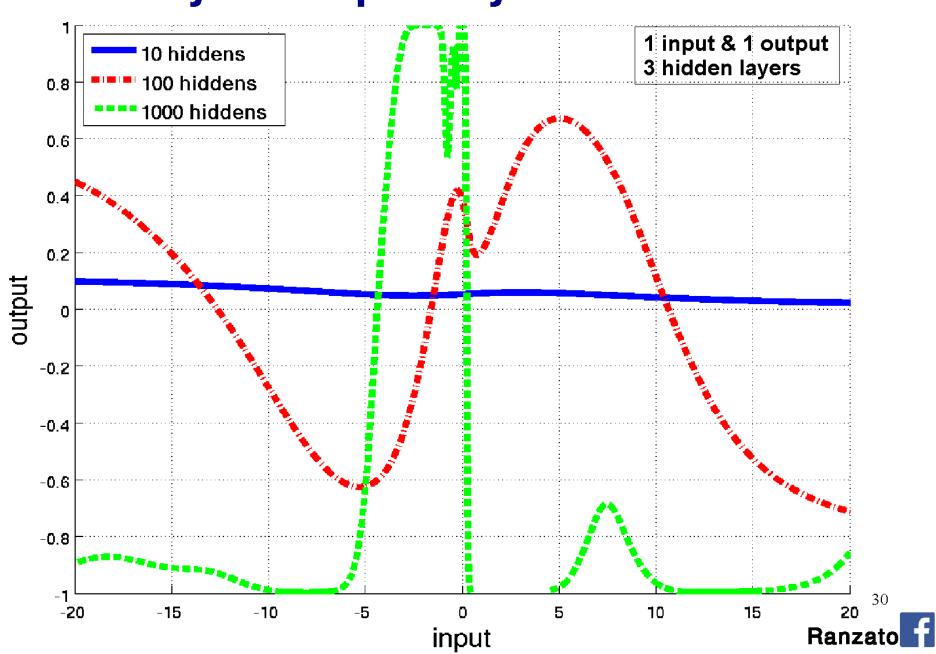
Toy Example: Synthetic Data



Toy Example: Synthetic Data



Toy Example: Synthetic Data



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- Convolutional Neural Networks
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This all seems pretty complicated. Why are we using Neural Networks? James's rough assessment:

Learning method	Ease of configuration
Neural Network	1
Nearest Neighbor	10
Linear SVM	10
Non-linear SVM	5
Decision Tree or Random Forest	4

This all seems pretty complicated. Why are we using Neural Networks? James's rough assessment:

Learning method	Ease of configuration	Ease of interpretation
Neural Network	1	1
Nearest Neighbor	10	10
Linear SVM	10	9
Non-linear SVM	5	4
Decision Tree or Random Forest	4	4

This all seems pretty complicated. Why are we using Neural Networks? James's rough assessment:

Learning method	Ease of configuration	Ease of interpretation	Speed / memory when training
Neural Network	1	1	1
Nearest Neighbor	10	10	8
Linear SVM	10	9	10
Non-linear SVM	5	4	2
Decision Tree or Random Forest	4	4	4

This all seems pretty complicated. Why are we using Neural Networks? James's rough assessment:

Learning method	Ease of configuration	Ease of interpretation	Speed / memory when training	Speed / memory at test time
Neural Network	1	1	1	6
Nearest Neighbor	10	10	8	4
Linear SVM	10	9	10	10
Non-linear SVM	5	4	2	2
Decision Tree or Random Forest	4	4	4	8

This all seems pretty complicated. Why are we using Neural Networks? James's rough assessment:

Learning method	Ease of configuration	Ease of interpretation	Speed / memory when training	Speed / memory at test time	Accuracy w/ lots of data
Neural Network	1	1	1	6	10
Nearest Neighbor	10	10	8	4	7
Linear SVM	10	9	10	10	5
Non-linear SVM	5	4	2	2	8
Decision Tree or Random Forest	4	4	4	8	7

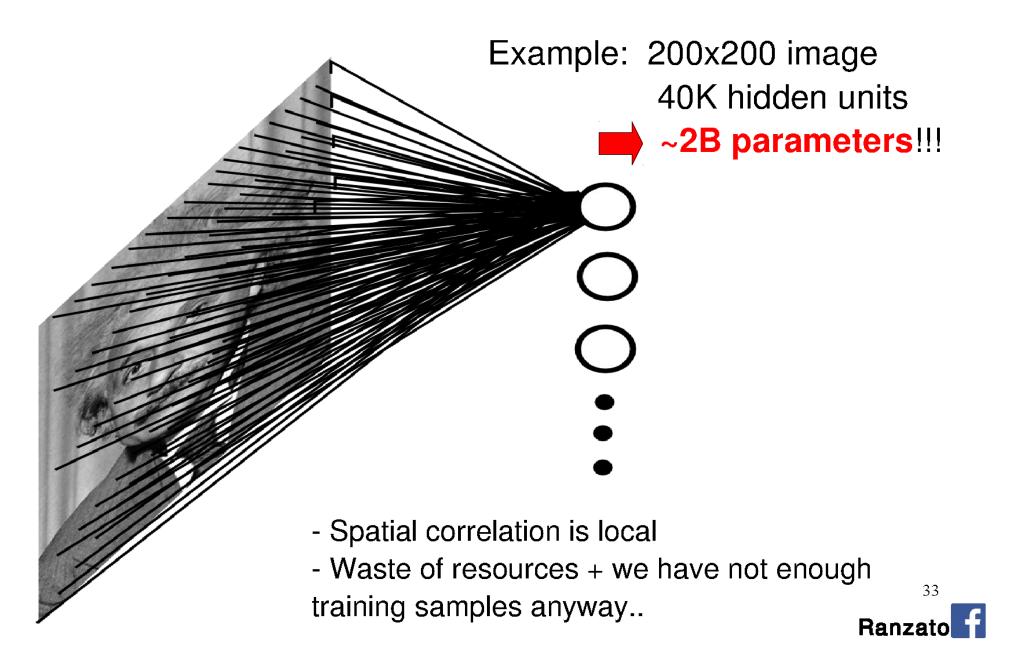
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Learning method	Ease of configu		Ease of interpretation	Speed / memory when training	Speed / memory at test time	Accuracy w/ lots of data		
Neural Network	1		1	1	6	10		
Nearest Neighbor	10		10	8	4	7		
Linear SVM	10	Representation design matters more for all of these						
Non-linear SVM	5							
Decision Tree or Random Forest	4							

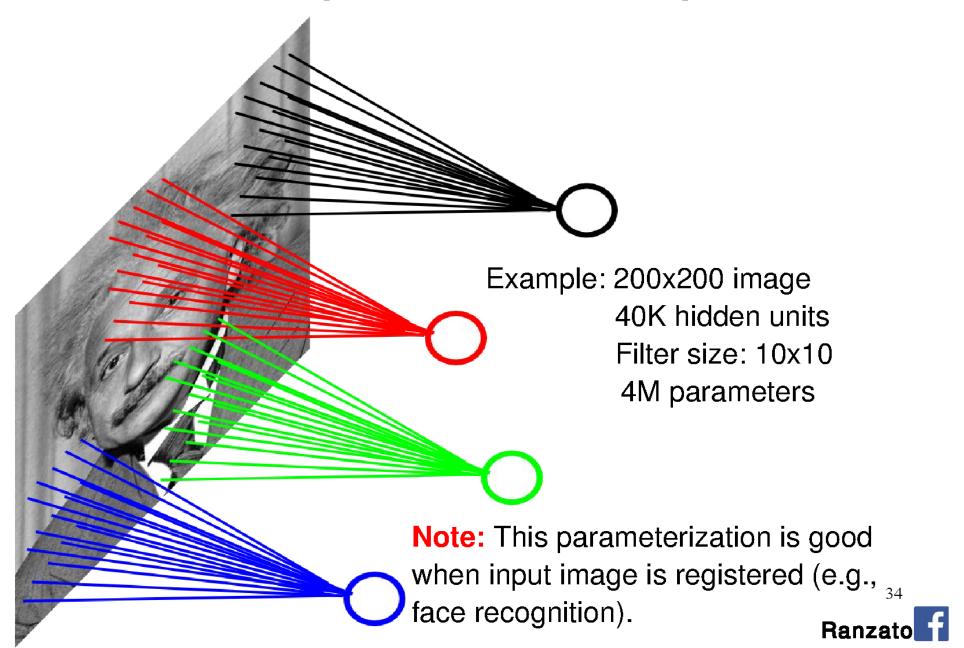
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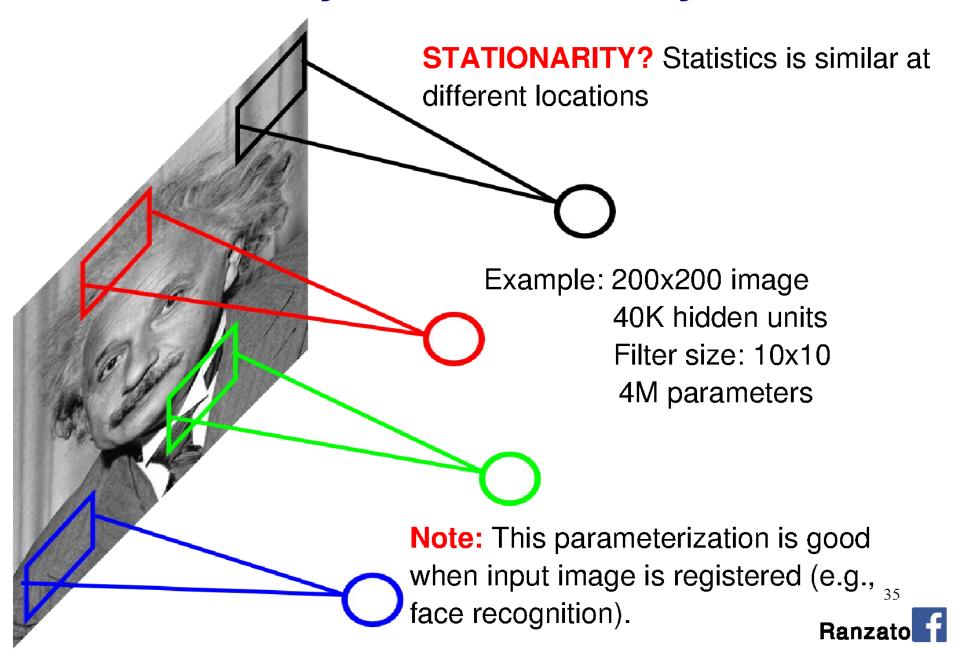
Fully Connected Layer

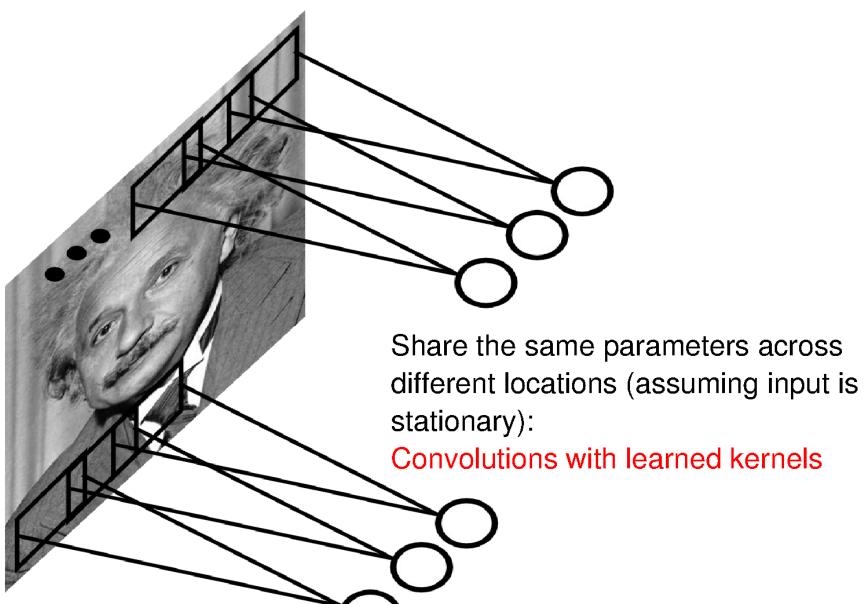


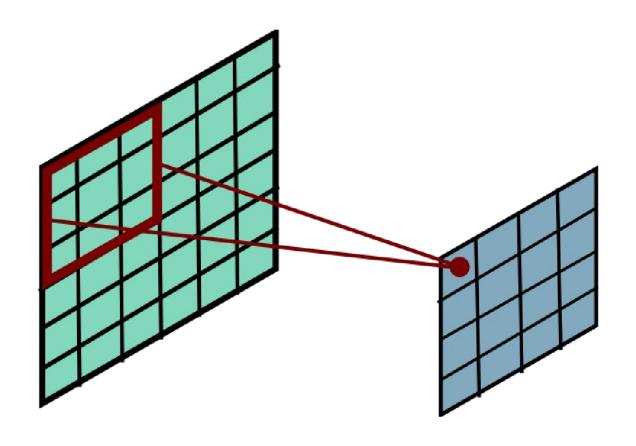
Locally Connected Layer



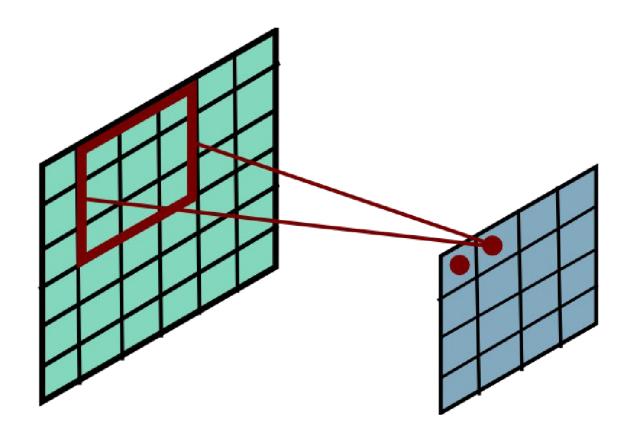
Locally Connected Layer



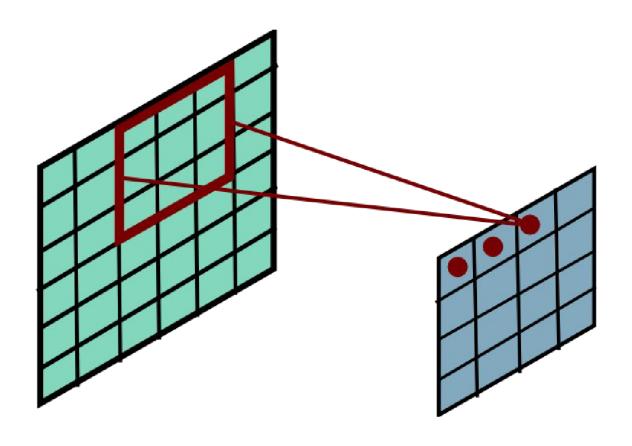




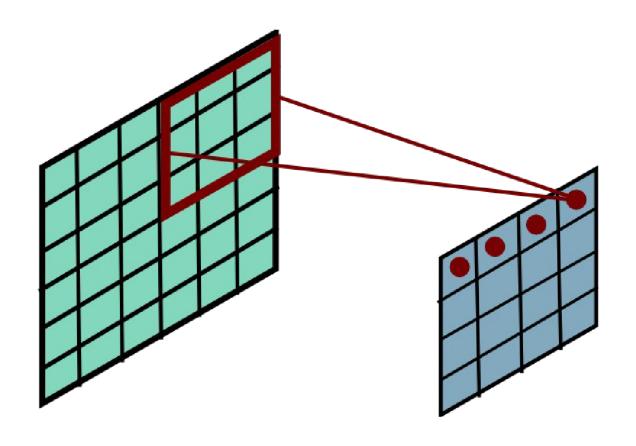




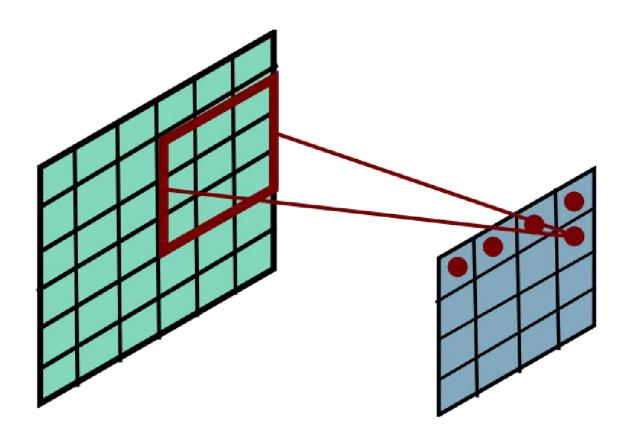




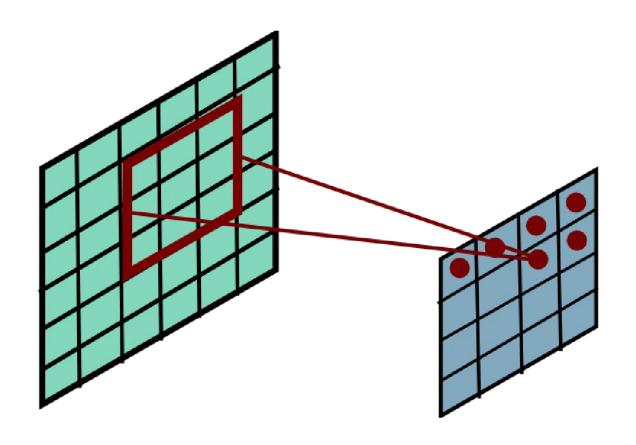




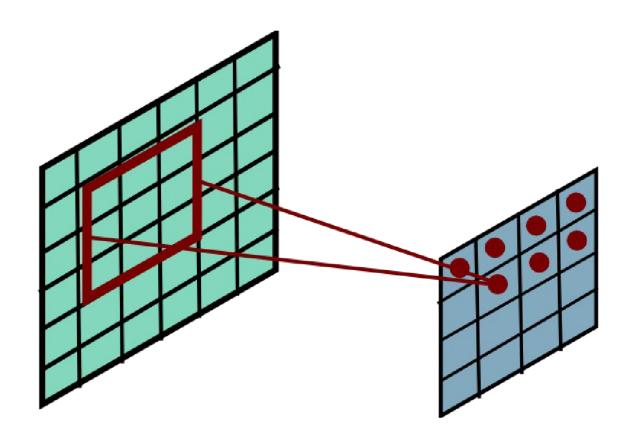




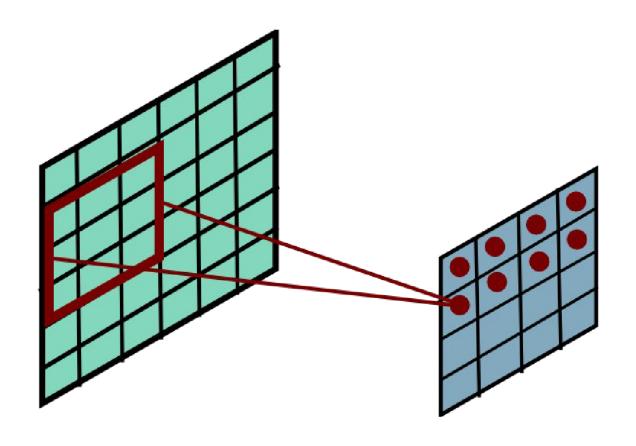




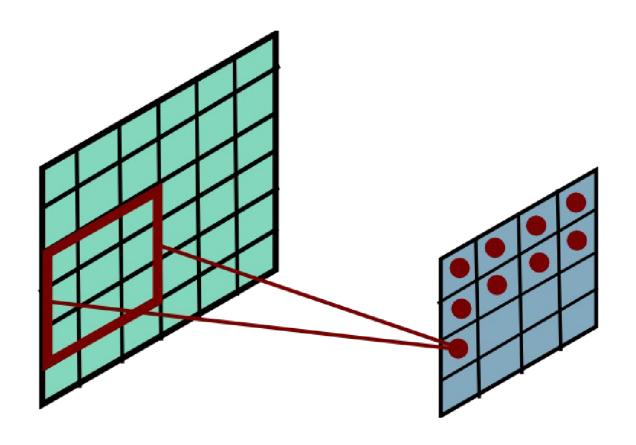




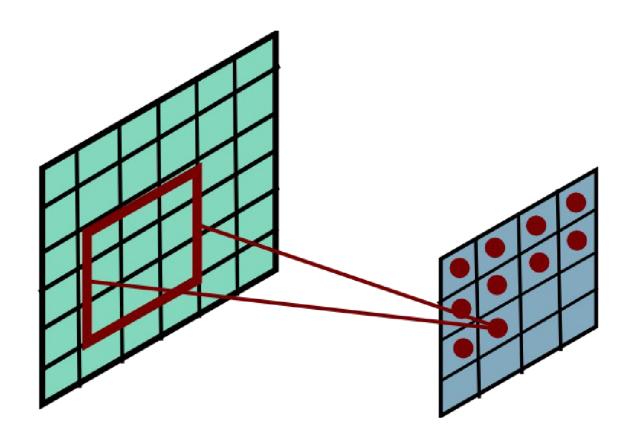




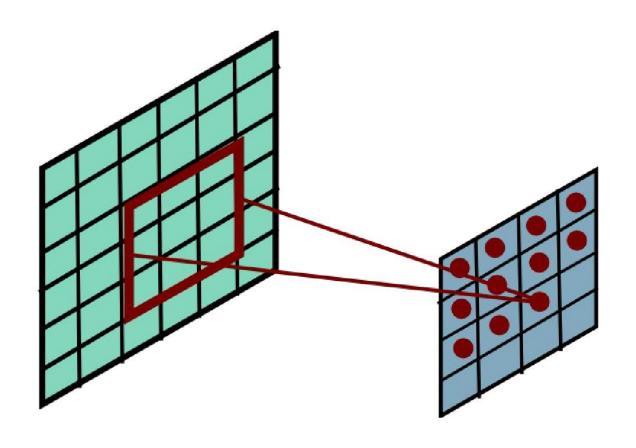




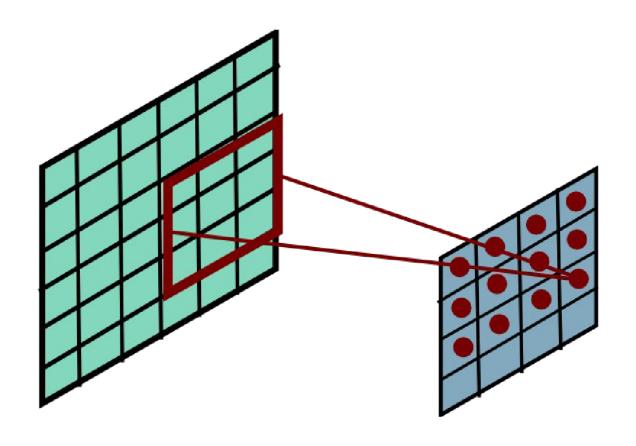




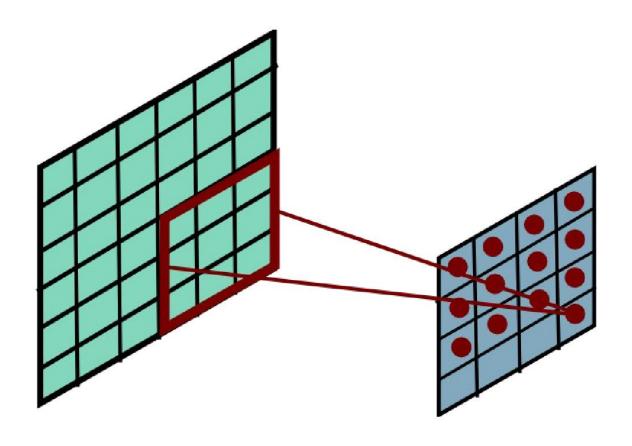




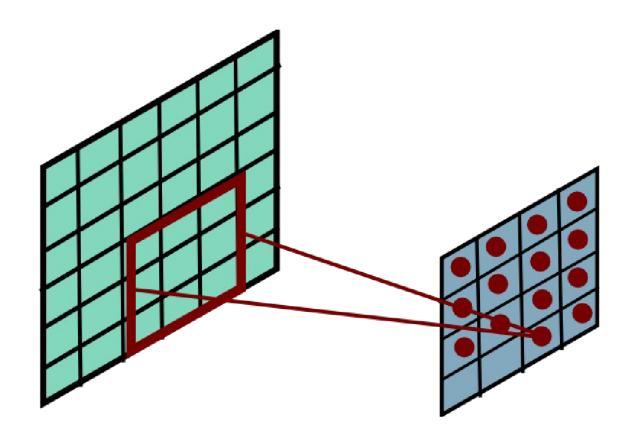




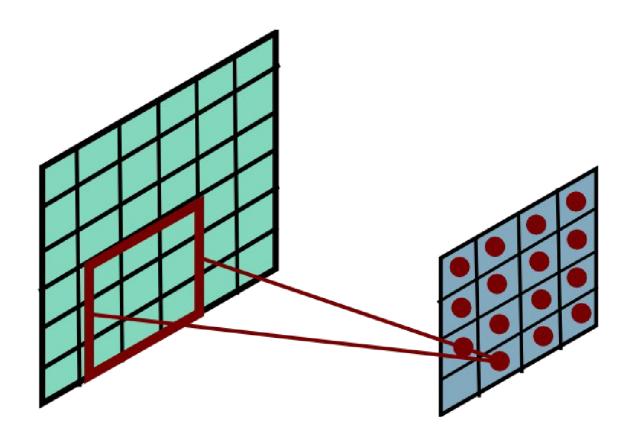




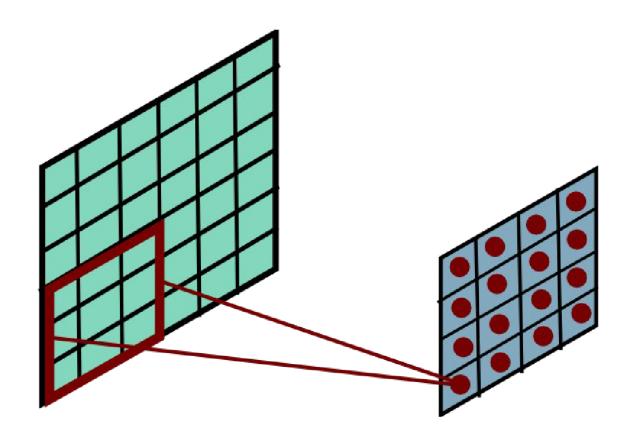




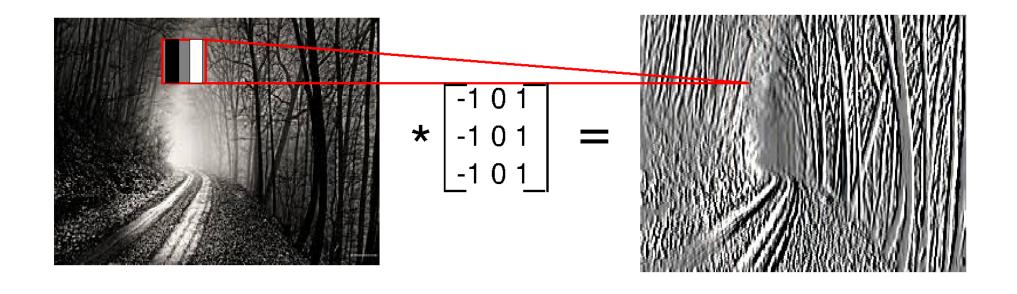


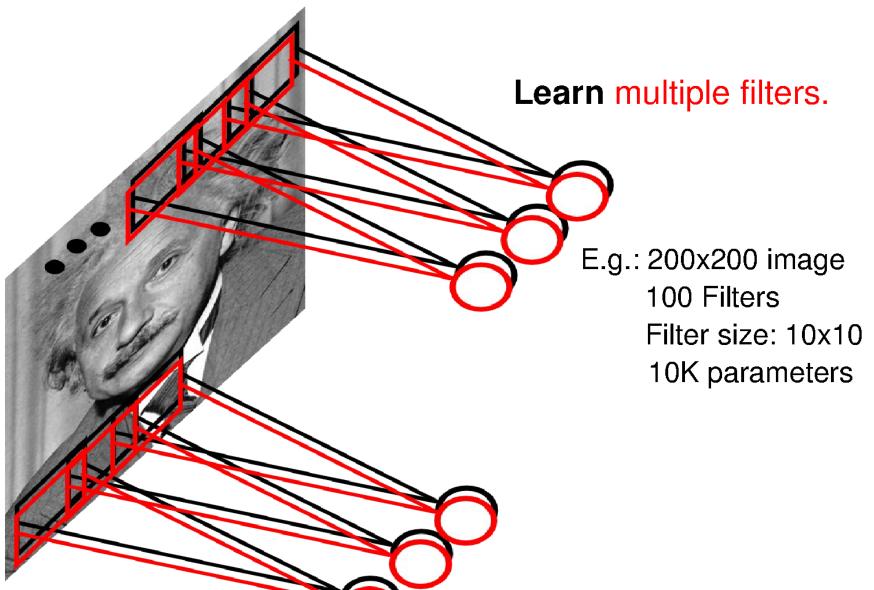


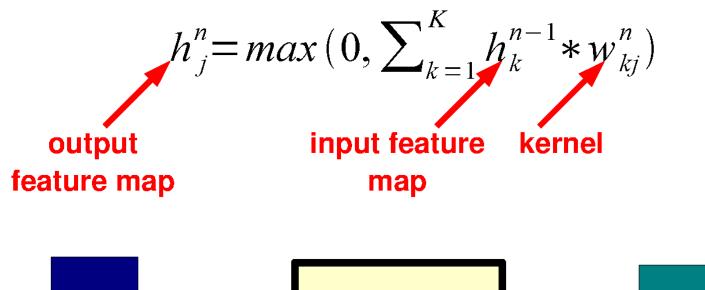


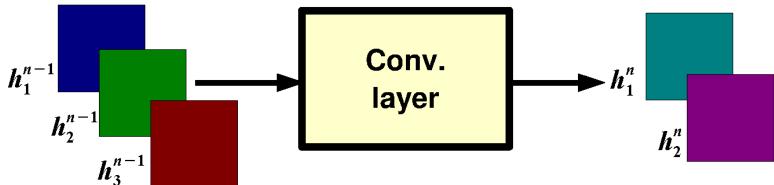


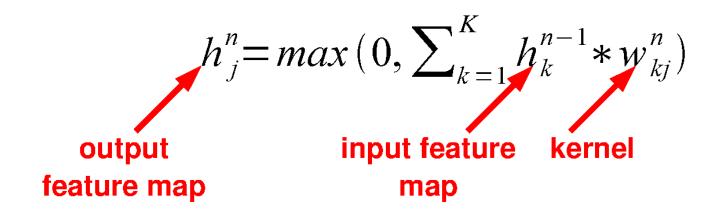


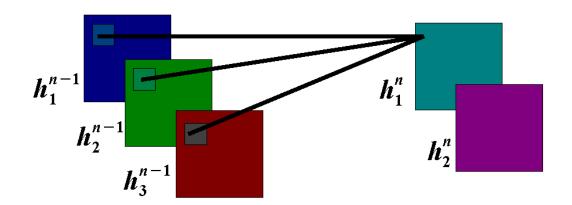


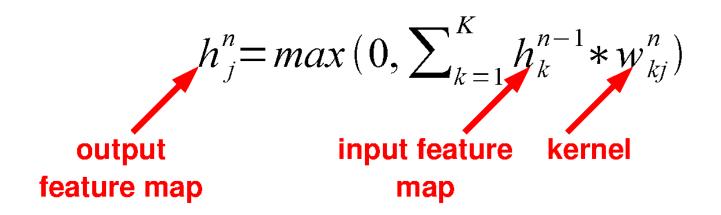


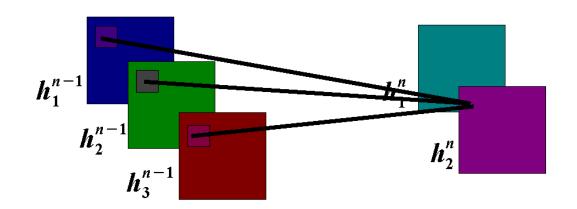












Question: What is the size of the output? What's the computational cost?

Answer: It is proportional to the number of filters and depends on the stride. If kernels have size KxK, input has size DxD, stride is 1, and there are M input feature maps and N output feature maps then:

- the input has size M@DxD
- the output has size N@(D-K+1)x(D-K+1)
- the kernels have MxNxKxK coefficients (which have to be learned)
- cost: M*K*K*N*(D-K+1)*(D-K+1)

Question: How many feature maps? What's the size of the filters?

Answer: Usually, there are more output feature maps than input feature maps. Convolutional layers can increase the number of hidden units by big factors (and are expensive to compute).

The size of the filters has to match the size/scale of the patterns wese want to detect (task dependent).

Key Ideas

A standard neural net applied to images:

- scales quadratically with the size of the input
- does not leverage stationarity

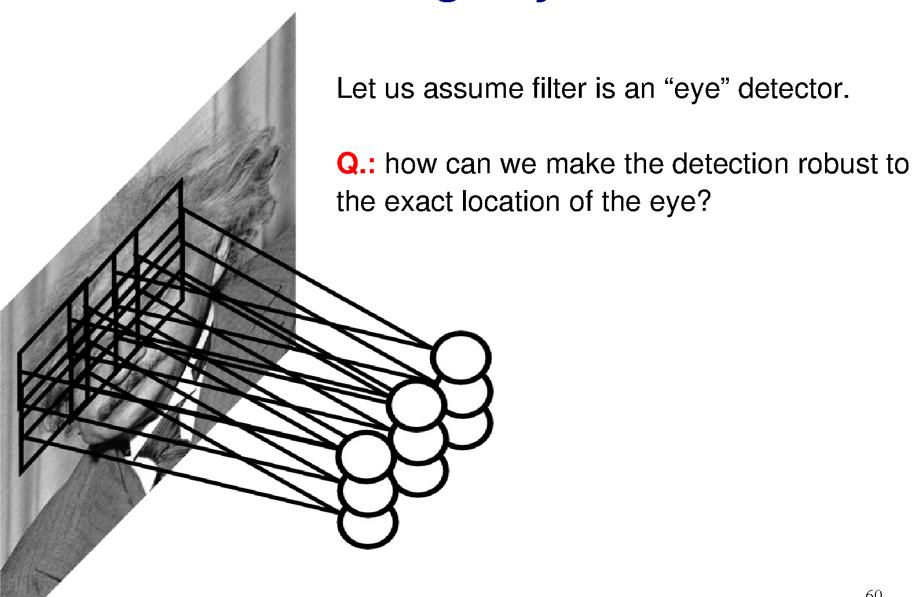
Solution:

- connect each hidden unit to a small patch of the input
- share the weight across space

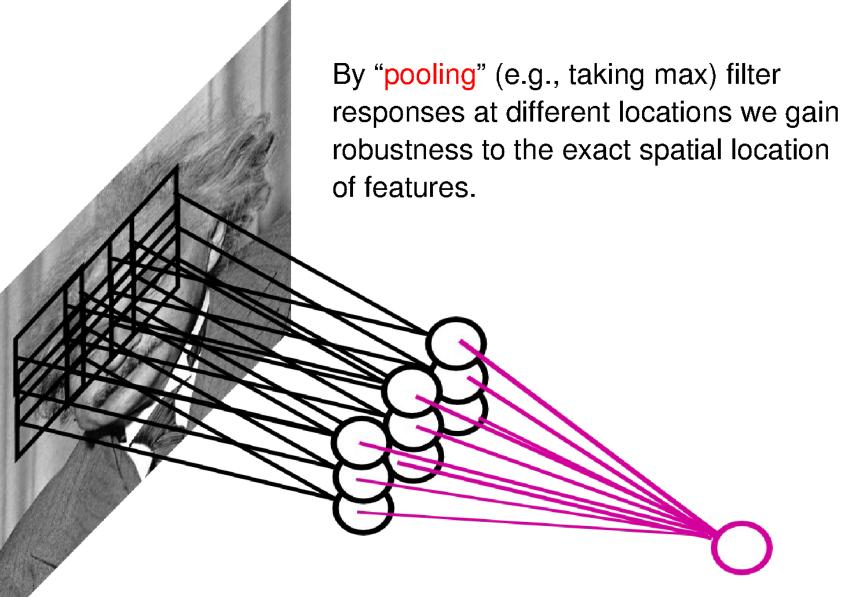
This is called: **convolutional layer.**

A network with convolutional layers is called convolutional network.

Pooling Layer



Pooling Layer



Pooling Layer: Examples

Max-pooling:

$$h_{j}^{n}(x, y) = max_{\bar{x} \in N(x), \bar{y} \in N(y)} h_{j}^{n-1}(\bar{x}, \bar{y})$$

Average-pooling:

$$h_{j}^{n}(x, y) = 1/K \sum_{\bar{x} \in N(x), \bar{y} \in N(y)} h_{j}^{n-1}(\bar{x}, \bar{y})$$

L2-pooling:

$$h_{j}^{n}(x, y) = \sqrt{\sum_{\bar{x} \in N(x), \bar{y} \in N(y)} h_{j}^{n-1}(\bar{x}, \bar{y})^{2}}$$

L2-pooling over features:

$$h_{j}^{n}(x, y) = \sqrt{\sum_{k \in N(j)} h_{k}^{n-1}(x, y)^{2}}$$



Pooling Layer

Question: What is the size of the output? What's the computational cost?

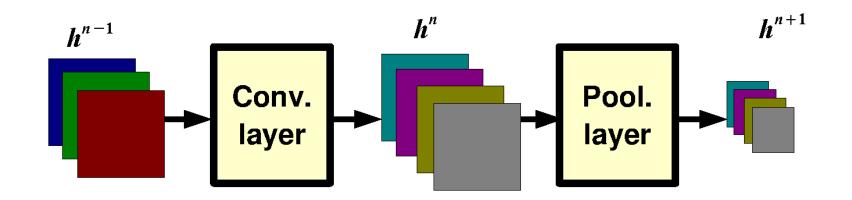
Answer: The size of the output depends on the stride between the pools. For instance, if pools do not overlap and have size KxK, and the input has size DxD with M input feature maps, then:

- output is M@(D/K)x(D/K)
- the computational cost is proportional to the size of the input (negligible compared to a convolutional layer)

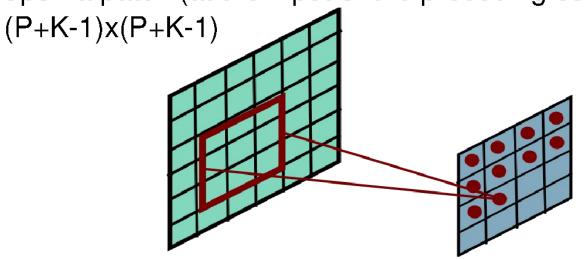
Question: How should I set the size of the pools?

Answer: It depends on how much "invariant" or robust to distortions we want the representation to be. It is best to pool slowly (via a few stacks of conv-pooling layers).

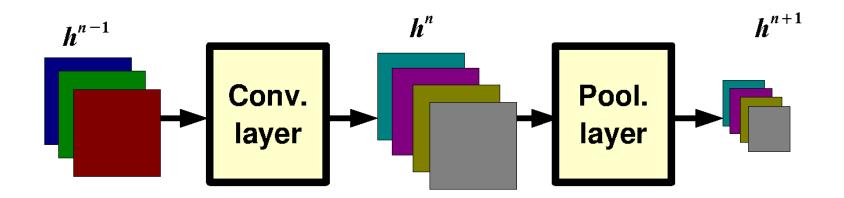
Pooling Layer: Receptive Field Size



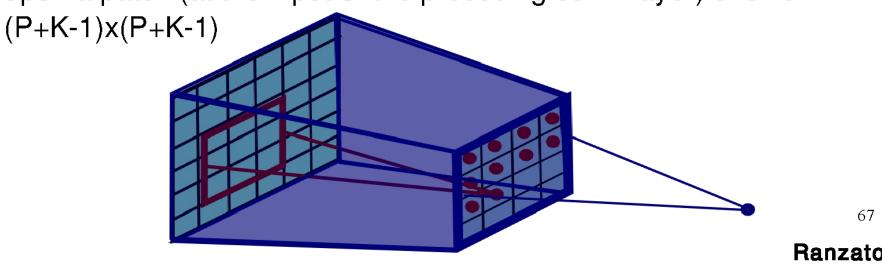
If convolutional filters have size KxK and stride 1, and pooling layer has pools of size PxP, then each unit in the pooling layer depends upon a patch (at the input of the preceding conv. layer) of size:



Pooling Layer: Receptive Field Size

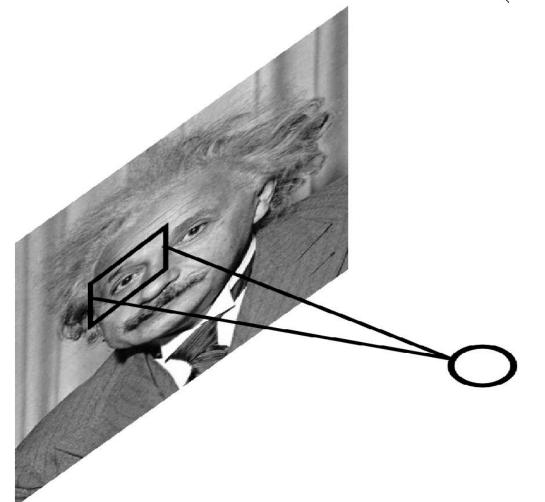


If convolutional filters have size KxK and stride 1, and pooling layer has pools of size PxP, then each unit in the pooling layer depends upon a patch (at the input of the preceding conv. layer) of size:

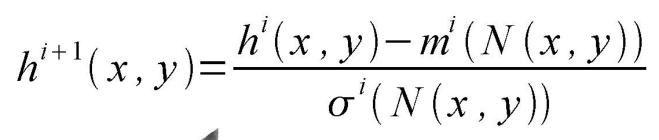


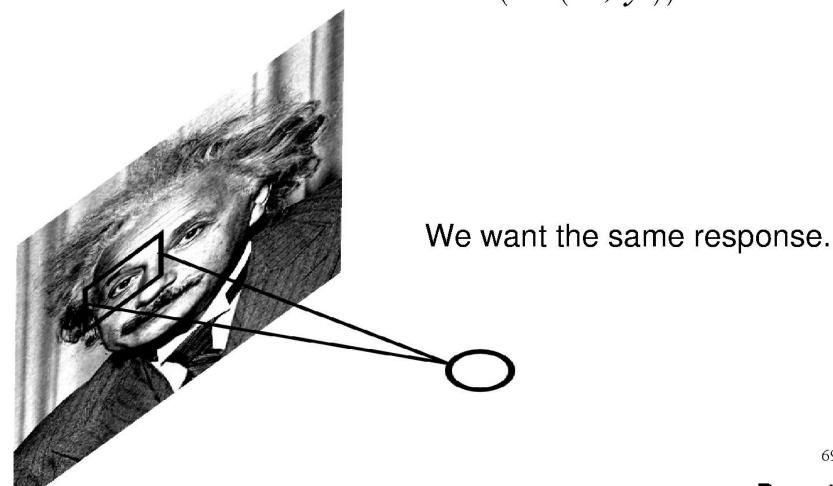
Local Contrast Normalization

$$h^{i+1}(x,y) = \frac{h^{i}(x,y) - m^{i}(N(x,y))}{\sigma^{i}(N(x,y))}$$

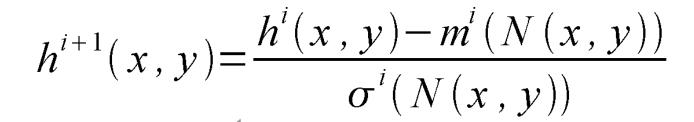


Local Contrast Normalization





Local Contrast Normalization



Performed also across features and in the higher layers..

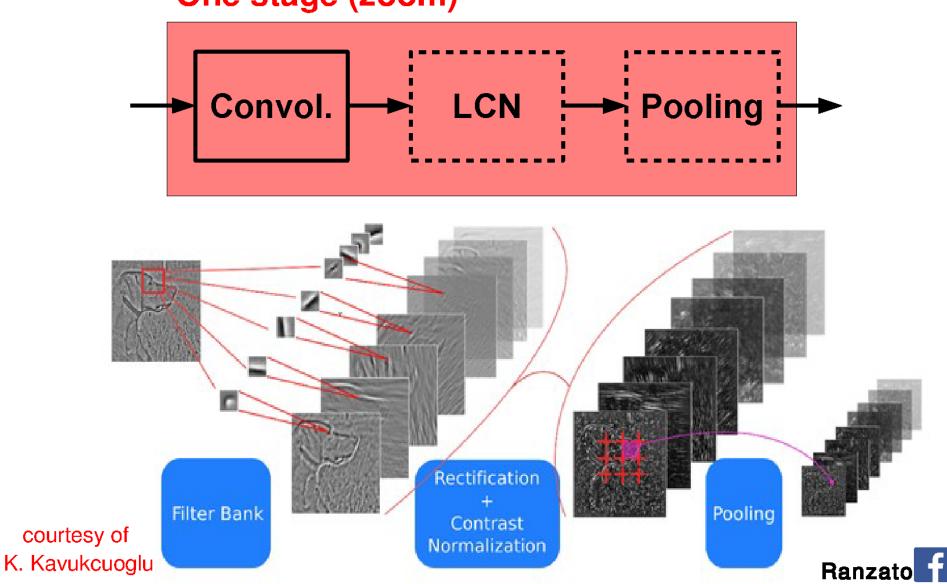
Effects:

- improves invariance
- improves optimization
- increases sparsity

Note: computational cost is negligible w.r.t. conv. layer.

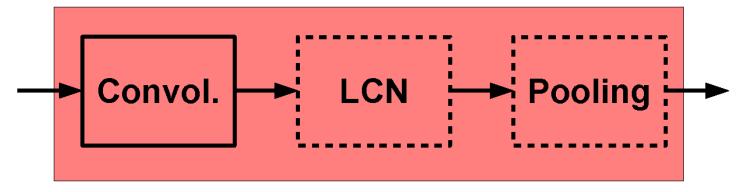
ConvNets: Typical Stage

One stage (zoom)



ConvNets: Typical Stage

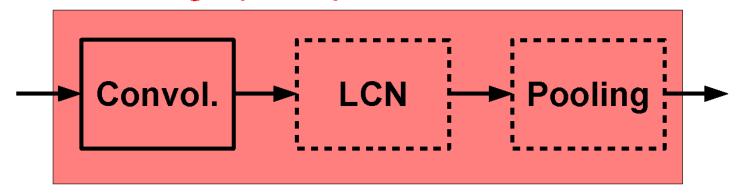
One stage (zoom)



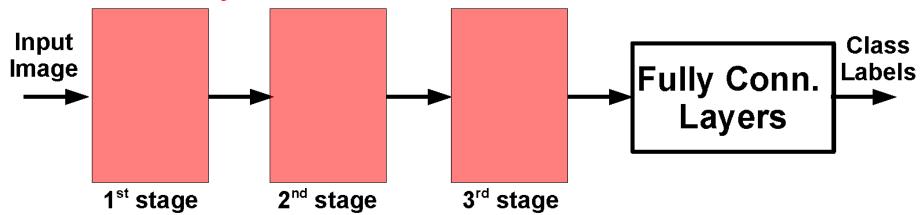
Conceptually similar to: SIFT, HoG, etc.

ConvNets: Typical Architecture

One stage (zoom)



Whole system



ConvNets: Typical Architecture

Whole system Input Image To stage 1st stage 2nd stage 3rd stage

Conceptually similar to:

SIFT \rightarrow K-Means \rightarrow Pyramid Pooling \rightarrow SVM

Lazebnik et al. "...Spatial Pyramid Matching..." CVPR 2006

SIFT \rightarrow Fisher Vect. \rightarrow Pooling \rightarrow SVM

Sanchez et al. "Image classifcation with F.V.: Theory and practice" IJCV 2012

ConvNets: Training

All layers are differentiable (a.e.). We can use standard back-propagation.

Algorithm:

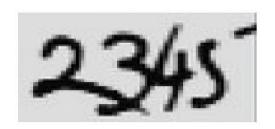
Given a small mini-batch

- F-PROP
- B-PROP
- PARAMETER UPDATE

Outline

- Supervised Neural Networks
- Convolutional Neural Networks
- Examples
- Tips

- OCR / House number & Traffic sign classification







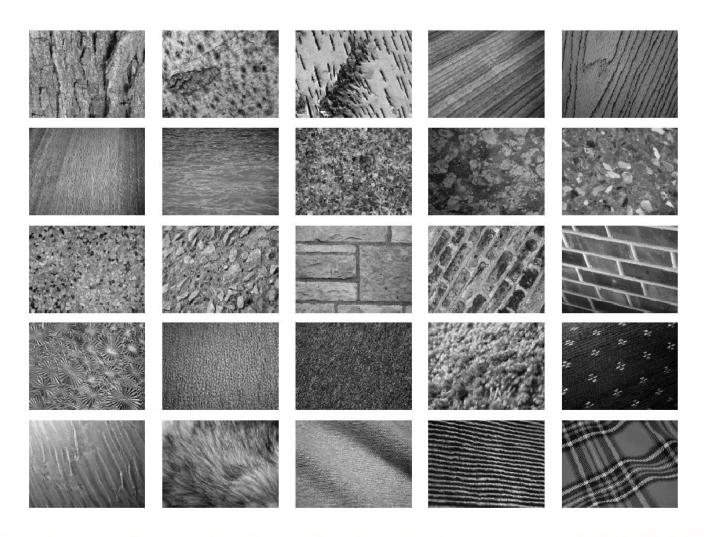
Ciresan et al. "MCDNN for image classification" CVPR 2012

Wan et al. "Regularization of neural networks using dropconnect" ICML 2013

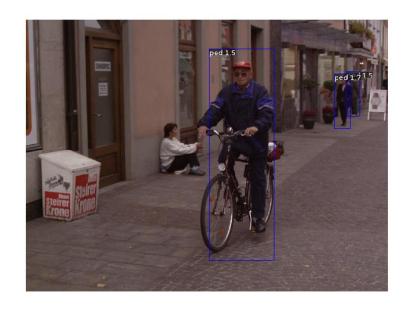
82

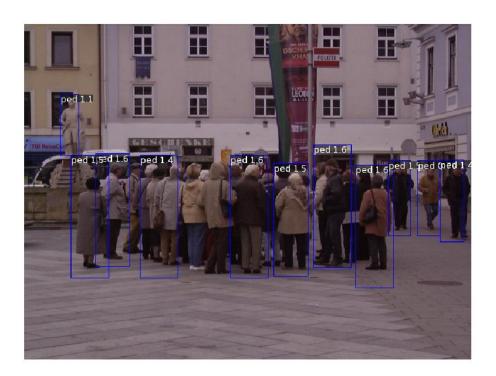
Jaderberg et al. "Synthetic data and ANN for natural scene text recognition" arXiv 2014

- Texture classification



- Pedestrian detection



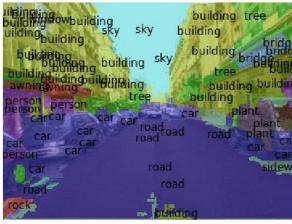


- Scene Parsing







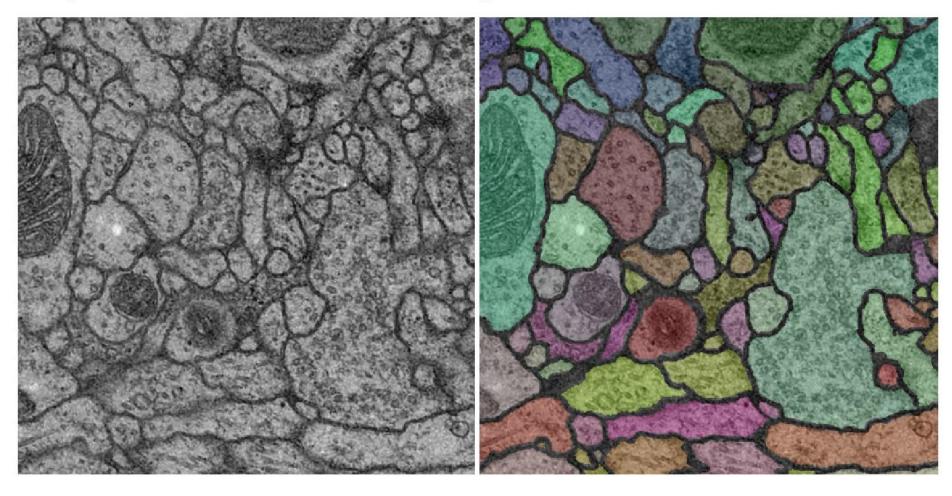


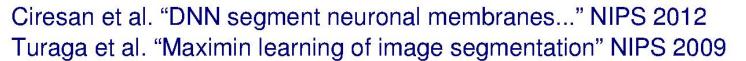




Ranzato

- Segmentation 3D volumetric images







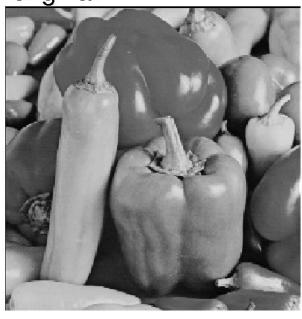
- Action recognition from videos



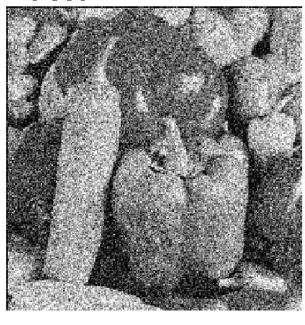
Taylor et al. "Convolutional learning of spatio-temporal features" ECCV 2010 Karpathy et al. "Large-scale video classification with CNNs" CVPR 2014 Simonyan et al. "Two-stream CNNs for action recognition in videos" arXiv 2014

- Denoising

original



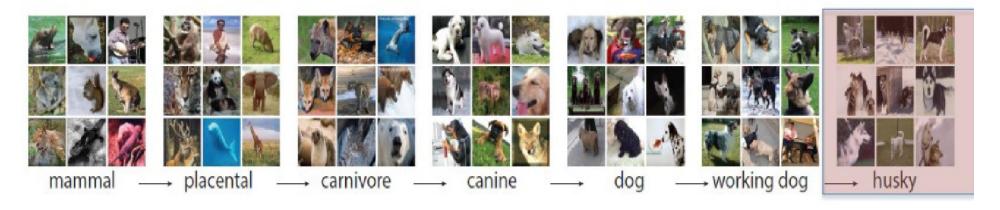
noised



denoised



Dataset: ImageNet 2012



- S: (n) Eskimo dog, husky (breed of heavy-coated Arctic sled dog)
 - o direct hypernym / inherited hypernym / sister term
 - S: (n) working dog (any of several breeds of usually large powerful dogs bred to work as draft animals and guard and guide dogs)
 - S: (n) dog, domestic dog, Canis familiaris (a member of the genus Canis (probably descended from the common wolf) that has been domesticated by man since prehistoric times; occurs in many breeds) "the dog barked all night"
 - S: (n) canine, canid (any of various fissiped mammals with nonretractile claws and typically long muzzles)
 - S: (n) carnivore (a terrestrial or aquatic flesh-eating mammal) "terrestrial carnivores have four or five clawed digits on each limb"
 - S: (n) placental, placental mammal, eutherian, eutherian mammal (mammals having a placenta; all mammals except monotremes and marsupials)
 - S: (n) mammal, mammalian (any warm-blooded vertebrate having the skin more or less covered with hair; young are born alive except for the small subclass of
 monotremes and nourished with milk)
 - S: (n) vertebrate, craniate (animals having a bony or cartilaginous skeleton with a segmented spinal column and a large brain enclosed in a skull or cranium)
 - S: (n) chordate (any animal of the phylum Chordata having a notochord or spinal column)
 - S: (n) animal, animate being, beast, brute, creature, fauna (a living organism characterized by voluntary movement)
 - S: (n) organism, being (a living thing that has (or can develop) the ability to act or function independently)
 - S: (n) living thing, animate thing (a living (or once living) entity)
 - S: (n) whole, unit (an assemblage of parts that is regarded as a single entity) "how big is that part compared to the
 whole?": "the team is a unit"
 - S: (n) object, physical object (a tangible and visible entity; an entity that can cast a shadow) "it was full of rackets, balls and other objects"
 - S: (n) physical entity (an entity that has physical existence)
 - S: (n) entity (that which is perceived or known or inferred to have its own distinct existence (living or nonliving))

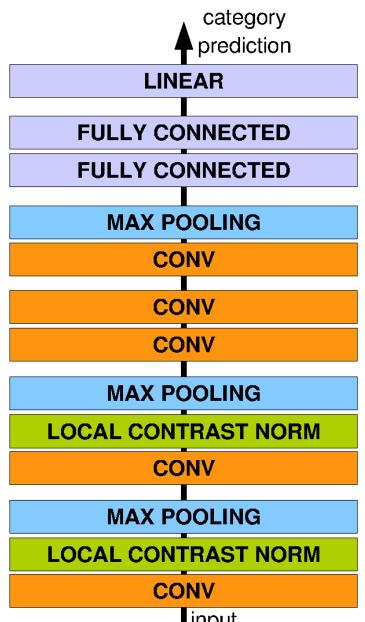
Deng et al. "Imagenet: a large scale hierarchical image database" CVPR 2009

ImageNet

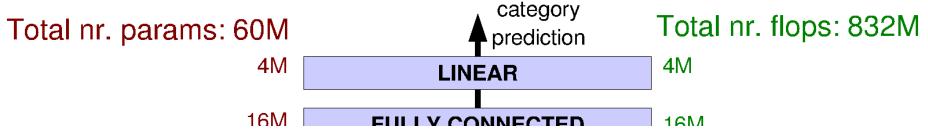
Examples of hammer:



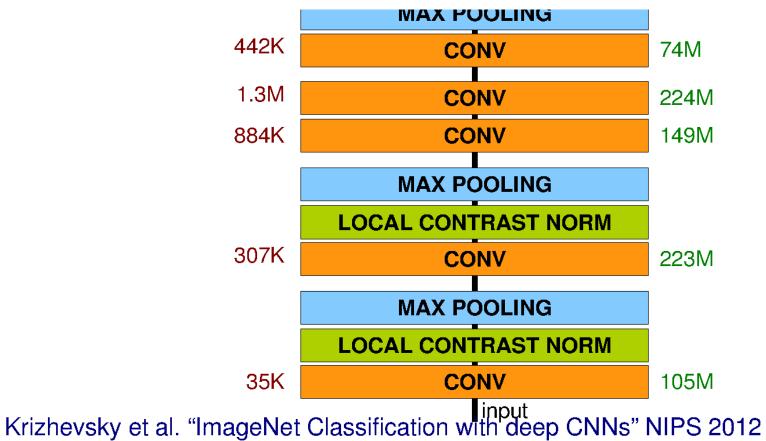
Architecture for Classification



Architecture for Classification



The first convolutional layer filters the $224 \times 224 \times 3$ input image with 96 kernels of size $11 \times 11 \times 3$ with a stride of 4 pixels (this is the distance between the receptive field centers of neighboring



96

Optimization

SGD with momentum:

- Learning rate = 0.01
- Momentum = 0.9

Improving generalization by:

- Weight sharing (convolution)
- Input distortions
- **■** Dropout = 0.5
- Weight decay = 0.0005

Results: ILSVRC 2012

