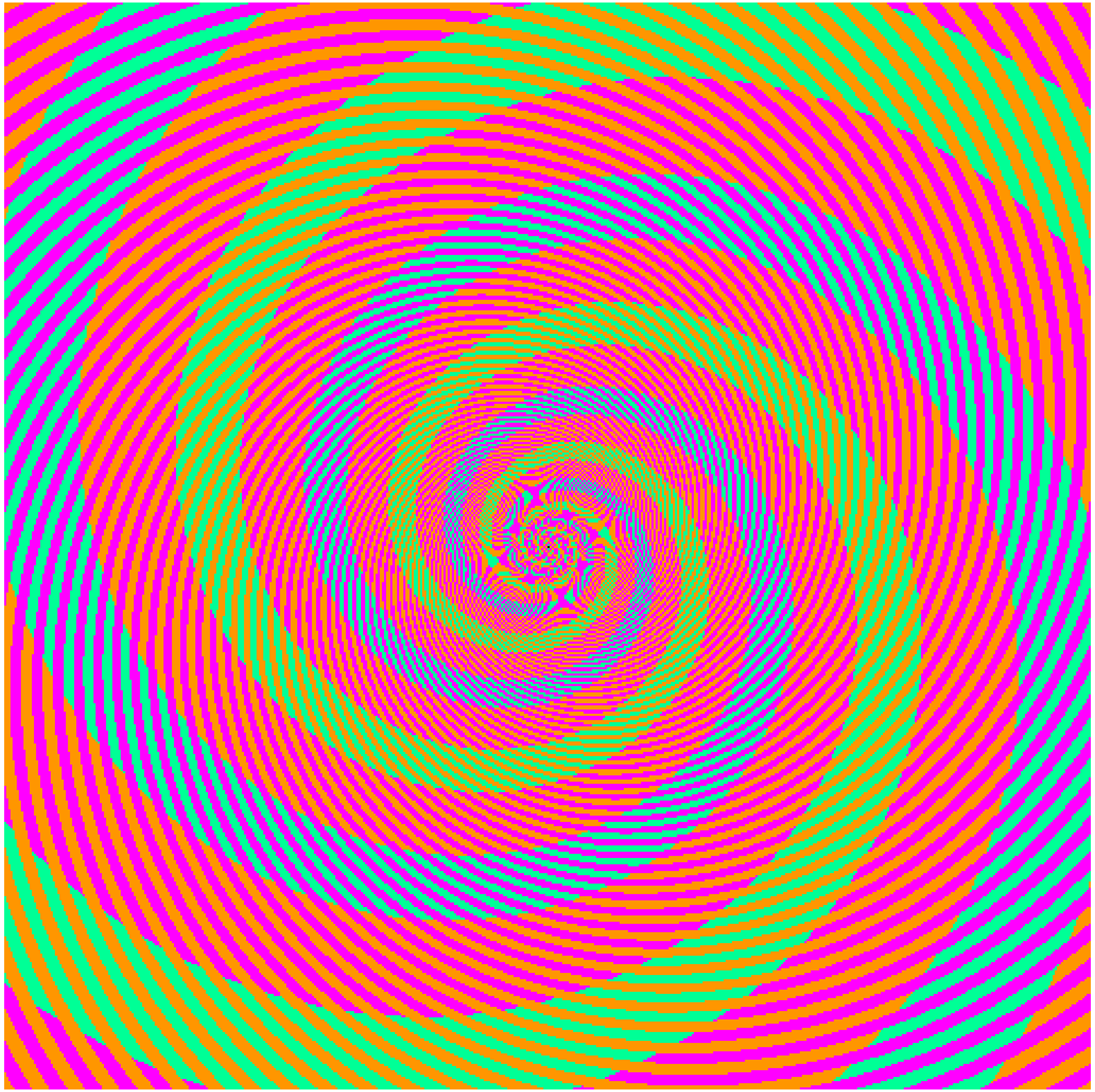
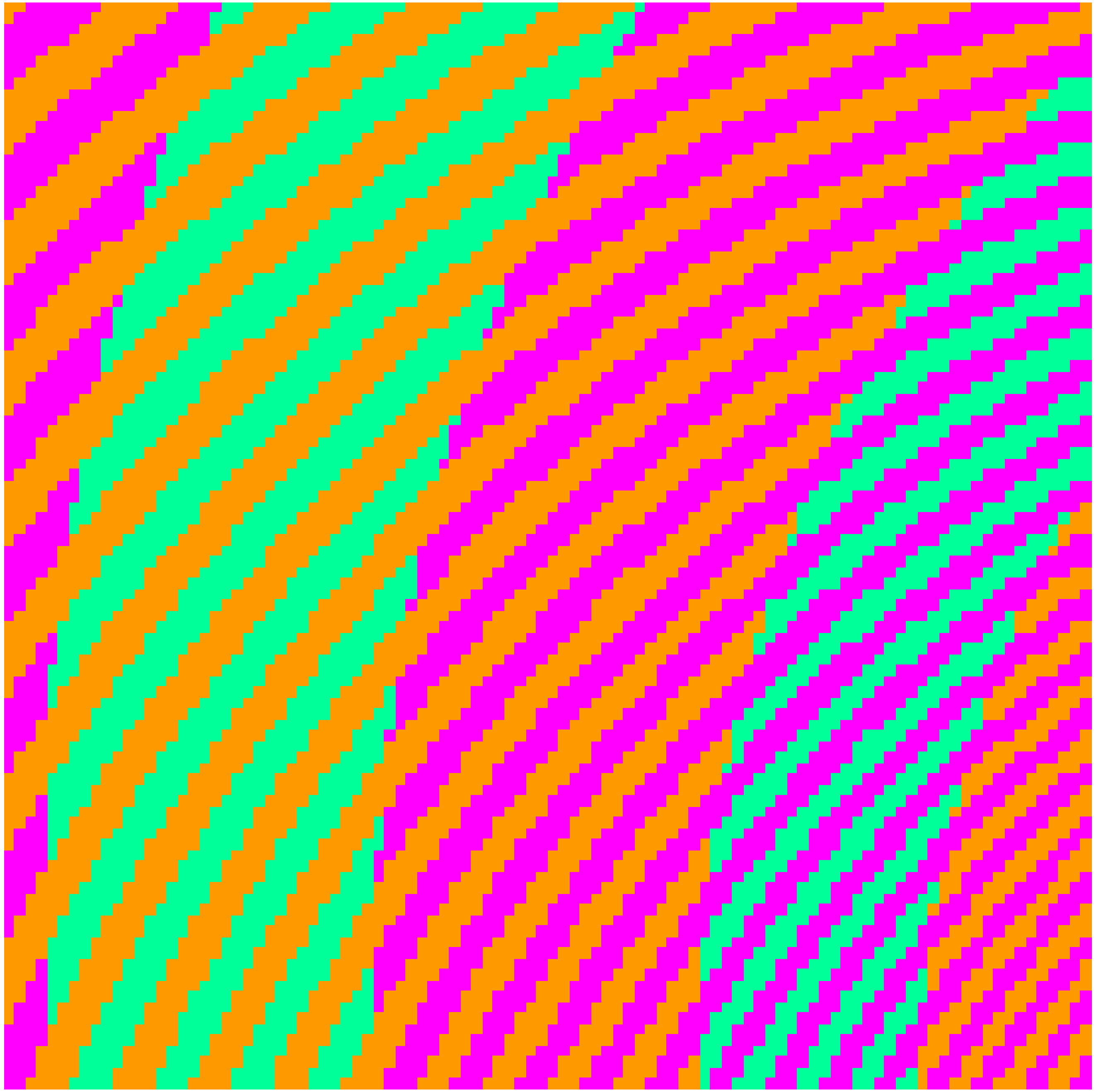
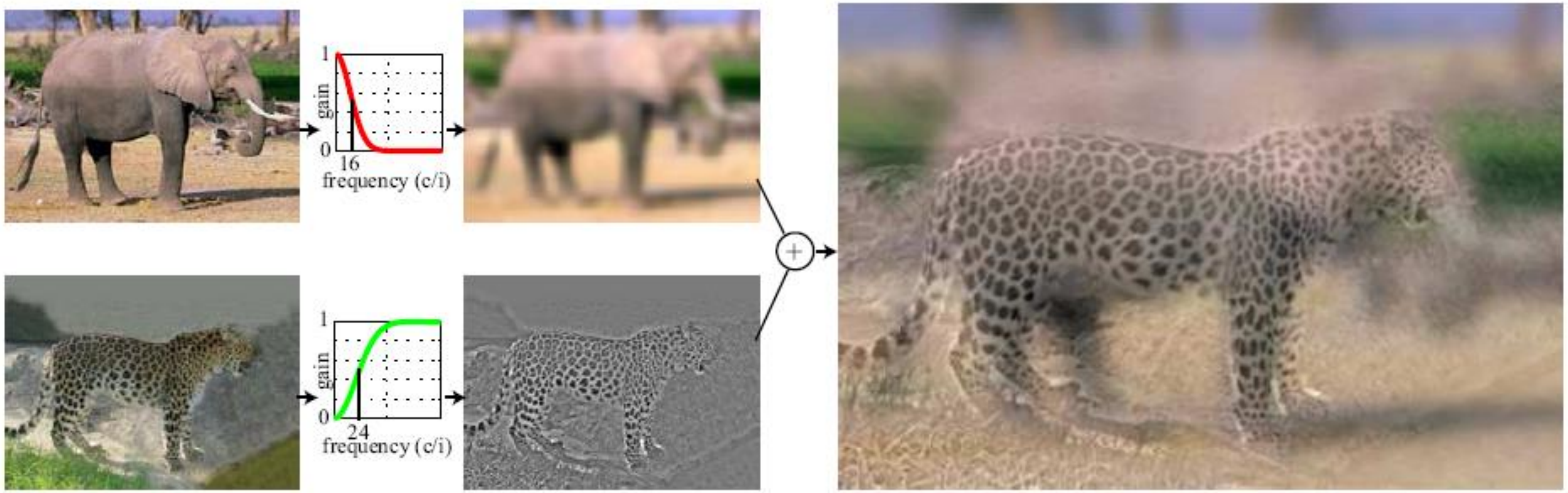


The blue and green colors are actually the same



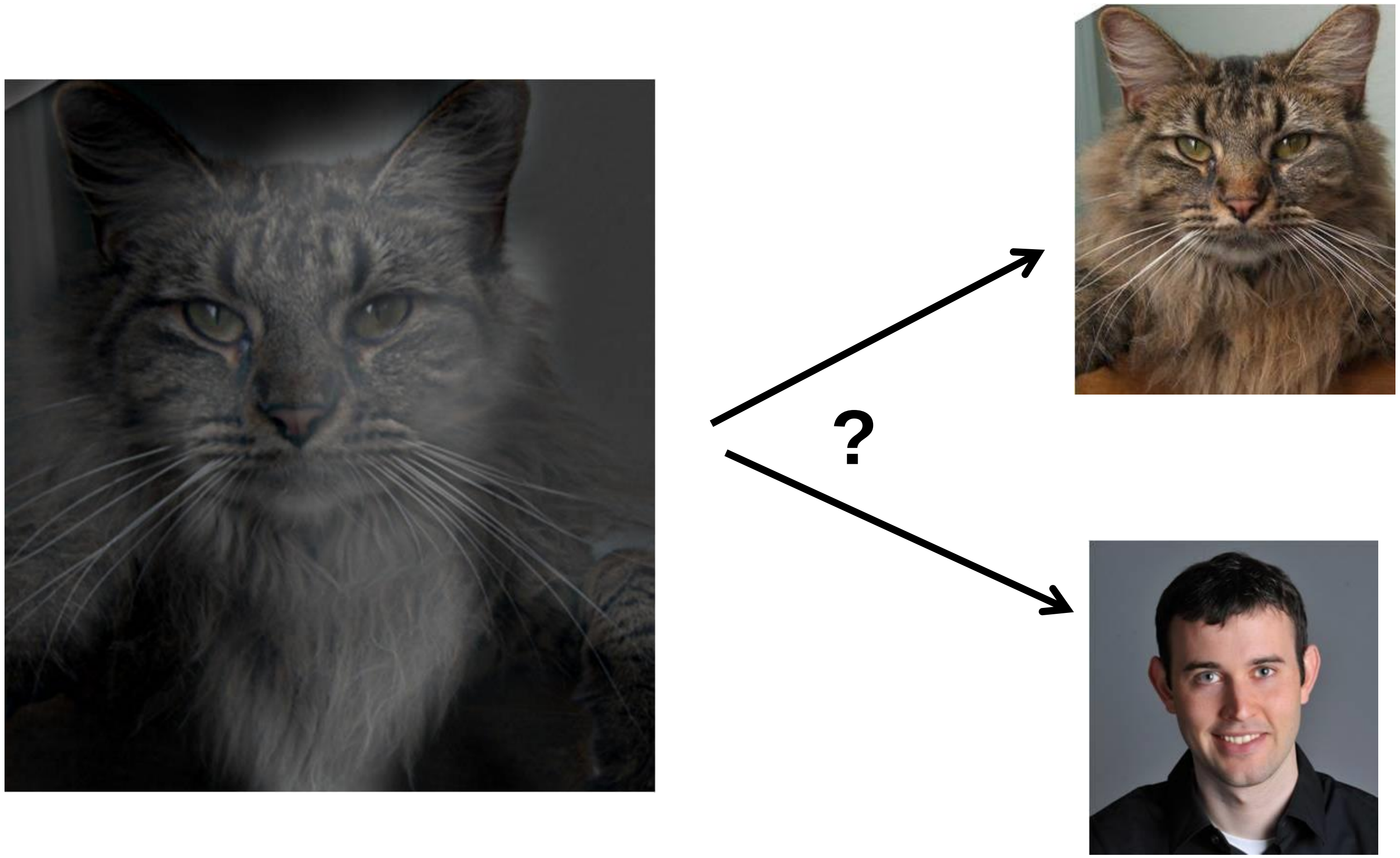
http://blogs.discovermagazine.com/badastronomy/2009/06/24/the-blue-and-the-green/

Hybrid Images



- A. Oliva, A. Torralba, P.G. Schyns, "Hybrid Images," SIGGRAPH 2006

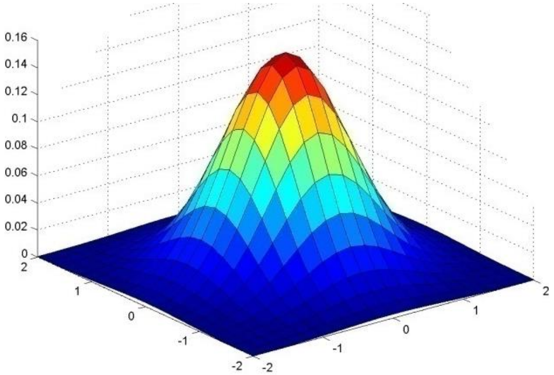
Why do we get different, distance-dependent interpretations of hybrid images?

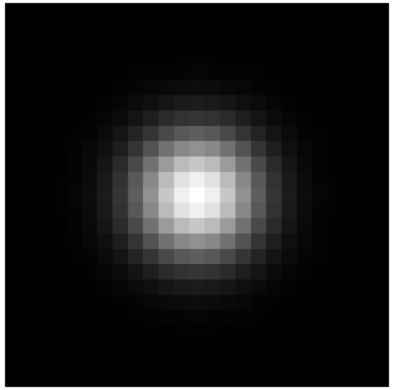


Slide: Hoiem

Important filter: Gaussian

- Weight contributions of neighboring pixels by nearness





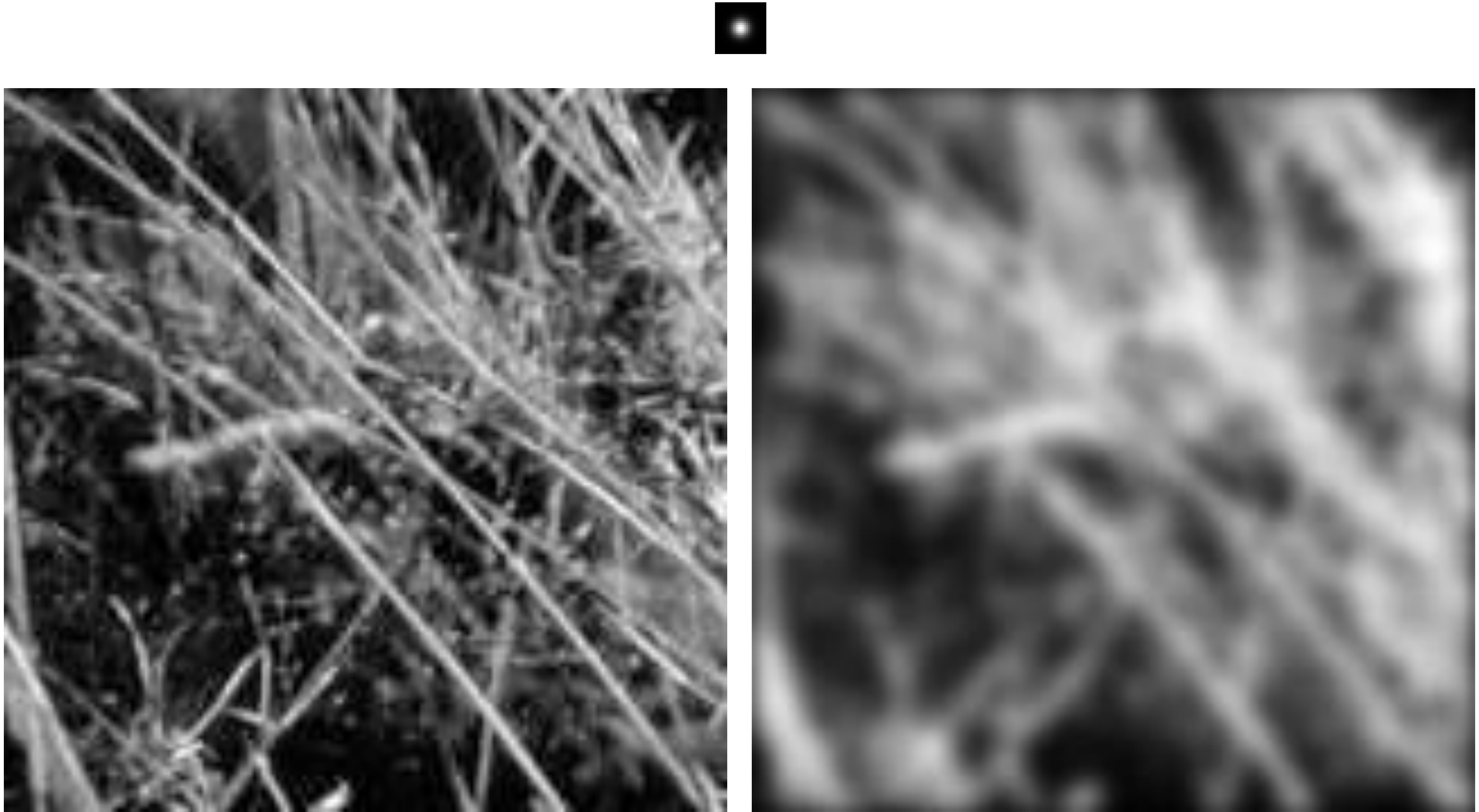
0.003	0.013	0.022	0.013	0.003
0.013	0.059	0.097	0.059	0.013
0.022	0.097	0.159	0.097	0.022
0.013	0.059	0.097	0.059	0.013
0.003	0.013	0.022	0.013	0.003

5 x 5, σ = 1

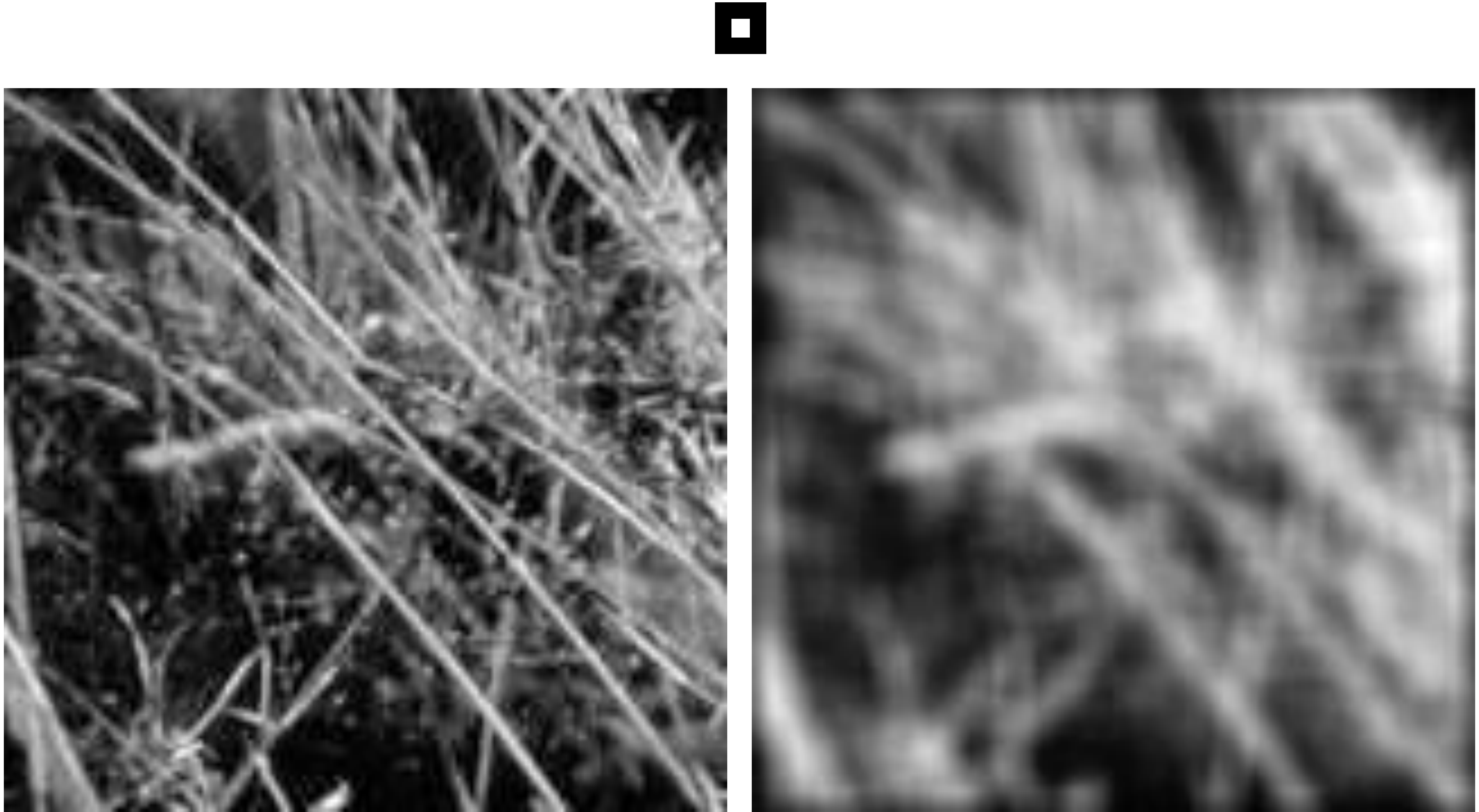
$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

Slide credit: Christopher Rasmussen

Smoothing with Gaussian filter



Smoothing with box filter



Gaussian filters

- Remove “high-frequency” components from the image (low-pass filter)
  - Images become more smooth
- Convolution with self is another Gaussian
  - So can smooth with small-width kernel, repeat, and get same result as larger-width kernel would have
  - Convolving two times with Gaussian kernel of width σ is same as convolving once with kernel of width $\sigma\sqrt{2}$
- *Separable* kernel
  - Factors into product of two 1D Gaussians

Source: K. Grauman

Separability of the Gaussian filter

$$G_\sigma(x, y) = \frac{1}{2\pi\sigma^2} \exp^{-\frac{x^2 + y^2}{2\sigma^2}}$$

$$= \left(\frac{1}{\sqrt{2\pi}\sigma} \exp^{-\frac{x^2}{2\sigma^2}} \right) \left(\frac{1}{\sqrt{2\pi}\sigma} \exp^{-\frac{y^2}{2\sigma^2}} \right)$$

The 2D Gaussian can be expressed as the product of two functions, one a function of *x* and the other a function of *y*

In this case, the two functions are the (identical) 1D Gaussian

Source: D. Lowe

Separability example

2D convolution
(center location only)

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \ast \begin{bmatrix} 2 & 3 & 3 \\ 3 & 5 & 5 \\ 4 & 4 & 6 \end{bmatrix}$$

The filter factors
into a product of 1D
filters:

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

Perform convolution
along rows:

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \ast \begin{bmatrix} 2 & 3 & 3 \\ 3 & 5 & 5 \\ 4 & 4 & 6 \end{bmatrix} = \begin{bmatrix} & 11 & \\ & 18 & \\ & 18 & \end{bmatrix}$$

Followed by convolution
along the remaining column:

Source: K. Grauman

Separability

- Why is separability useful in practice?

Some practical matters

Practical matters

How big should the filter be?

- Values at edges should be near zero
- Rule of thumb for Gaussian: set filter half-width to about 3 σ

Practical matters

- What about near the edge?
  - – the filter window falls off the edge of the image
  - – need to extrapolate
  - – methods:
    - clip filter (black)
    - wrap around
    - copy edge
    - reflect across edge

Source: S. Marschner

Recap of Filtering

- Linear filtering is dot product at each position
  - Not a matrix multiplication
  - Can smooth, sharpen, translate (among many other uses)

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\1 & 1 & 1\\1 & 1 & 1\end{bmatrix}$$

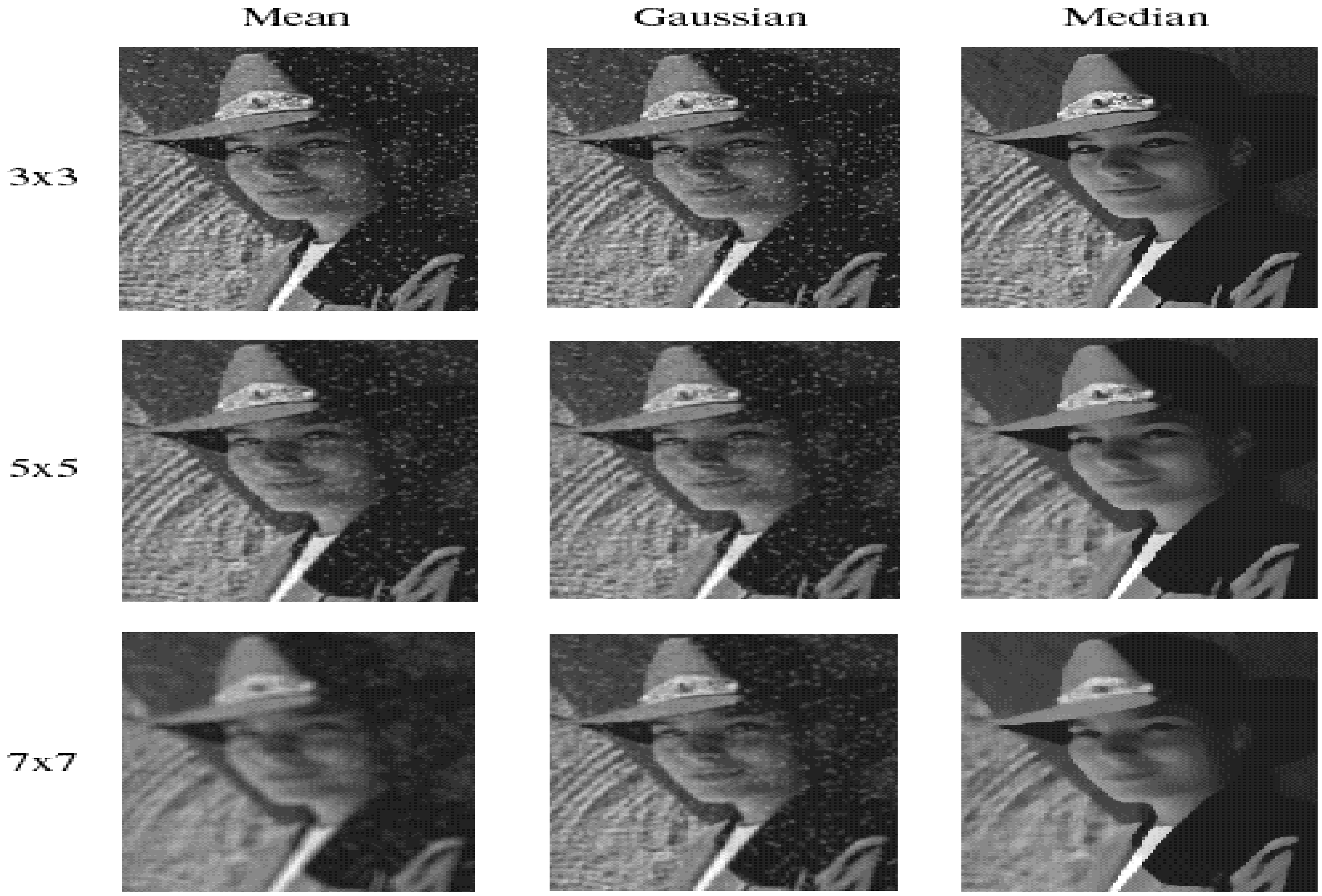
- Be aware of details for filter size, extrapolation, cropping

Median filters

- A **Median Filter** operates over a window by selecting the median intensity in the window.
- What advantage does a median filter have over a mean filter?
- Is a median filter a kind of convolution?

Slide by Steve Seitz

Comparison: salt and pepper noise



Slide by Steve Seitz

Review: questions

1. Write down a 3x3 filter that returns a positive value if the average value of the 4-adjacent neighbors is less than the center and a negative value otherwise

2. Write down a filter that will compute the gradient in the x-direction:

```
gradx(y,x) = im(y,x+1)-im(y,x) for each x, y
```

Slide: Hoiem

Review: questions

3. Fill in the blanks:

Filtering Operator

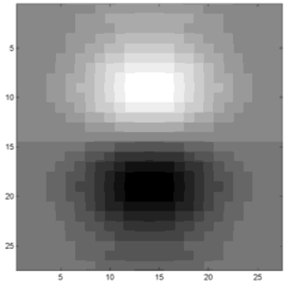
a) _ = D * B

b) A = _ * _

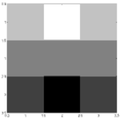
c) F = D * _

d) _ = D * D

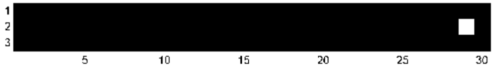
A



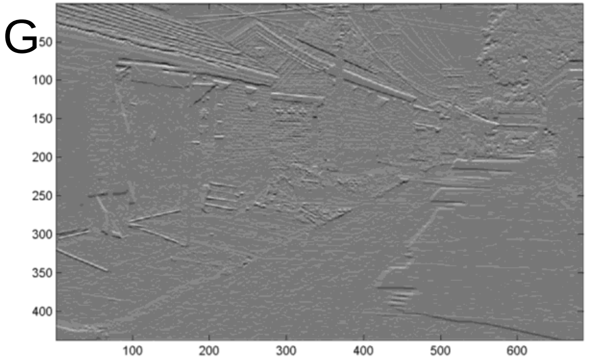
B



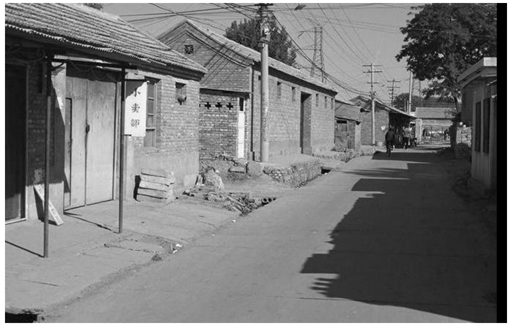
E



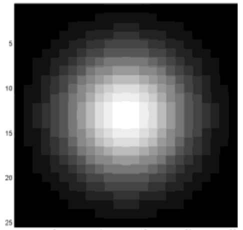
G



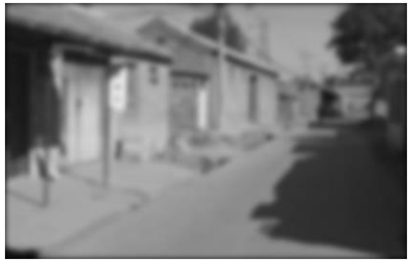
F



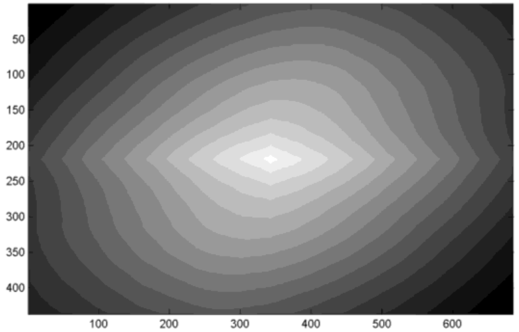
C



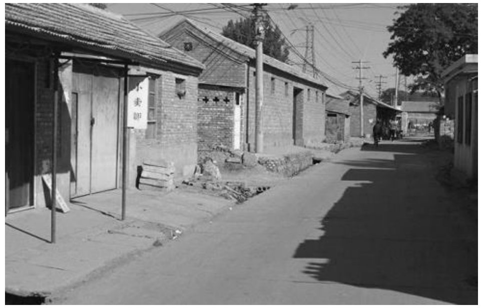
H



I

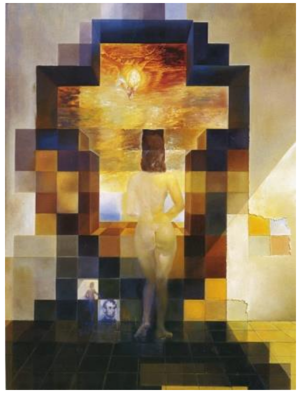


D



Slide: Hoiem

Thinking in Frequency



Slides: Hoiem, Efros, and others

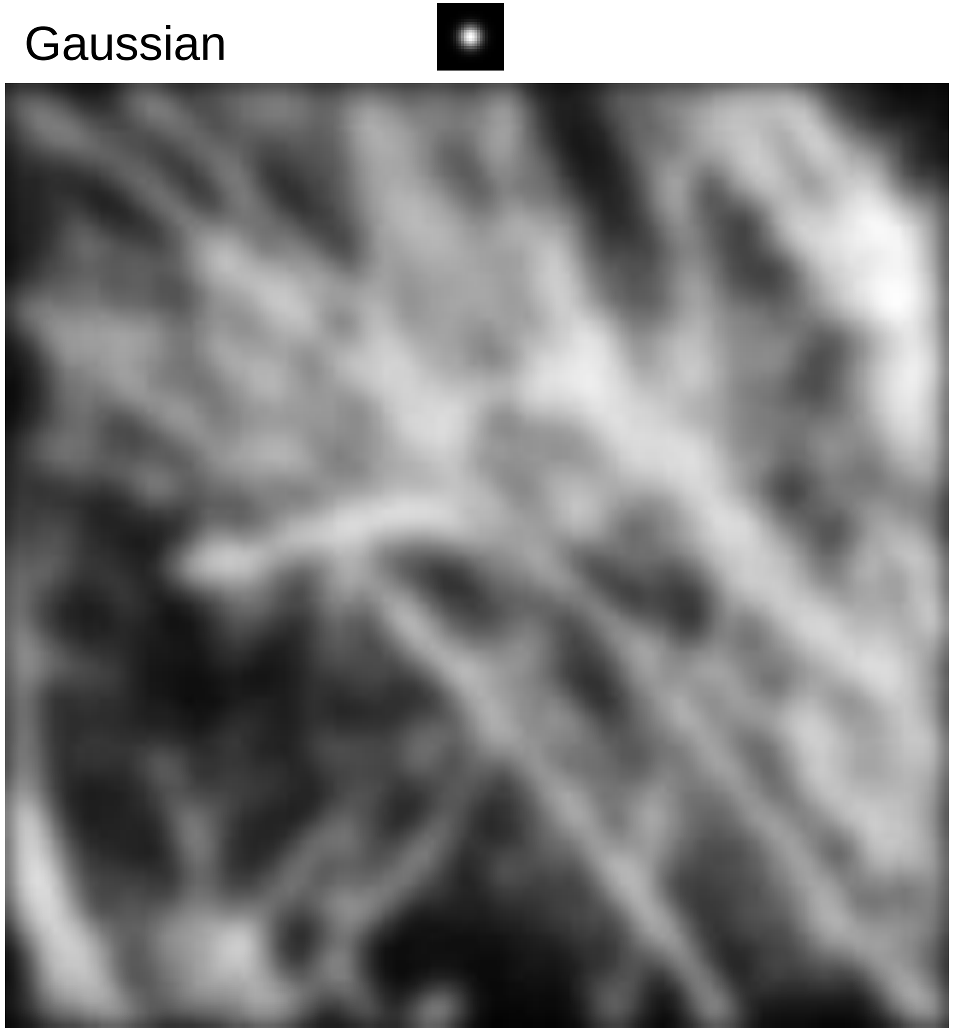
This lecture

- Fourier transform and frequency domain
  - Frequency view of filtering
- Reminder: Read your textbook
  - Today’s lecture covers material in 3.4

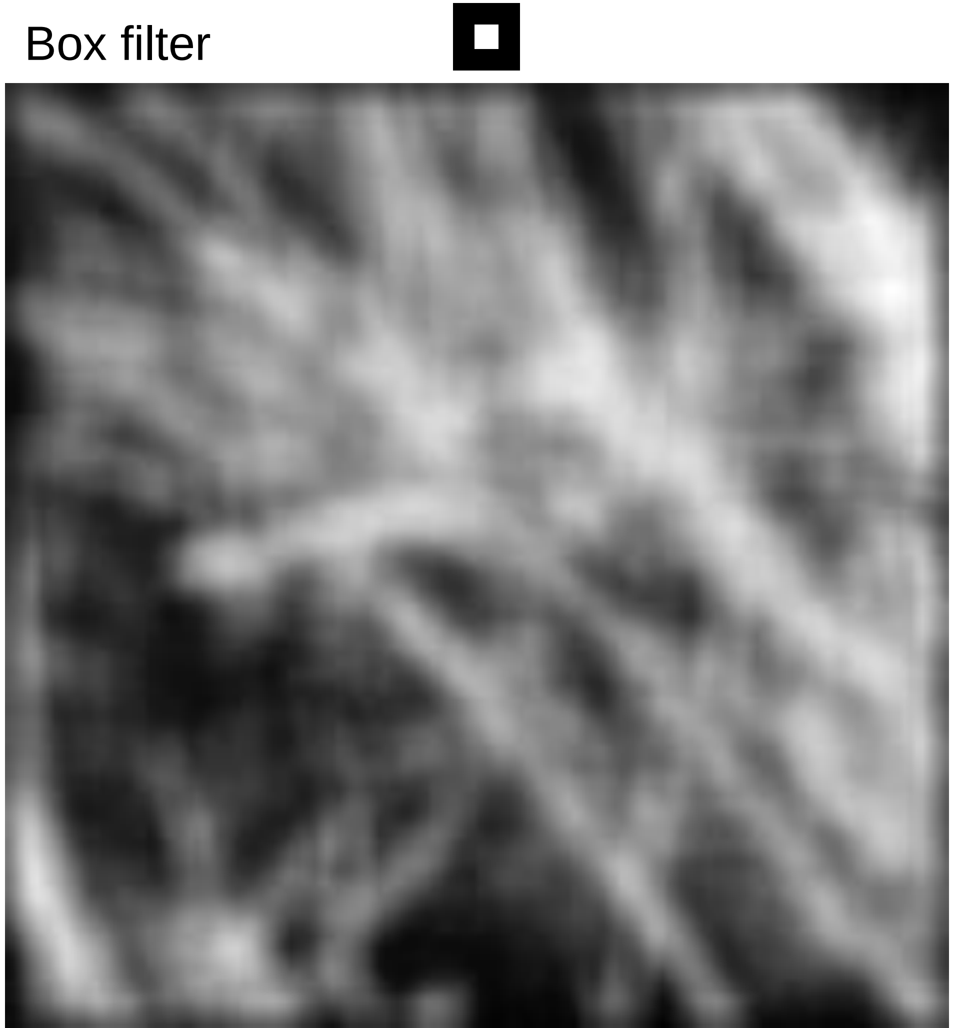
Slide: Hoiem

Why does the Gaussian give a nice smooth image, but the square filter give edgy artifacts?

Gaussian



Box filter



Why does a lower resolution image still make sense to us? What do we lose?

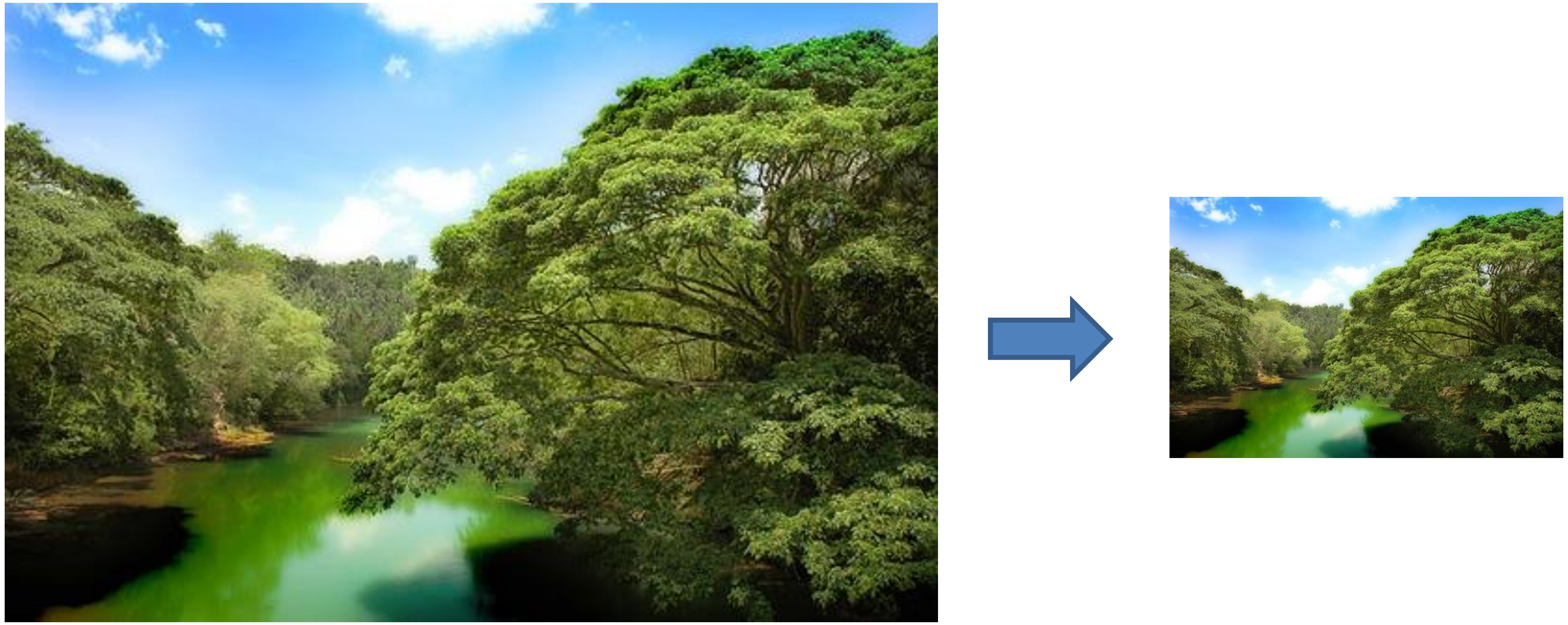
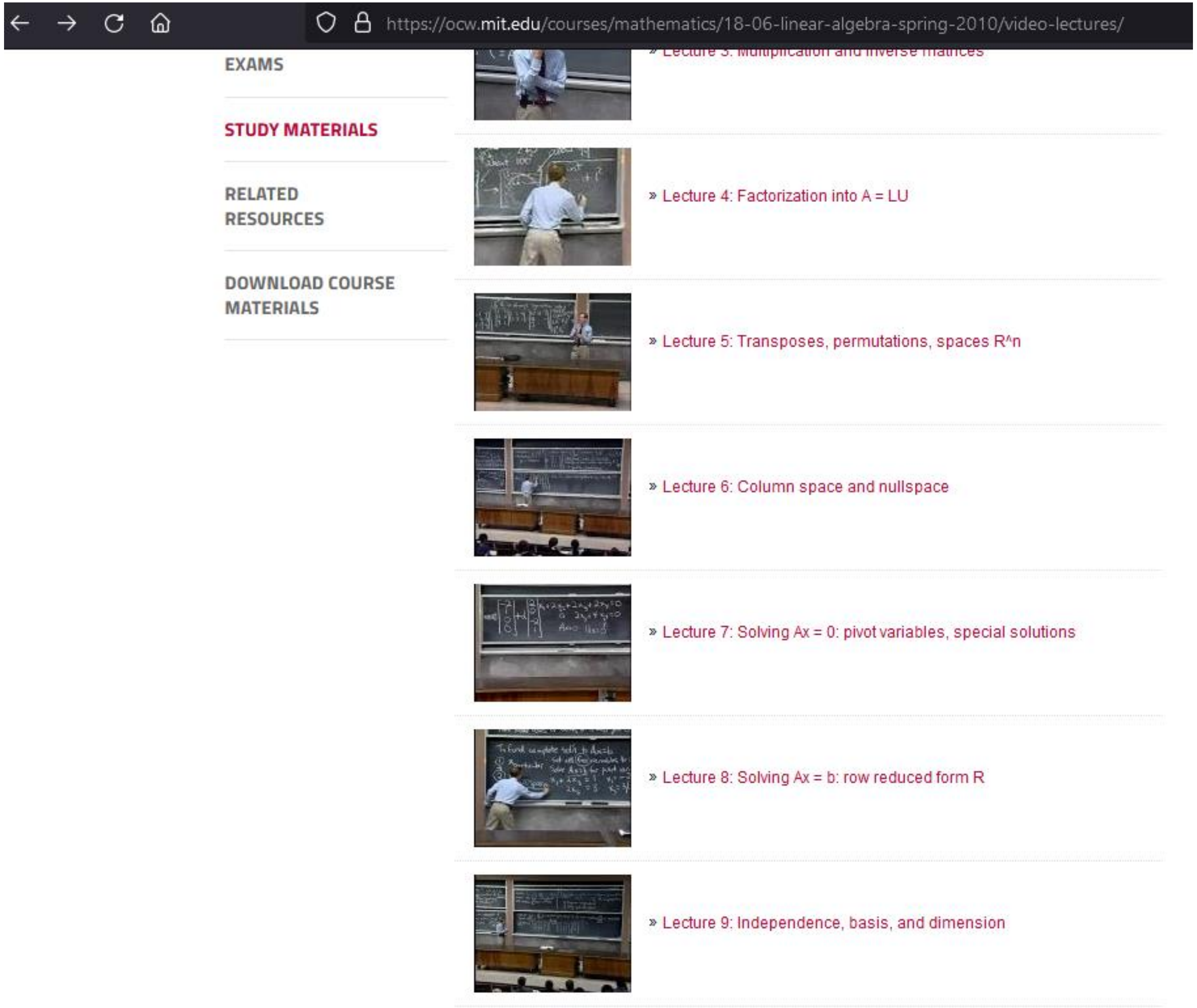


Image: http://www.flickr.com/photos/igorms/136916757/ Slide: Hoiem

Thinking in terms of frequency

Background: Change of Basis



https://ocw.mit.edu/courses/mathematics/18-06-linear-algebra-spring-2010/video-lectures/

EXAMS

STUDY MATERIALS

RELATED RESOURCES

DOWNLOAD COURSE MATERIALS

» Lecture 3: Multiplication and inverse matrices

» Lecture 4: Factorization into A = LU

» Lecture 5: Transposes, permutations, spaces R^n

» Lecture 6: Column space and nullspace

» Lecture 7: Solving Ax = 0: pivot variables, special solutions

» Lecture 8: Solving Ax = b: row reduced form R

» Lecture 9: Independence, basis, and dimension

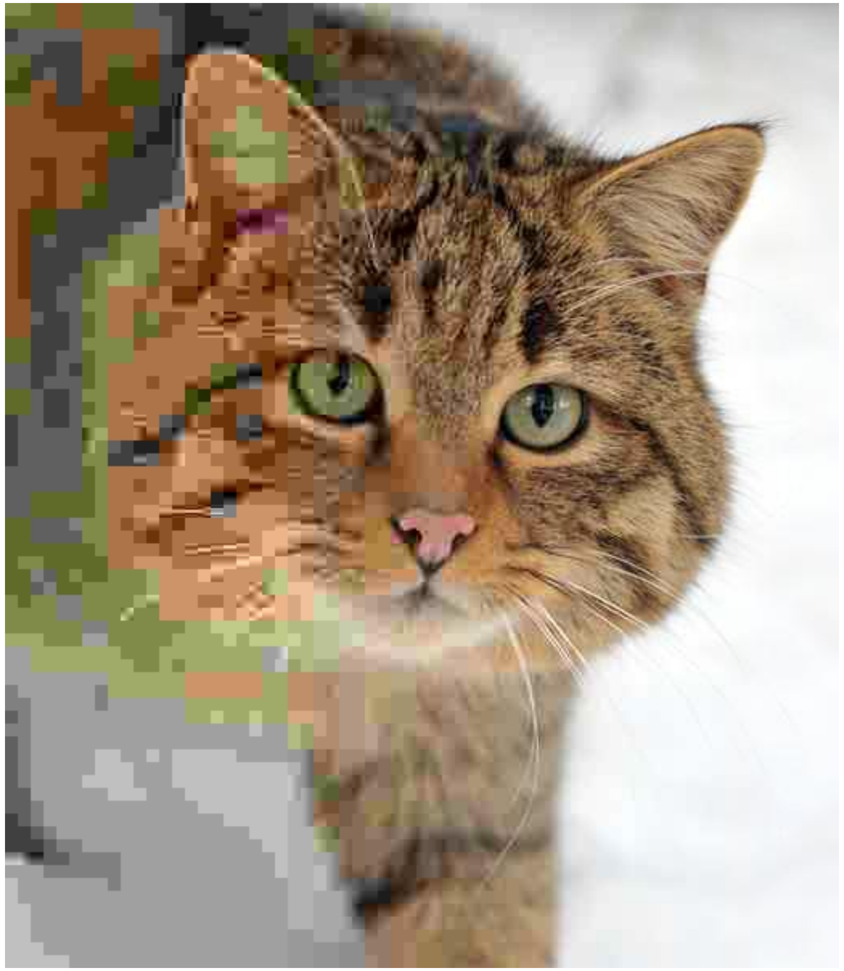
Background: Change of Basis

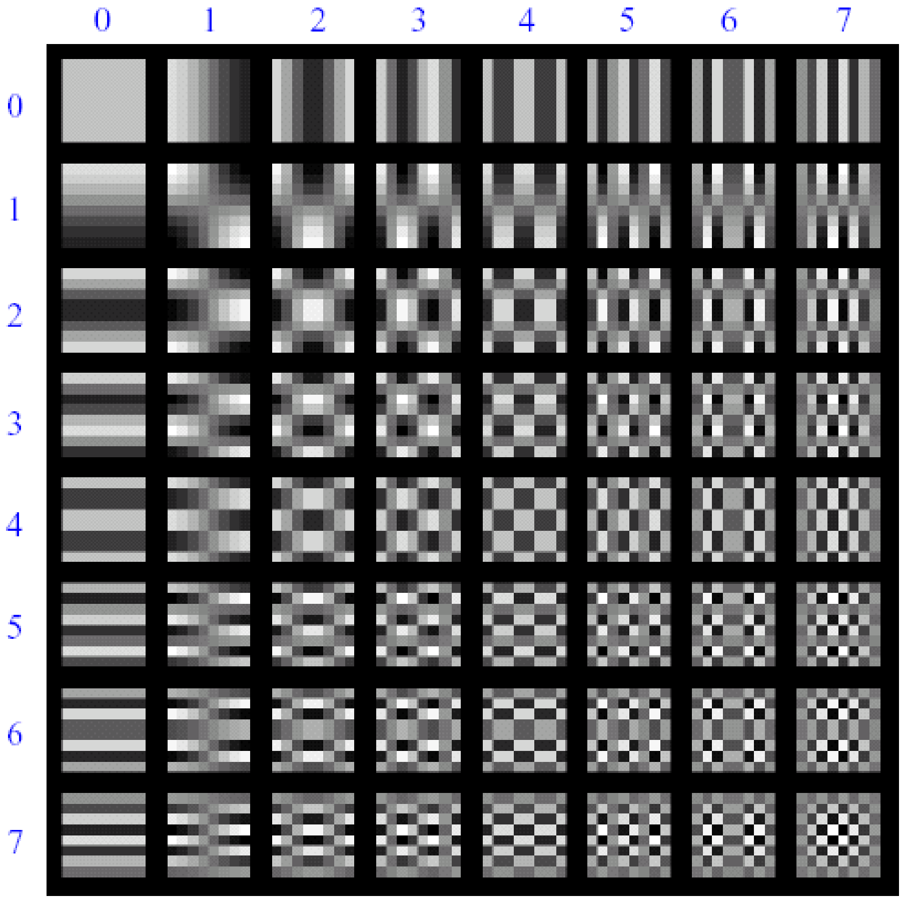
For vectors and for image patches

Related concept: Image Compression

How is it that a 4MP image can be compressed to a few hundred KB without a noticeable change?

Lossy Image Compression (JPEG)



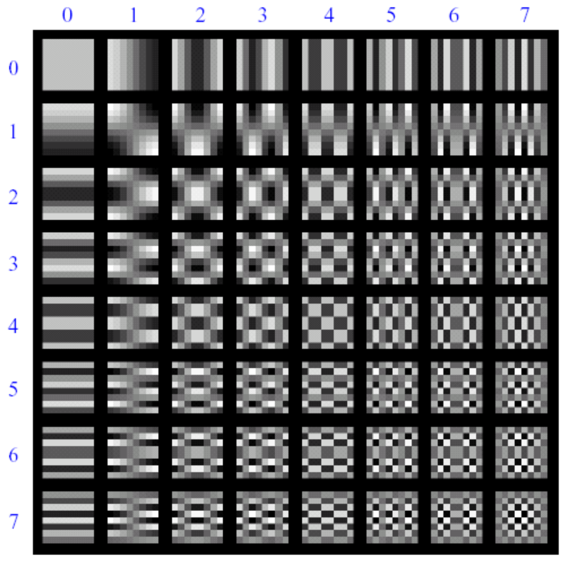


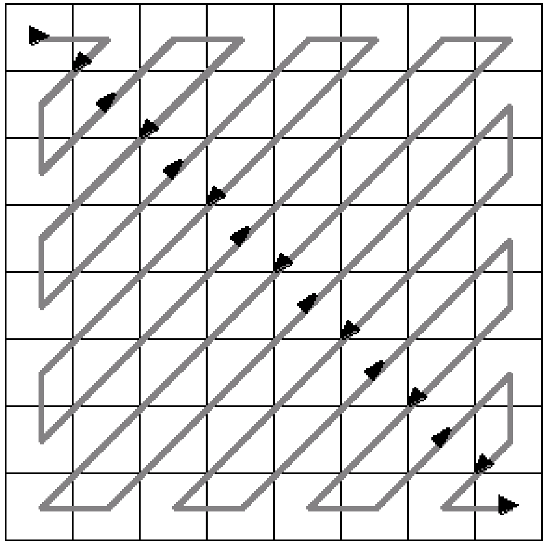
Block-based Discrete Cosine Transform (DCT)

https://en.wikipedia.org/wiki/JPEG

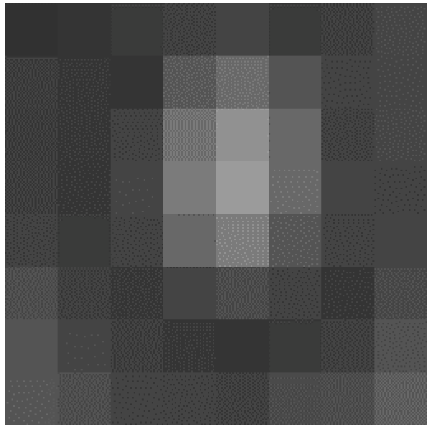
Using DCT in JPEG

- The first coefficient $B(0,0)$ is the DC component, the average intensity
- The top-left coeffs represent low frequencies, the bottom right – high frequencies

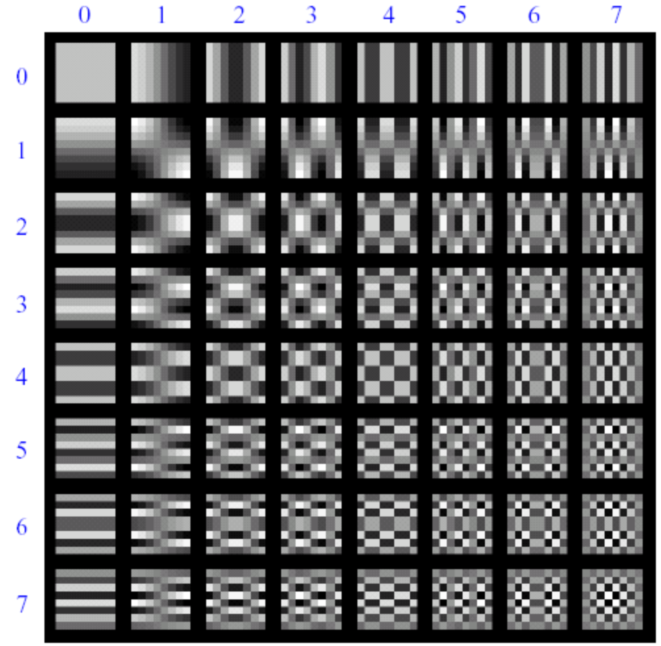




Lossy Image Compression (JPEG)



8x8 image patch



DCT bases

$$G = \overset{\xrightarrow{u}}{\begin{bmatrix} -415.38 & -30.19 & -61.20 & 27.24 & 56.13 & -20.10 & -2.39 & 0.46 \\ 4.47 & -21.86 & -60.76 & 10.25 & 13.15 & -7.09 & -8.54 & 4.88 \\ -46.83 & 7.37 & 77.13 & -24.56 & -28.91 & 9.93 & 5.42 & -5.65 \\ -48.53 & 12.07 & 34.10 & -14.76 & -10.24 & 6.30 & 1.83 & 1.95 \\ 12.12 & -6.55 & -13.20 & -3.95 & -1.88 & 1.75 & -2.79 & 3.14 \\ -7.73 & 2.91 & 2.38 & -5.94 & -2.38 & 0.94 & 4.30 & 1.85 \\ -1.03 & 0.18 & 0.42 & -2.42 & -0.88 & -3.02 & 4.12 & -0.66 \\ -0.17 & 0.14 & -1.07 & -4.19 & -1.17 & -0.10 & 0.50 & 1.68 \end{bmatrix}} \Big\downarrow v$$

Patch representation after projecting on to DCT bases

Image compression using DCT

- Quantize
  - More coarsely for high frequencies (which also tend to have smaller values)
  - Many quantized high frequency values will be zero
- Encode
  - Can decode with inverse dct

Filter responses

$$G = \overset{\displaystyle \overset{u}{\longrightarrow}}{\begin{bmatrix} -415.38 & -30.19 & -61.20 & 27.24 & 56.13 & -20.10 & -2.39 & 0.46 \\ 4.47 & -21.86 & -60.76 & 10.25 & 13.15 & -7.09 & -8.54 & 4.88 \\ -46.83 & 7.37 & 77.13 & -24.56 & -28.91 & 9.93 & 5.42 & -5.65 \\ -48.53 & 12.07 & 34.10 & -14.76 & -10.24 & 6.30 & 1.83 & 1.95 \\ 12.12 & -6.55 & -13.20 & -3.95 & -1.88 & 1.75 & -2.79 & 3.14 \\ -7.73 & 2.91 & 2.38 & -5.94 & -2.38 & 0.94 & 4.30 & 1.85 \\ -1.03 & 0.18 & 0.42 & -2.42 & -0.88 & -3.02 & 4.12 & -0.66 \\ -0.17 & 0.14 & -1.07 & -4.19 & -1.17 & -0.10 & 0.50 & 1.68 \end{bmatrix}} \Big\downarrow v$$

Quantization table

$$Q = \begin{bmatrix} 16 & 11 & 10 & 16 & 24 & 40 & 51 & 61 \\ 12 & 12 & 14 & 19 & 26 & 58 & 60 & 55 \\ 14 & 13 & 16 & 24 & 40 & 57 & 69 & 56 \\ 14 & 17 & 22 & 29 & 51 & 87 & 80 & 62 \\ 18 & 22 & 37 & 56 & 68 & 109 & 103 & 77 \\ 24 & 35 & 55 & 64 & 81 & 104 & 113 & 92 \\ 49 & 64 & 78 & 87 & 103 & 121 & 120 & 101 \\ 72 & 92 & 95 & 98 & 112 & 100 & 103 & 99 \end{bmatrix}$$

Quantized values

$$B = \begin{bmatrix} -26 & -3 & -6 & 2 & 2 & -1 & 0 & 0 \\ 0 & -2 & -4 & 1 & 1 & 0 & 0 & 0 \\ -3 & 1 & 5 & -1 & -1 & 0 & 0 & 0 \\ -3 & 1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

JPEG Compression Summary

1. Convert image to YCrCb
2. Subsample color by factor of 2
   - People have bad resolution for color
3. Split into blocks (8x8, typically), subtract 128
4. For each block
   a. Compute DCT coefficients
   b. Coarsely quantize
      - Many high frequency components will become zero
   c. Encode (e.g., with Huffman coding)

http://en.wikipedia.org/wiki/YCbCr
http://en.wikipedia.org/wiki/JPEG

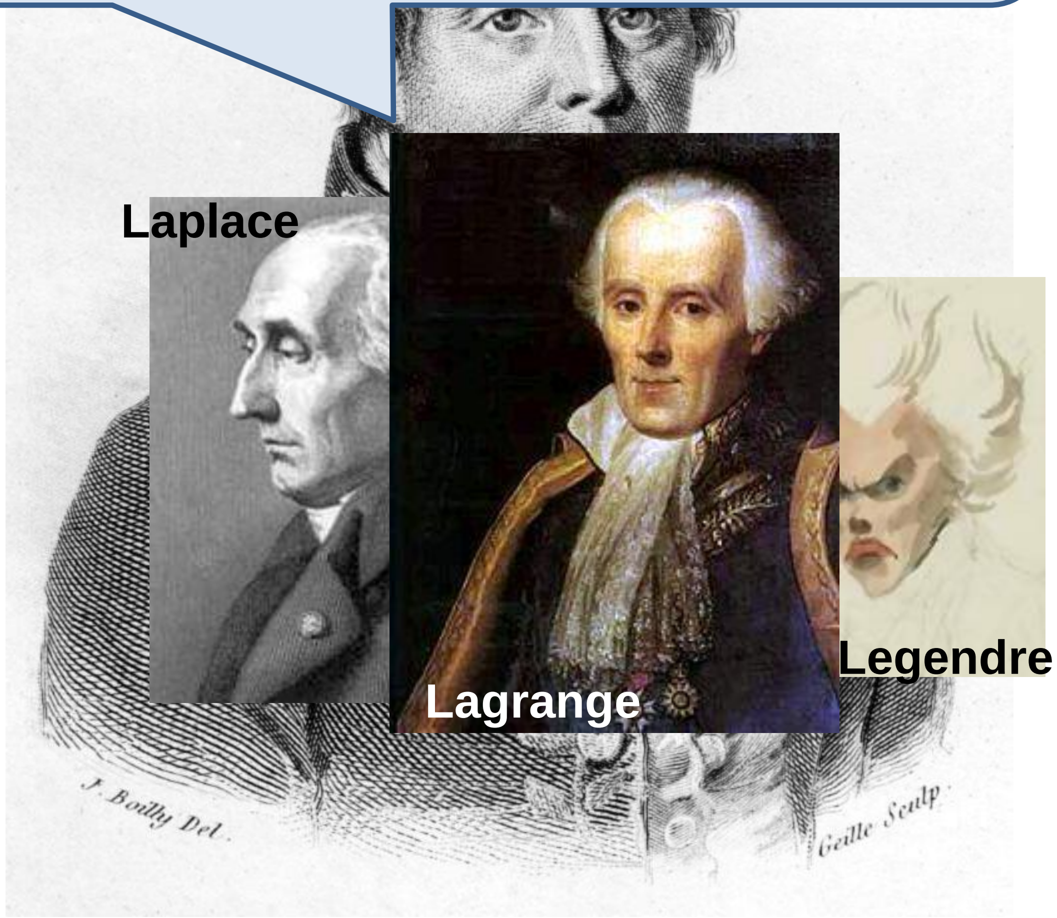
Jean Baptiste Joseph Fourier (1768-1830)

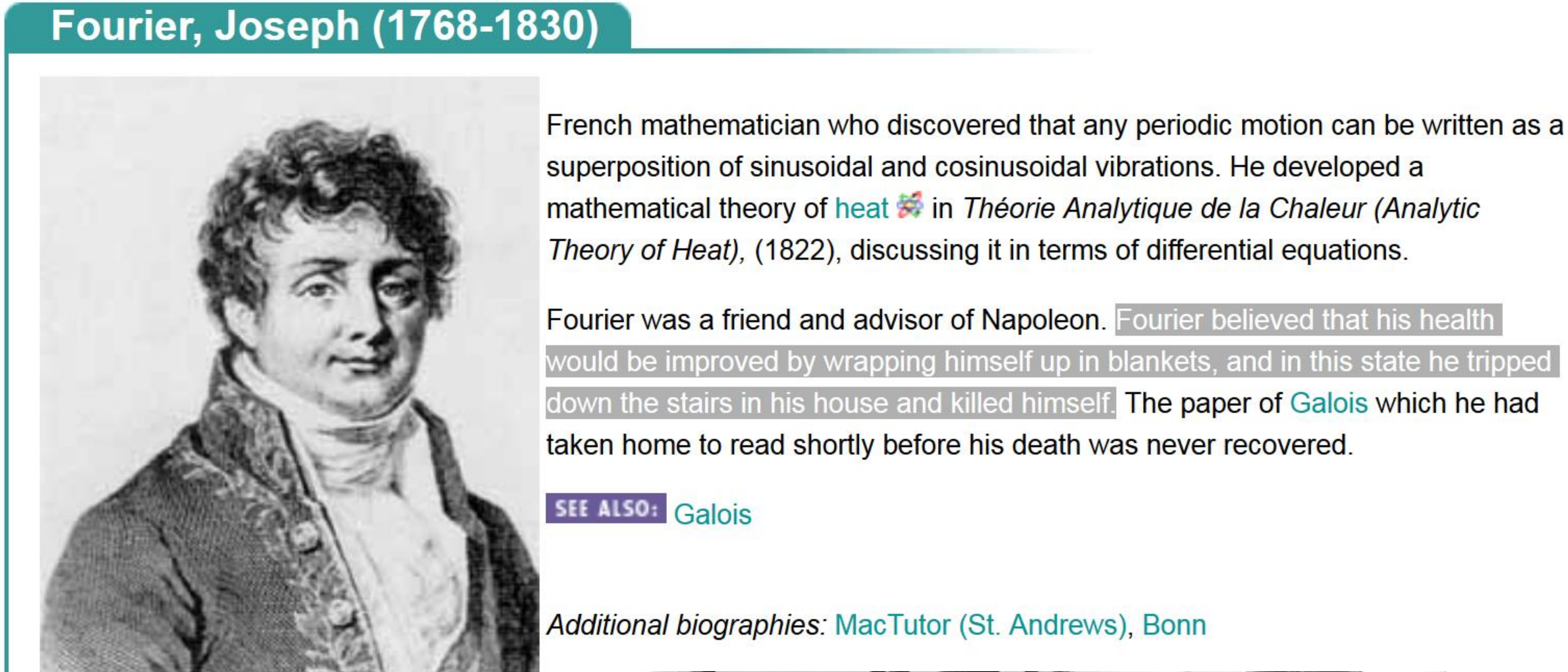
had crazy idea (1807):

***Any** univariate function can*
rewritten as a weighted sum
sines and cosines of different
frequencies.

> *...the manner in which the author arrives at these equations is not exempt of difficulties and...his analysis to integrate them still leaves something to be desired on the score of generality and even rigour.*

- Don’t believe it?
  - Neither did Lagrange, Laplace, Poisson and other big wigs
  - Not translated into English until 1878!
- But it’s (mostly) true!
  - called Fourier Series
  - there are some subtle restrictions





Fourier, Joseph (1768-1830)

French mathematician who discovered that any periodic motion can be written as a superposition of sinusoidal and cosinusoidal vibrations. He developed a mathematical theory of heat in *Théorie Analytique de la Chaleur (Analytic Theory of Heat),* (1822), discussing it in terms of differential equations.

Fourier was a friend and advisor of Napoleon. Fourier believed that his health would be improved by wrapping himself up in blankets, and in this state he tripped down the stairs in his house and killed himself. The paper of Galois which he had taken home to read shortly before his death was never recovered.

SEE ALSO: Galois

Additional biographies: MacTutor (St. Andrews), Bonn

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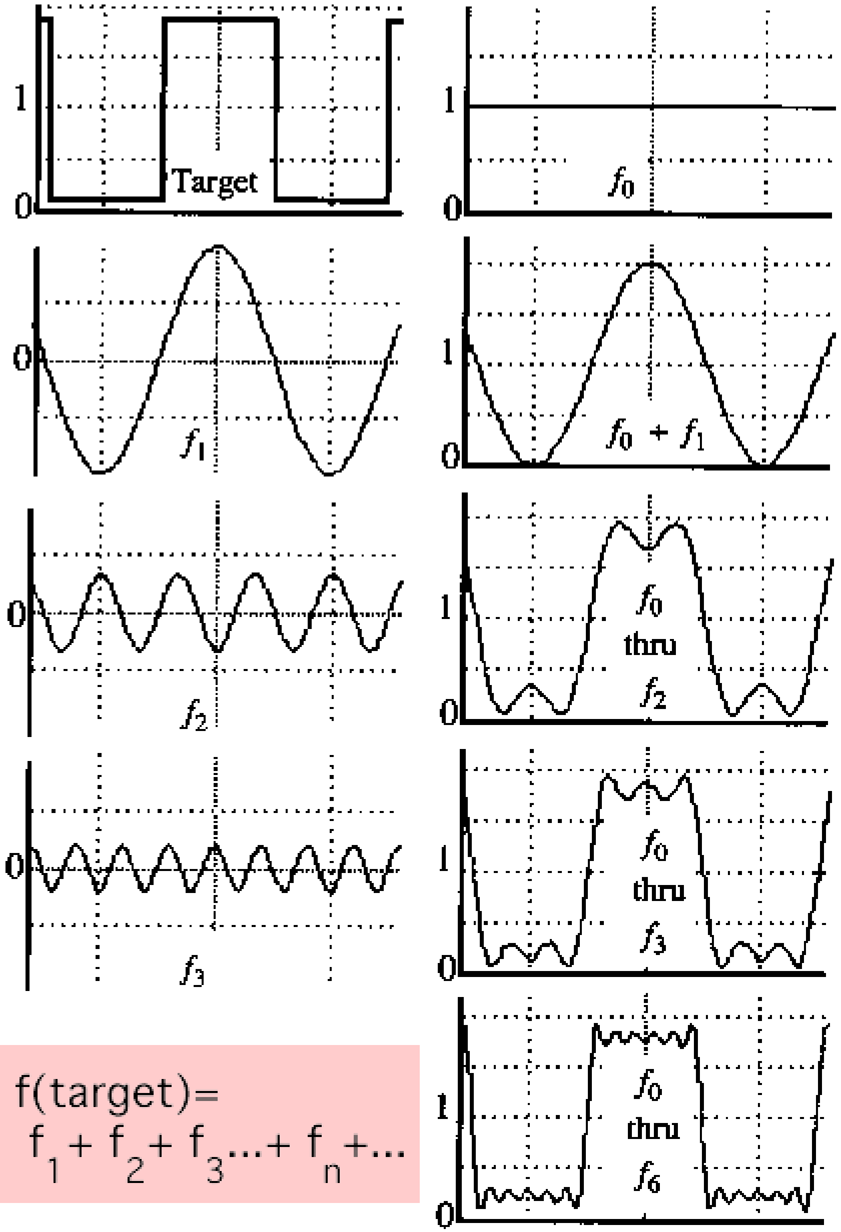
How would math have changed if the Slanket or Snuggie had been invented?

A sum of sines

Our building block:

$$A\sin(\omega x + \phi)$$

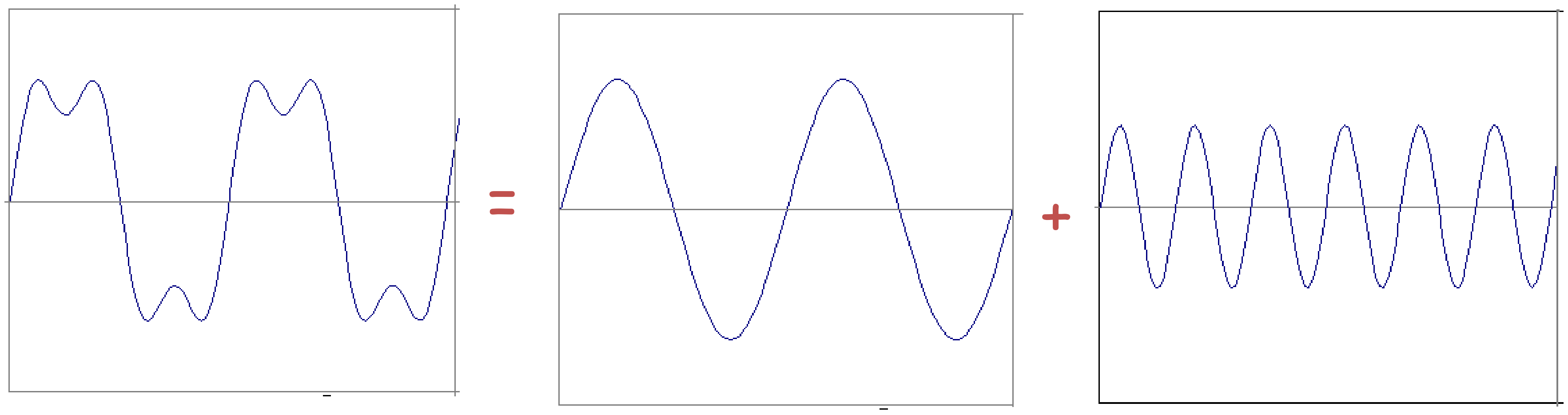
Add enough of them to get any signal *g(x)* you want!

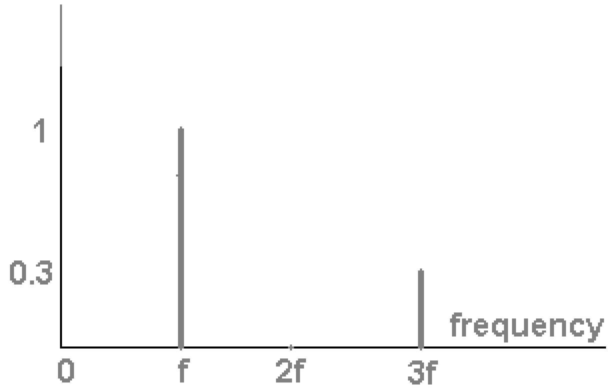


f(target)= $f_1 + f_2 + f_3 ... + f_n + ...$

Frequency Spectra

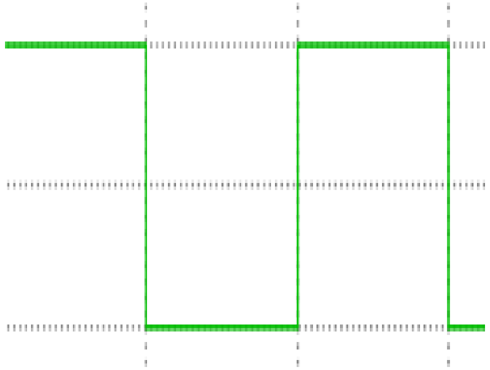
- example : $g(t) = \sin(2\pi f\, t) + (1/3)\sin(2\pi(3f)\, t)$



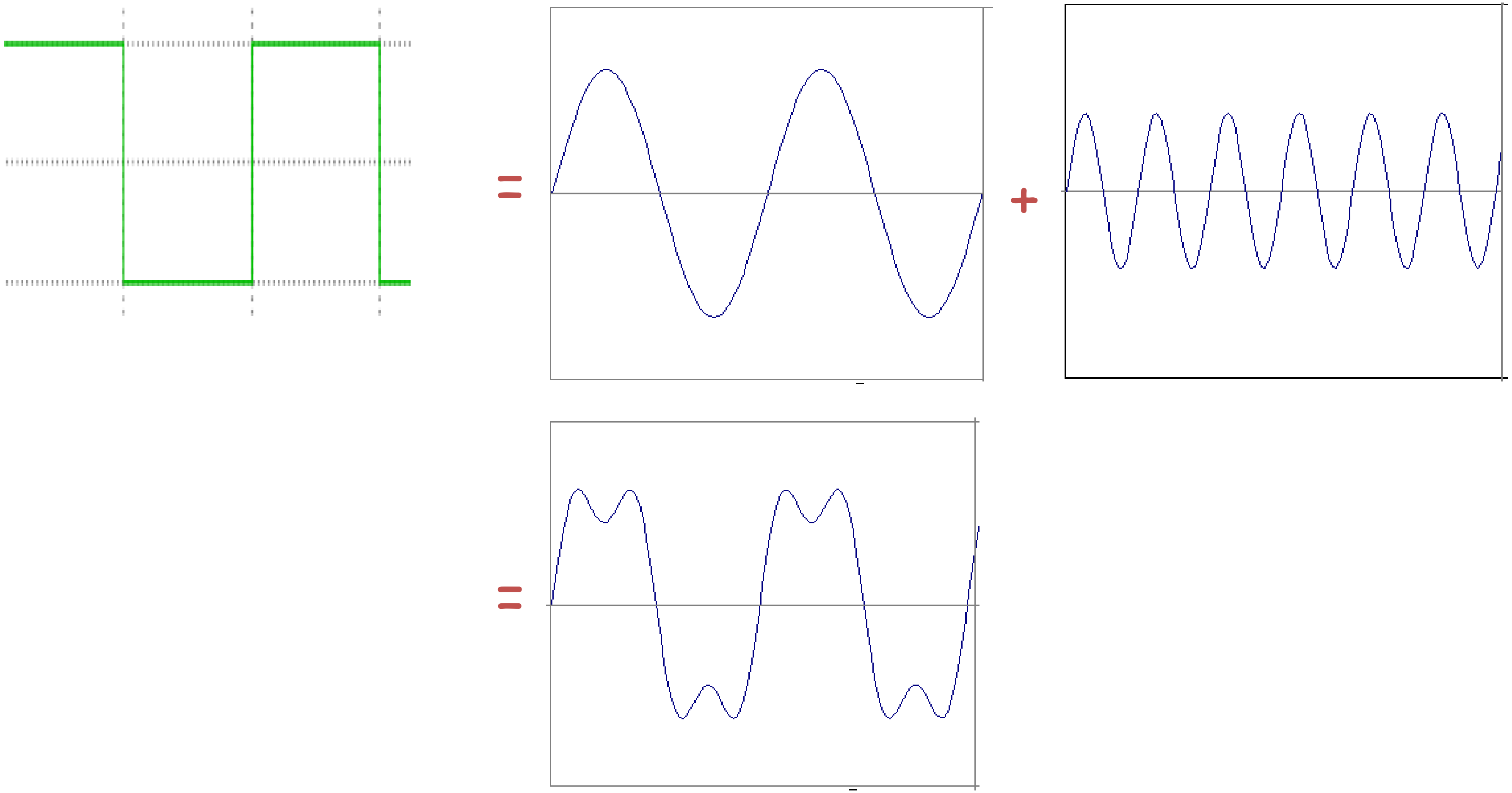


Slides: Efros

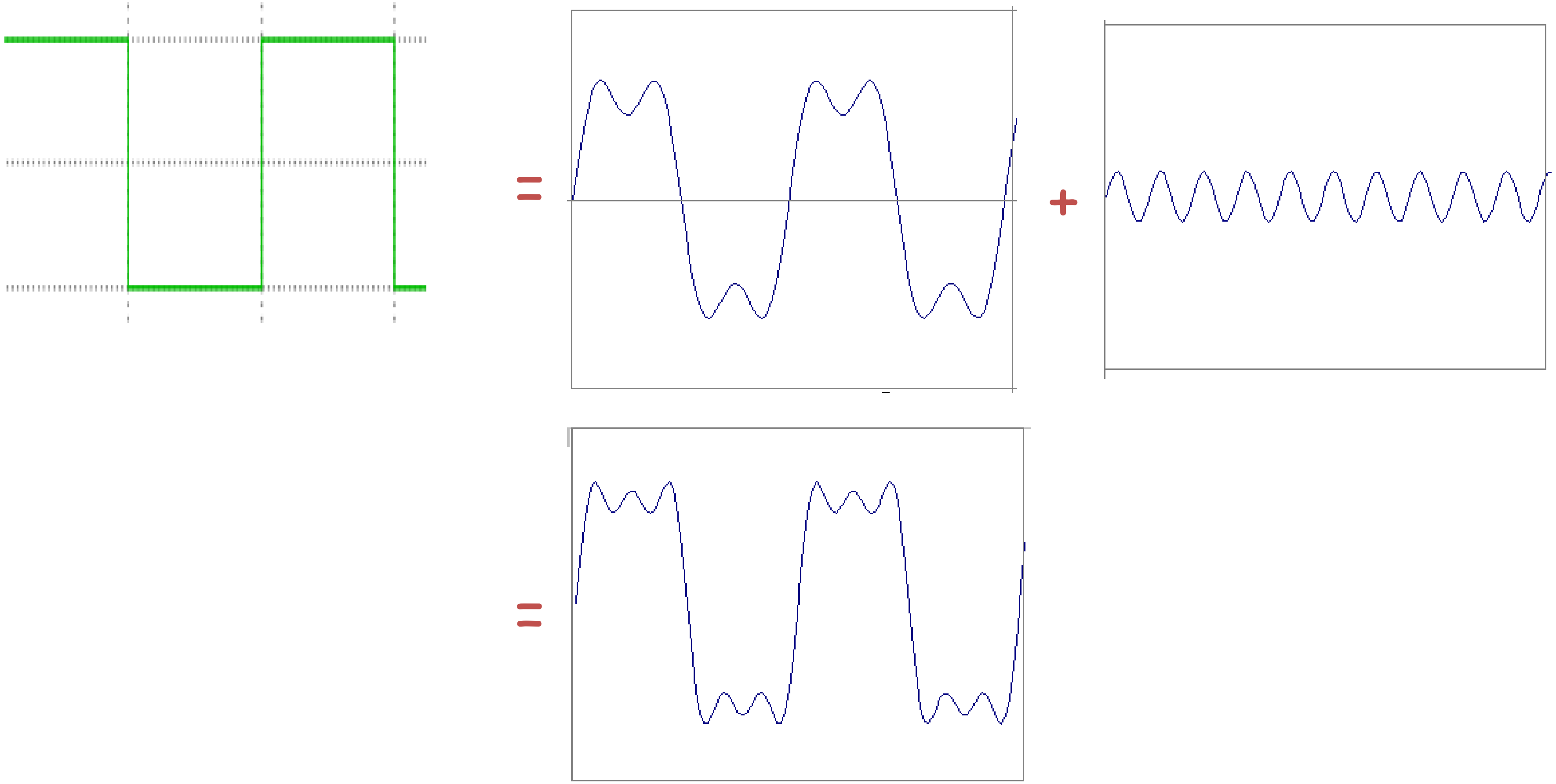
Frequency Spectra



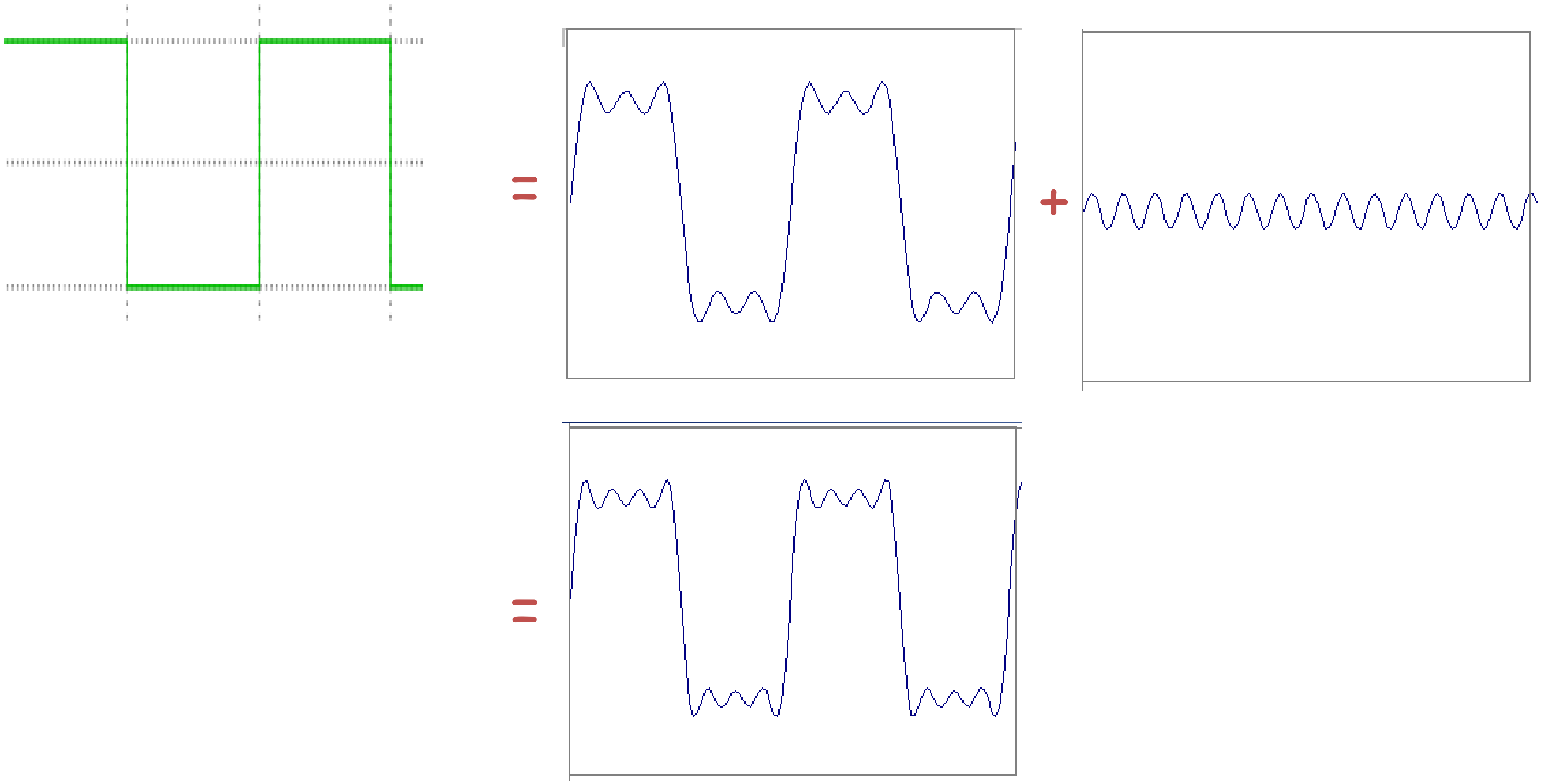
Frequency Spectra



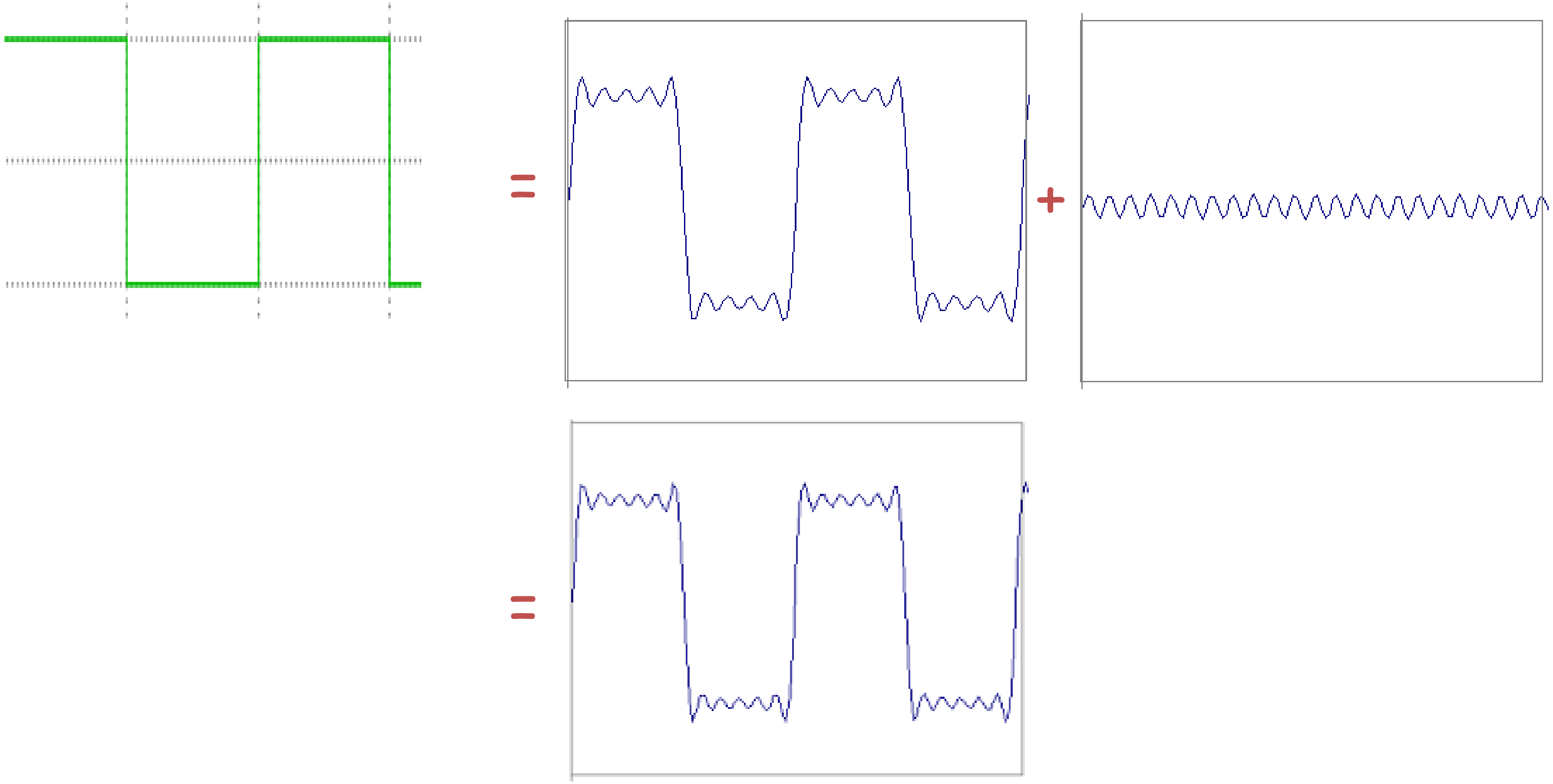
Frequency Spectra



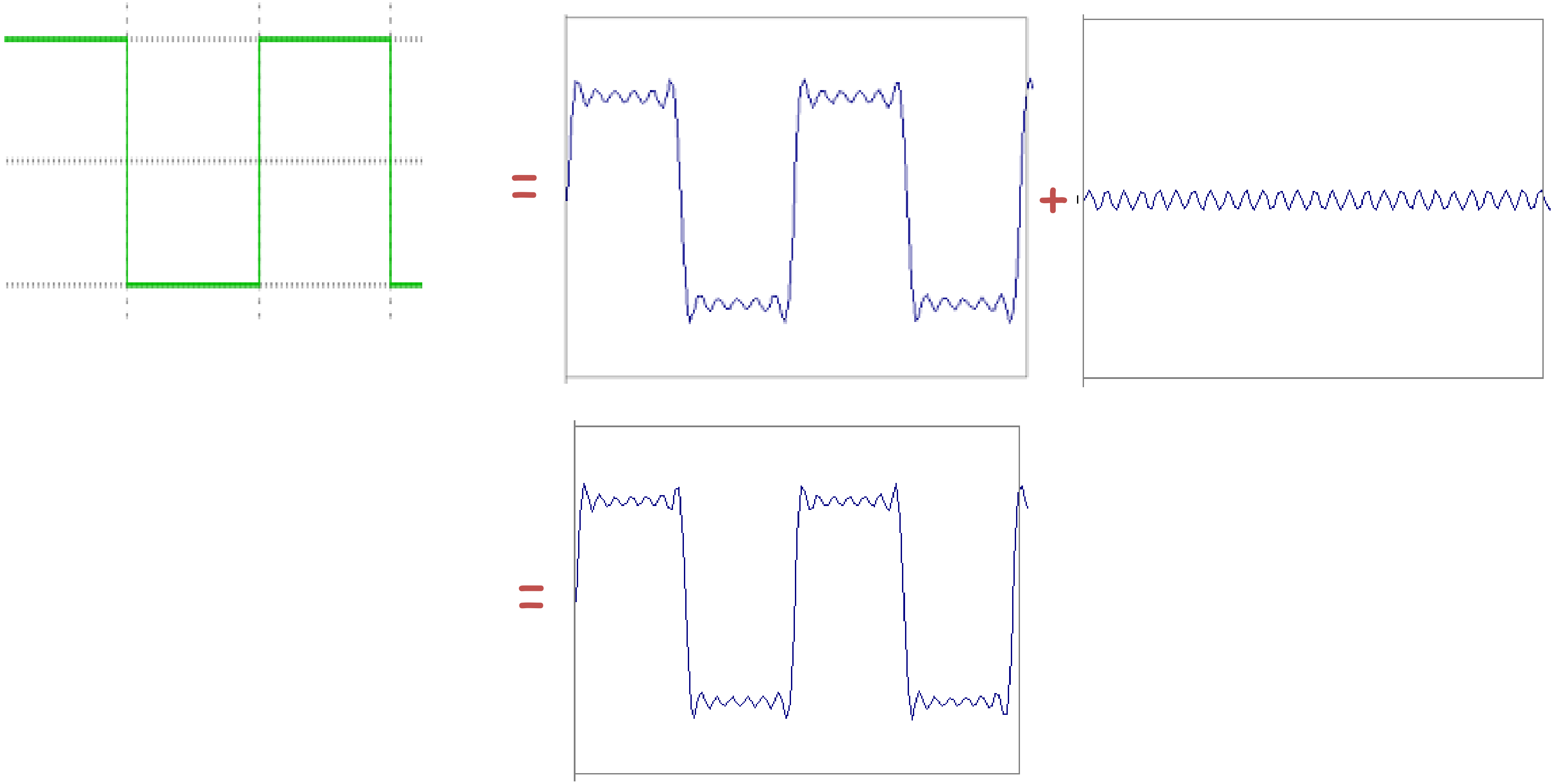
Frequency Spectra



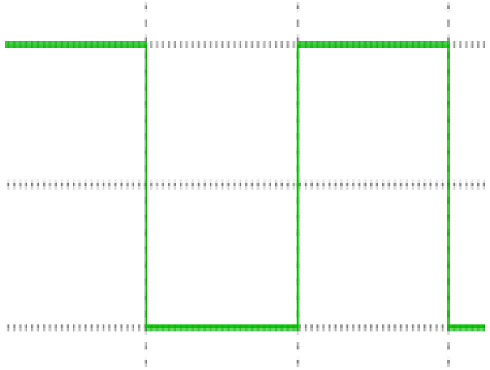
Frequency Spectra



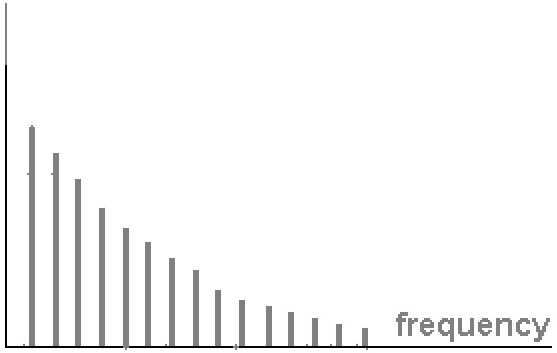
Frequency Spectra



Frequency Spectra

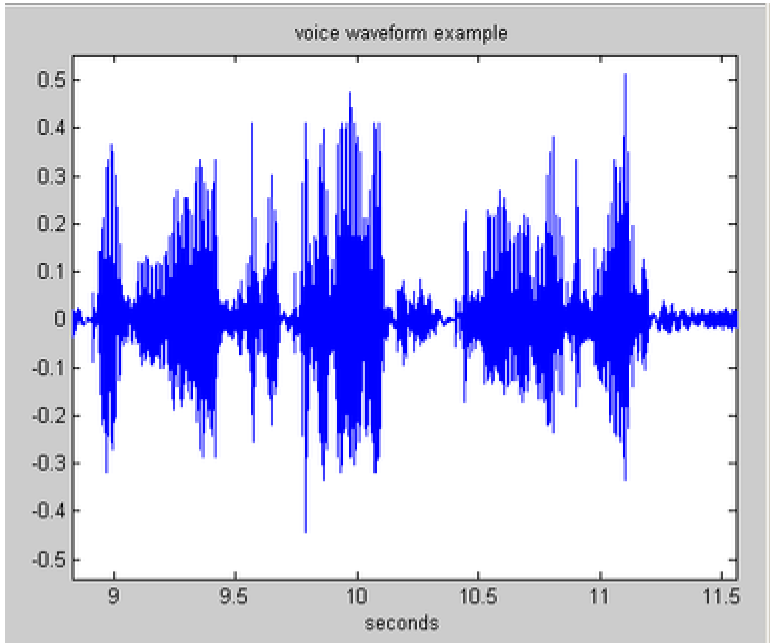


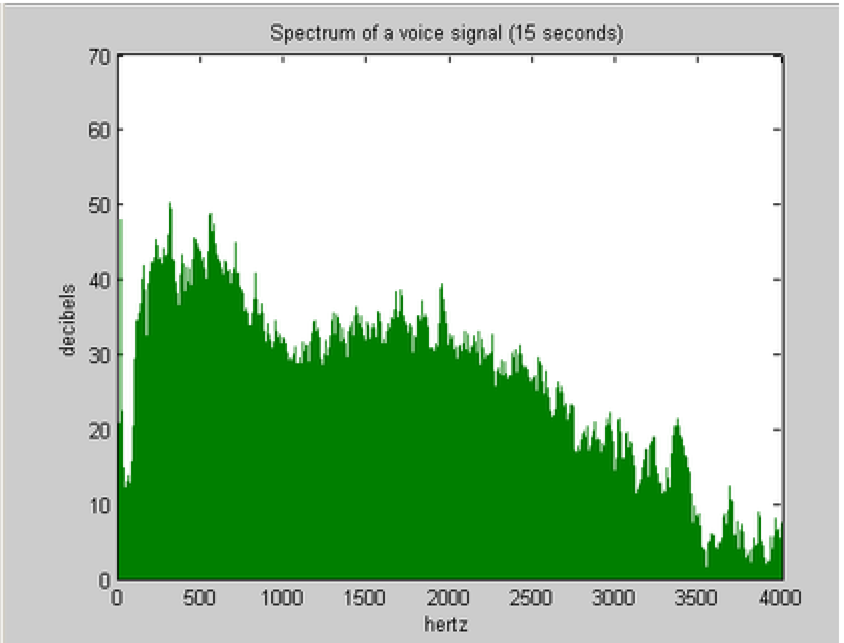
$$= \boxed{A\sum_{k=1}^{\infty}\frac{1}{k}\sin(2\pi kt)}$$



Example: Music

- We think of music in terms of frequencies at different magnitudes

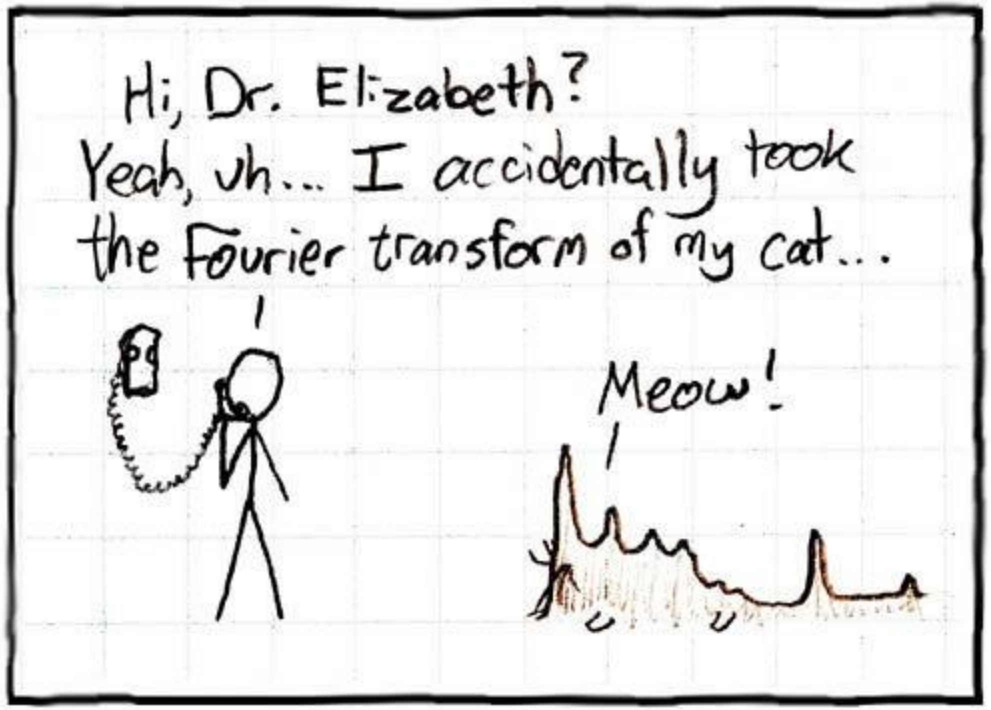




Slide: Hoiem

Other signals

- We can also think of all kinds of other signals the same way

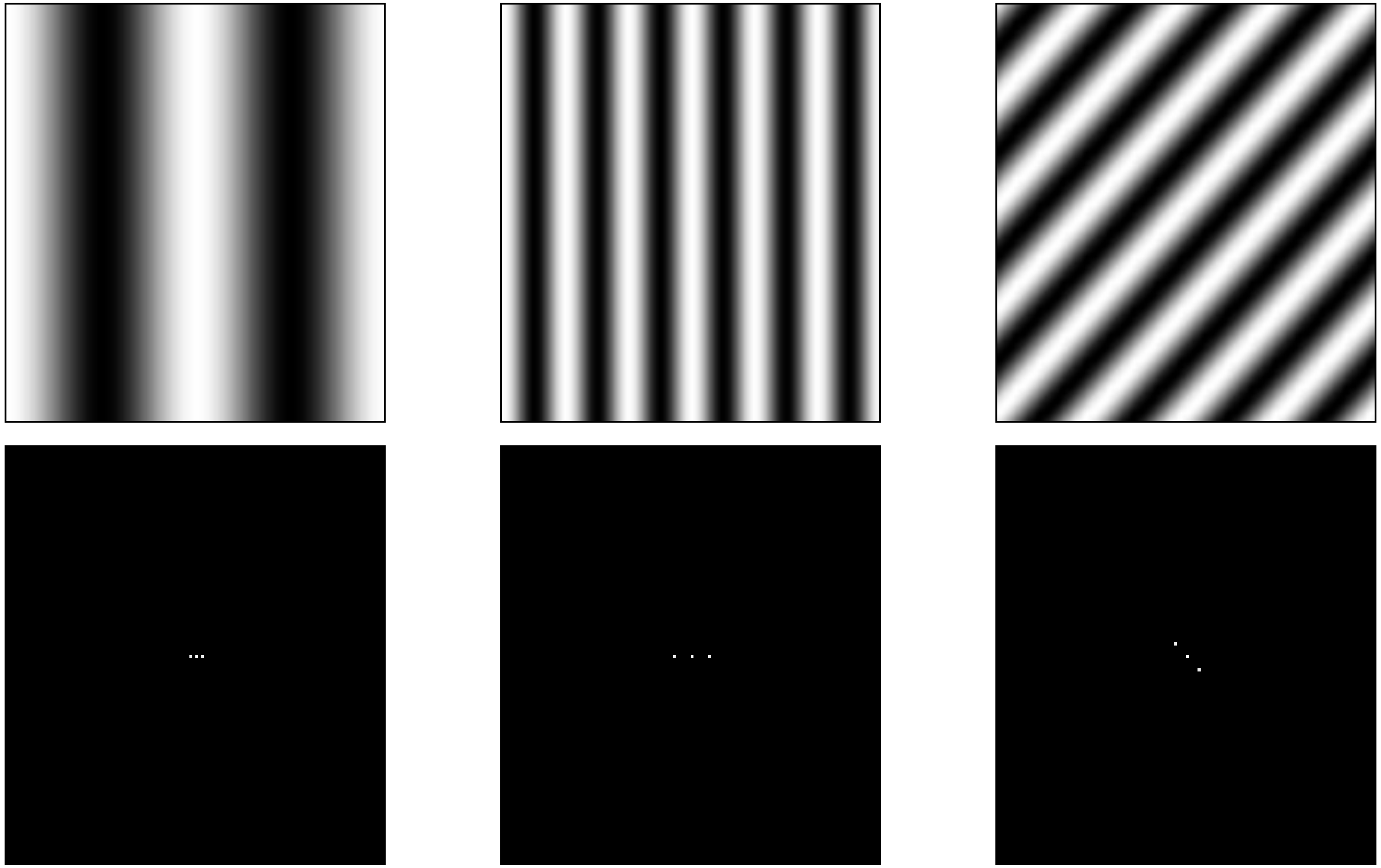

Hi, Dr. Elizabeth?
Yeah, uh... I accidentally took the Fourier transform of my cat...
Meow!

xkcd.com

Fourier analysis in images

Intensity Image

Fourier Image



http://sharp.bu.edu/~slehar/fourier/fourier.html#filtering

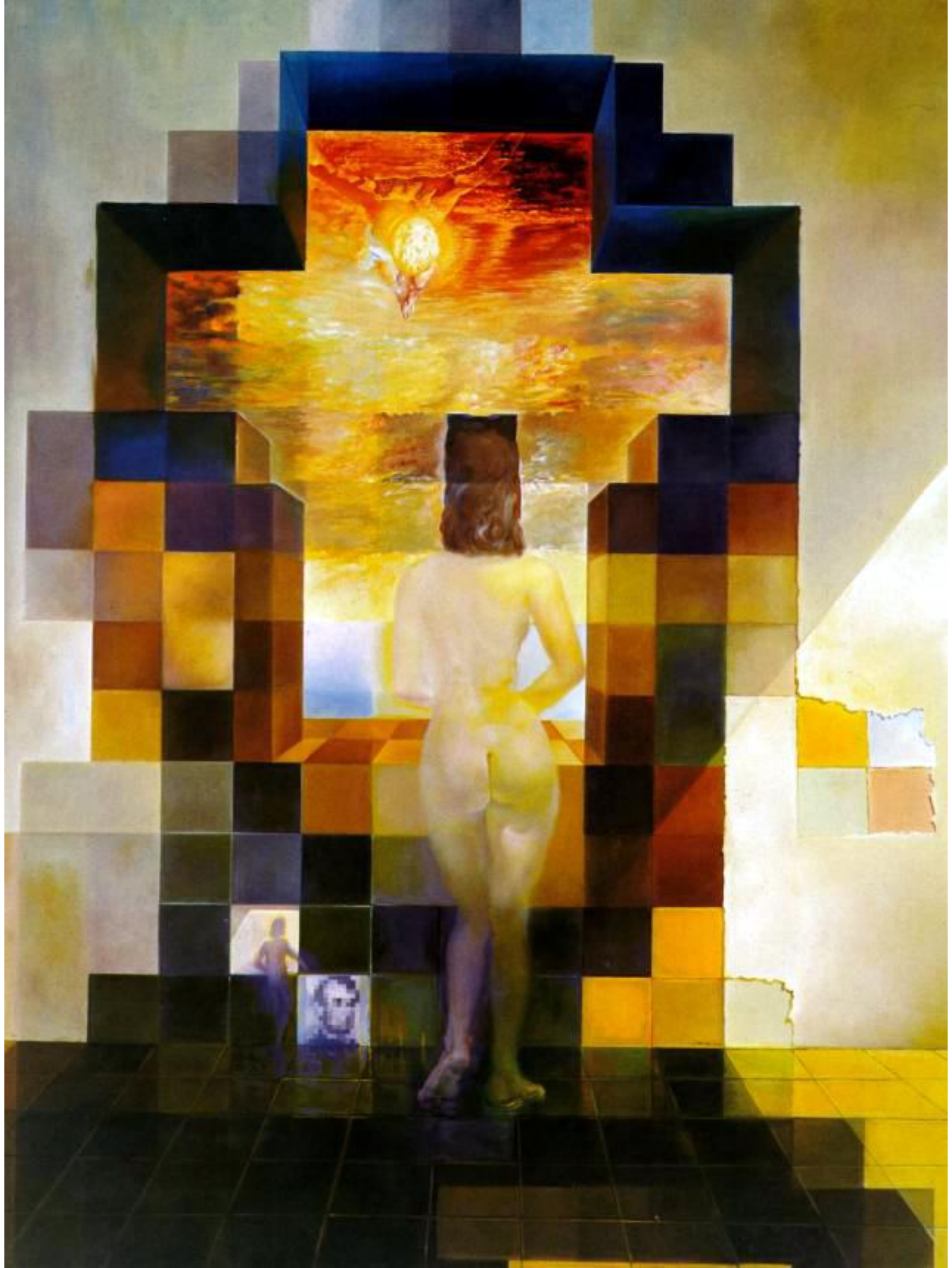
Fourier Transform

- Fourier transform stores the magnitude and phase at each frequency
  - Magnitude encodes how much signal there is at a particular frequency
  - Phase encodes spatial information (indirectly)
  - For mathematical convenience, this is often notated in terms of real and complex numbers

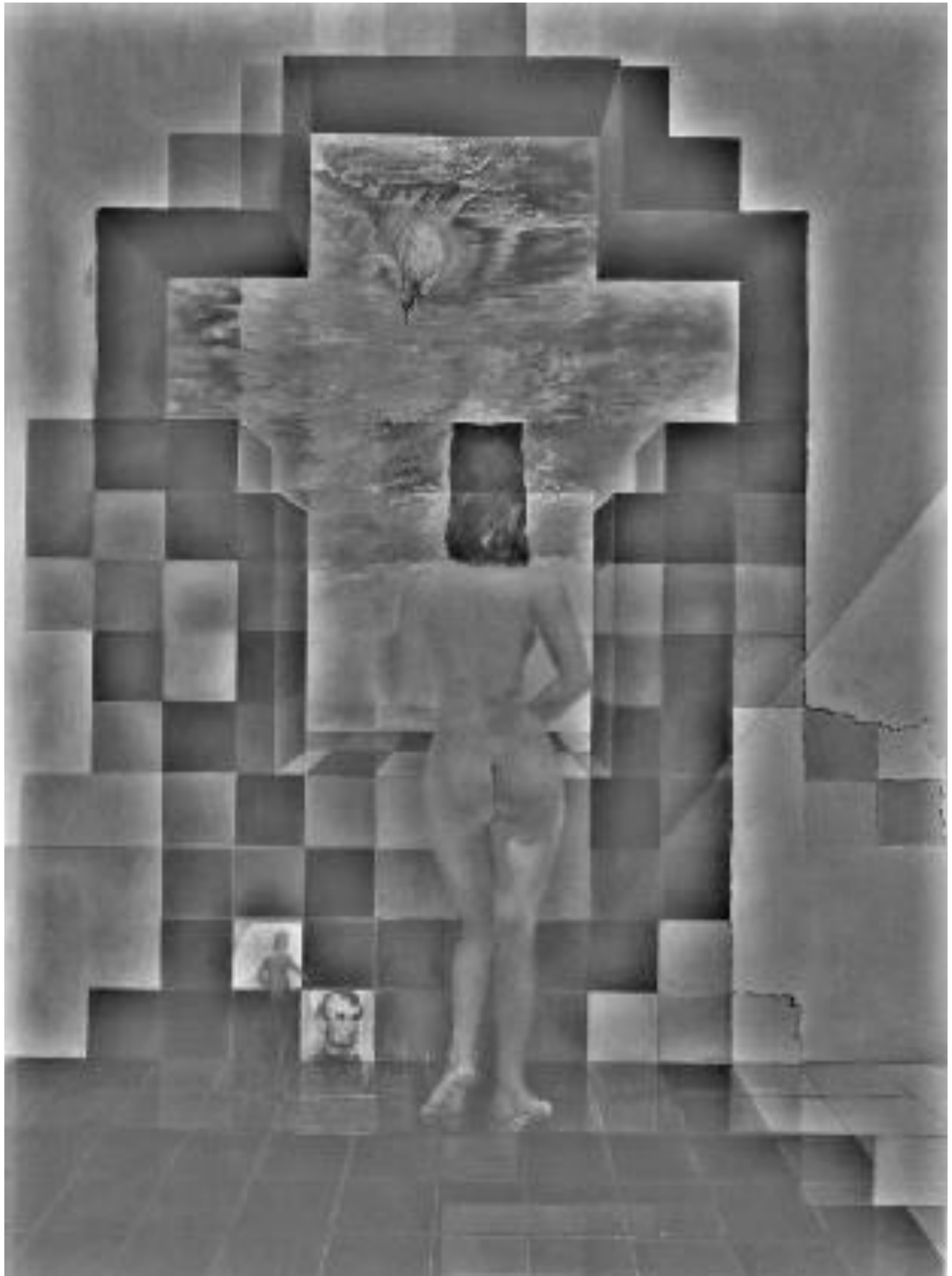
Amplitude: $A = \pm\sqrt{R(\omega)^2 + I(\omega)^2}$

Phase: $\phi = \tan^{-1}\frac{I(\omega)}{R(\omega)}$

Salvador Dali invented Hybrid Images?

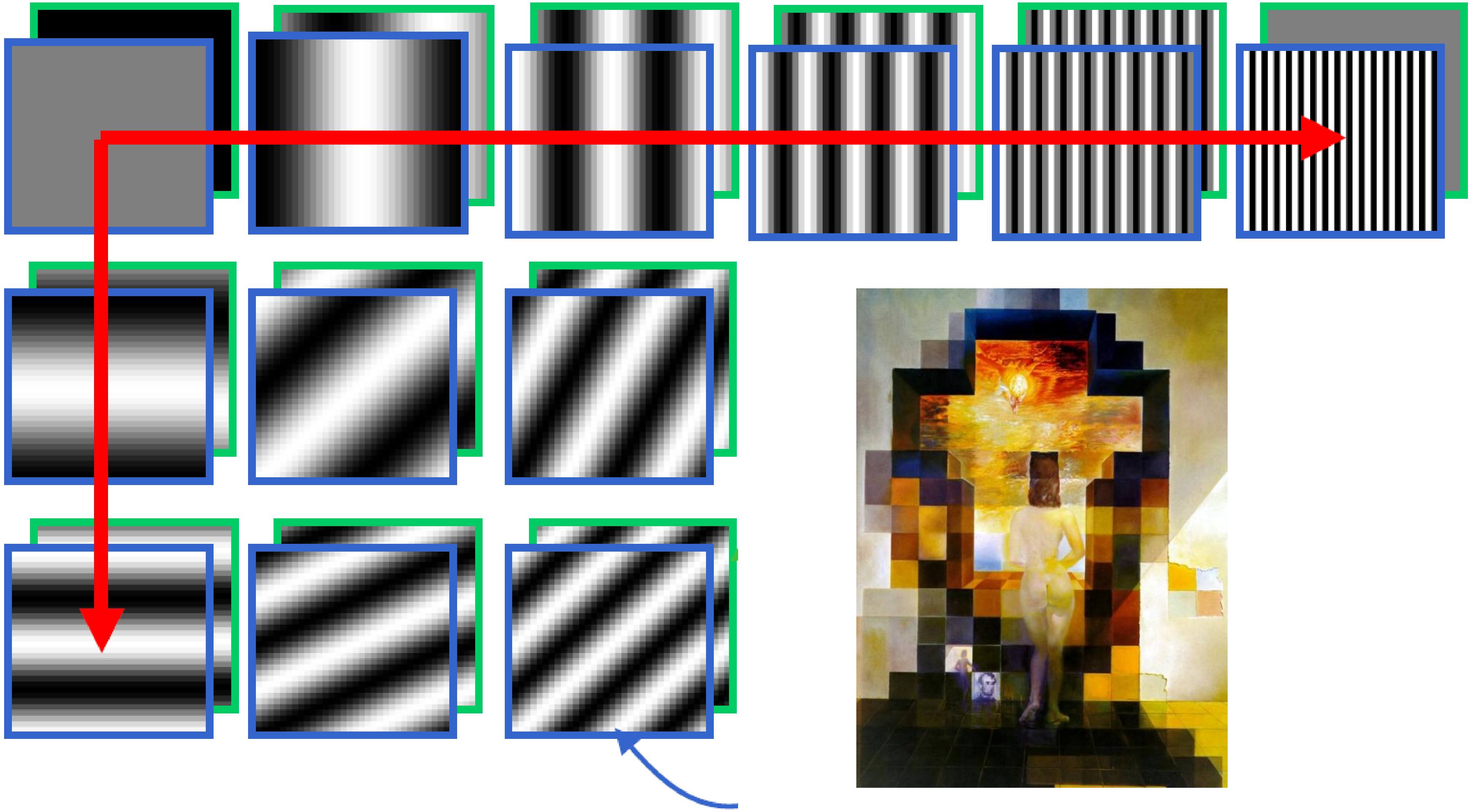


Salvador Dali
“Gala Contemplating the Mediterranean Sea, which at 20 meters becomes the portrait of Abraham Lincoln”, 1976



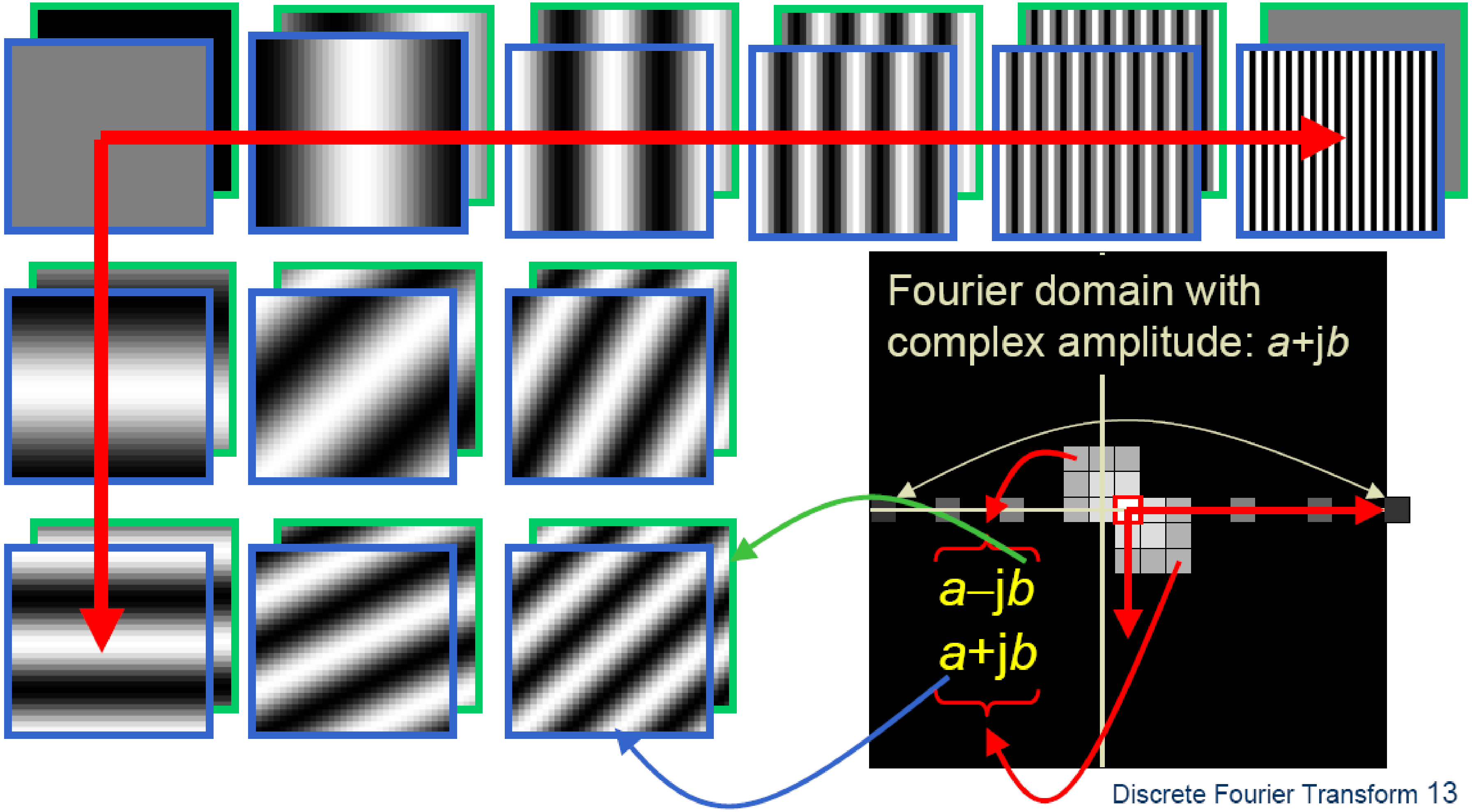
Fourier Bases

Teases away fast vs. slow changes in the image.



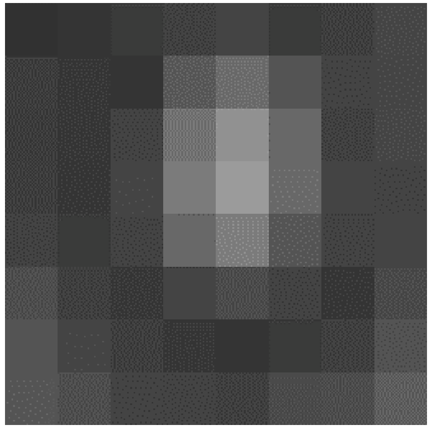
This change of basis is the Fourier Transform

Fourier Bases

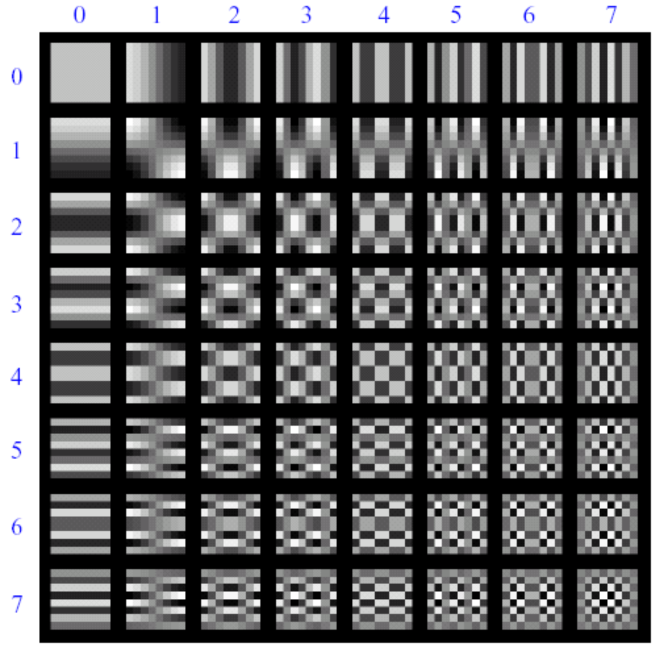


Discrete Fourier Transform 13

This looks a lot like DCT in JPEG compression



8x8 image patch

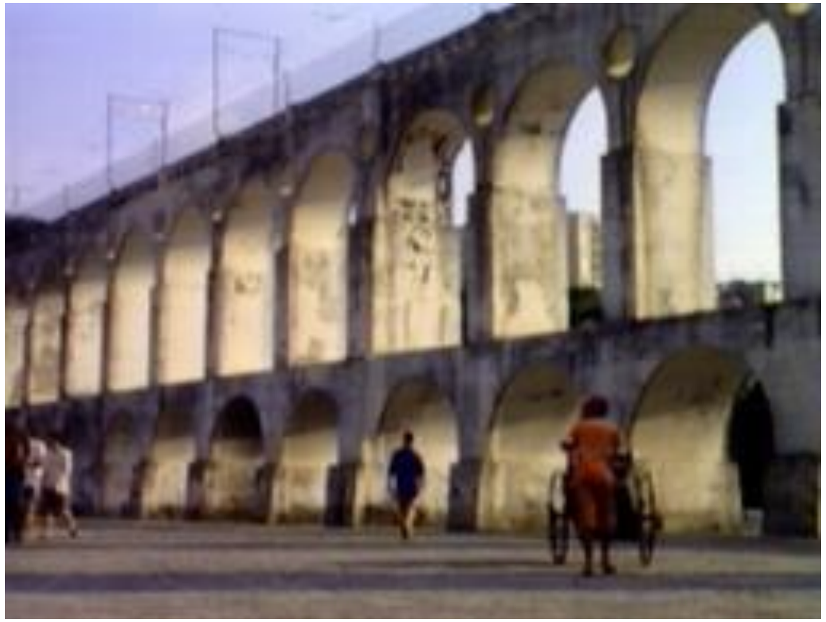


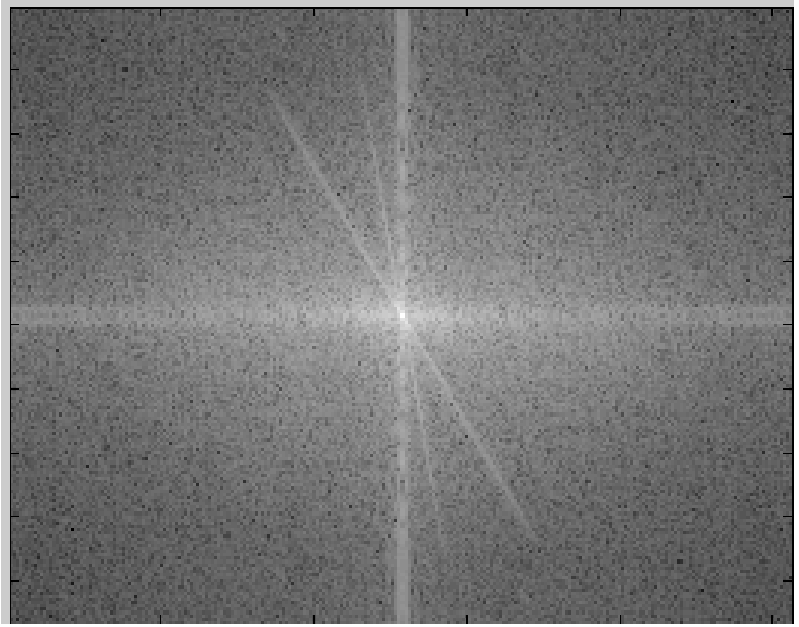
DCT bases

$$G = \overset{\overset{u}{\longrightarrow}}{\begin{bmatrix} -415.38 & -30.19 & -61.20 & 27.24 & 56.13 & -20.10 & -2.39 & 0.46 \\ 4.47 & -21.86 & -60.76 & 10.25 & 13.15 & -7.09 & -8.54 & 4.88 \\ -46.83 & 7.37 & 77.13 & -24.56 & -28.91 & 9.93 & 5.42 & -5.65 \\ -48.53 & 12.07 & 34.10 & -14.76 & -10.24 & 6.30 & 1.83 & 1.95 \\ 12.12 & -6.55 & -13.20 & -3.95 & -1.88 & 1.75 & -2.79 & 3.14 \\ -7.73 & 2.91 & 2.38 & -5.94 & -2.38 & 0.94 & 4.30 & 1.85 \\ -1.03 & 0.18 & 0.42 & -2.42 & -0.88 & -3.02 & 4.12 & -0.66 \\ -0.17 & 0.14 & -1.07 & -4.19 & -1.17 & -0.10 & 0.50 & 1.68 \end{bmatrix}} \Big\downarrow v$$

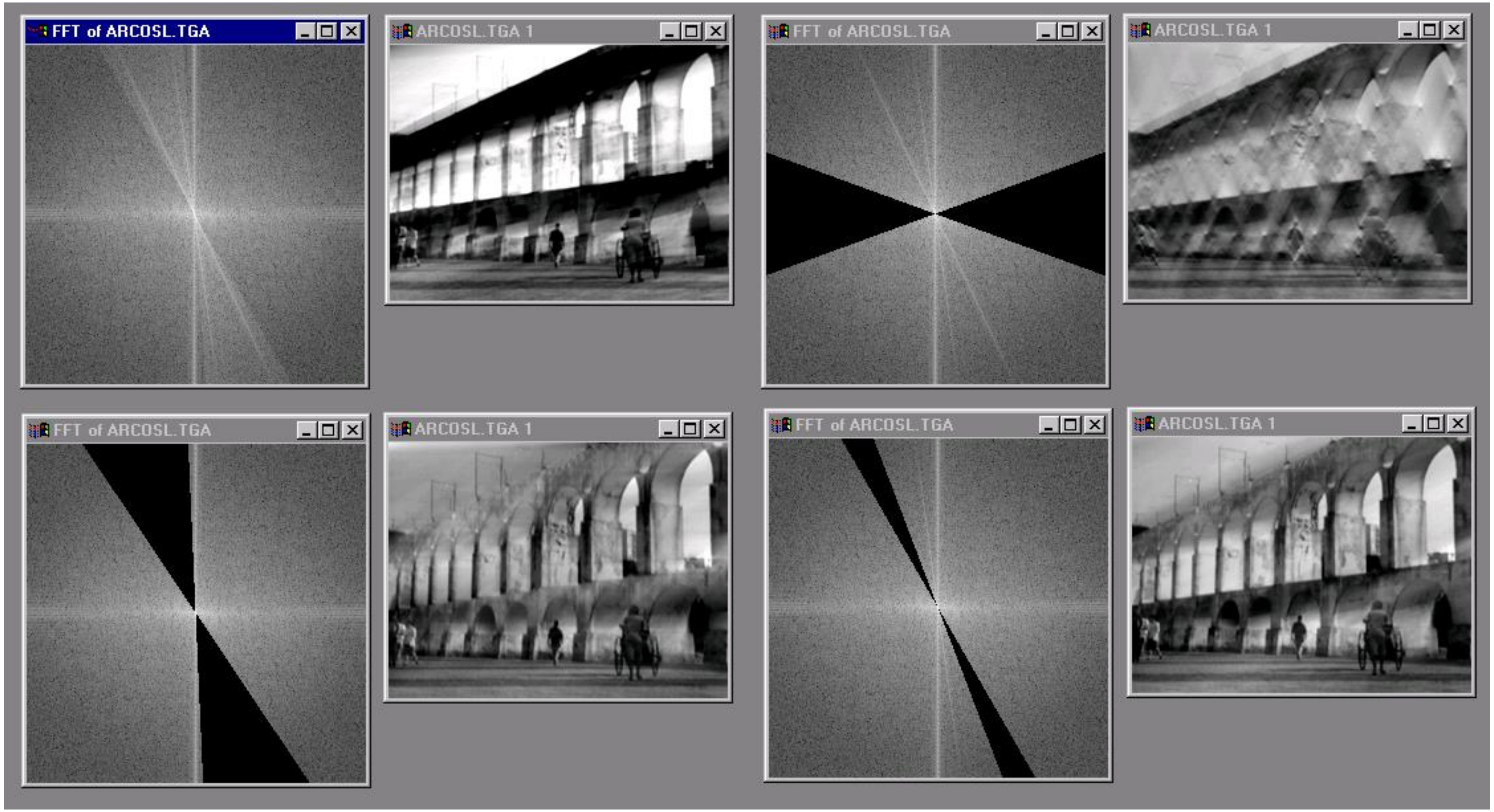
Patch representation after projecting on to DCT bases

Man-made Scene

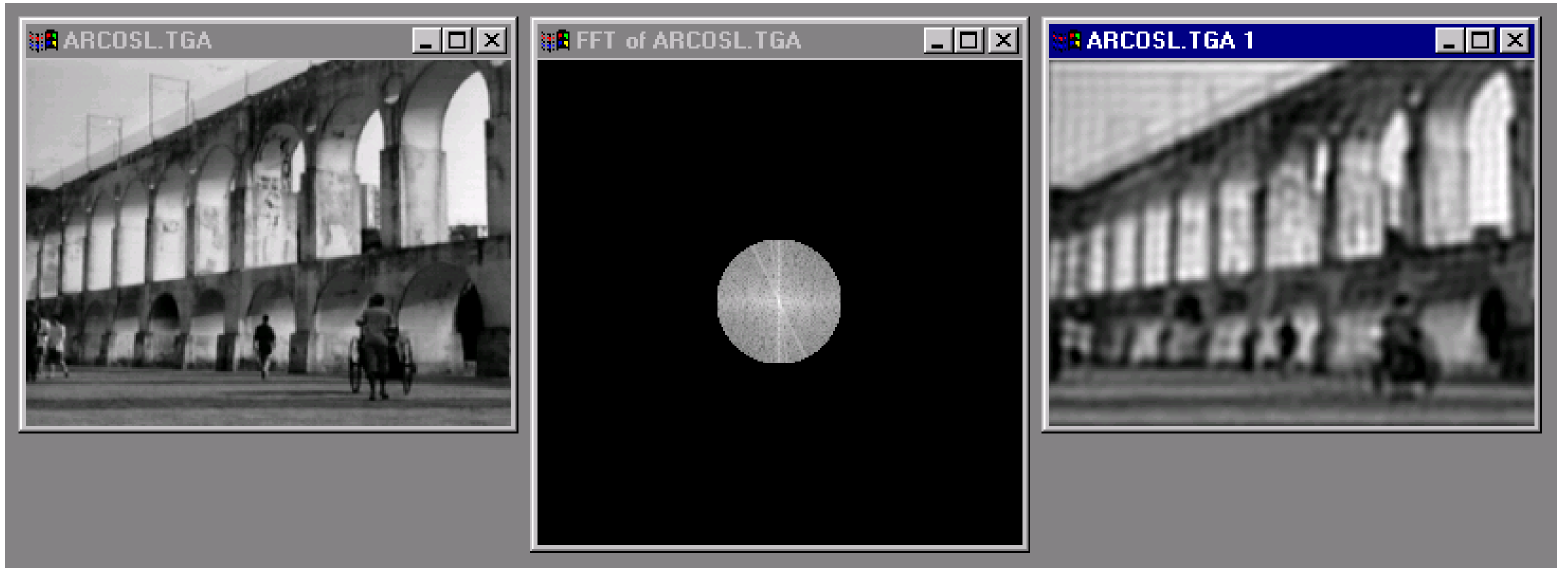


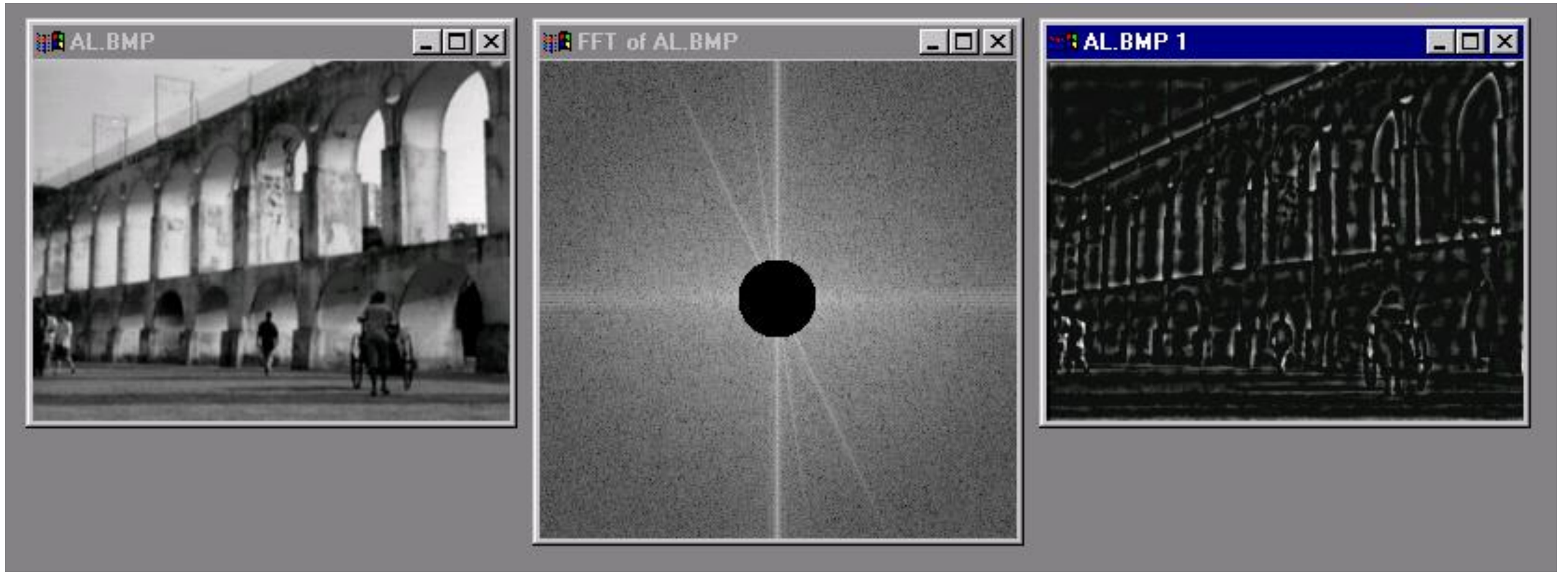


Can change spectrum, then reconstruct



Low and High Pass filtering





Computing the Fourier Transform

$$H(\omega) = \mathcal{F}\left\{h(x)\right\} = Ae^{j\phi}$$

Continuous

$$H(\omega) = \int_{-\infty}^{\infty} h(x) e^{-j\omega x} dx$$

Discrete

$$H(k) = \frac{1}{N} \sum_{x=0}^{N-1} h(x) e^{-j\frac{2\pi kx}{N}}$$

k = -N/2..N/2

Fast Fourier Transform (FFT): NlogN

The Convolution Theorem

- The Fourier transform of the convolution of two functions is the product of their Fourier transforms

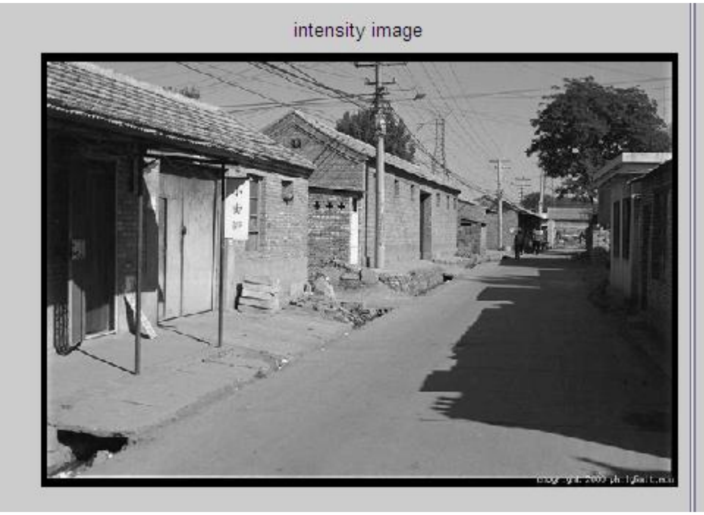
$$\mathrm{F}[g * h] = \mathrm{F}[g]\,\mathrm{F}[h]$$

- **Convolution** in spatial domain is equivalent to **multiplication** in frequency domain!

$$g * h = \mathrm{F}^{-1}[\mathrm{F}[g]\,\mathrm{F}[h]]$$

Filtering in spatial domain

1	0	-1
2	0	-2
1	0	-1



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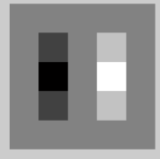

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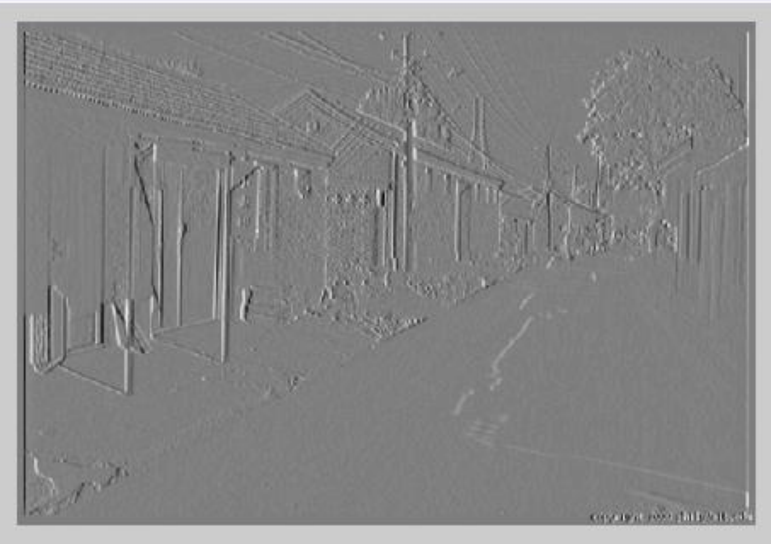
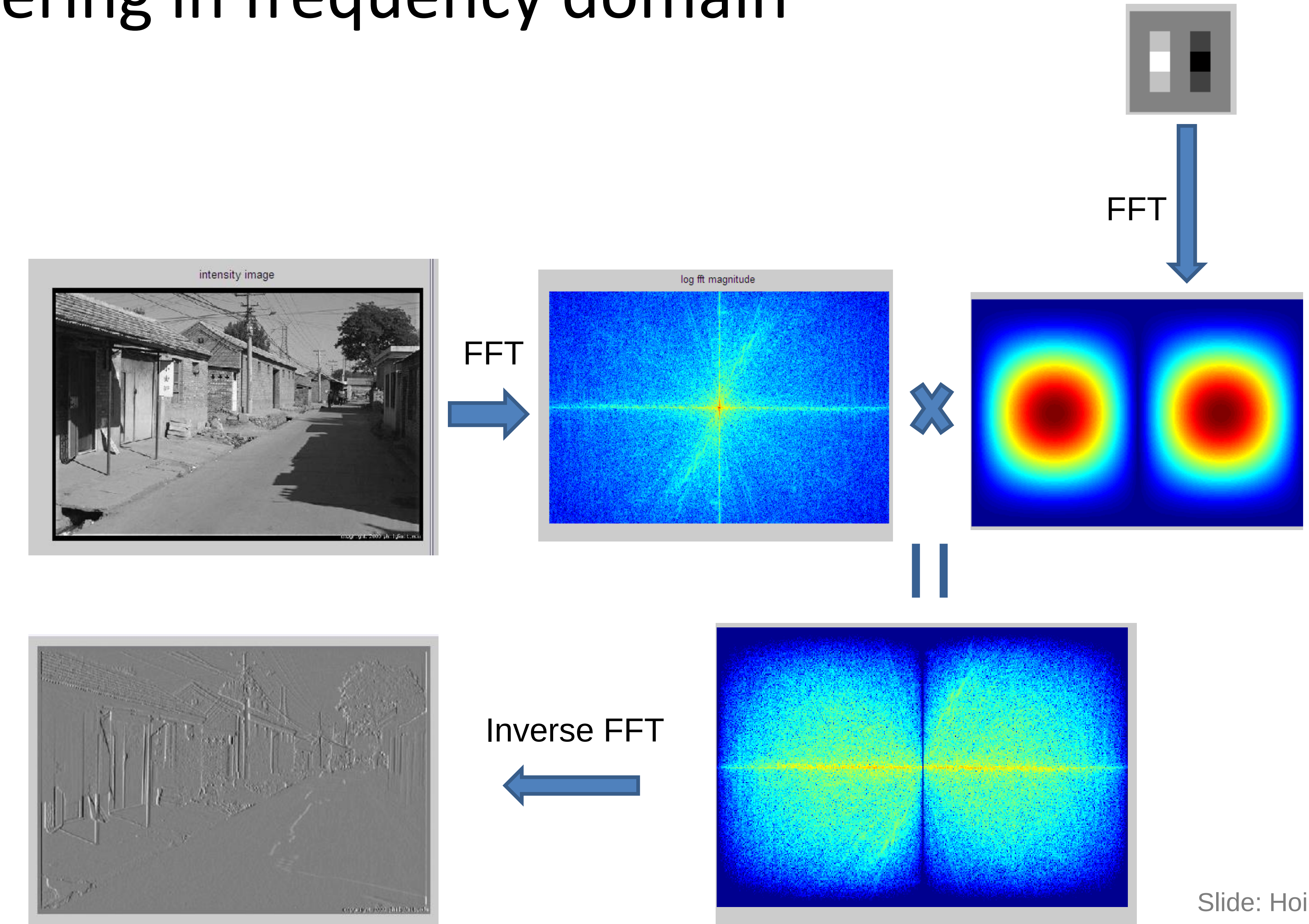


Filtering in frequency domain



FFT

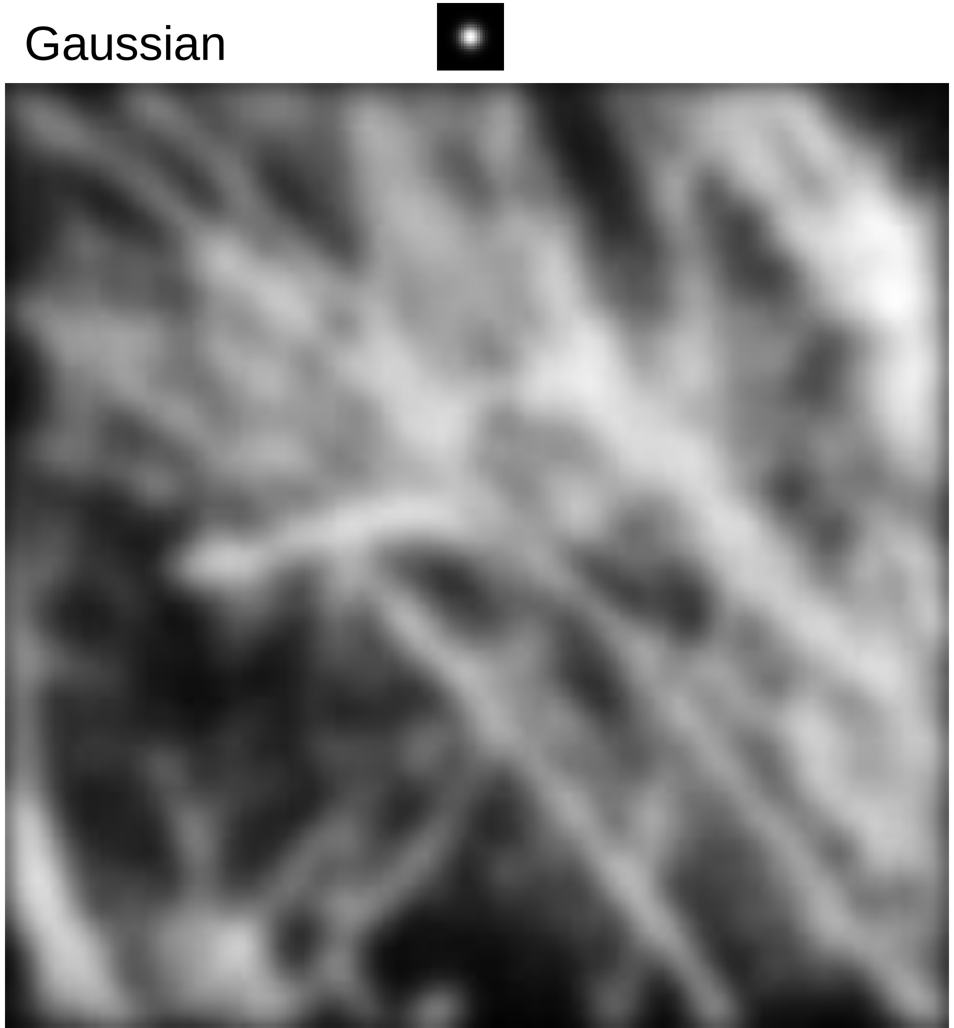
FFT

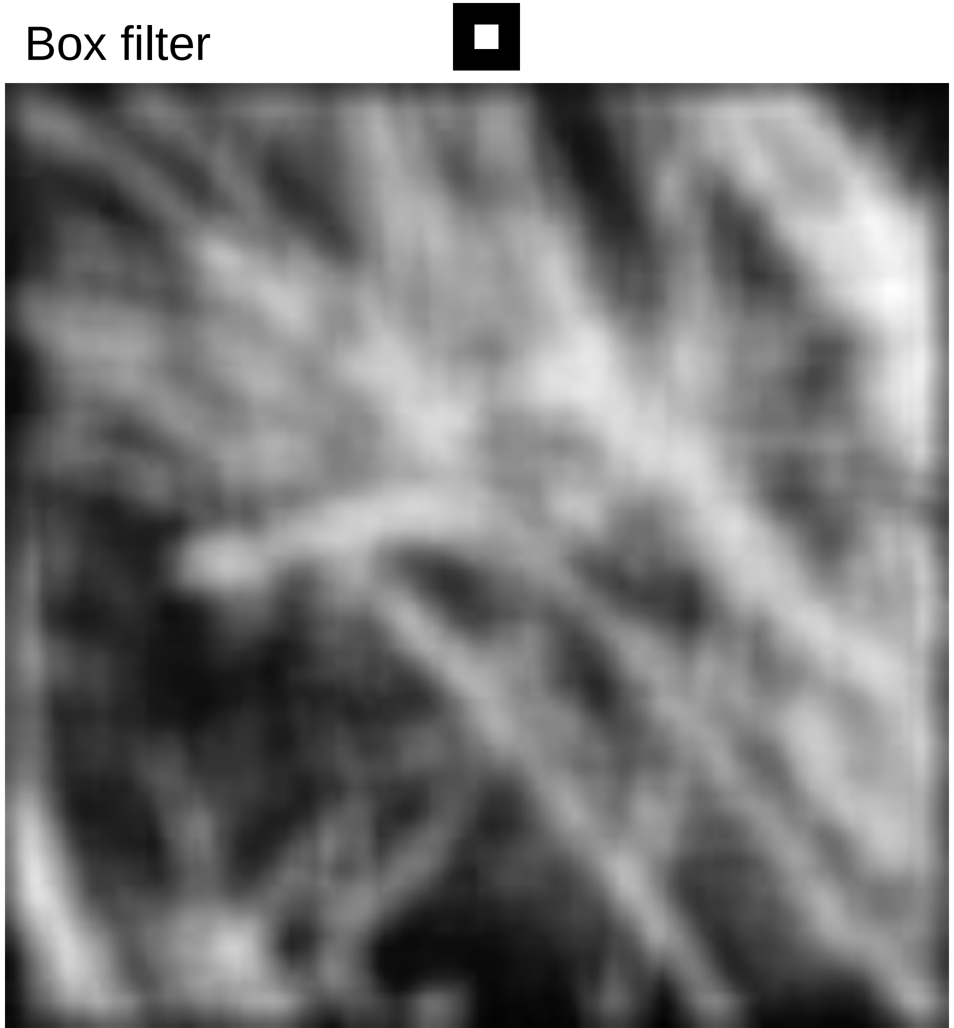
Inverse FFT

Slide: Hoiem

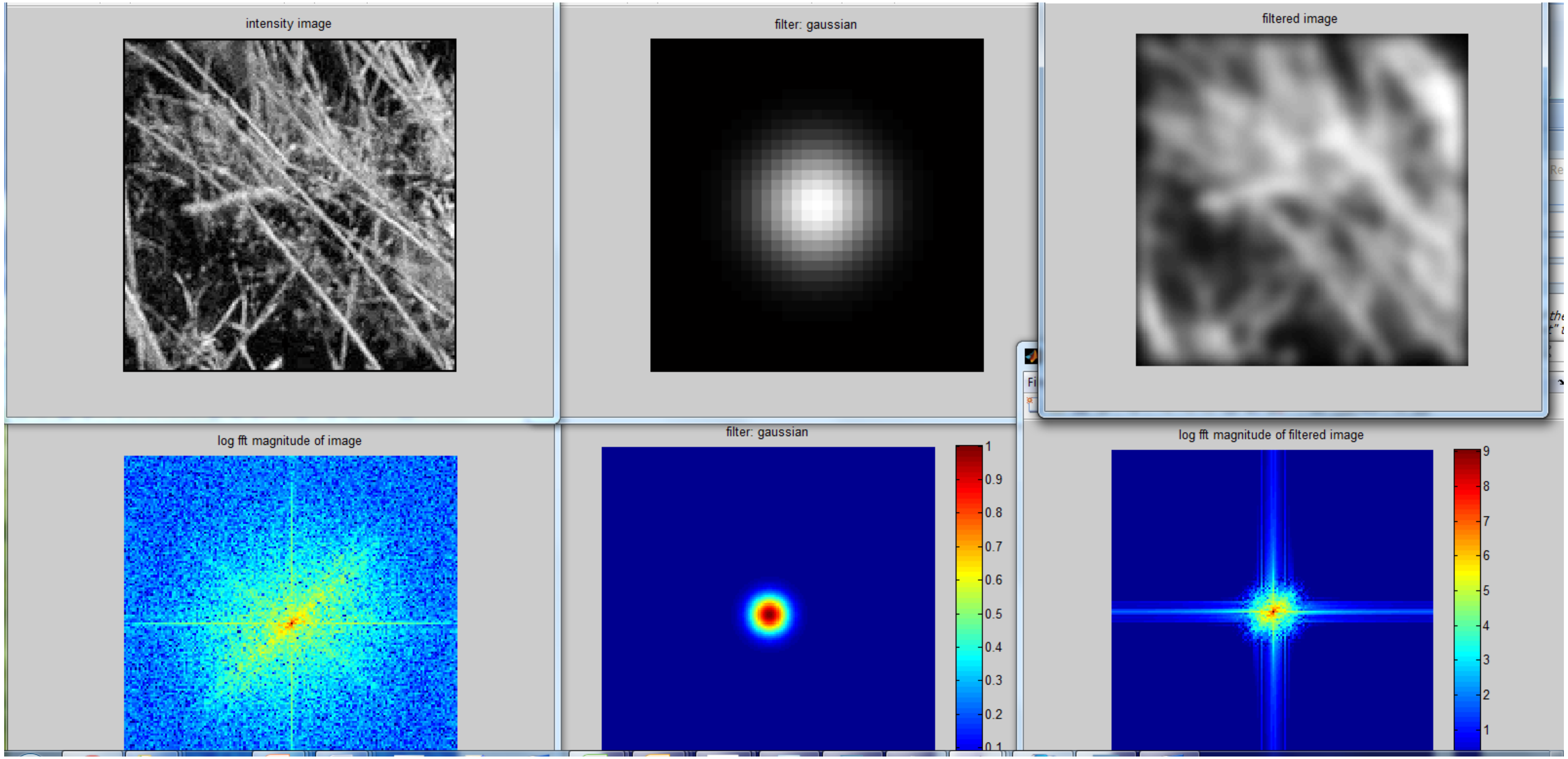
Filtering

Why does the Gaussian give a nice smooth image, but the square filter give edgy artifacts?

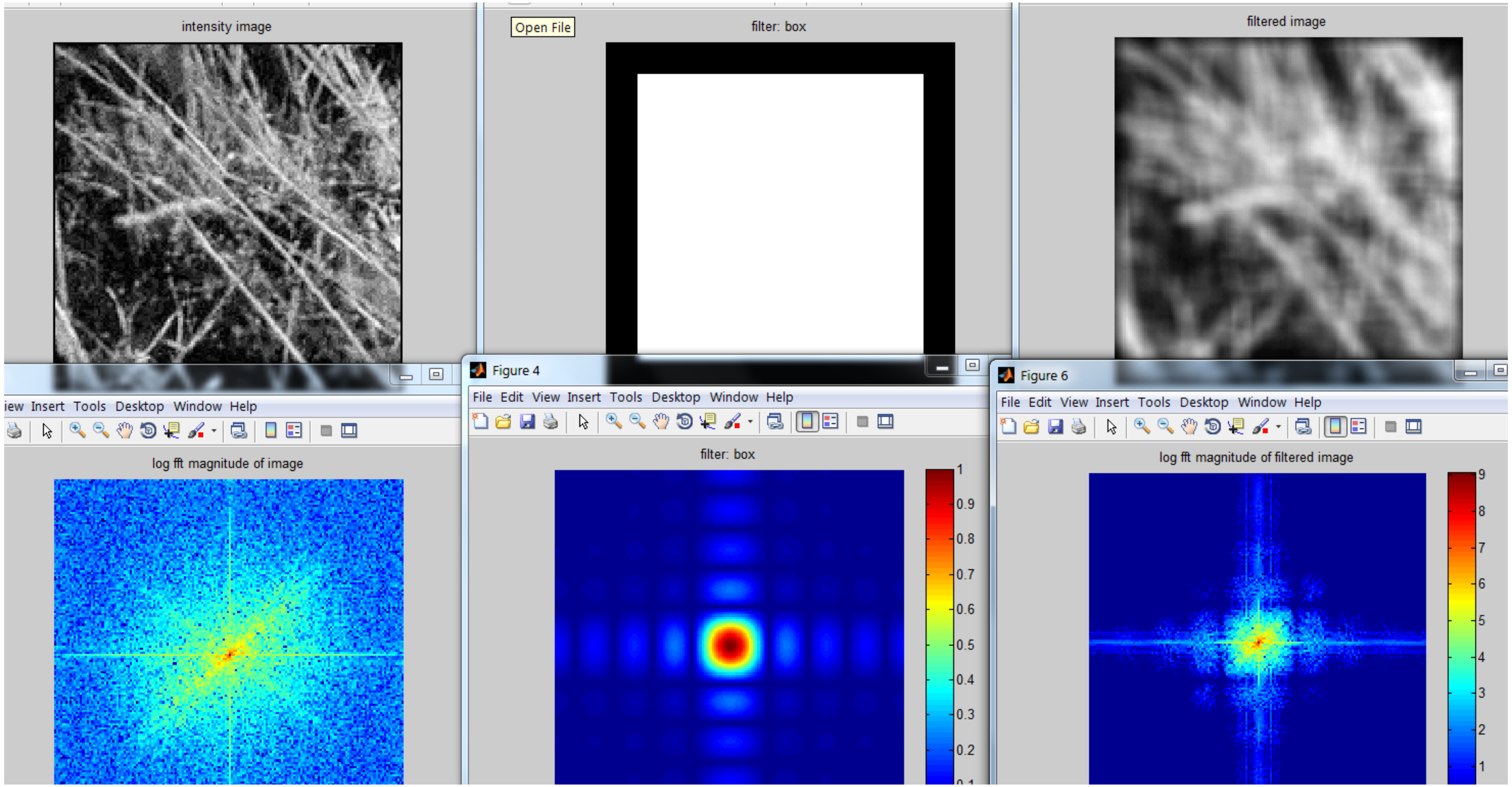

Gaussian


Box filter

Gaussian


intensity image
filter: gaussian
filtered image
log fft magnitude of image
filter: gaussian
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
log fft magnitude of filtered image
9
8
7
6
5
4
3
2
1

Box Filter

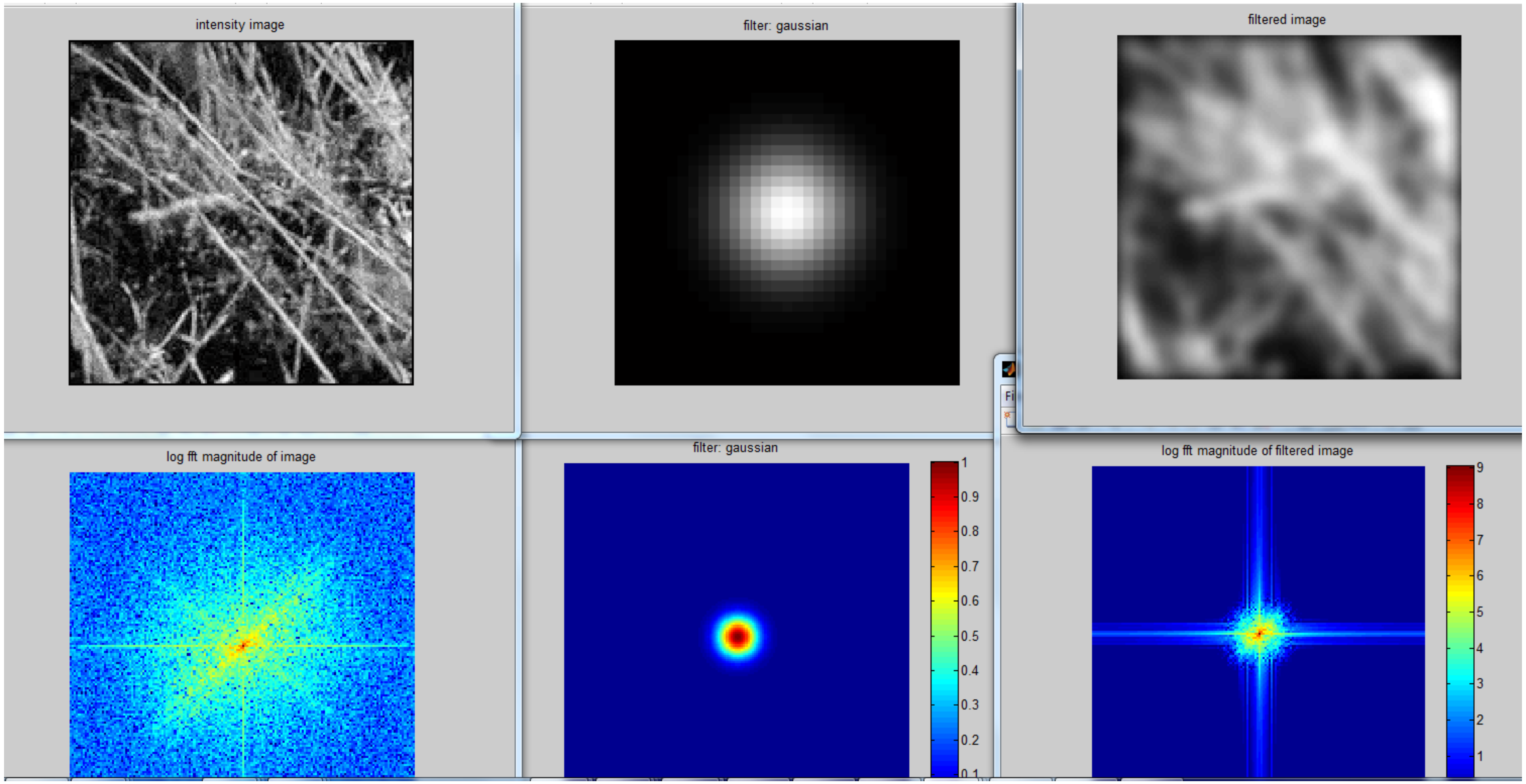

intensity image
Open File
filter: box
filtered image
Figure 4
Figure 6
iew Insert Tools Desktop Window Help
File Edit View Insert Tools Desktop Window Help
File Edit View Insert Tools Desktop Window Help
log fft magnitude of image
filter: box
log fft magnitude of filtered image
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
9
8
7
6
5
4
3
2
1

Is convolution invertible?

- If convolution is just multiplication in the Fourier domain, isn’t deconvolution just division?
- Sometimes, it clearly is invertible (e.g. a convolution with an identity filter)
- In one case, it clearly isn’t invertible (e.g. convolution with an all zero filter)
- What about for common filters like a Gaussian?

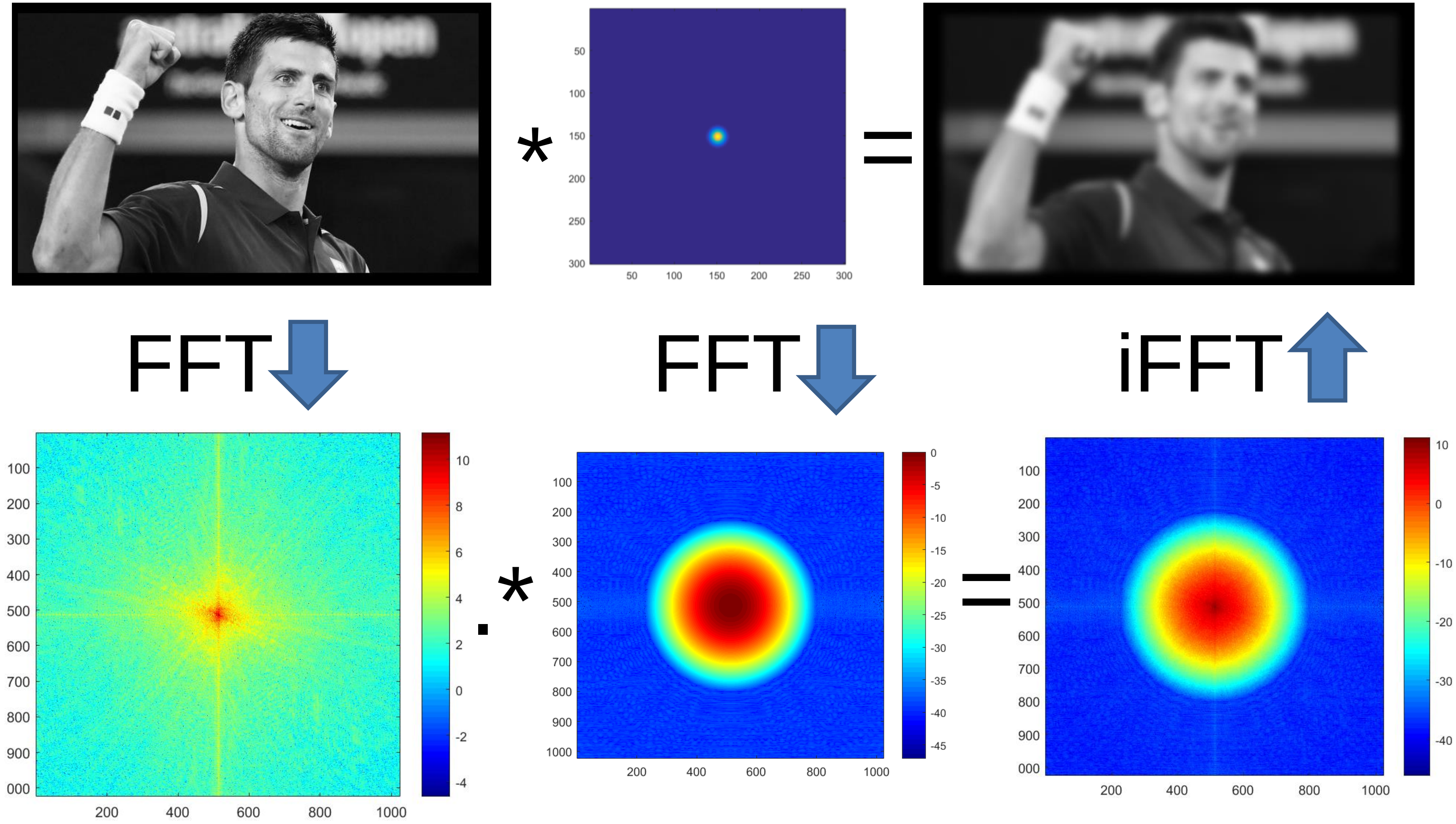
But you can’t invert multiplication by 0

- But it’s not quite zero, is it…



Let’s experiment on Novak

Convolution



*

=

FFT

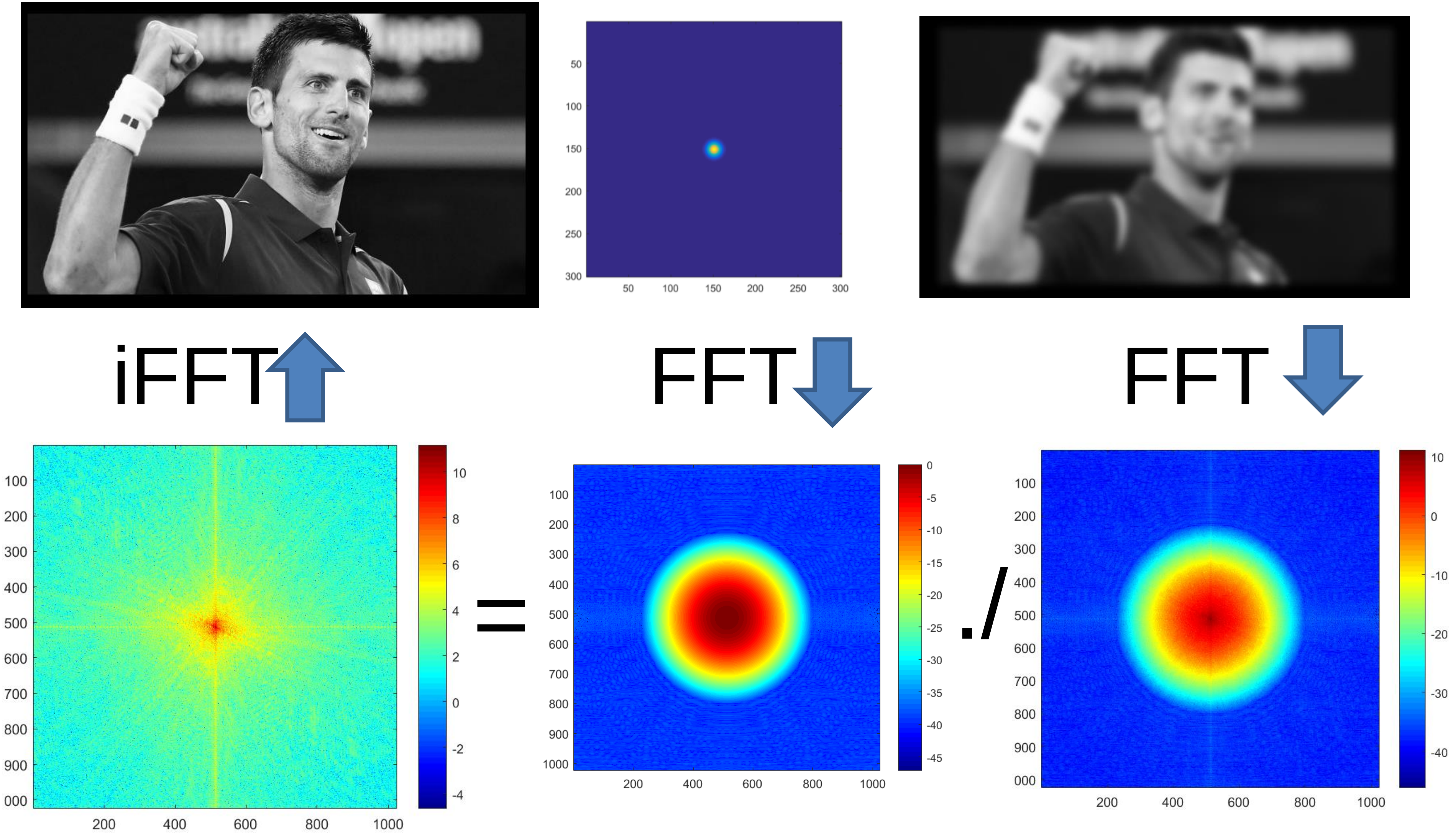
FFT

iFFT

.*

=

Deconvolution?



iFFT

FFT

FFT

=

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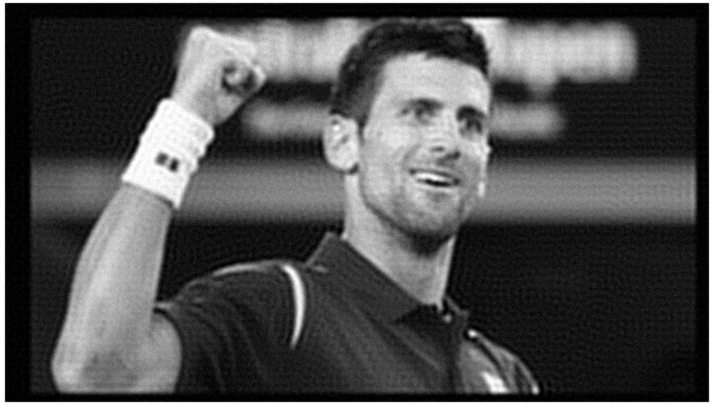
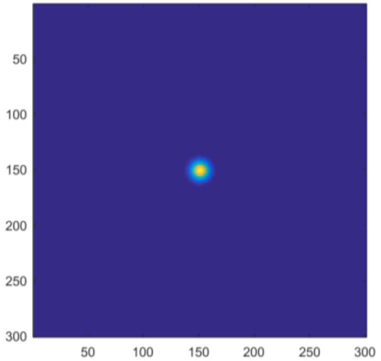
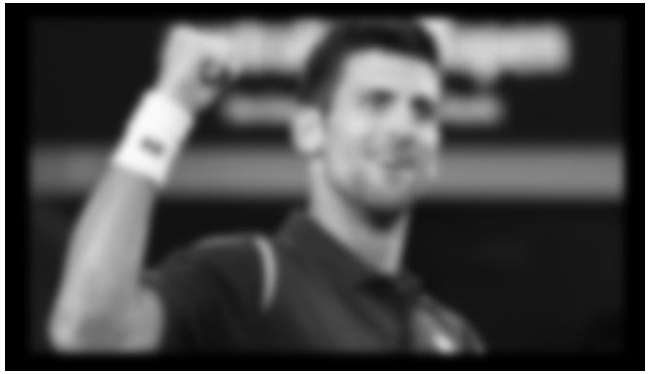
But under more realistic conditions





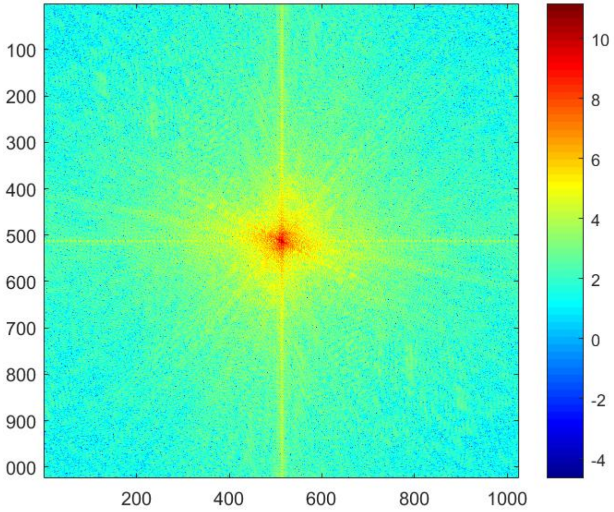
Random noise, .000001 magnitude



iFFT

FFT

FFT



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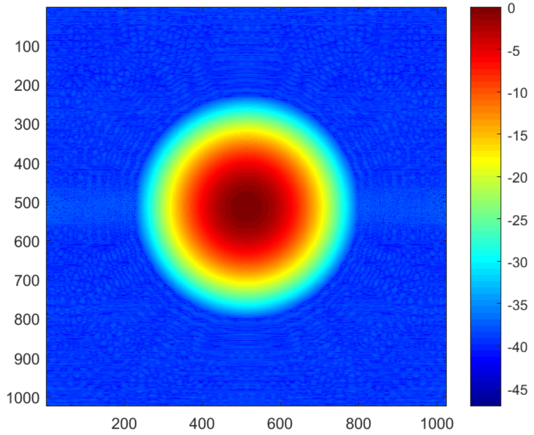

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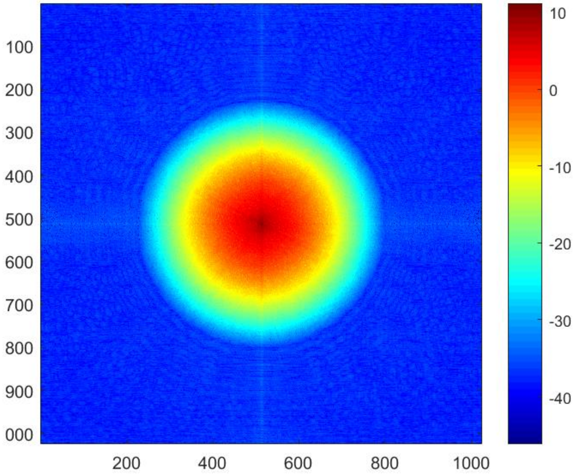
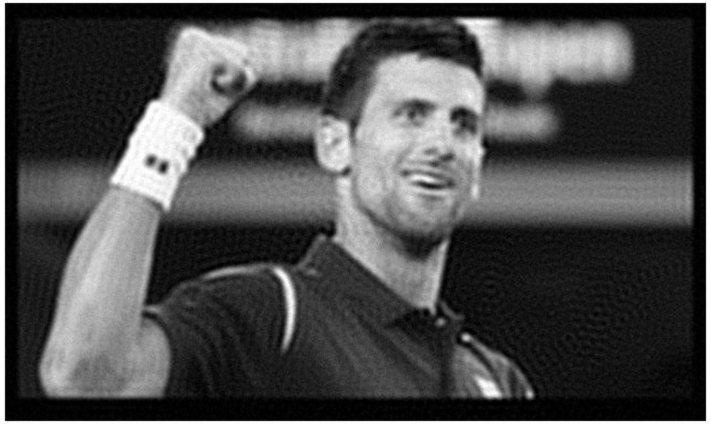
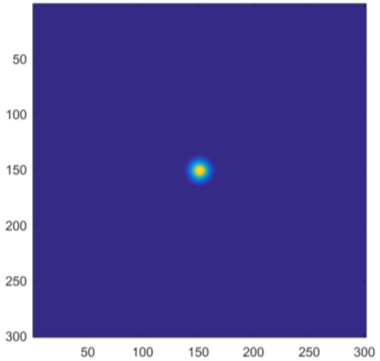
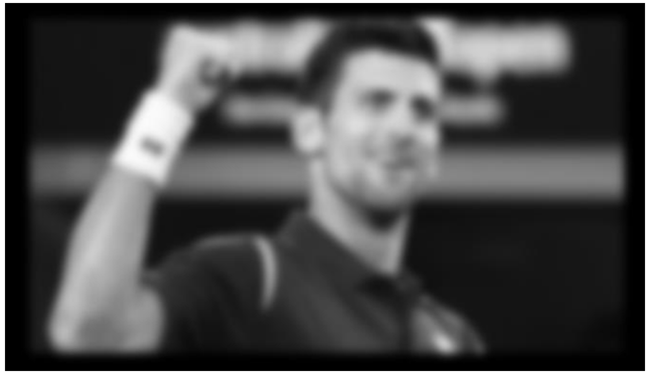


But under more realistic conditions





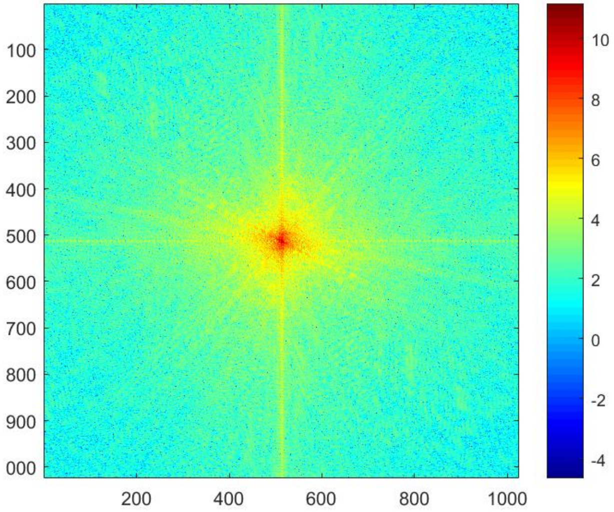
Random noise, .0001 magnitude



iFFT

FFT

FFT



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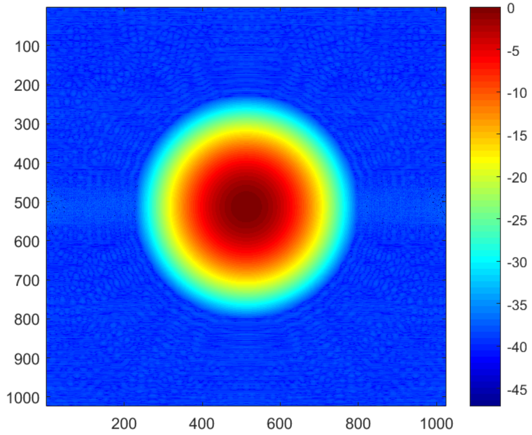

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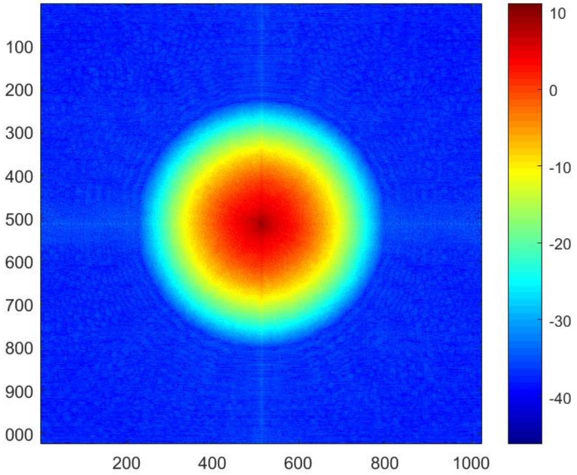
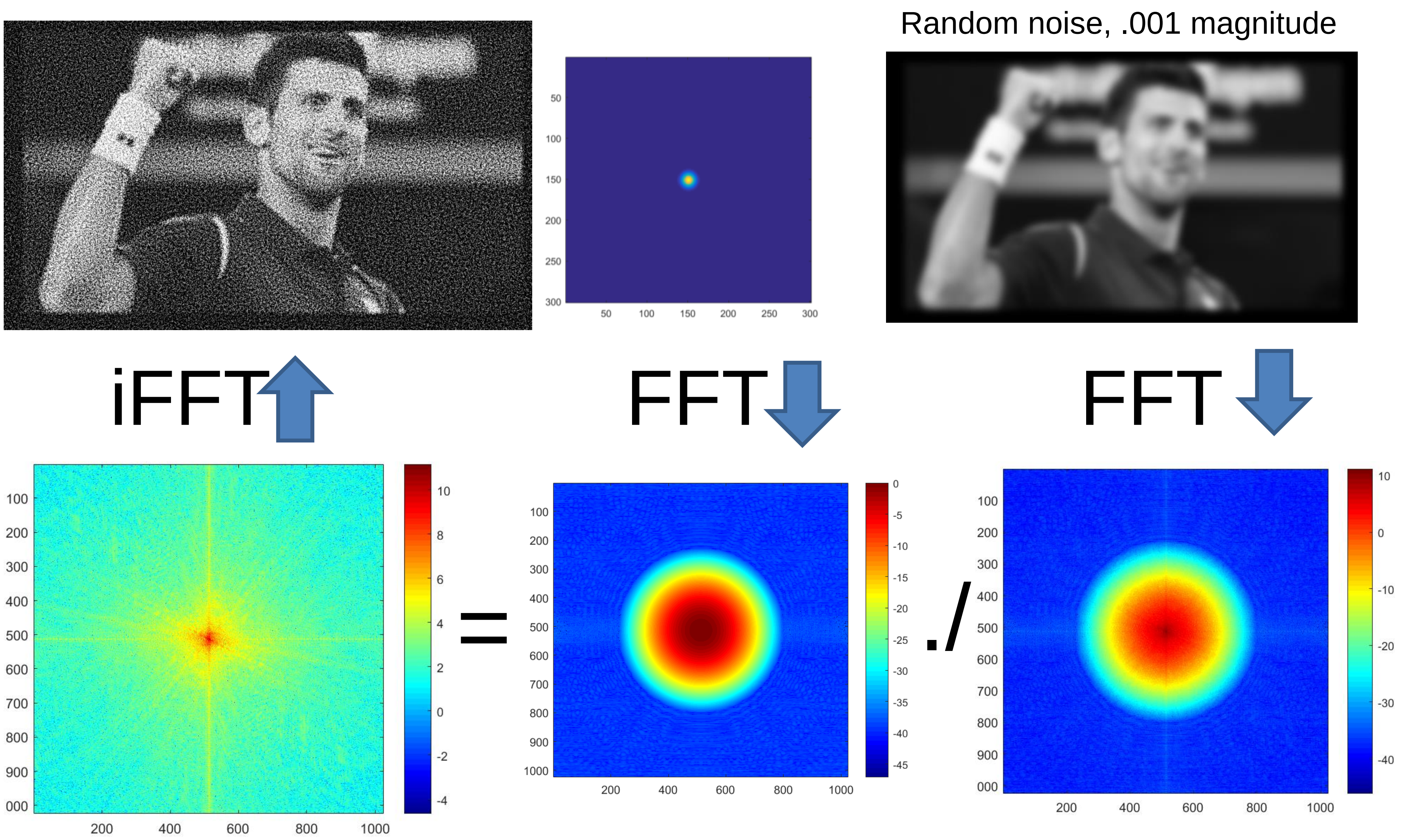


But under more realistic conditions



Random noise, .001 magnitude

iFFT

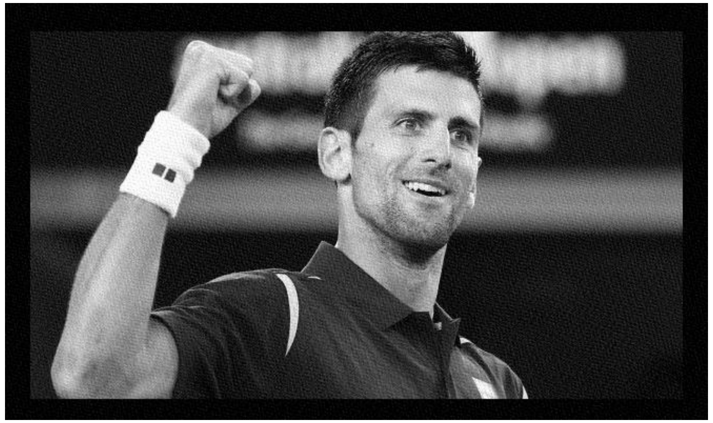
FFT

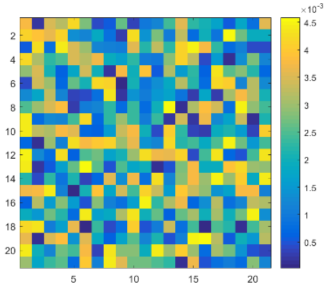
FFT

=

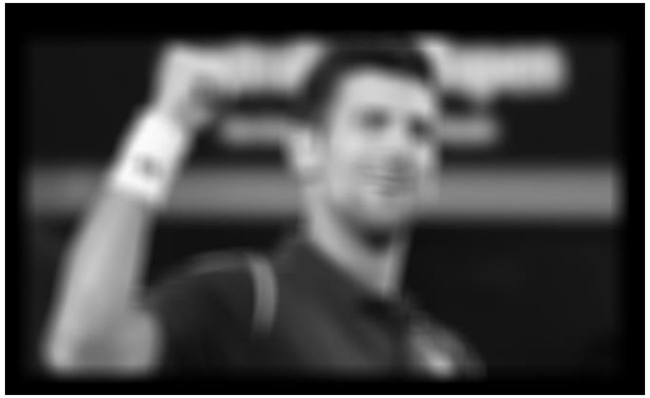
./

With a random filter…

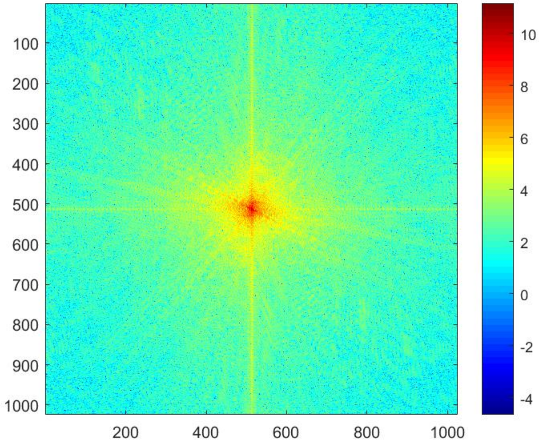




Random noise, .001 magnitude



iFFT ↑ FFT ↓ FFT ↓



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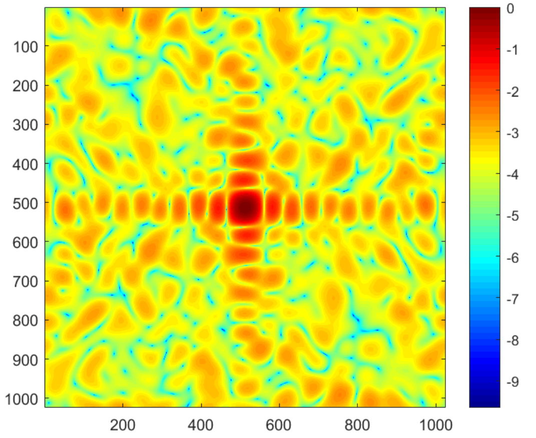

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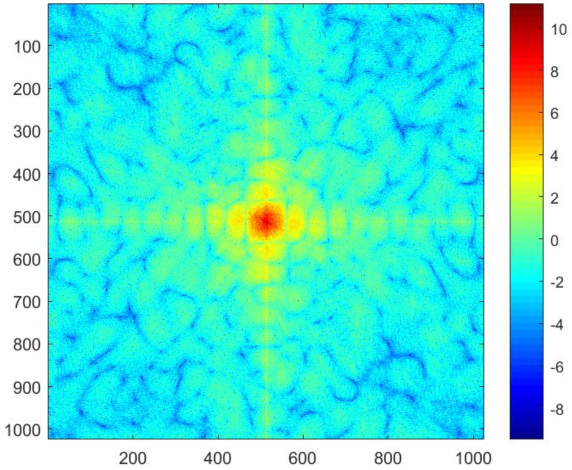


Deconvolution is hard

- Active research area.
- Even if you know the filter (non-blind deconvolution), it is still very hard and requires strong *regularization.*
- If you don’t know the filter (blind deconvolution) it is harder still.

Blind Deconvolution Example

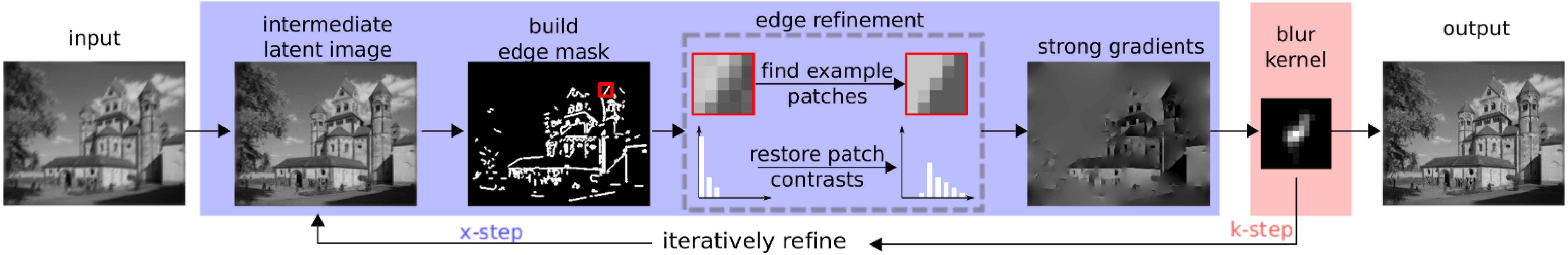
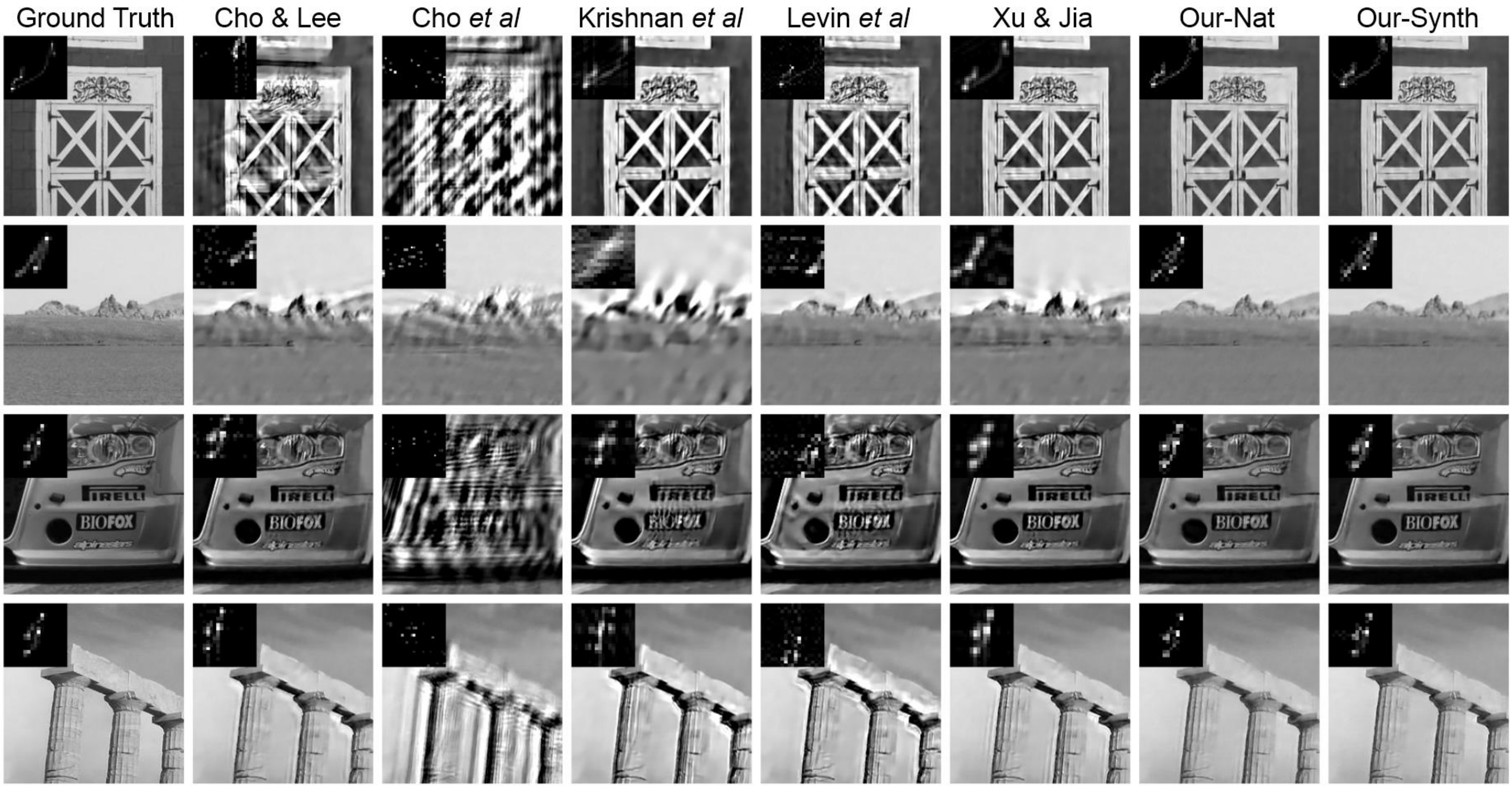


Figure 1. Algorithm pipeline. Our algorithm iterates between x-step and k-step with the help of a patch prior for edge refinement process. In particular, we coerce edges to become sharp and increase local contrast for edge patches. The blur kernel is then updated using the strong gradients from the restored latent image. After kernel estimation, the method of [20] is used for final non-blind deconvolution.

Edge-based Blur Kernel Estimation Using Patch Priors.
Libin Sun, Sunghyun Cho, Jue Wang, and James Hays.
IEEE International Conference on Computational Photography 2013.



Ground Truth Cho & Lee Cho *et al* Krishnan *et al* Levin *et al* Xu & Jia Our-Nat Our-Synth

Edge-based Blur Kernel Estimation Using Patch Priors.
Libin Sun, Sunghyun Cho, Jue Wang, and James Hays.
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