

Welcome back!

- Optional project 0 is out.
- Project 1 will be out soon.
- Read Szeliski 2.1, especially 2.1.4
- Today
 - Image projection
 - Filtering

From the 3D to 2D

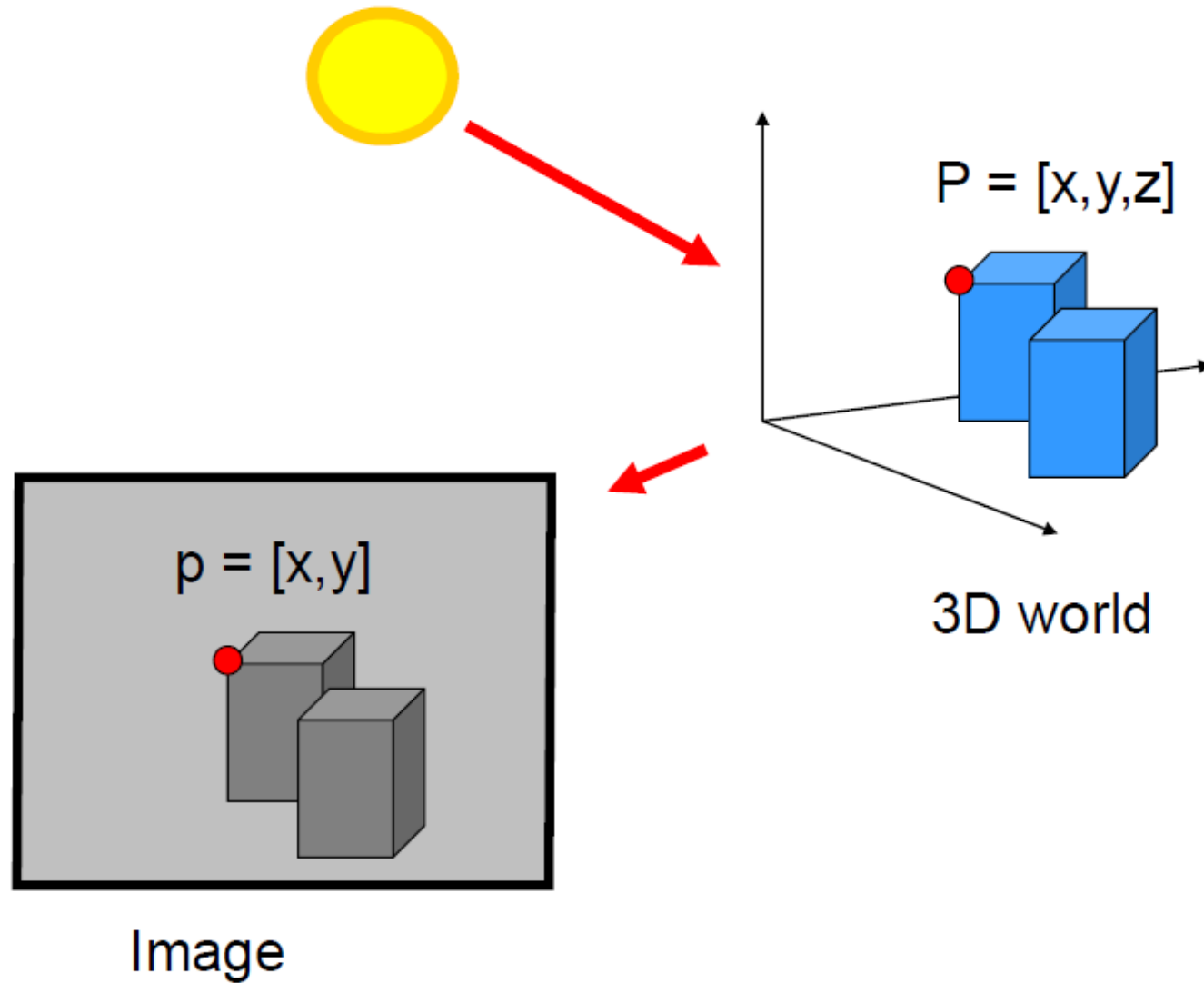
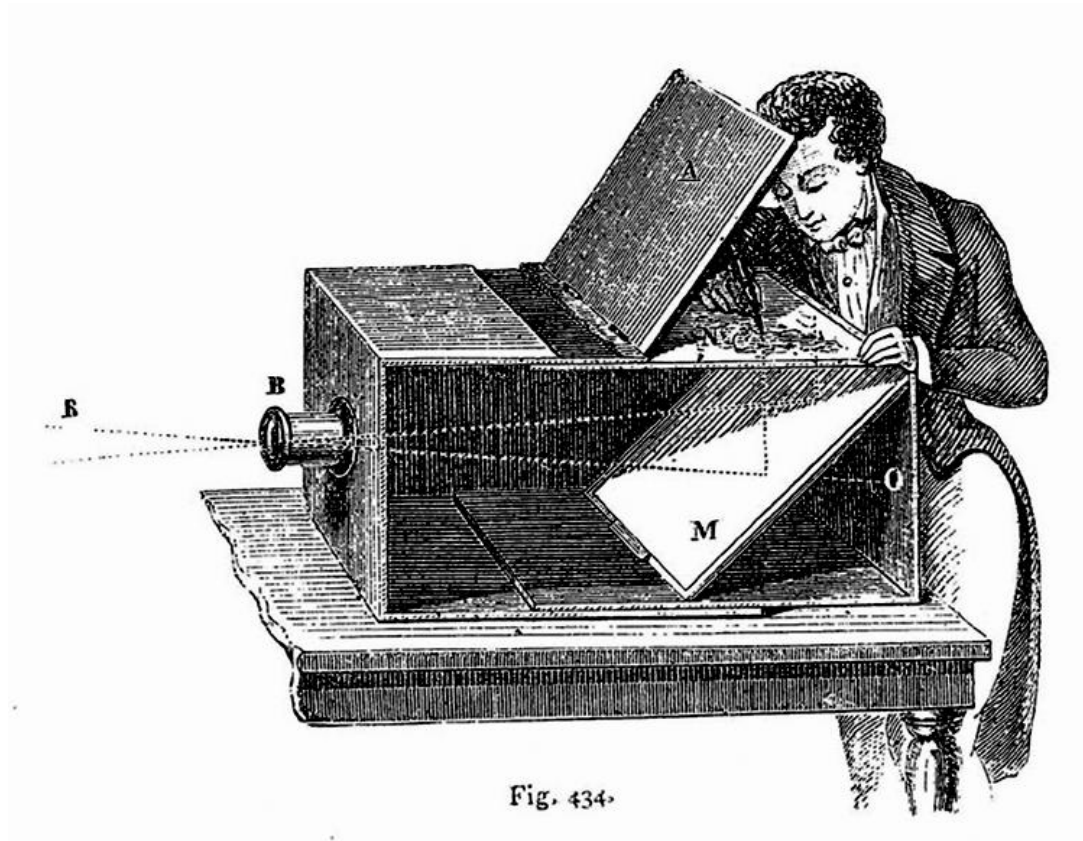


Image formation

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Camera Obscura used for Tracing



Lens Based Camera Obscura, 1568

First Photograph

Oldest surviving photograph
– Took 8 hours on pewter plate



Joseph Niepce, 1826

Photograph of the first photograph



Stored at UT Austin

Niepce later teamed up with Daguerre, who eventually created Daguerrotypes



“Louis Daguerre—the inventor of daguerreotype—shot what is not only the world's oldest photograph of Paris, but also the first photo with humans. The 10-minute long exposure was taken in 1839 in Place de la République and it's just possible to make out two blurry figures in the left-hand corner.”



Great history lesson on the chemistry and engineering challenges of early photography from the “Technology Connections” YouTube channel.

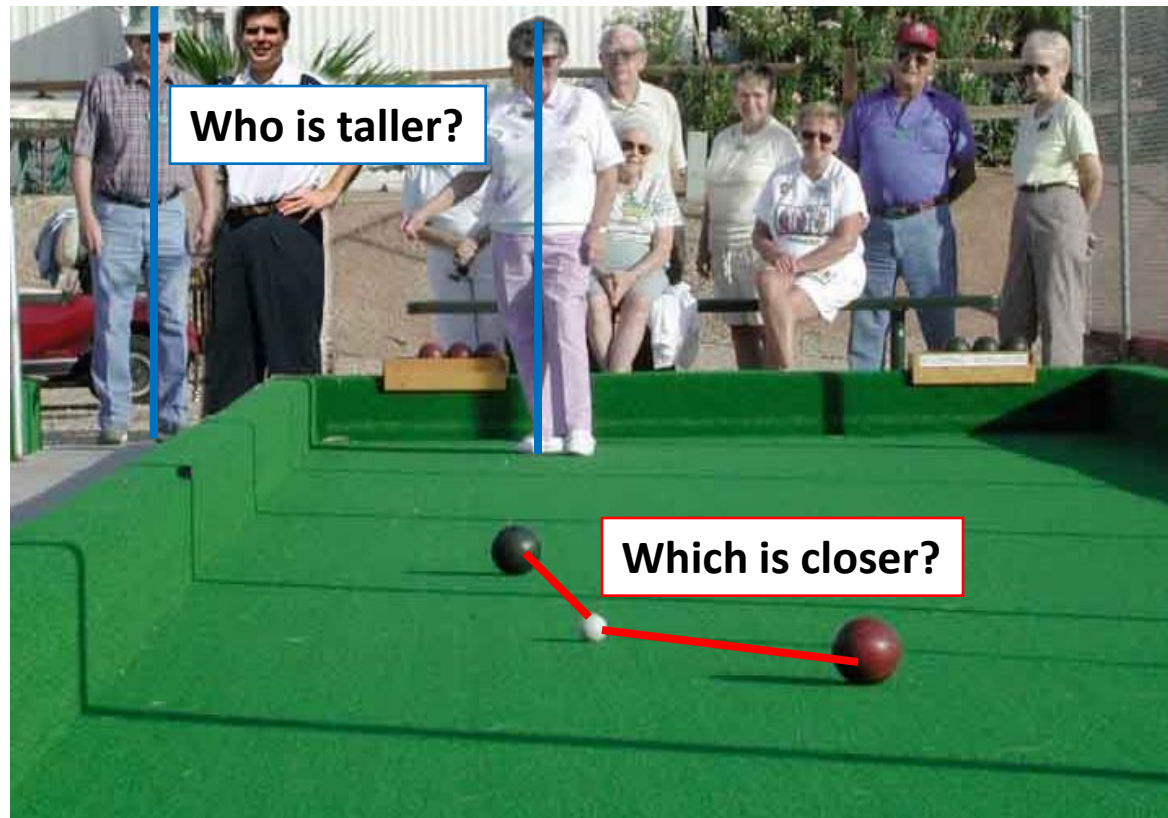


https://www.youtube.com/watch?v=wbbH77rYaa8&list=PLv0jwu7G_DFV6yW240e6CbiwCLaZ0Z6PV

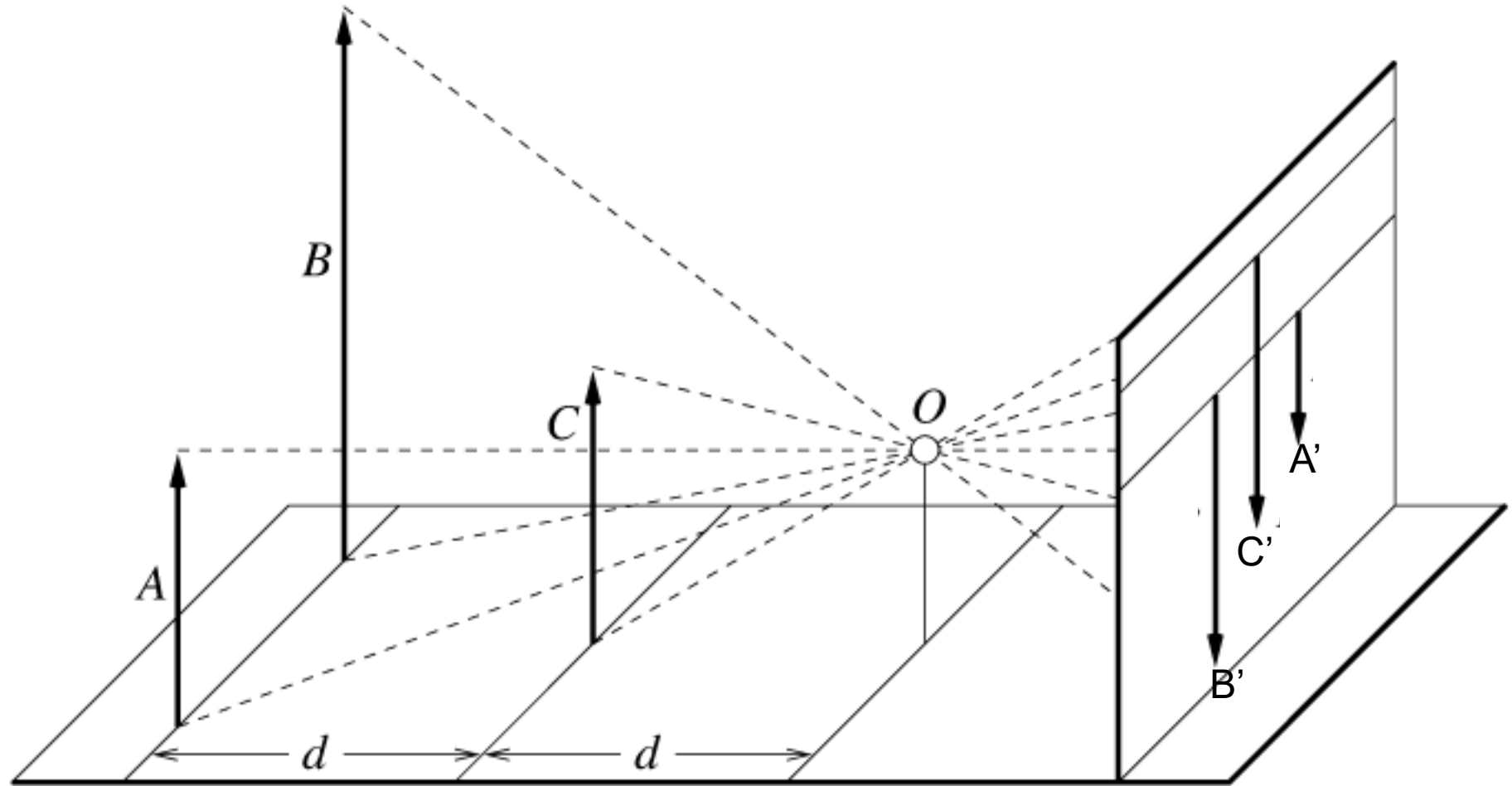
Projective Geometry

What is lost?

- Length



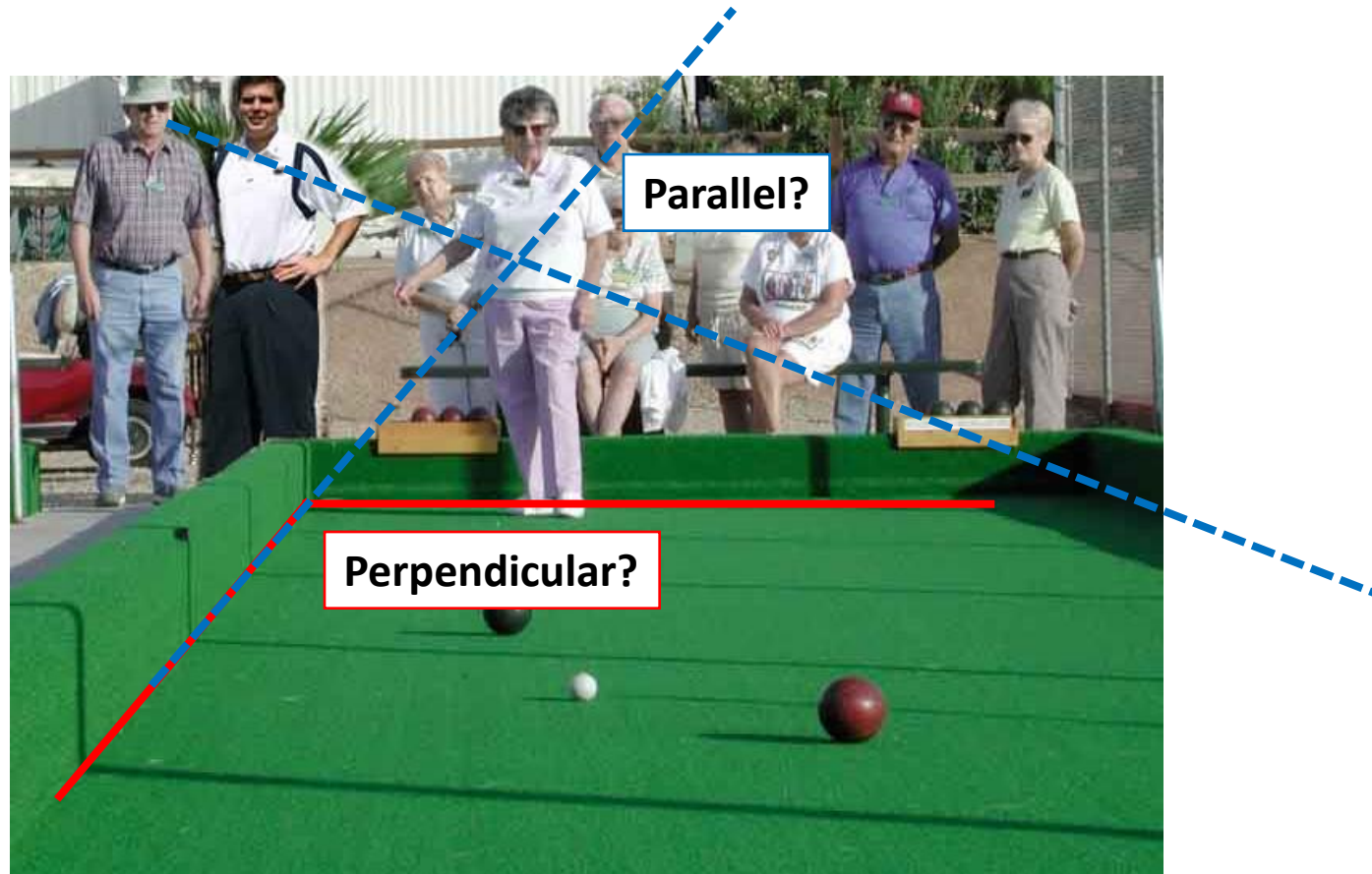
Length and area are not preserved



Projective Geometry

What is lost?

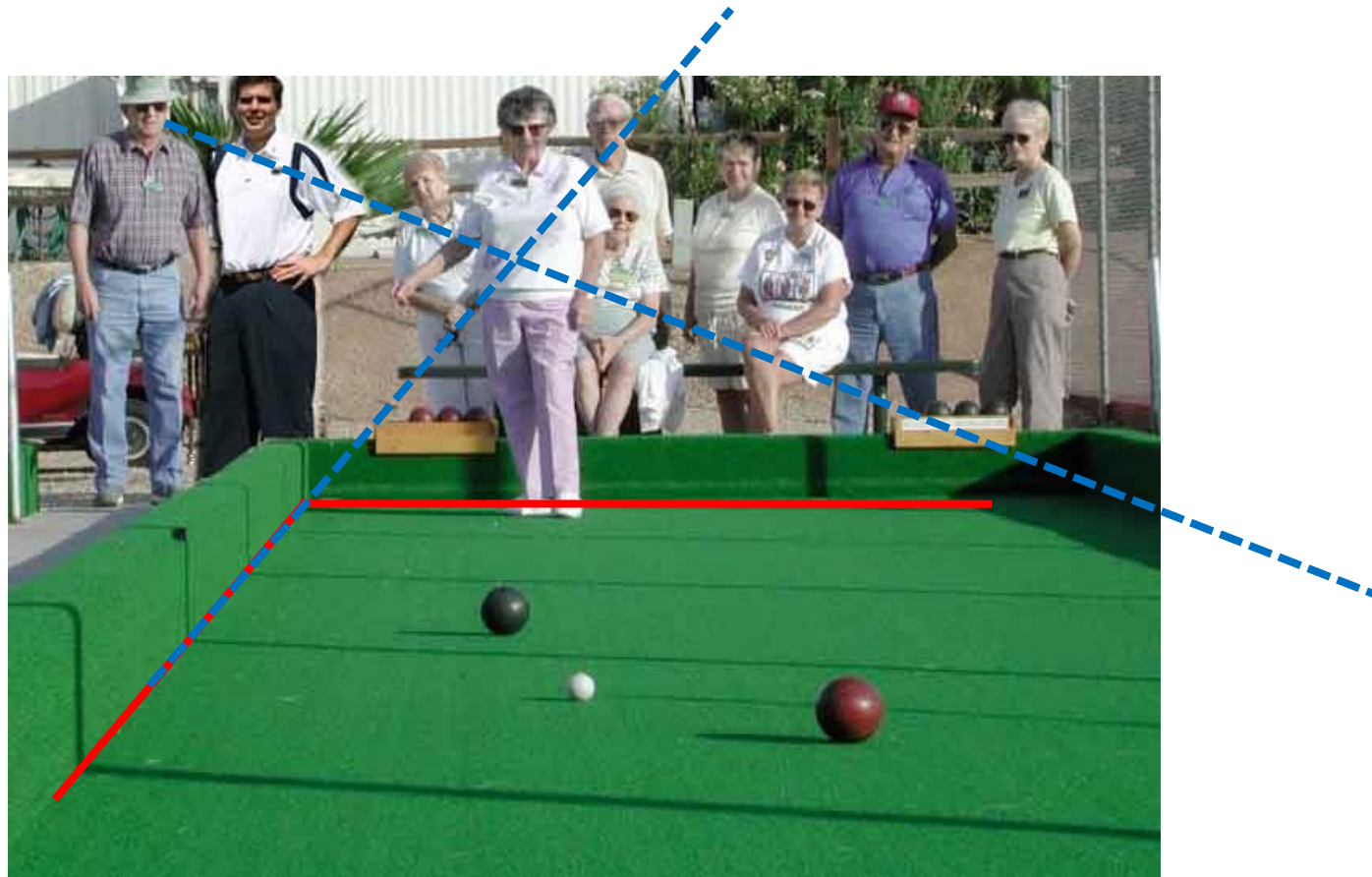
- Length
- Angles



Projective Geometry

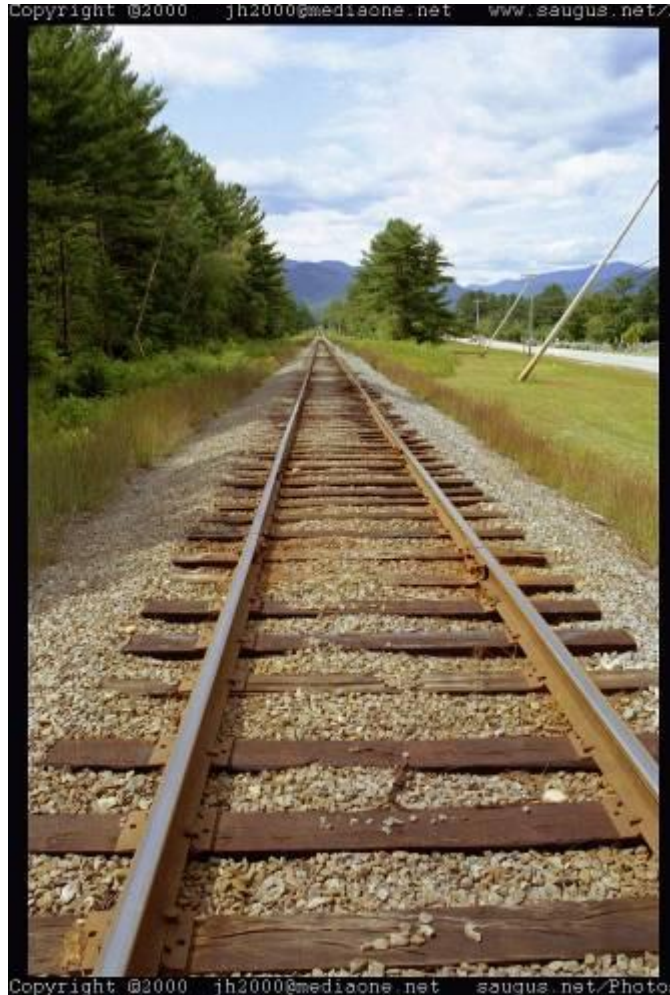
What is preserved?

- Straight lines are still straight

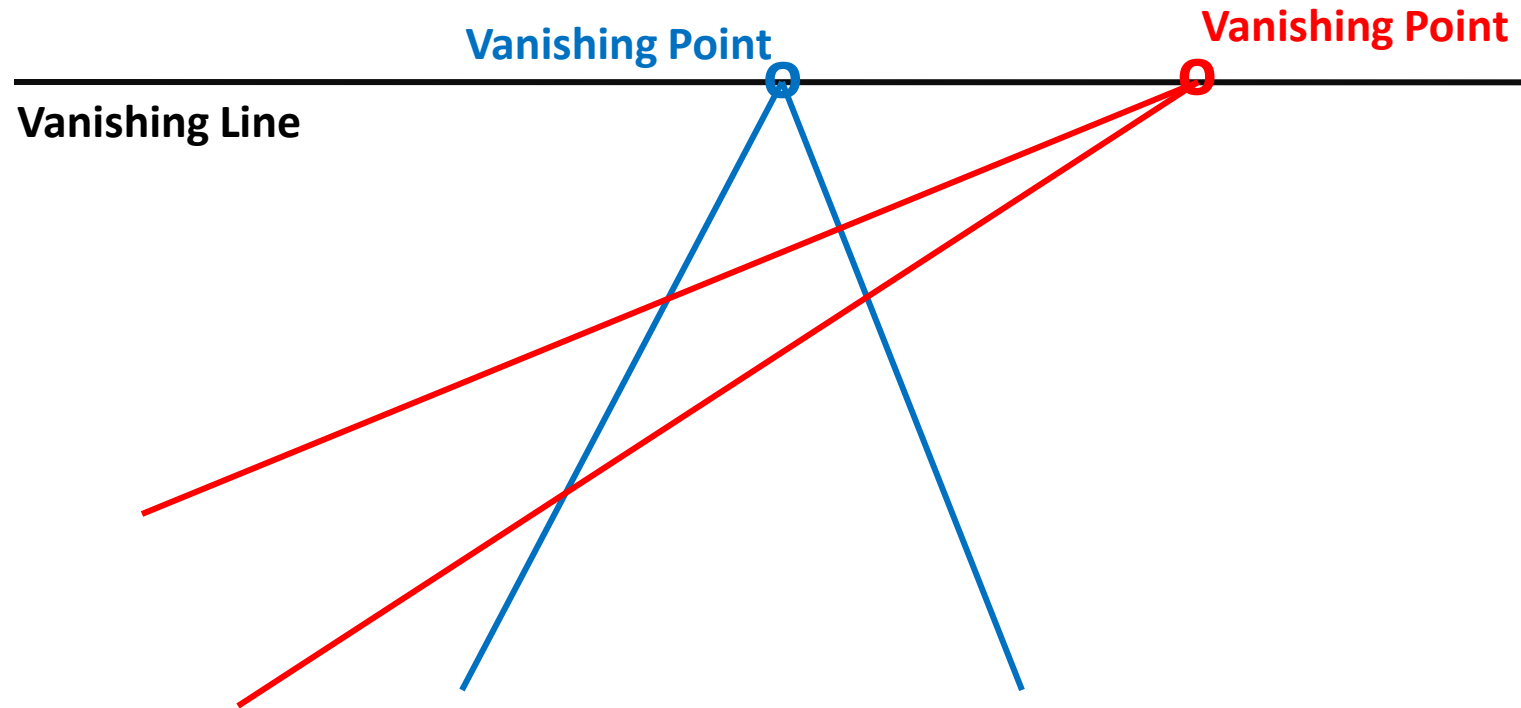


Vanishing points and lines

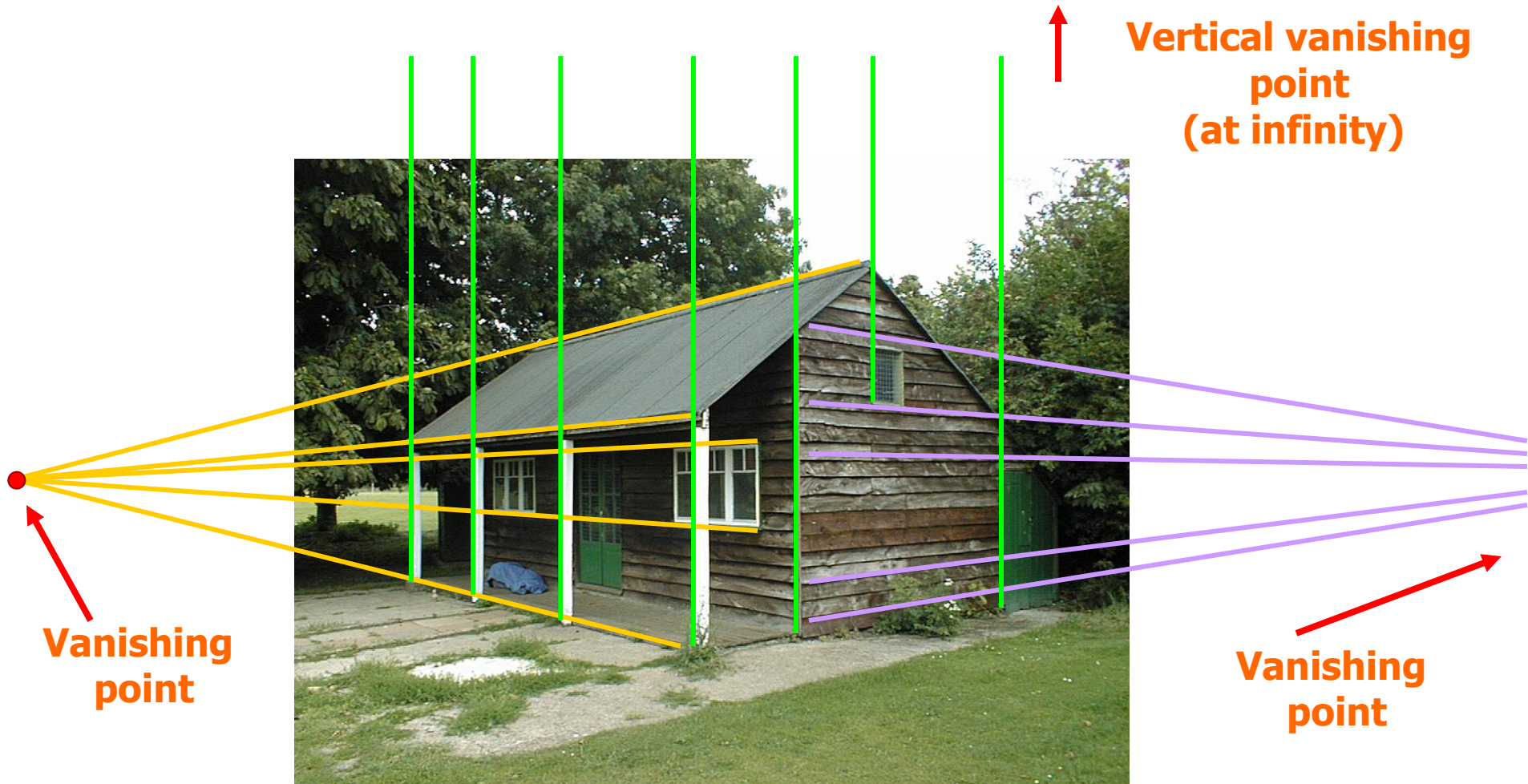
Parallel lines in the world intersect in the image at a “vanishing point”



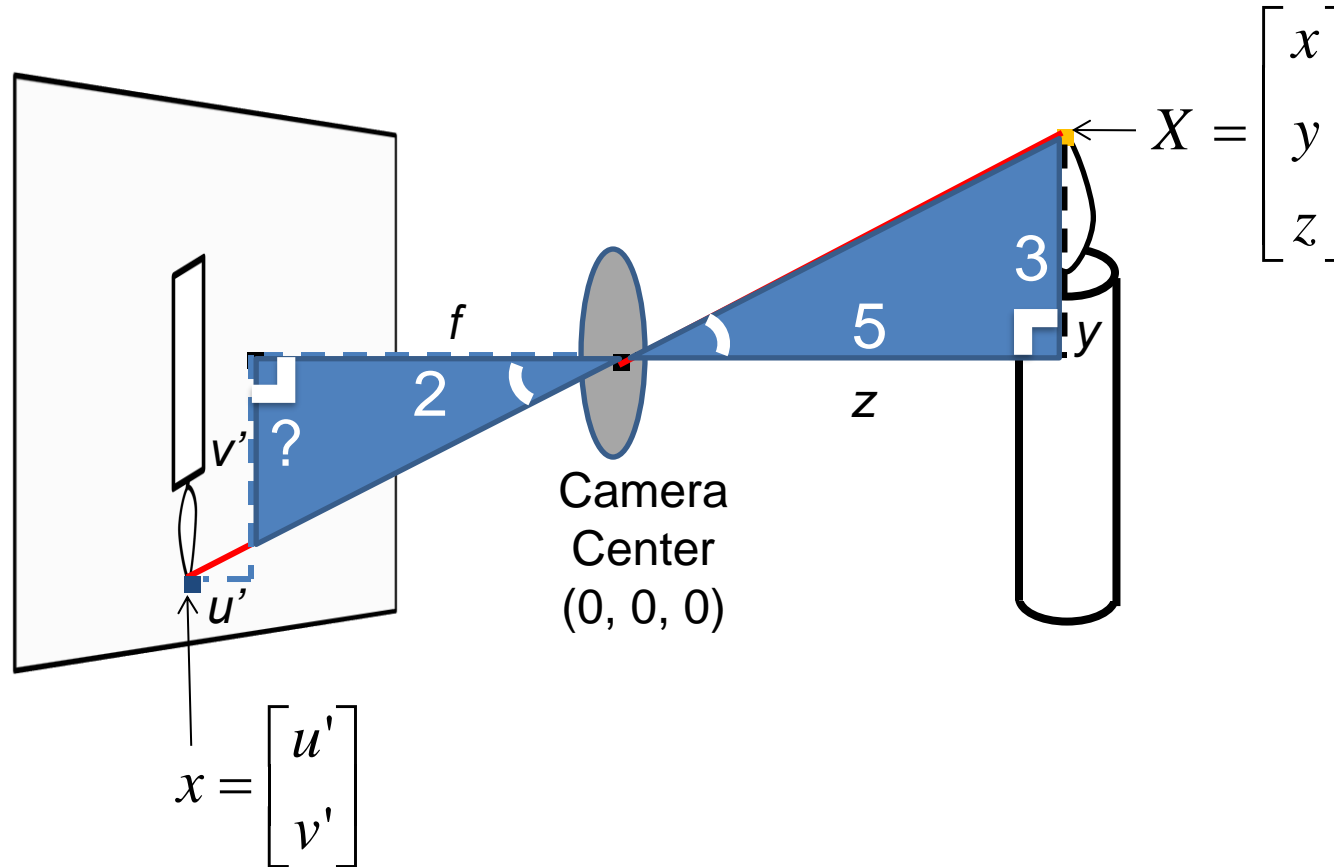
Vanishing points and lines



Vanishing points and lines



Projection: world coordinates \rightarrow image coordinates



If $x = 2$, $y = 3$, $z = 5$, and $f = 2$

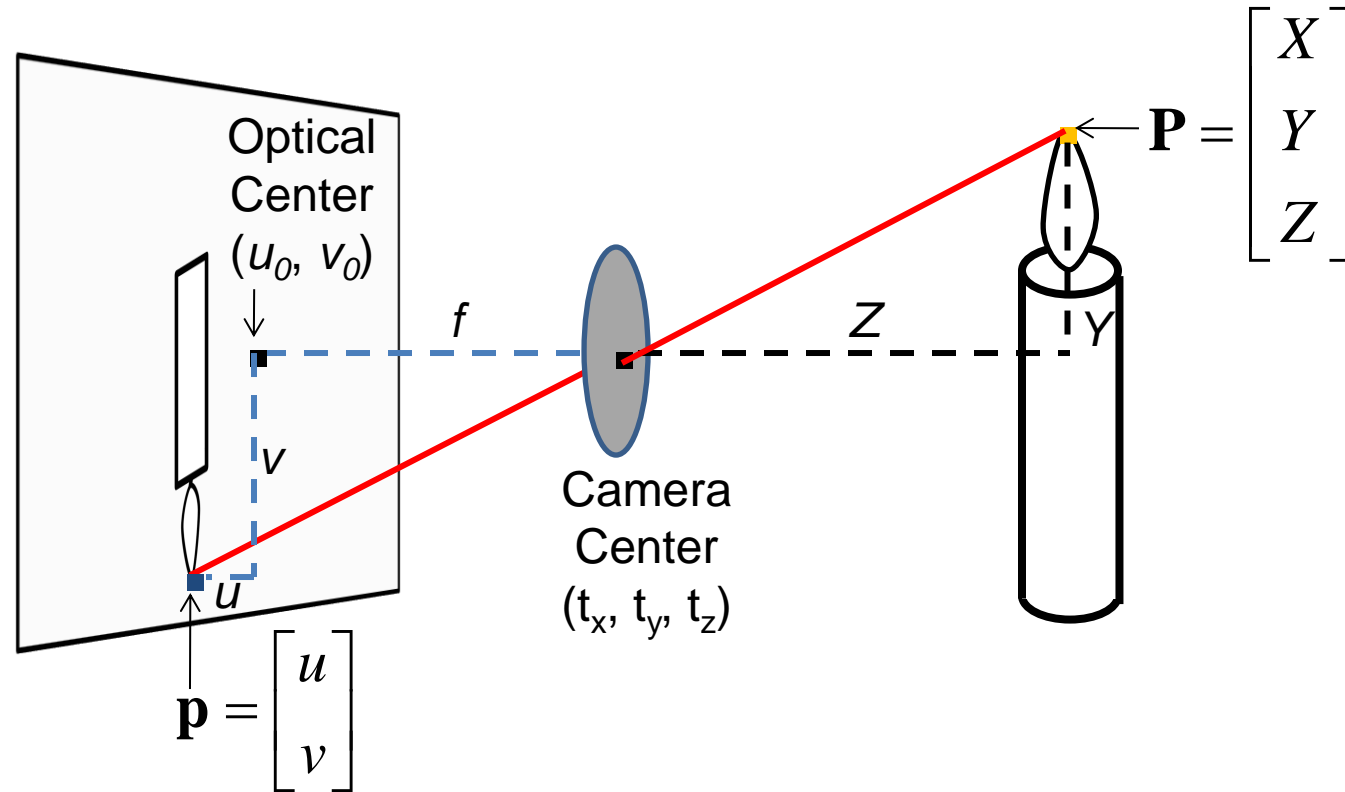
What are u' and v' ?

$$\frac{v'}{-f} = \frac{y}{z}$$

$$u' = -x * \frac{f}{z}$$
$$v' = -y * \frac{f}{z}$$

$$u' = -2 * \frac{2}{5}$$
$$v' = -3 * \frac{2}{5}$$

Projection: world coordinates \rightarrow image coordinates



How do we handle the general case?

Interlude: why does this matter?

Relating multiple views



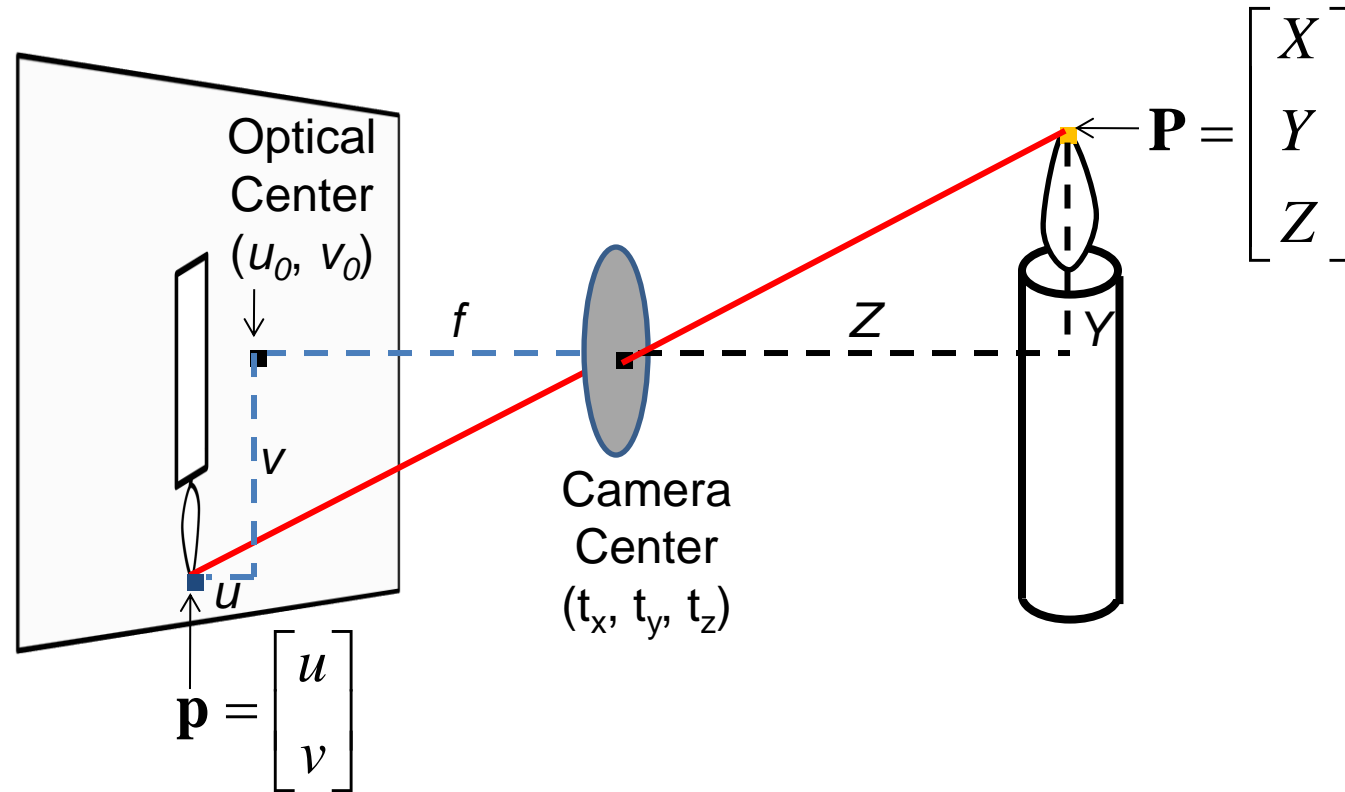
Photo Tourism

Exploring photo collections in 3D

Noah Snavely Steven M. Seitz Richard Szeliski
University of Washington *Microsoft Research*

SIGGRAPH 2006

Projection: world coordinates \rightarrow image coordinates



How do we handle the general case?

Homogeneous coordinates

Conversion

Converting to *homogeneous* coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image
coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene
coordinates

Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Homogeneous coordinates

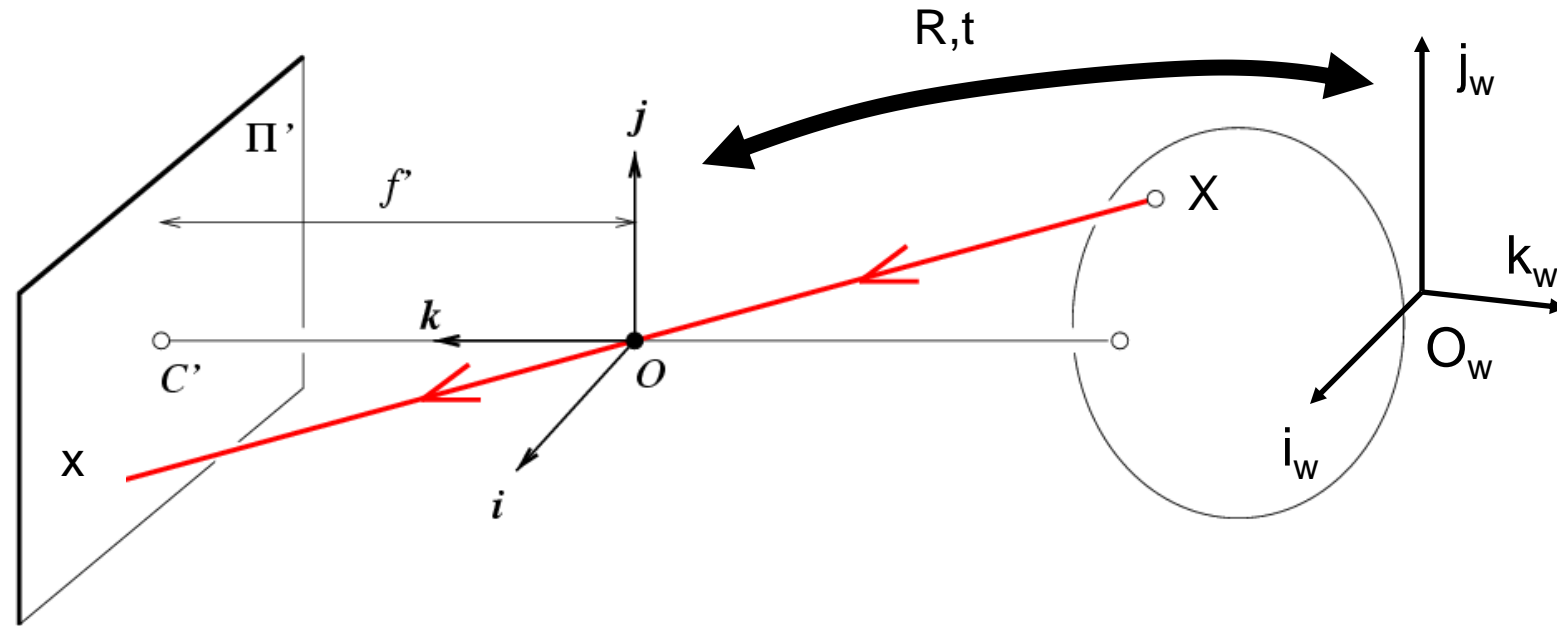
Invariant to scaling

$$k \begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} kx \\ ky \\ kw \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{kx}{kw} \\ \frac{ky}{kw} \\ \frac{kw}{kw} \end{bmatrix} = \begin{bmatrix} \frac{x}{w} \\ \frac{y}{w} \\ 1 \end{bmatrix}$$

Homogeneous Coordinates Cartesian Coordinates

Point in Cartesian is ray in Homogeneous

Projection matrix



$$\mathbf{x} = \mathbf{K} \begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix} \mathbf{X}$$

\mathbf{x} : Image Coordinates: $(u, v, 1)$

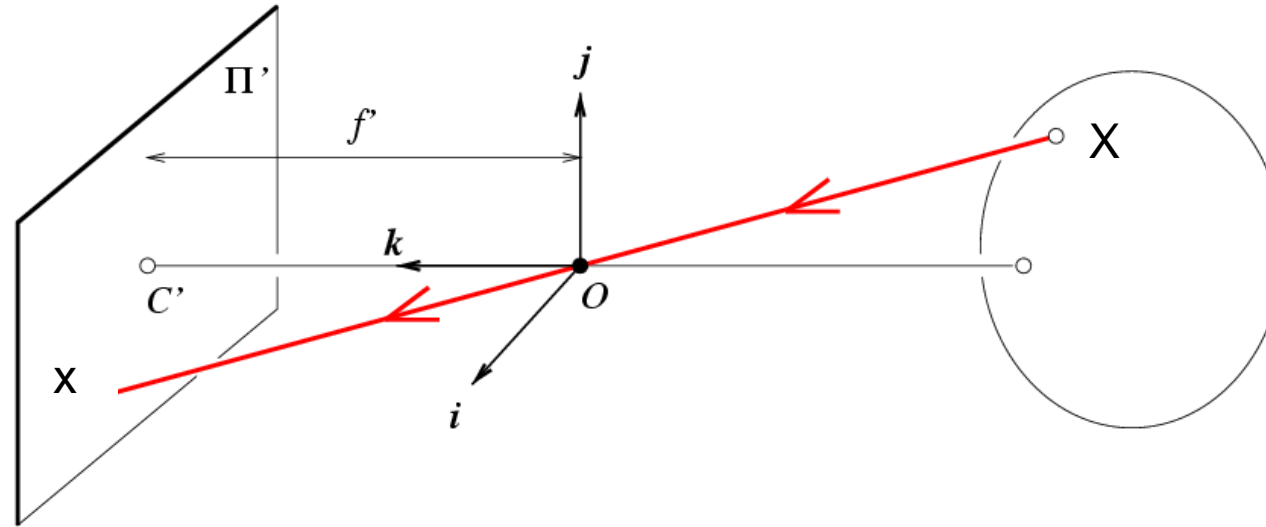
\mathbf{K} : Intrinsic Matrix (3×3)

\mathbf{R} : Rotation (3×3)

\mathbf{t} : Translation (3×1)

\mathbf{X} : World Coordinates: $(X, Y, Z, 1)$

Projection matrix



Intrinsic Assumptions

- Unit aspect ratio
- Optical center at (0,0)
- No skew

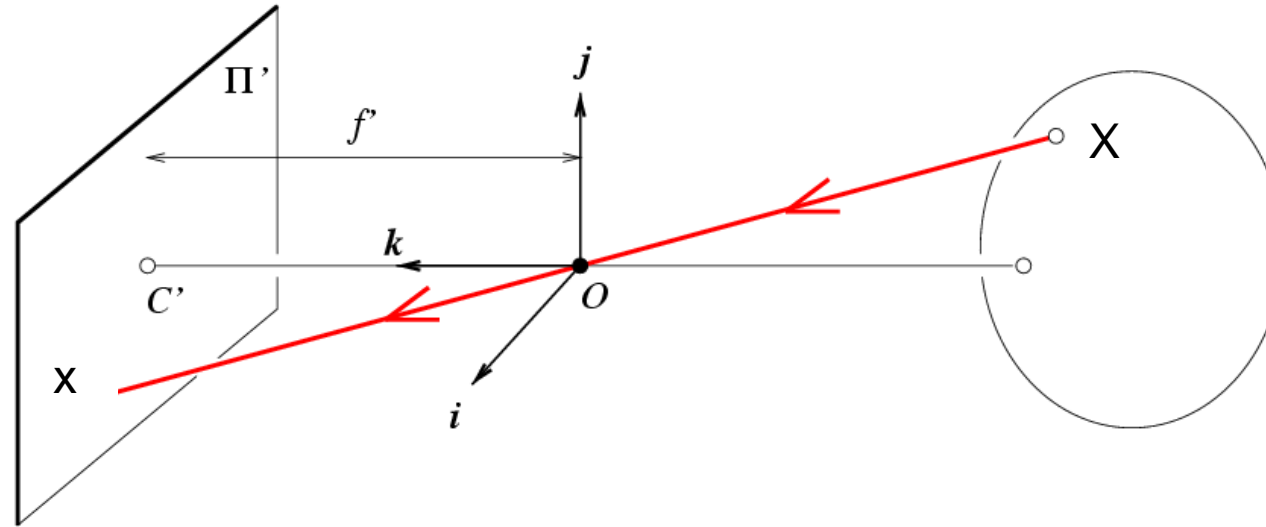
Extrinsic Assumptions

- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \mathbf{X} \Rightarrow w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

K

Projection matrix



Intrinsic Assumptions

- Unit aspect ratio
- Optical center at $(0,0)$
- No skew

Extrinsic Assumptions

- No rotation
- Camera at $(0,0,0)$

$$\mathbf{x} = \mathbf{K} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \mathbf{X} \Rightarrow w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Remove assumption: known optical center

Intrinsic Assumptions Extrinsic Assumptions

- Unit aspect ratio
- No skew

- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \mathbf{X} \quad \Rightarrow \quad w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Remove assumption: square pixels

Intrinsic Assumptions

- No skew

Extrinsic Assumptions

- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \mathbf{X} \Rightarrow w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & 0 & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Remove assumption: non-skewed pixels

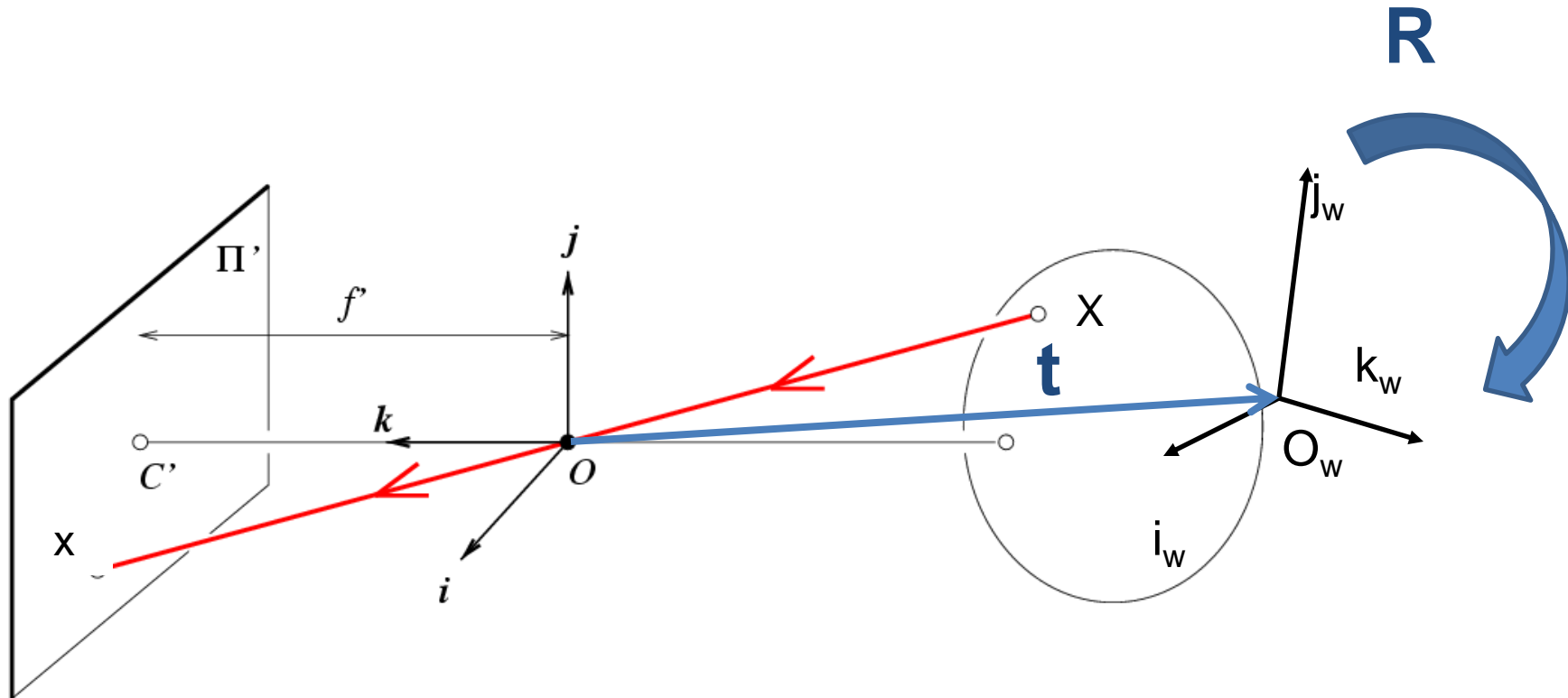
Intrinsic Assumptions Extrinsic Assumptions

- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \mathbf{X} \Rightarrow w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Note: different books use different notation for parameters

Oriented and Translated Camera



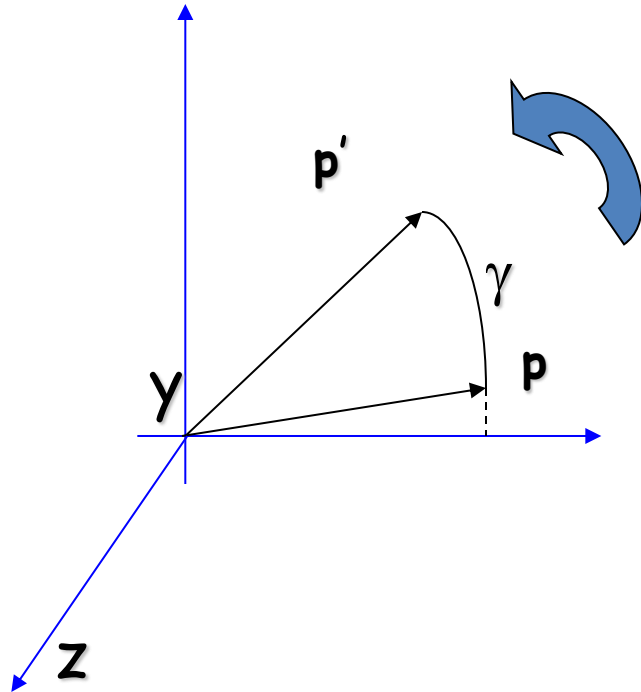
Allow camera translation

Intrinsic Assumptions Extrinsic Assumptions
• No rotation

$$\mathbf{x} = \mathbf{K} \begin{bmatrix} \mathbf{I} & \mathbf{t} \end{bmatrix} \mathbf{X} \quad \Rightarrow \quad w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & 0 & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3D Rotation of Points

Rotation around the coordinate axes, **counter-clockwise**:



$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix}$$

$$R_y(\beta) = \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix}$$

$$R_z(\gamma) = \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

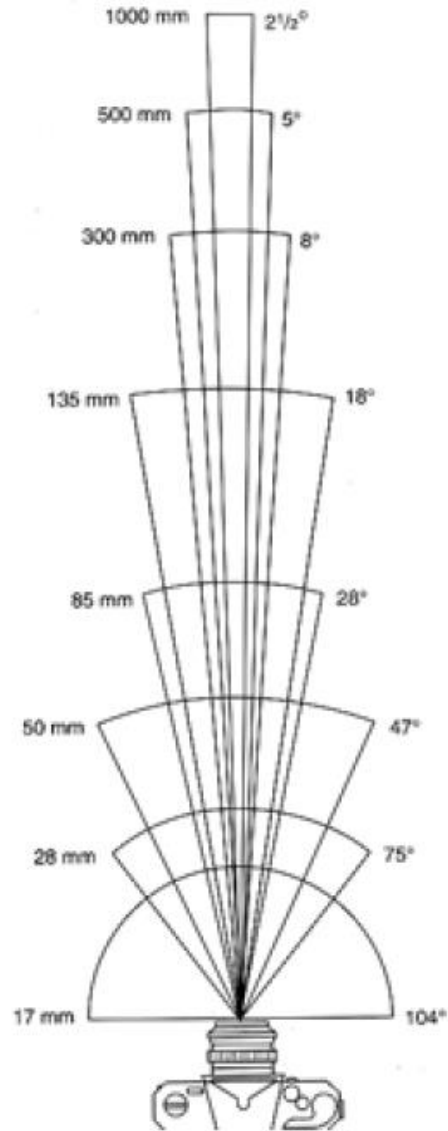
Allow camera rotation

$$\mathbf{x} = \mathbf{K}[\mathbf{R} \quad \mathbf{t}] \mathbf{X}$$



$$w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} t_x \\ t_y \\ t_z \\ 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Field of View (Zoom, focal length)



17mm



28mm



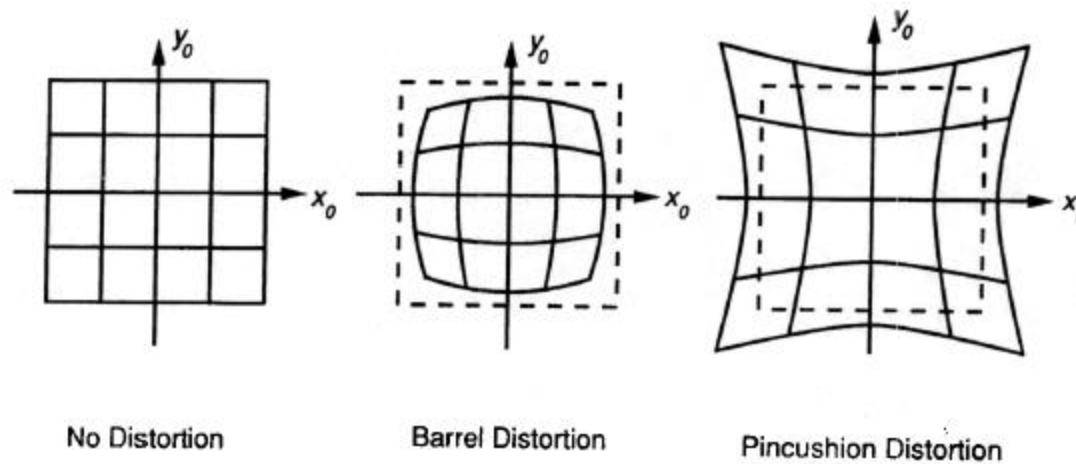
50mm



85mm

From London and Upton

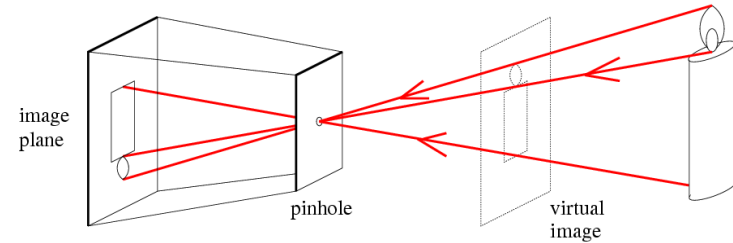
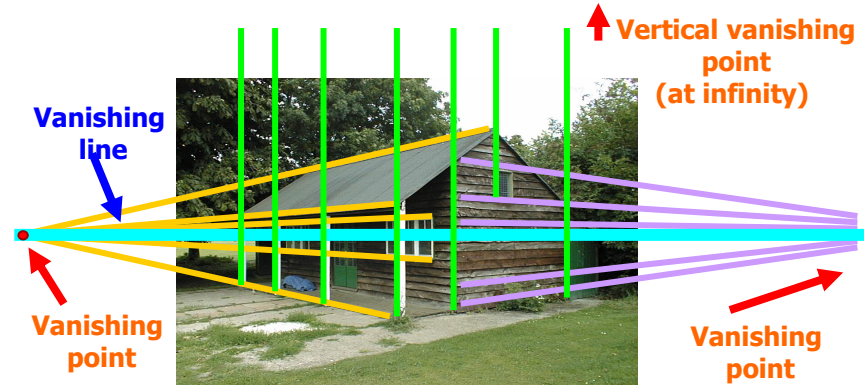
Beyond Pinholes: Radial Distortion



Corrected Barrel Distortion

Things to remember

- Vanishing points and vanishing lines
- Pinhole camera model and camera projection matrix
- Homogeneous coordinates



$$\mathbf{x} = \mathbf{K}[\mathbf{R} \quad \mathbf{t}] \mathbf{X}$$

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Reminder: read your book

- Lectures have assigned readings
- Szeliski 2.1 and especially 2.1.4 cover the geometry of image formation

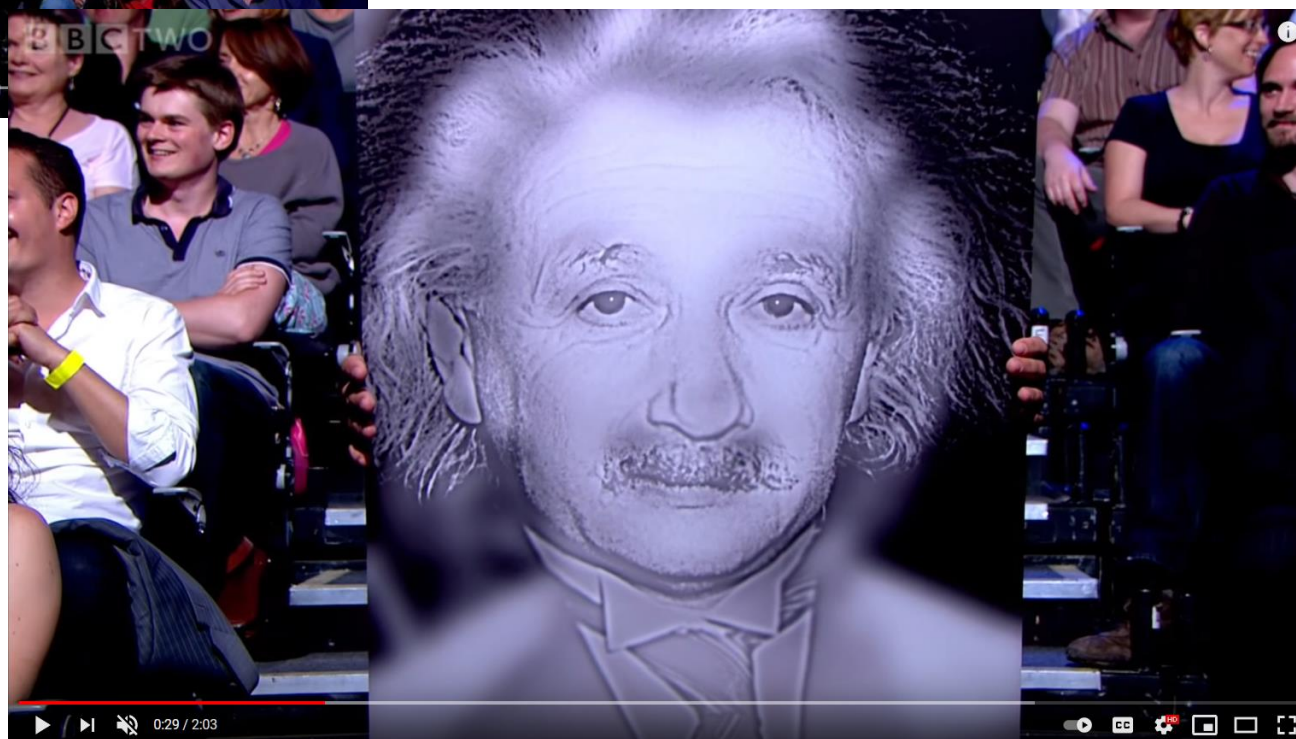
Image Filtering



Computer Vision

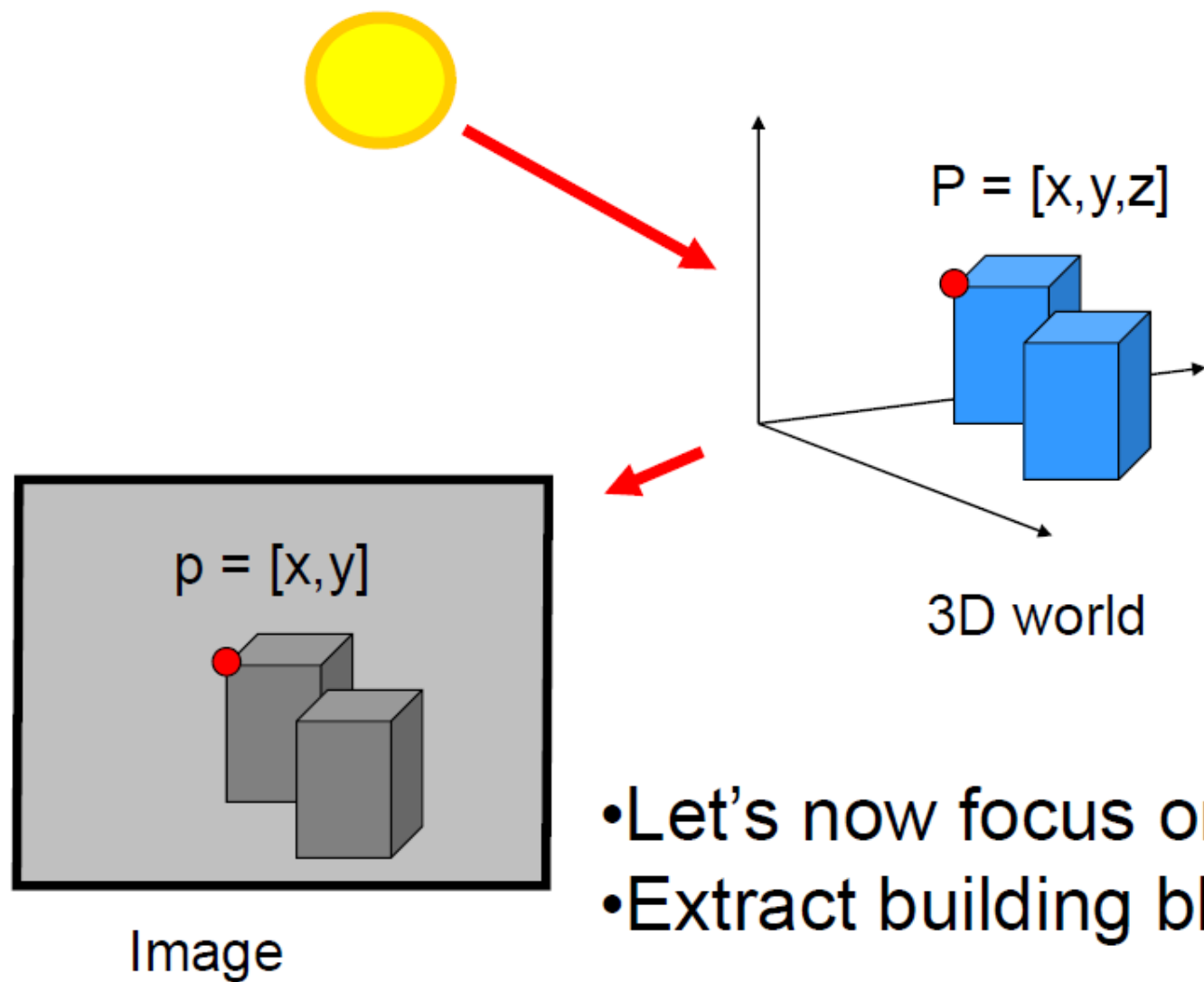
James Hays

Many slides by Derek Hoiem



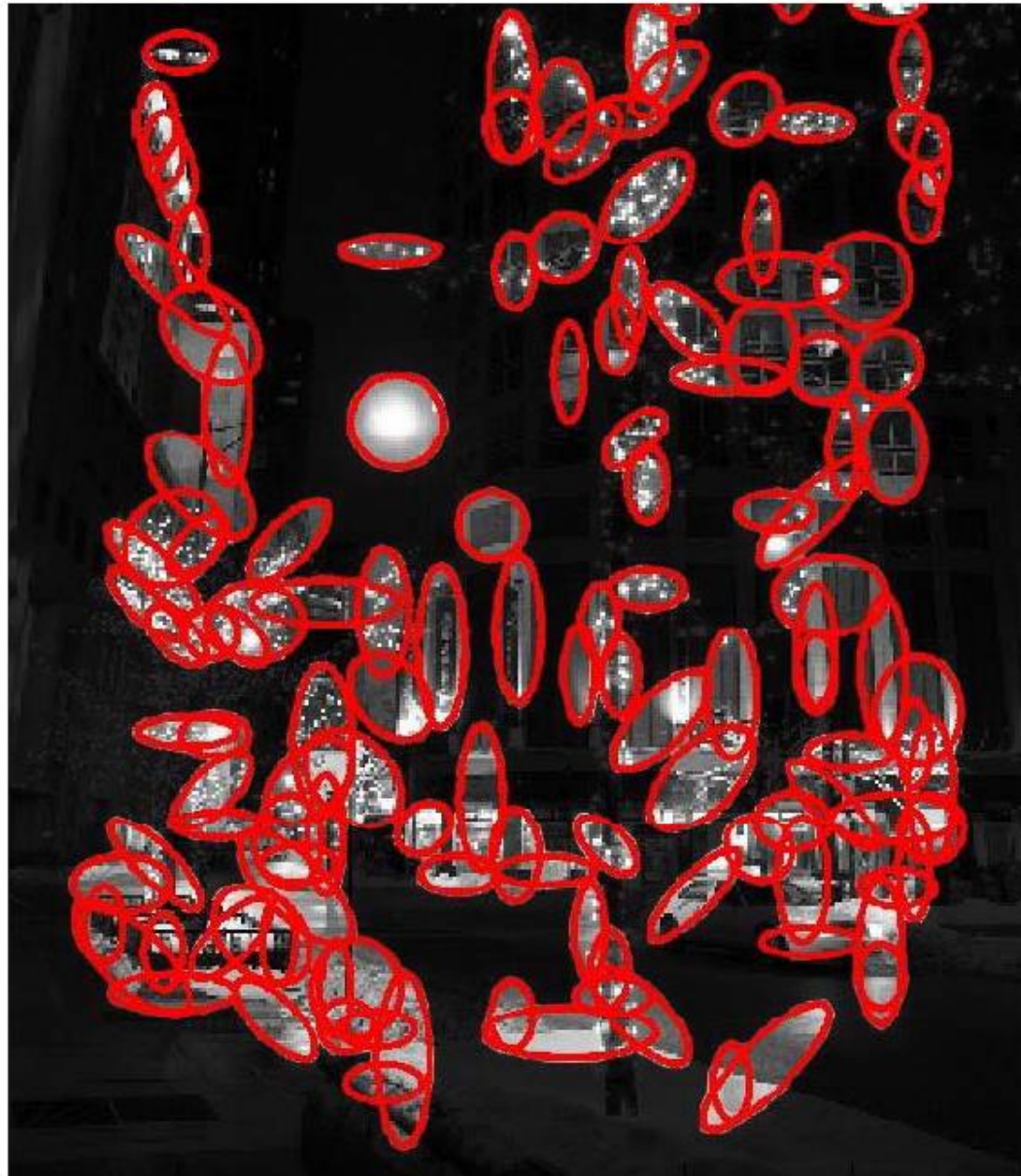
BBC Clip: <https://www.youtube.com/watch/OlumoQ05gS8>

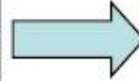
From the 3D to 2D



- Let's now focus on 2D
- Extract building blocks

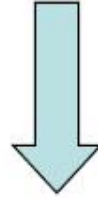
Extract useful building blocks





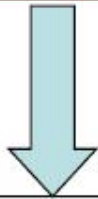
**Feature
Detection**

e.g. DoG



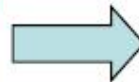
**Feature
Description**

e.g. SIFT

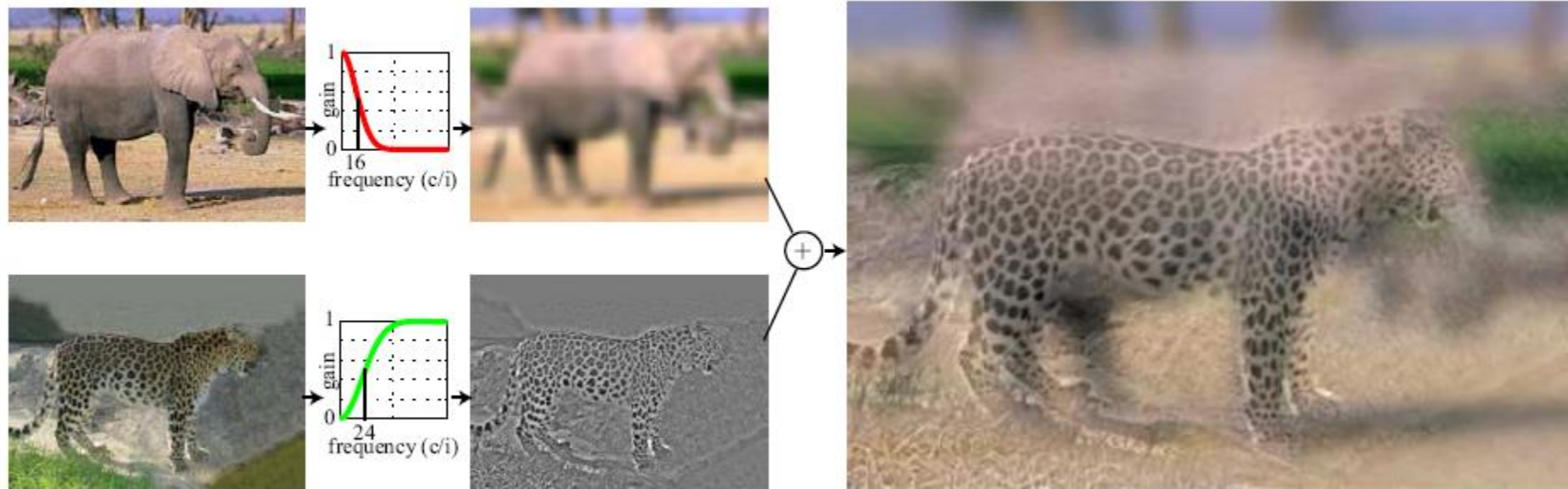


Matching /
Indexing /
Detection

*database of local
descriptors*



Hybrid Images



- A. Oliva, A. Torralba, P.G. Schyns, [“Hybrid Images,”](#) SIGGRAPH 2006

Upcoming classes: two views of filtering

- Image filters in spatial domain
 - Filter is a mathematical operation of a grid of numbers
 - Smoothing, sharpening, measuring texture
- Image filters in the frequency domain
 - Filtering is a way to modify the frequencies of images
 - Denoising, sampling, image compression

Image filtering (or convolution)

- Image filtering: compute function of local neighborhood at each position
- Really important!
 - Enhance images
 - Denoise, resize, increase contrast, etc.
 - Extract information from images
 - Texture, edges, distinctive points, etc.
 - Detect patterns
 - Template matching
 - Deep Convolutional Networks

Example: box filter

$g[\cdot, \cdot]$

	1	1	1
1	1	1	1
9	1	1	1

Image filtering

$$g[\cdot, \cdot] \frac{1}{9}$$

1	1	1
1	1	1
1	1	1

$f[\cdot, \cdot]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[\cdot, \cdot]$

$$h[m, n] = \sum_{k, l} g[k, l] f[m + k, n + l]$$

Image filtering

$$g[\cdot, \cdot] \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$f[\cdot, \cdot]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[\cdot, \cdot]$

	0	10							

$$h[m, n] = \sum_{k, l} g[k, l] f[m + k, n + l]$$

Image filtering

$$g[\cdot, \cdot] \frac{1}{9}$$

1	1	1
1	1	1
1	1	1

$f[\cdot, \cdot]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[\cdot, \cdot]$

	0	10	20						

$$h[m, n] = \sum_{k, l} g[k, l] f[m + k, n + l]$$

Image filtering

$$g[\cdot, \cdot] \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$f[\cdot, \cdot]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[\cdot, \cdot]$

	0	10	20	30					

$$h[m, n] = \sum_{k, l} g[k, l] f[m + k, n + l]$$

Image filtering

$$g[\cdot, \cdot] \frac{1}{9}$$

1	1	1
1	1	1
1	1	1

$f[\cdot, \cdot]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[\cdot, \cdot]$

	0	10	20	30	30				

$$h[m, n] = \sum_{k, l} g[k, l] f[m + k, n + l]$$

Image filtering

$$g[\cdot, \cdot] \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$f[\cdot, \cdot]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[\cdot, \cdot]$

	0	10	20	30	30				
							?		
					50				

$$h[m, n] = \sum_{k, l} g[k, l] f[m + k, n + l]$$

Image filtering

$$g[\cdot, \cdot] = \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$f[\cdot, \cdot]$$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$h[\cdot, \cdot]$$

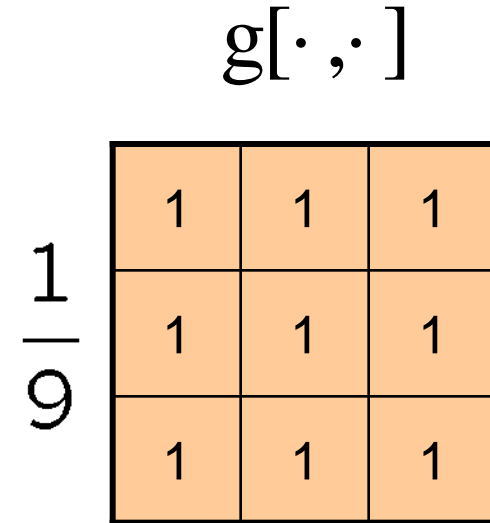
	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

$$h[m, n] = \sum_{k, l} g[k, l] f[m + k, n + l]$$

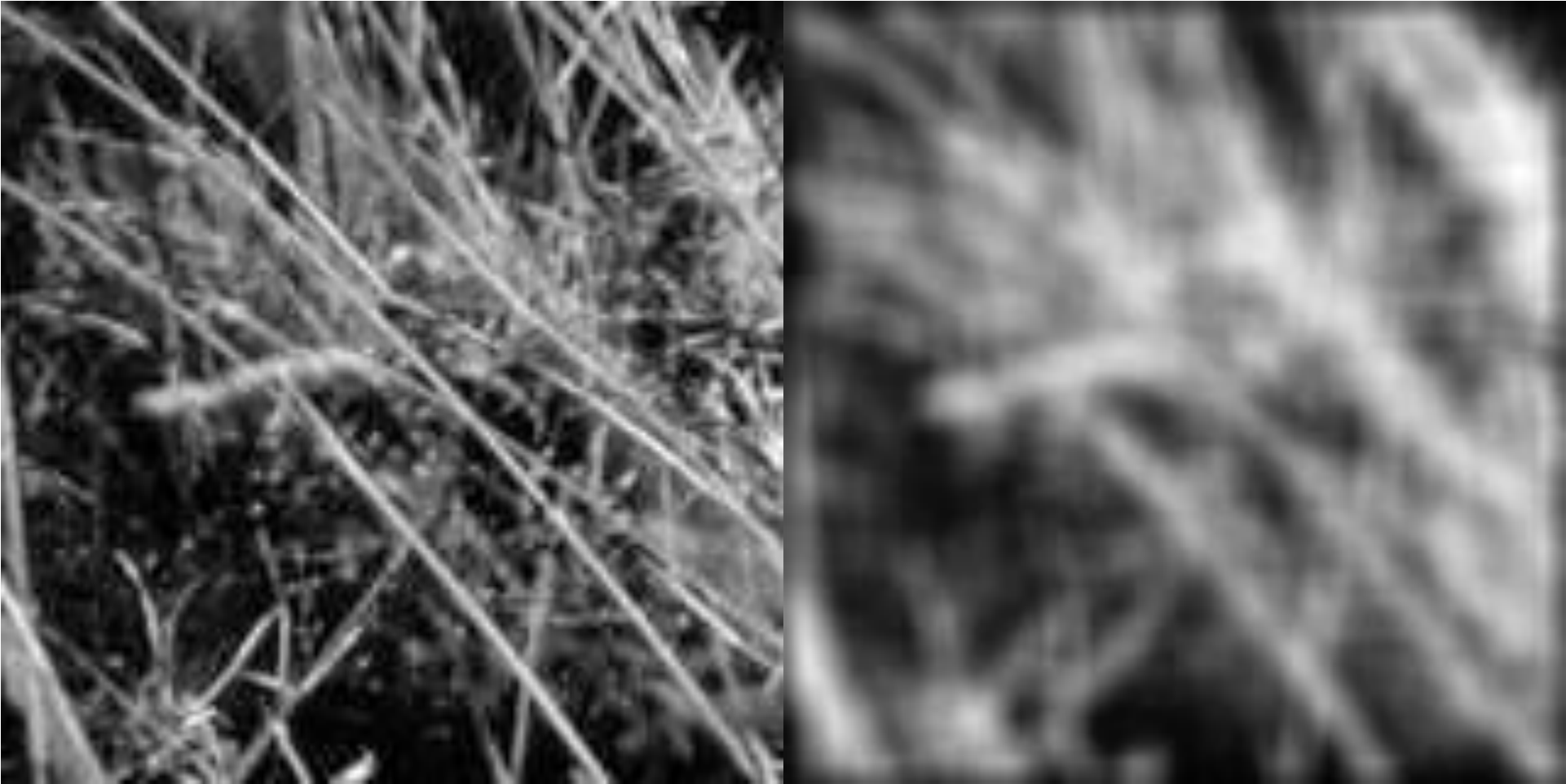
Box Filter

What does it do?

- Replaces each pixel with an average of its neighborhood
- Achieve smoothing effect (remove sharp features)



Smoothing with box filter



Practice with linear filters



Original

0	0	0
0	1	0
0	0	0

?

Practice with linear filters



Original

0	0	0
0	1	0
0	0	0



Filtered
(no change)

Practice with linear filters



Original

0	0	0
0	0	1
0	0	0

?

Practice with linear filters



Original

0	0	0
0	0	1
0	0	0



Shifted left
By 1 pixel

Practice with linear filters



Original

0	0	0
0	2	0
0	0	0

-

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

?

(Note that filter sums to 1)

Practice with linear filters



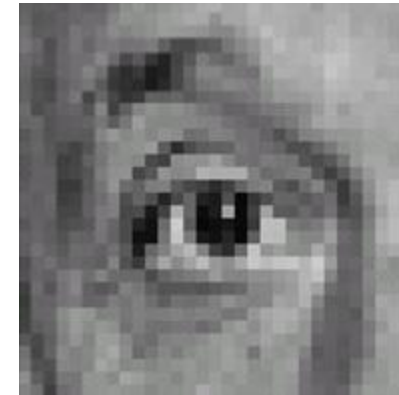
Original

0	0	0
0	2	0
0	0	0

−

$\frac{1}{9}$

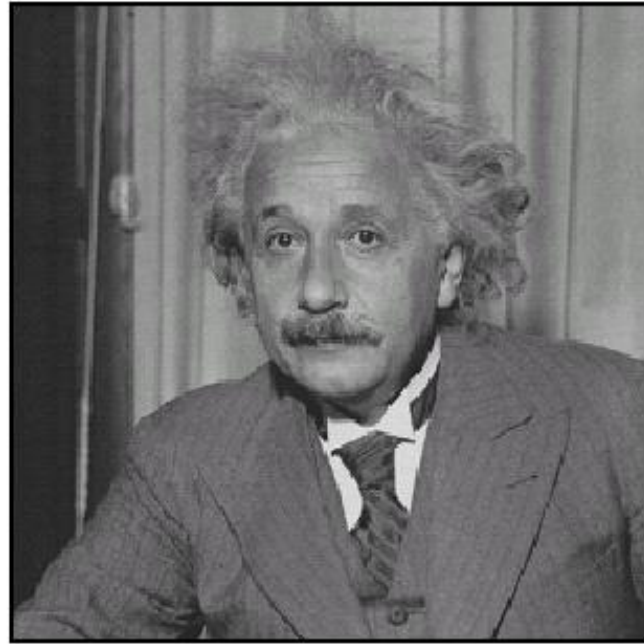
1	1	1
1	1	1
1	1	1



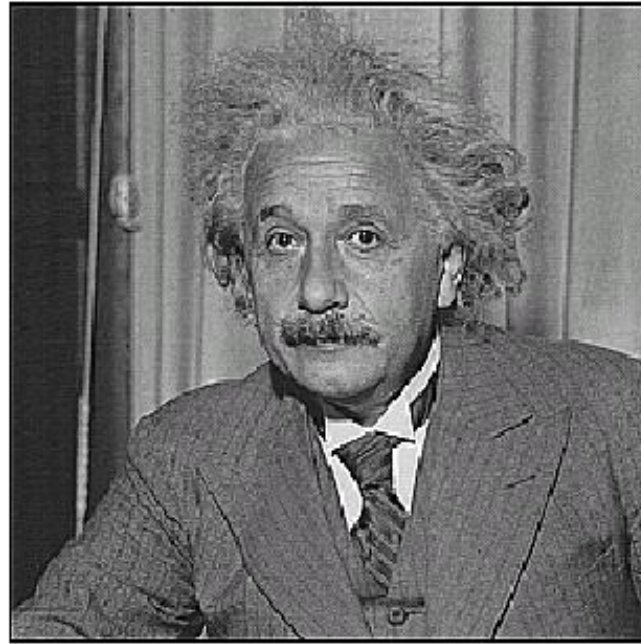
Sharpening filter

- Accentuates differences with local average

Sharpening

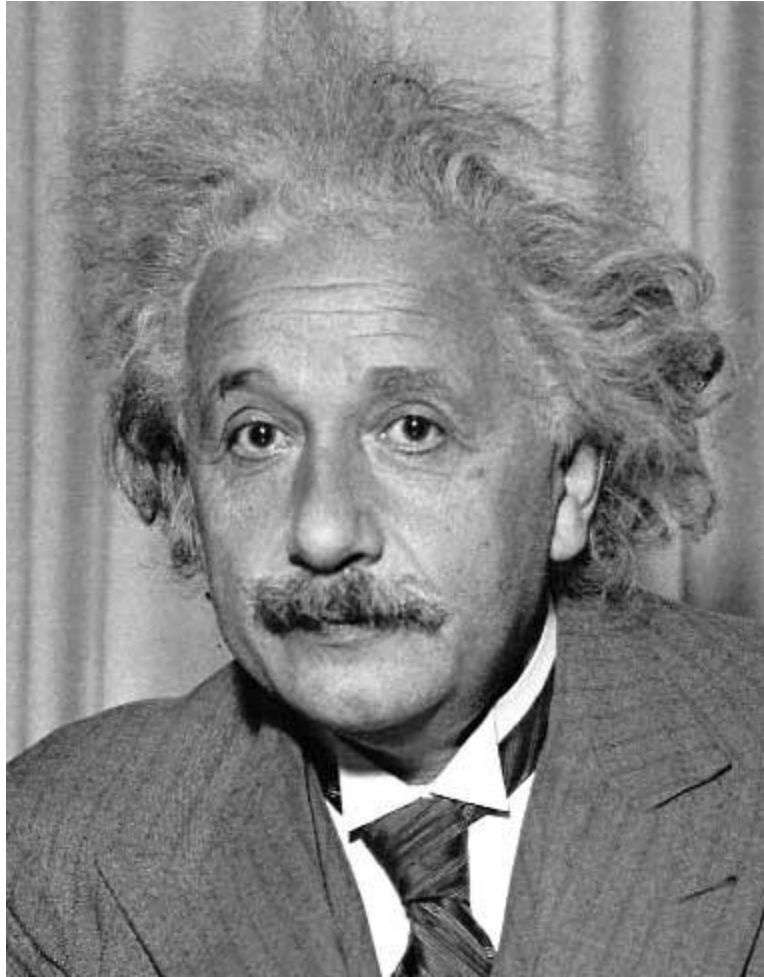


before



after

Other filters



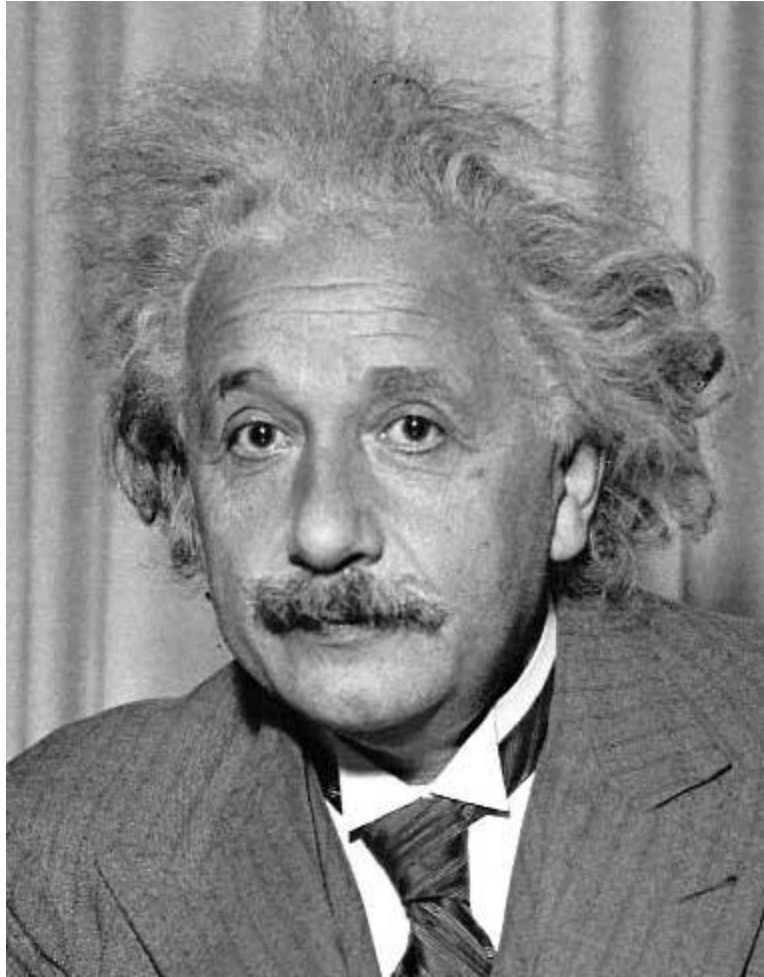
1	0	-1
2	0	-2
1	0	-1

Sobel



Vertical Edge
(absolute value)

Other filters



1	2	1
0	0	0
-1	-2	-1

Sobel



Horizontal Edge
(absolute value)

Next class: Light and Color and Thinking in Frequency

