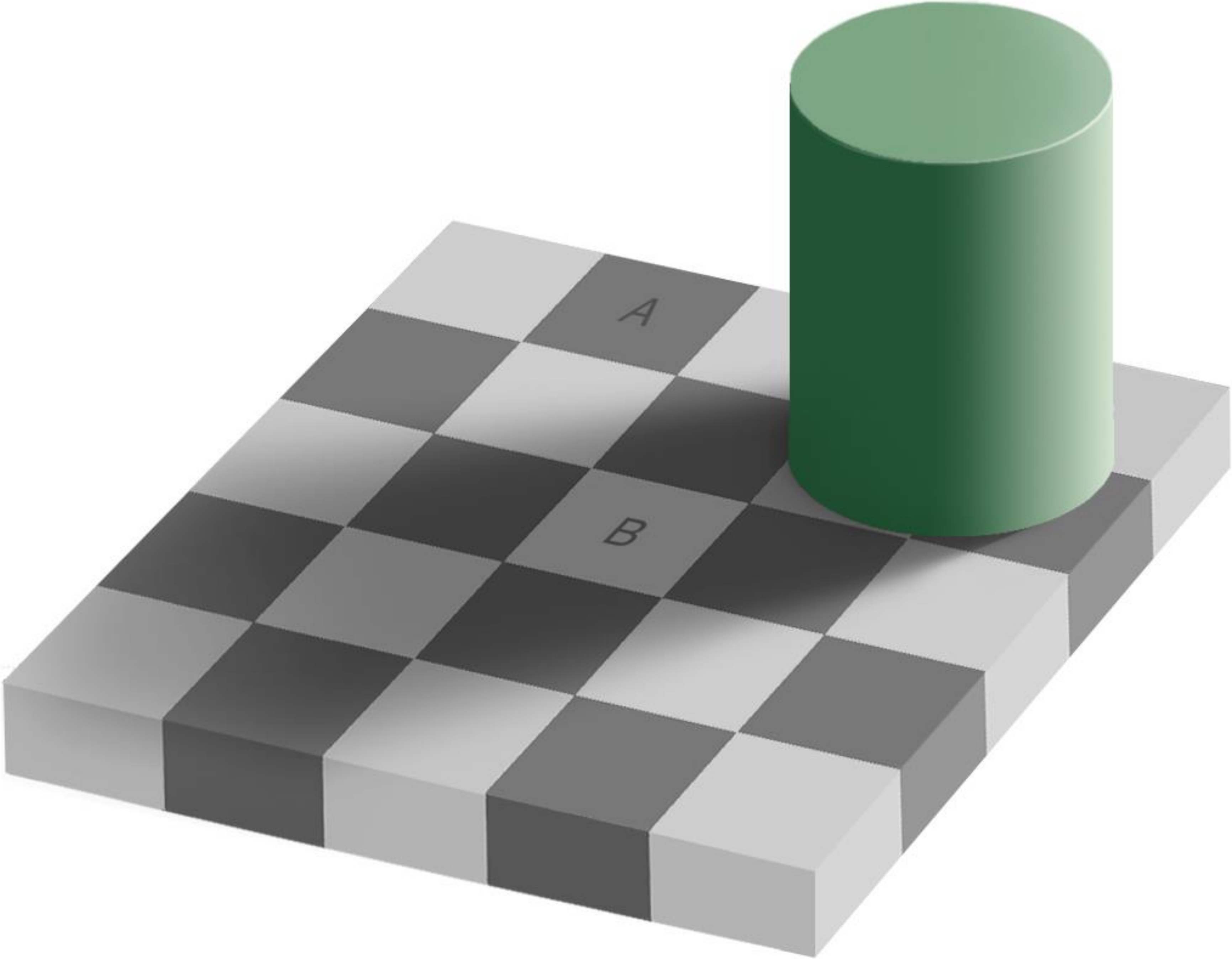
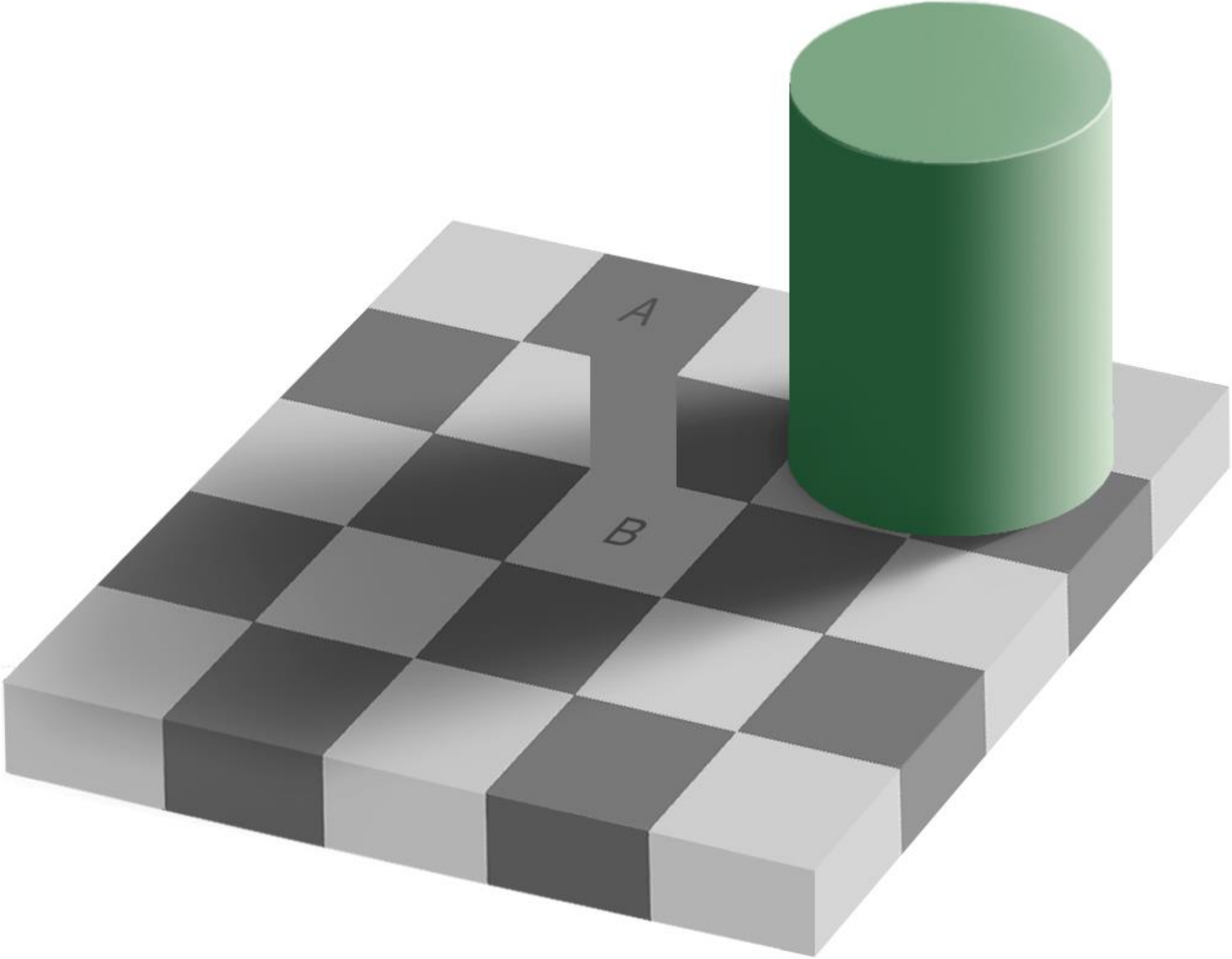
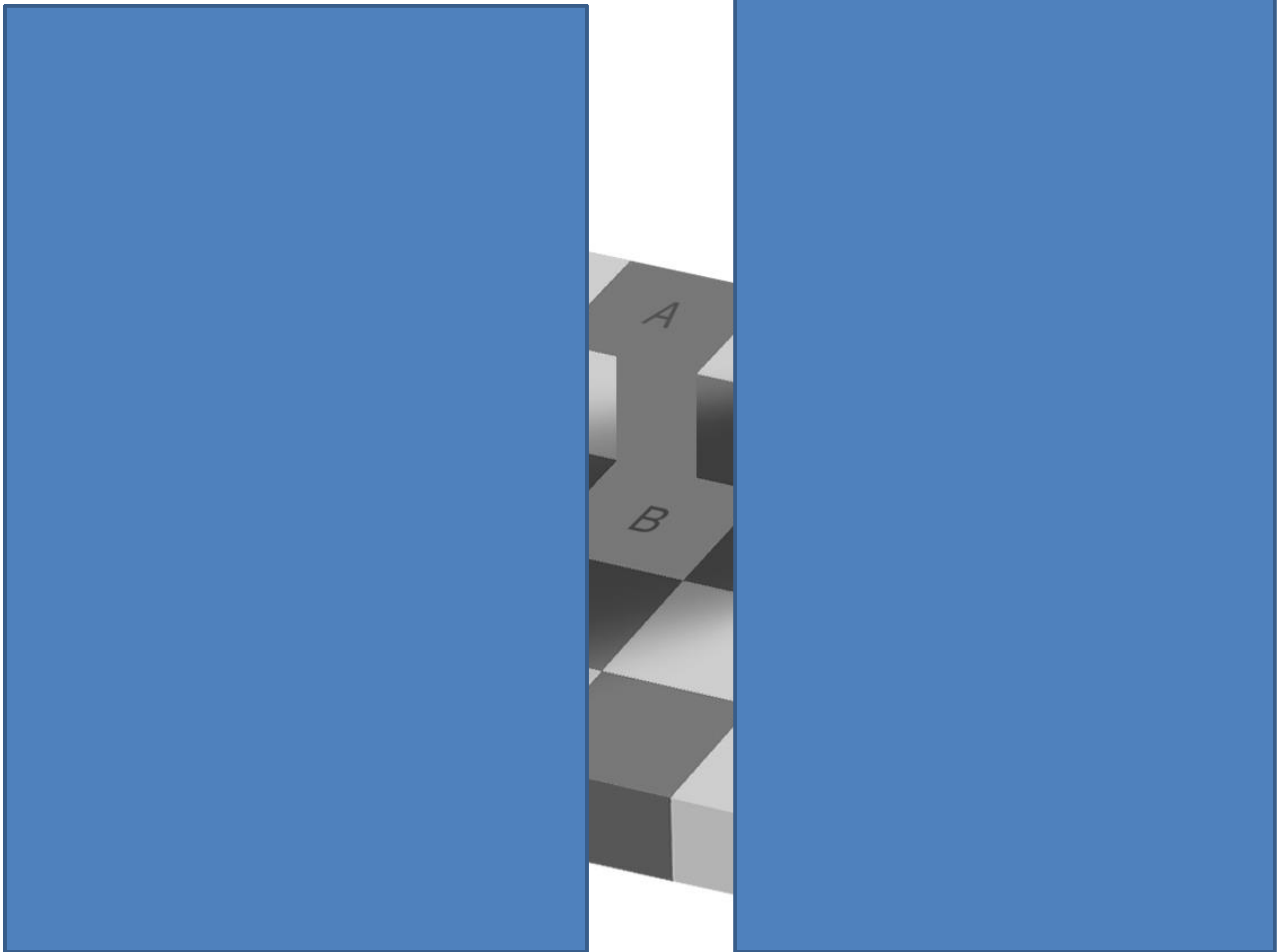
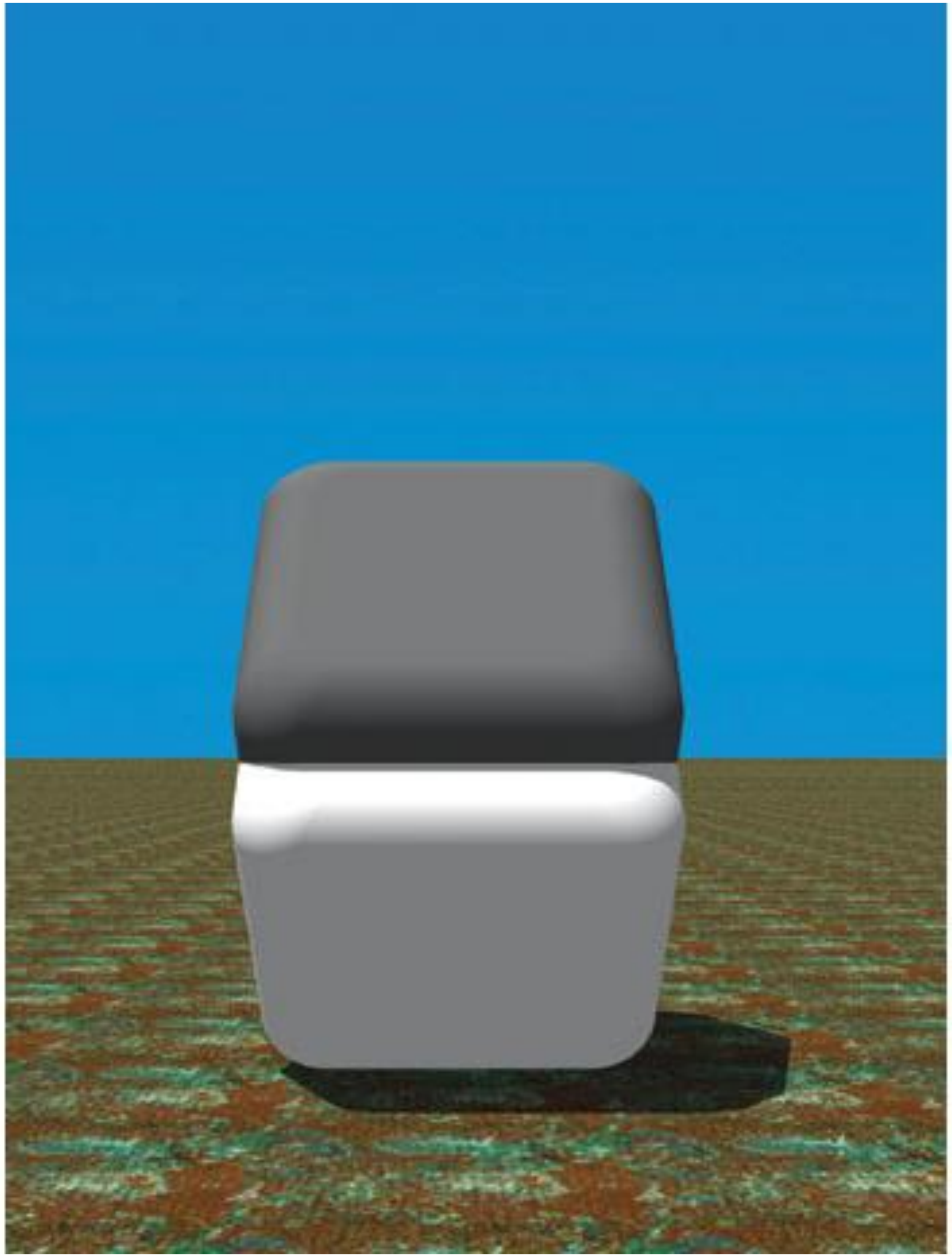

A
B


A
B


A
B



RANSAC, ICP, Fitting and Alignment

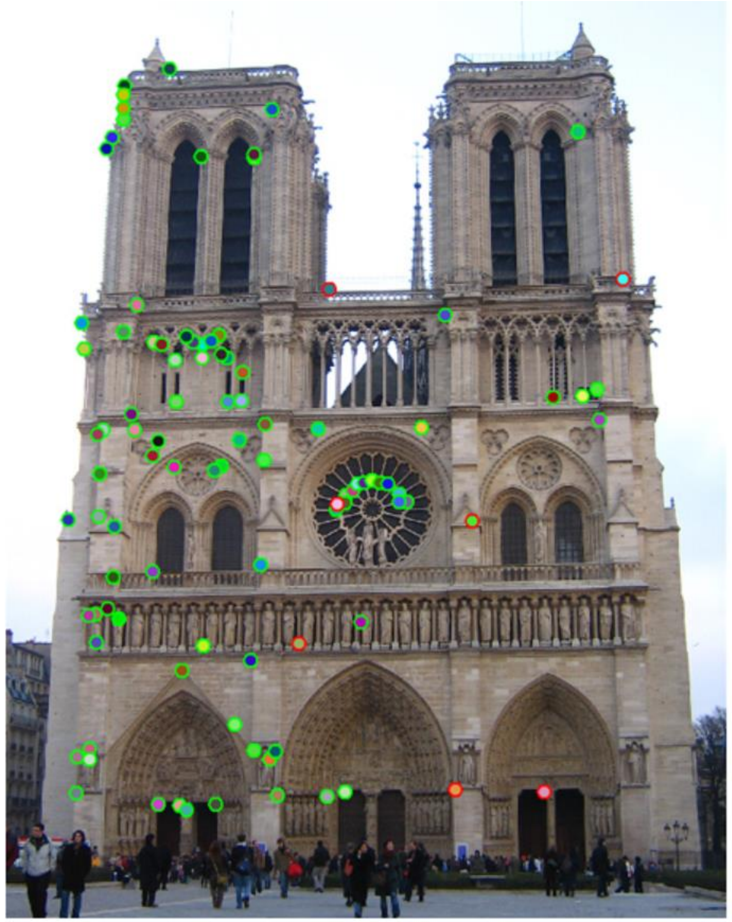
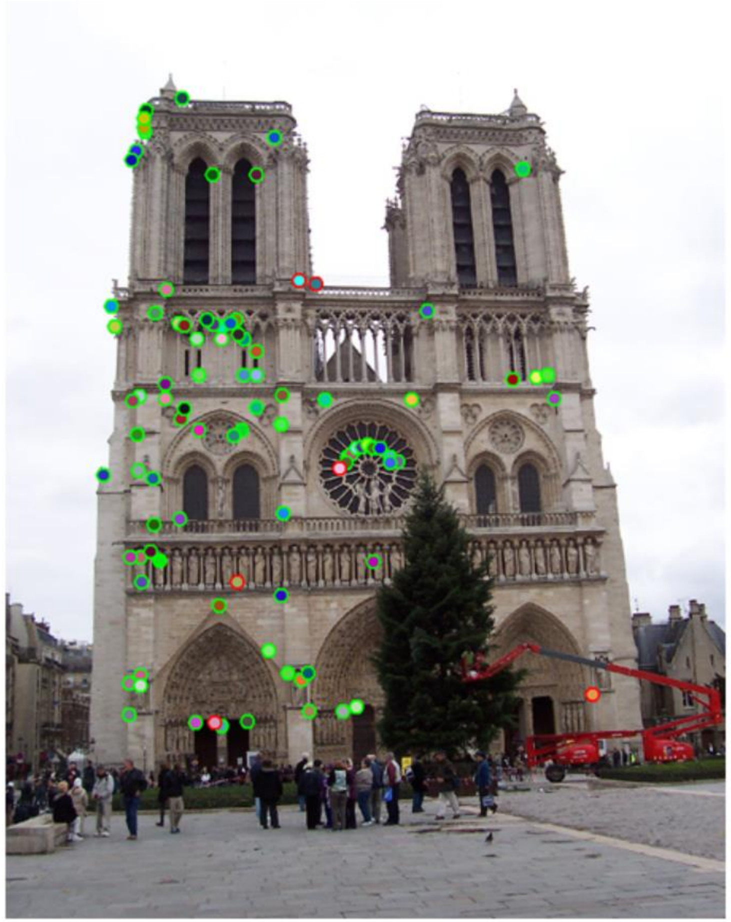
Computer Vision

Szeliski 2.1 and 8.1

James Hays

Acknowledgment: Many slides from Derek Hoiem, Lana Lazebnik, and Grauman&Leibe 2008 AAAI Tutorial

Project 2

The top 100 most confident local feature matches from a baseline implementation of project 2. In this case, 93 were correct (highlighted in green) and 7 were incorrect (highlighted in red).

Project 2: Local Feature Matching

Fitting and Alignment: Methods

- ~~Global optimization / Search for parameters~~
  - ~~Least squares fit~~
  - ~~Robust least squares~~
  - ~~Other parameter search methods~~
- ~~Hypothesize and test~~
  - ~~Hough transform~~
  - RANSAC
- Iterative Closest Points (ICP)

Review: Hough Transform

1. Create a grid of parameter values

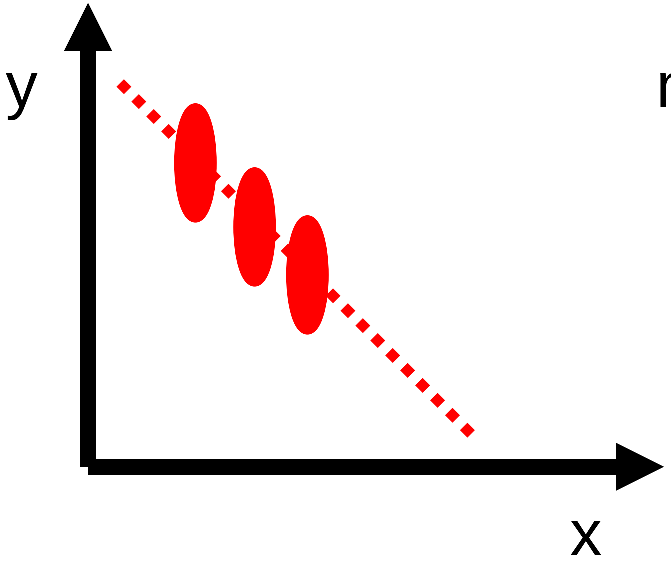
2. Each point (or correspondence) votes for a set of parameters, incrementing those values in grid

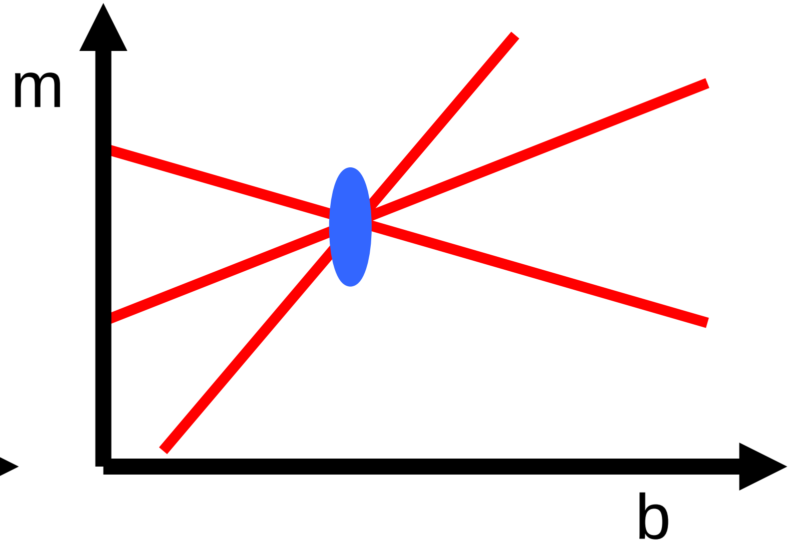
3. Find maximum or local maxima in grid

Review: Hough transform

P.V.C. Hough, *Machine Analysis of Bubble Chamber Pictures,* Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959

Given a set of points, find the curve or line that explains the data points best



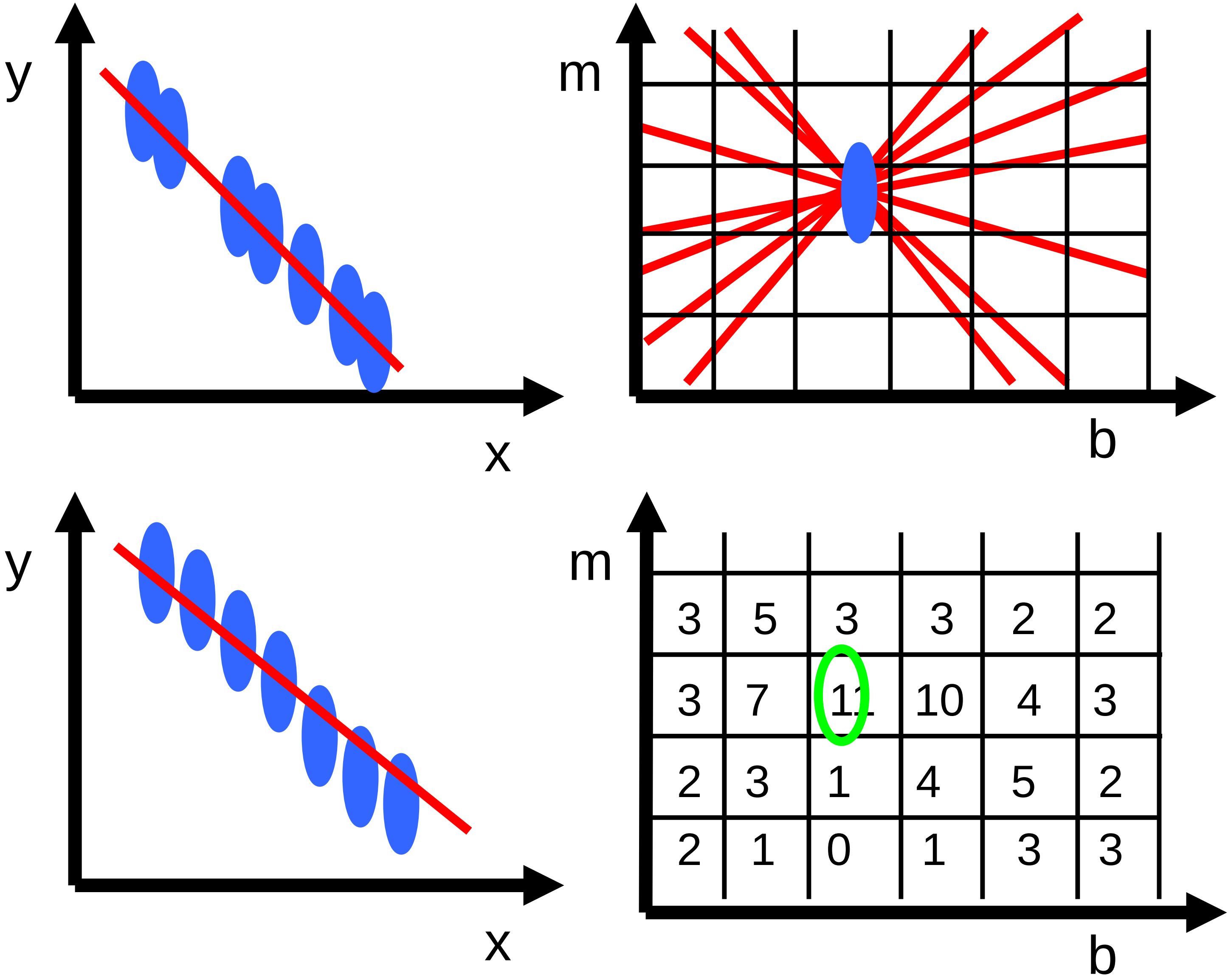


Hough space

$$y = m\,x + b$$

Slide from S. Savarese

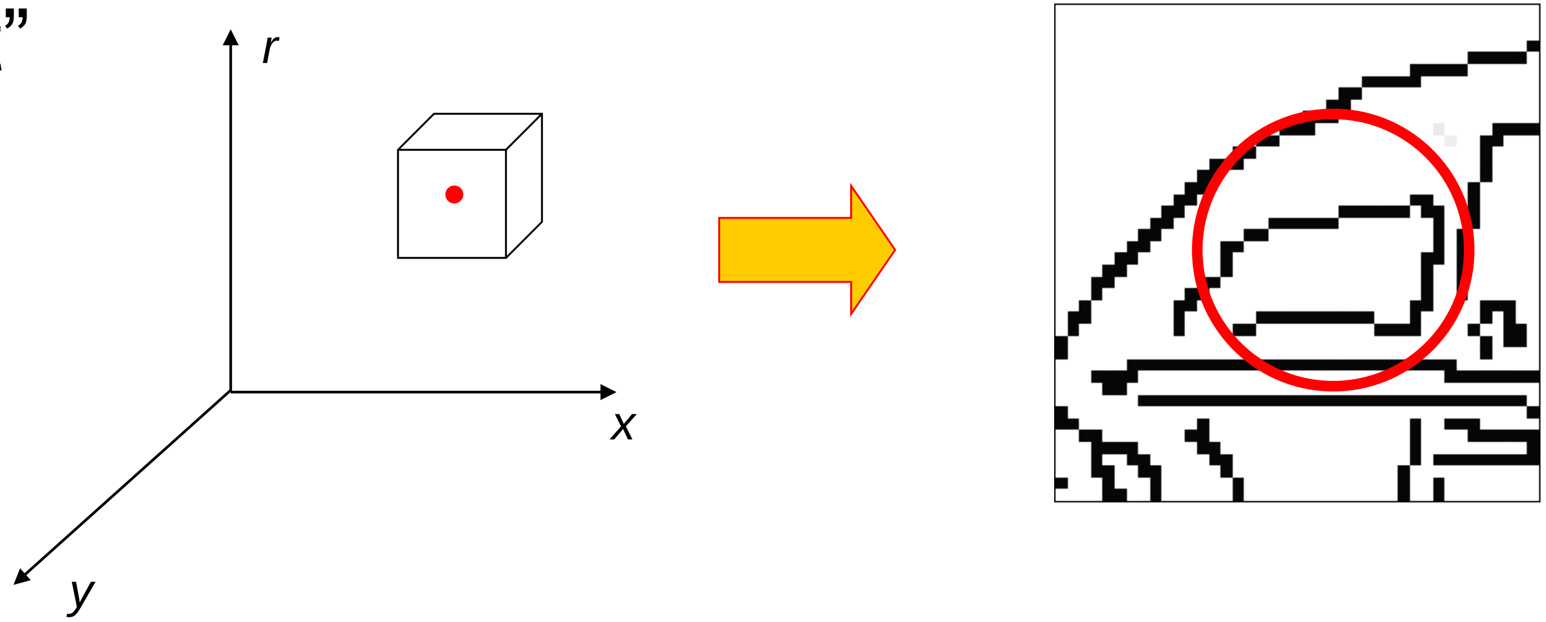
Review: Hough transform



Slide from S. Savarese

Hough transform for circles

- Conceptually equivalent (but maybe less efficient) grid search procedure: for each (x,y,r), draw the corresponding circle in the image and compute its “support”



Is this more or less efficient than voting with features?

Hough Transform

- How would we find circles?
  - Of fixed radius
  - Of unknown radius
  - Of unknown radius but with known edge orientation

Hough transform for circles

- Circle: center (a,b) and radius r

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$

- For an unknown radius r

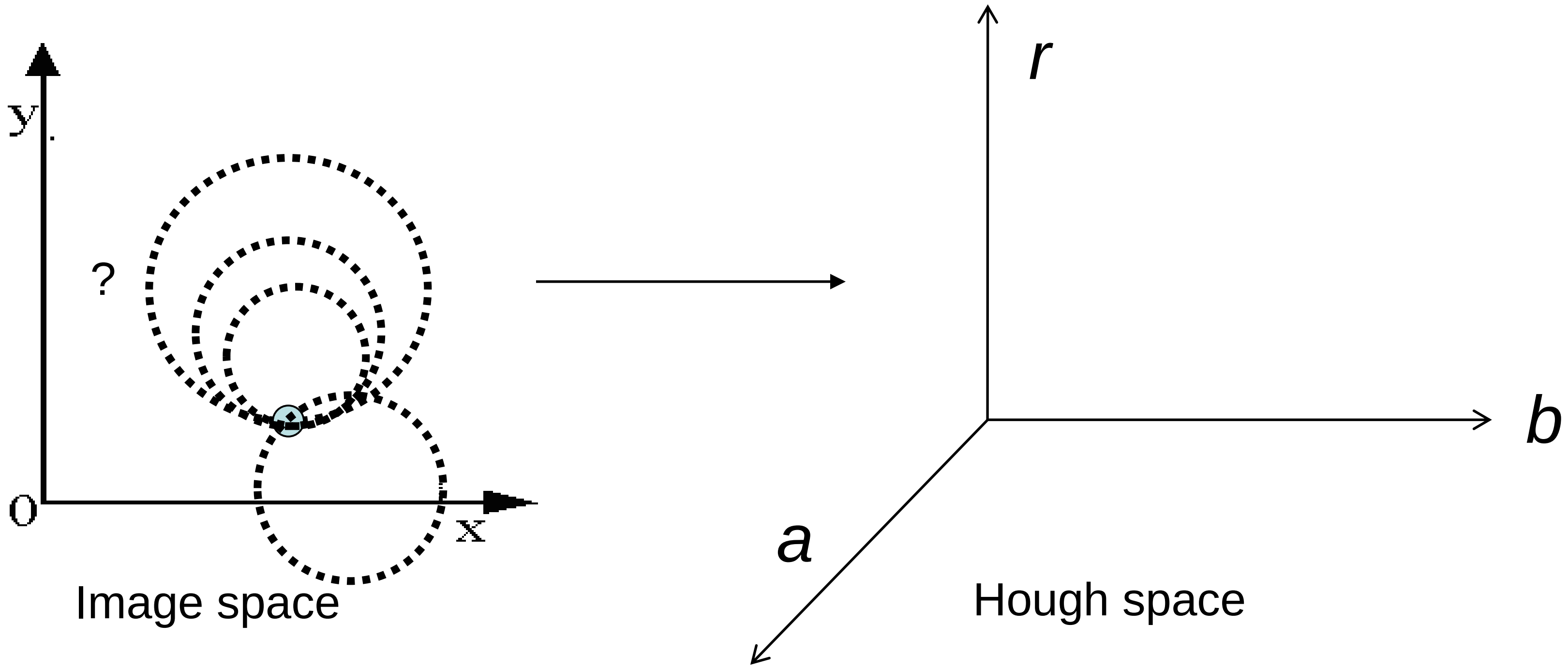


Image space

Hough space

Kristen Grauman

Hough transform for circles

- Circle: center (a,b) and radius r

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$

- For an unknown radius r

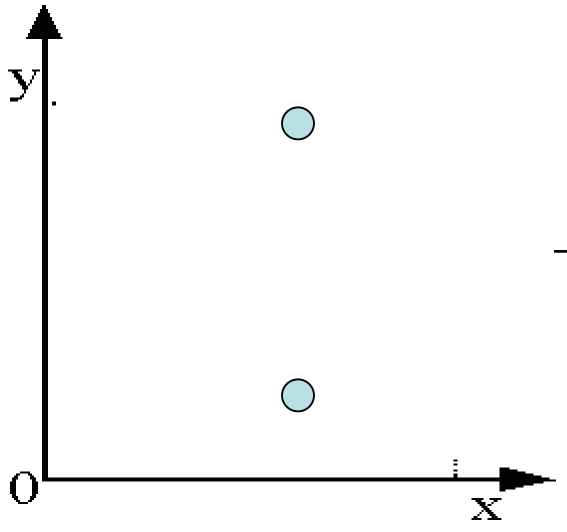
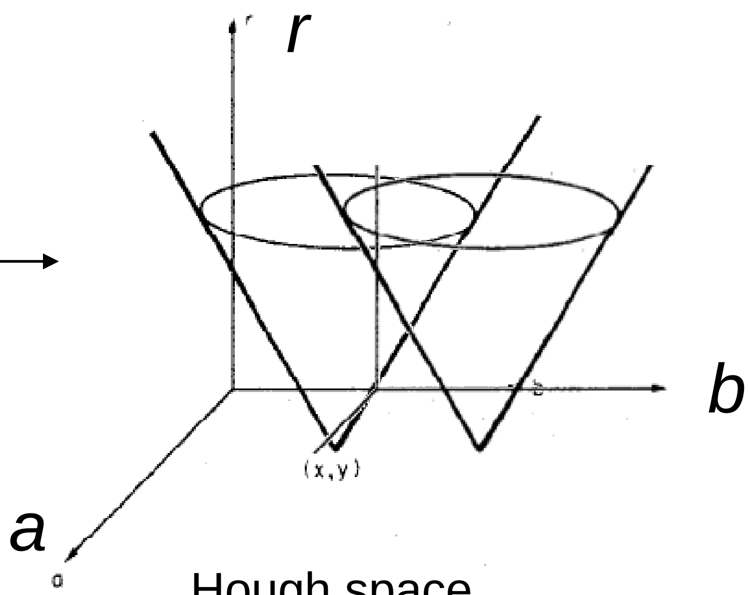


Image space



Hough space

Kristen Grauman

Hough Transform

- How would we find circles?
  - Of fixed radius
  - Of unknown radius
  - Of unknown radius but with known edge orientation

Hough transform for circles

- Circle: center (a,b) and radius r

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$

- For an unknown radius r, **known** gradient direction

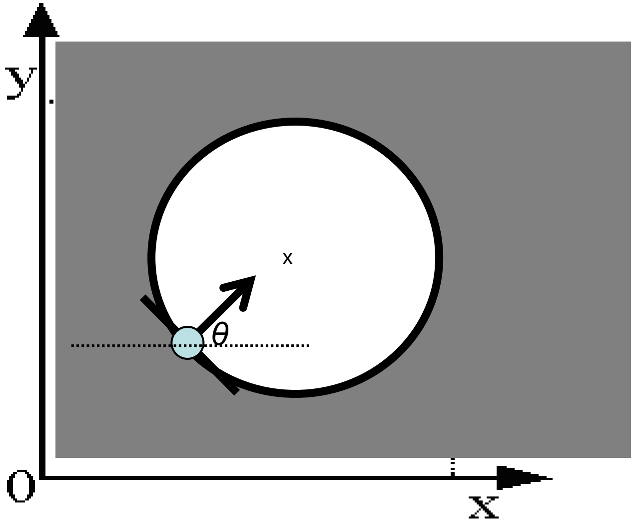
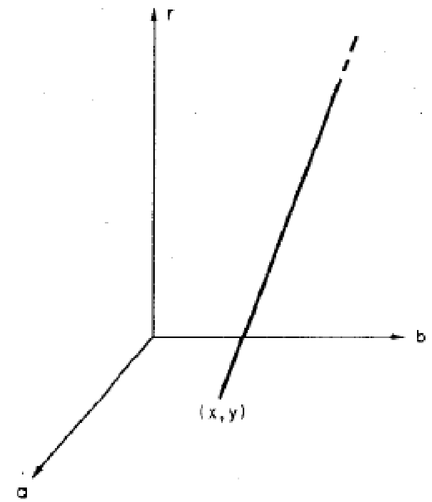


Image space



Hough space

Kristen Grauman

Hough transform for circles

For every edge pixel (x,y) :

For each possible radius value r:

For each possible gradient direction θ:

// or use estimated gradient at (x,y)

$a = x - r\cos(\theta)$ // column

$b = y + r\sin(\theta)$ // row

$H[a,b,r]$ += 1

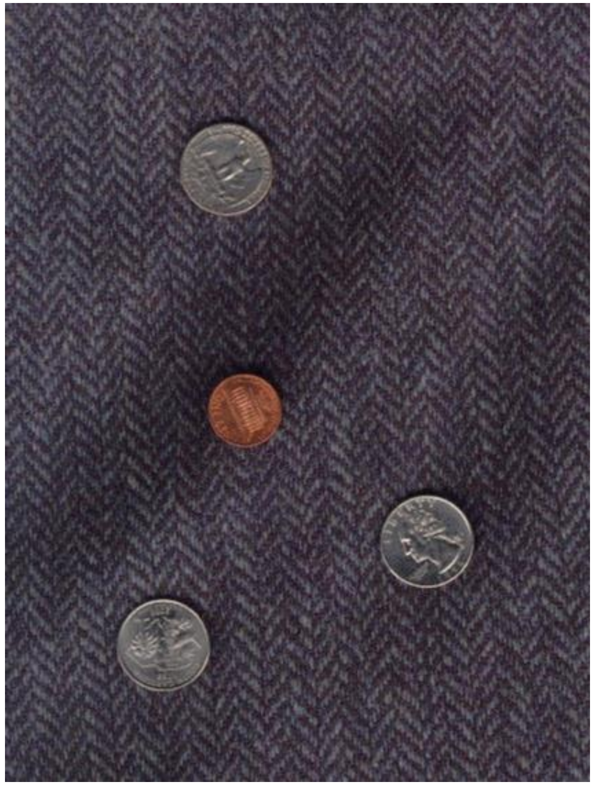
end

end

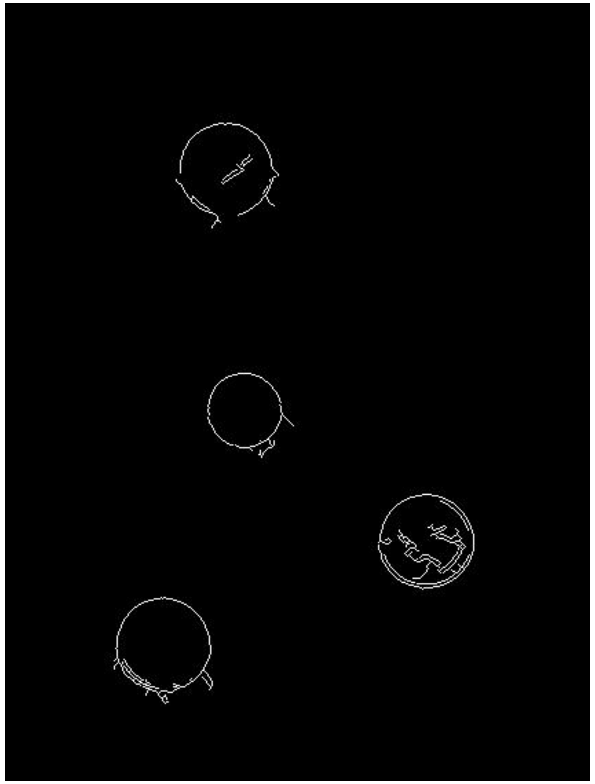
Kristen Grauman

Example: detecting circles with Hough

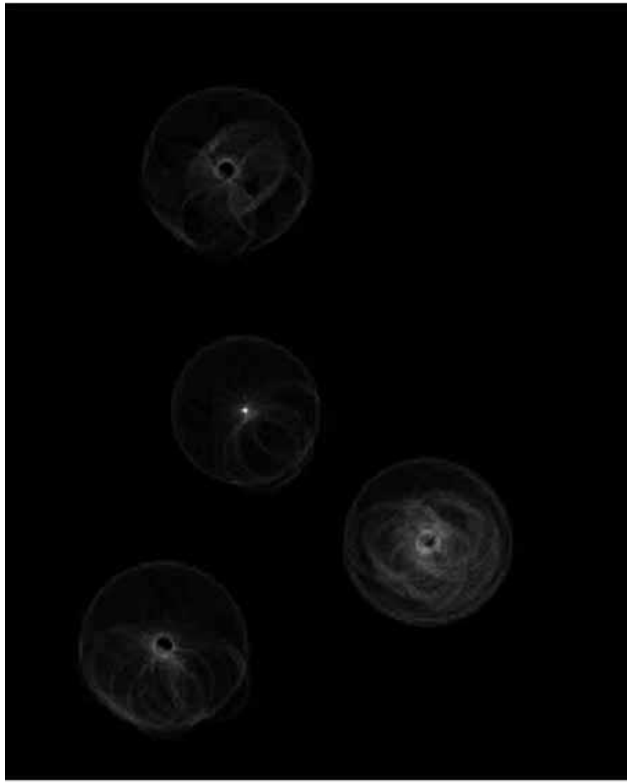
Original



Edges



Votes: Penny



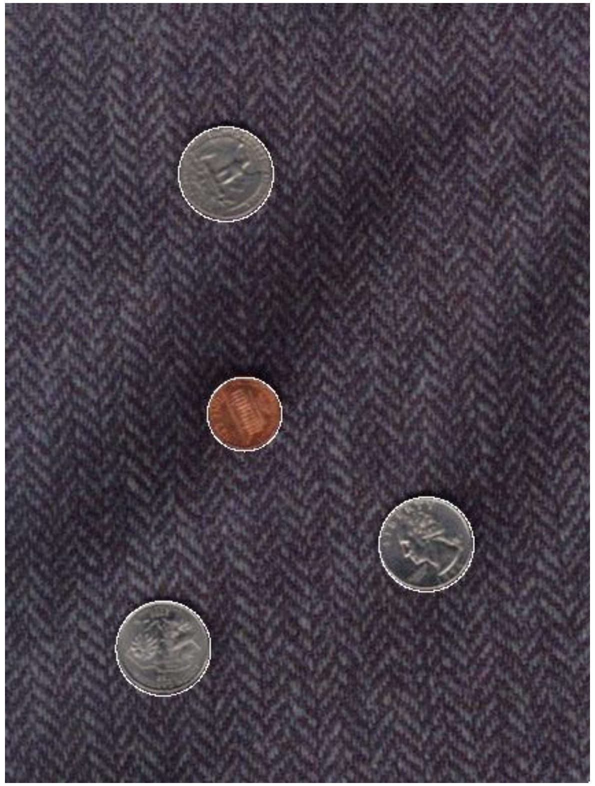
Note: a different Hough transform (with separate accumulators) was used for each circle radius (quarters vs. penny).

Slide credit: Kristen Grauman

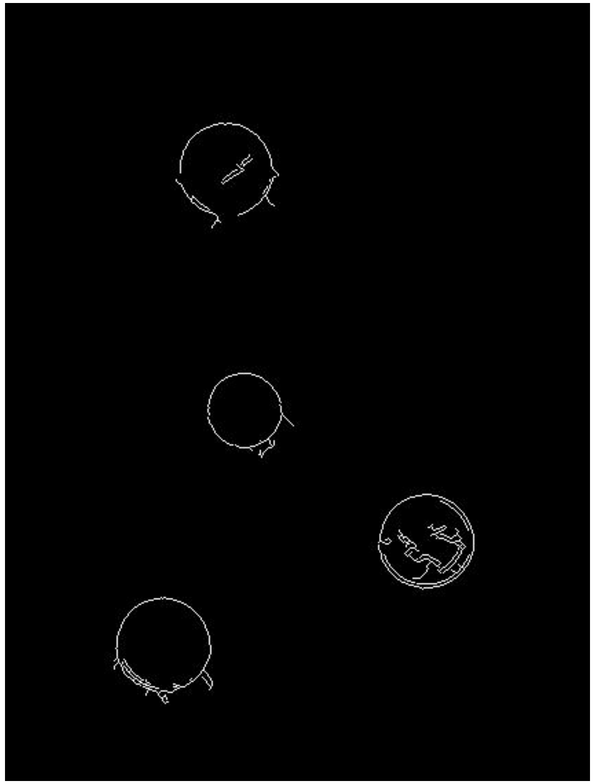
Example: detecting circles with Hough

Combined detections

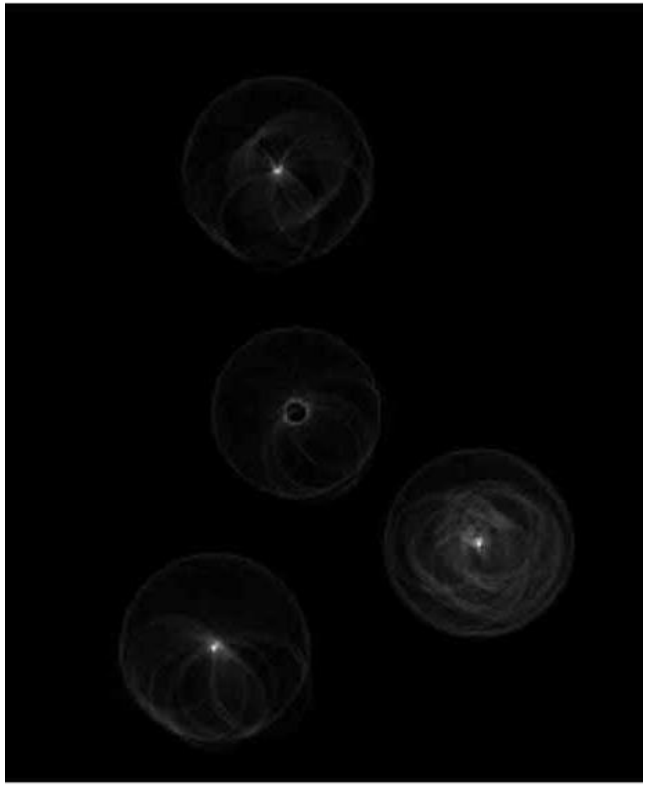
Original



Edges



Votes: Quarter



Slide credit: Kristen Grauman

Coin finding sample images from: Vivek Kwatra

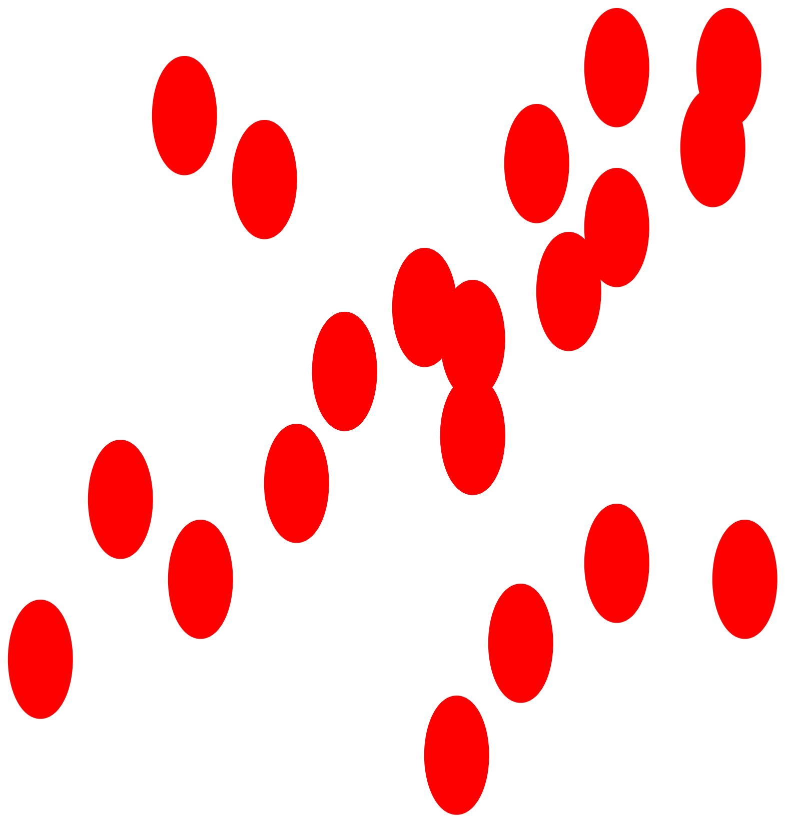
Fitting and Alignment: Methods

- ~~Global optimization / Search for parameters~~
  - ~~Least squares fit~~
  - ~~Robust least squares~~
  - ~~Other parameter search methods~~
- Hypothesize and test
  - ~~Hough transform~~
  - RANSAC
- Iterative Closest Points (ICP)

RANSAC

(RANdom SAmple Consensus) :

Fischler & Bolles in ‘81.



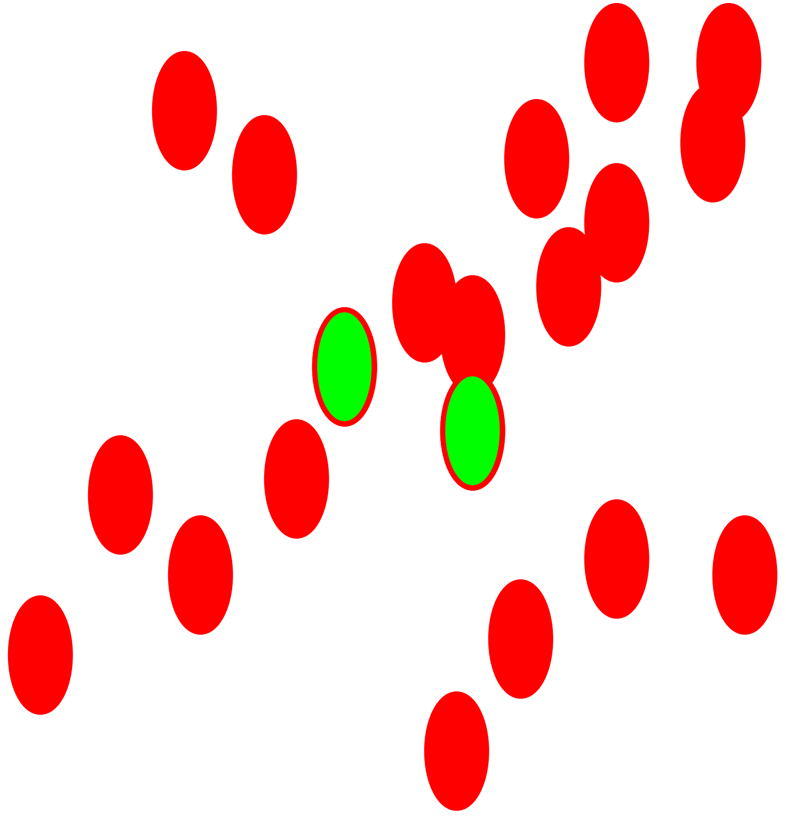
Algorithm:

1. **Sample** (randomly) the number of points required to fit the model
2. **Solve** for model parameters using samples
3. **Score** by the fraction of inliers within a preset threshold of the model

Repeat 1-3 until the best model is found with high confidence

RANSAC

Line fitting example



Algorithm:

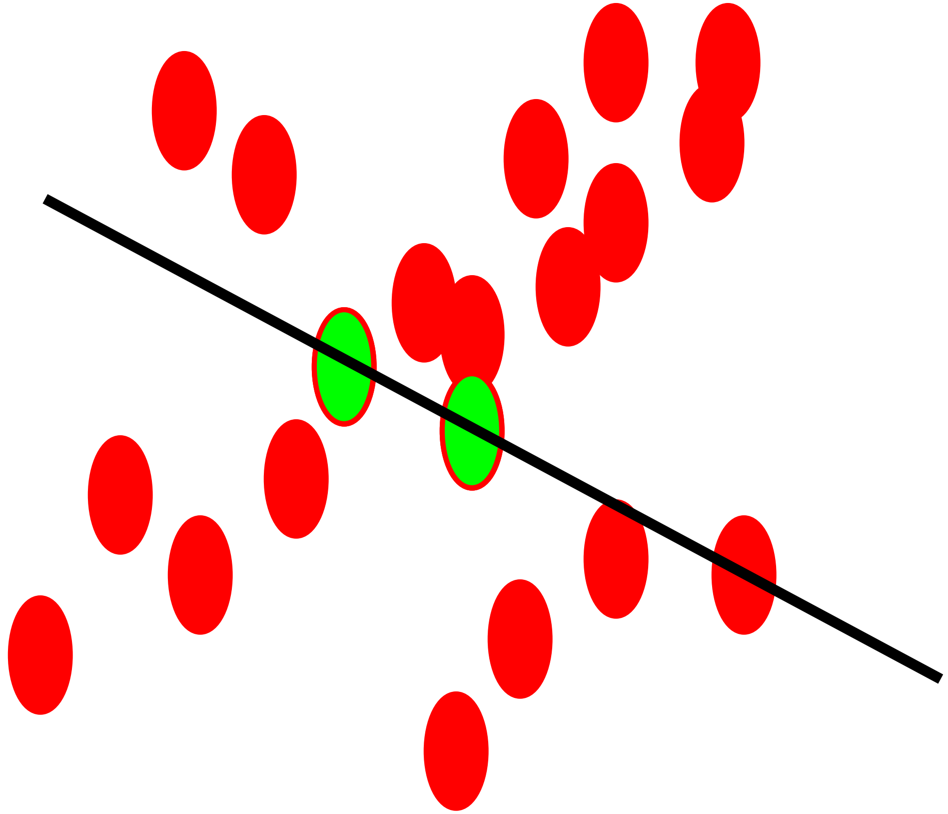
1. **Sample** (randomly) the number of points required to fit the model (#=2)
2. **Solve** for model parameters using samples
3. **Score** by the fraction of inliers within a preset threshold of the model

Repeat 1-3 until the best model is found with high confidence

Illustration by Savarese

RANSAC

Line fitting example



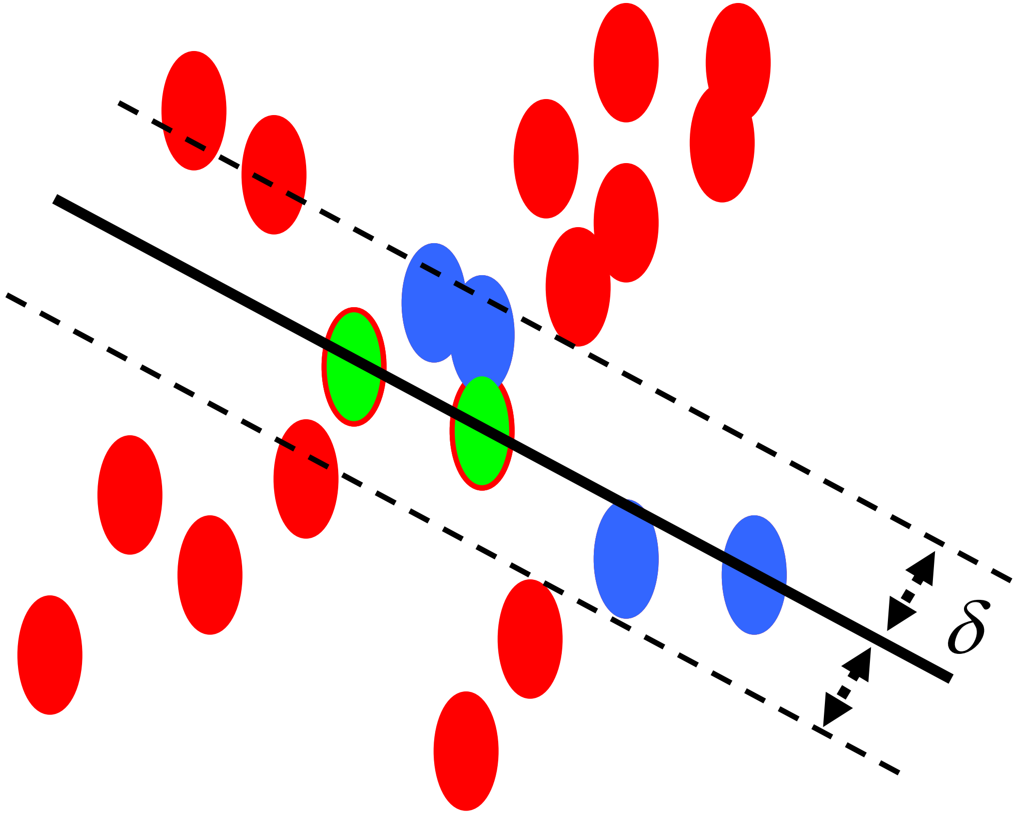
Algorithm:

1. **Sample** (randomly) the number of points required to fit the model (#=2)
2. **Solve** for model parameters using samples
3. **Score** by the fraction of inliers within a preset threshold of the model

Repeat 1-3 until the best model is found with high confidence

RANSAC

Line fitting example



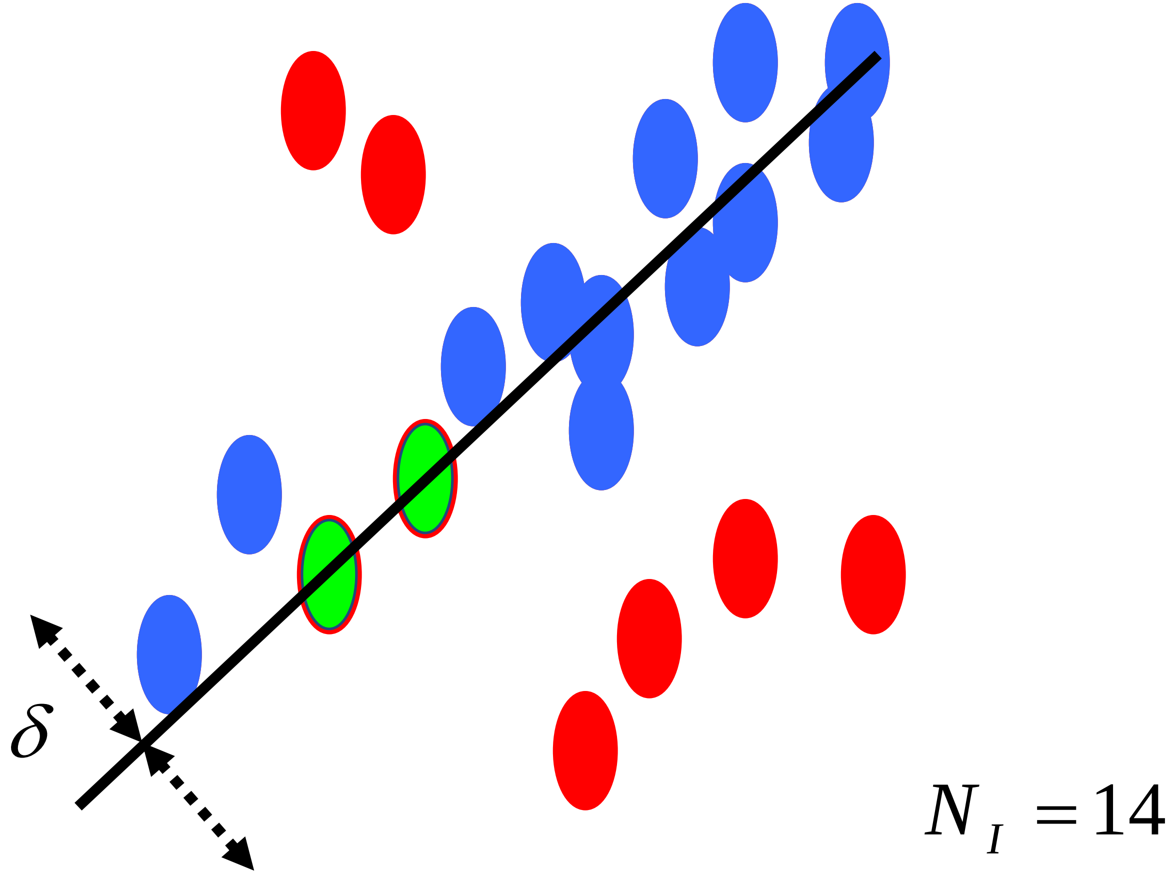
$$N_I = 6$$

Algorithm:

1. **Sample** (randomly) the number of points required to fit the model (#=2)
2. **Solve** for model parameters using samples
3. **Score** by the fraction of inliers within a preset threshold of the model

Repeat 1-3 until the best model is found with high confidence

RANSAC



Algorithm:

1. **Sample** (randomly) the number of points required to fit the model (#=2)
2. **Solve** for model parameters using samples
3. **Score** by the fraction of inliers within a preset threshold of the model

Repeat 1-3 until the best model is found with high confidence

How to choose parameters?

- Number of samples *N*
  - Choose *N* so that, with probability *p*, at least one random sample is free from outliers (e.g. *p*=0.99) (outlier ratio: *e*)
- Number of sampled points *s*
  - Minimum number needed to fit the model
- Distance threshold δ
  - Choose δ so that a good point with noise is likely (e.g., prob=0.95) within threshold

$$\boxed{N = \log(1-p) / \log\left(1-(1-e)^s\right)}$$

	proportion of outliers *e*						
s	5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

For p = 0.99

modified from M. Pollefeys

RANSAC conclusions

Good

- Robust to outliers
- Applicable for larger number of model parameters than Hough transform
- Optimization parameters are easier to choose than Hough transform

Bad

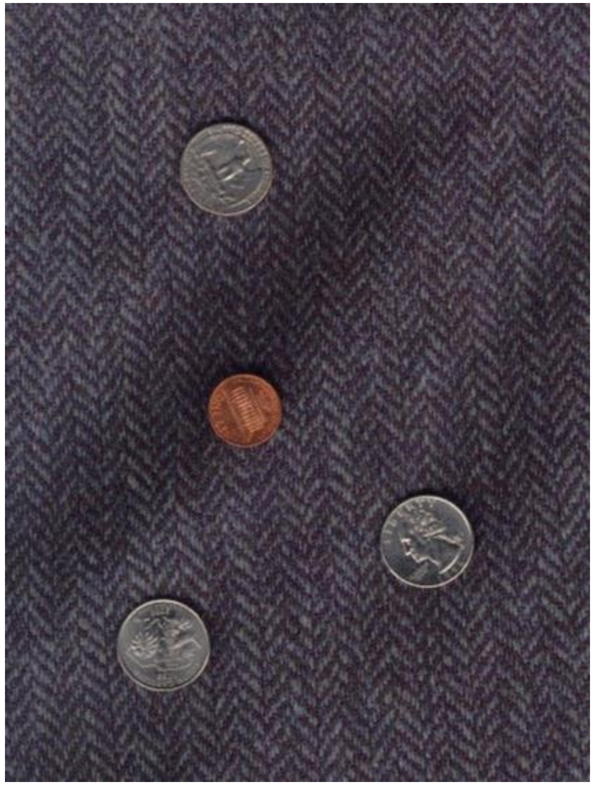
- Computational time grows quickly with fraction of outliers and number of parameters
- Not good for getting multiple fits

Common applications

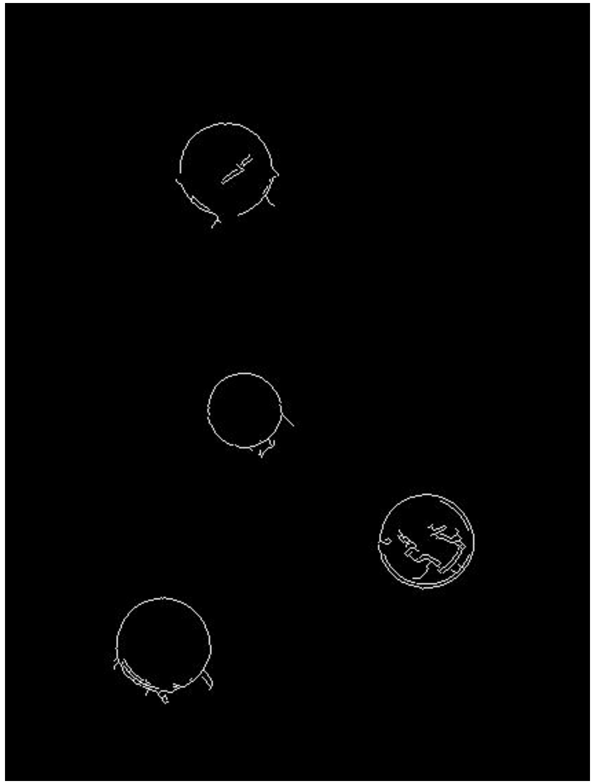
- Computing a homography (e.g., image stitching)
- Estimating fundamental matrix (relating two views)

Can we use RANSAC instead of Hough transform?

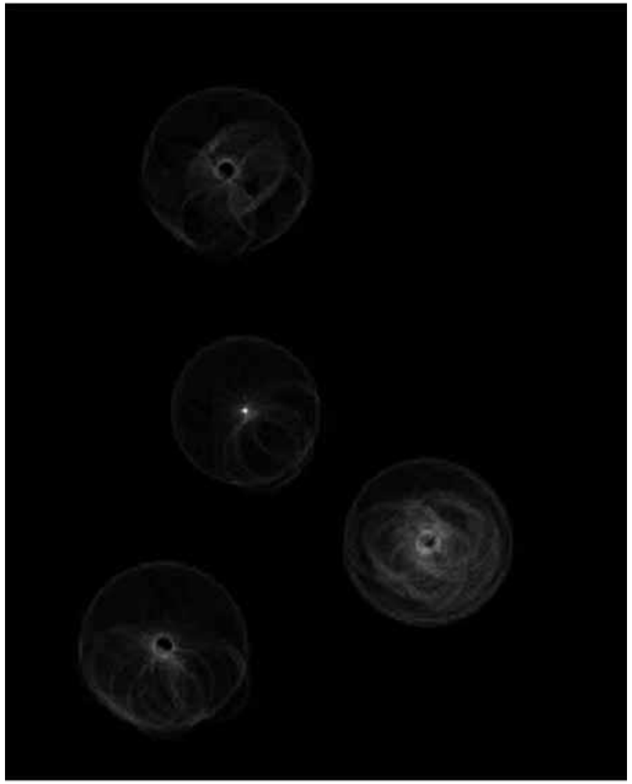
Original



Edges



Votes: Penny

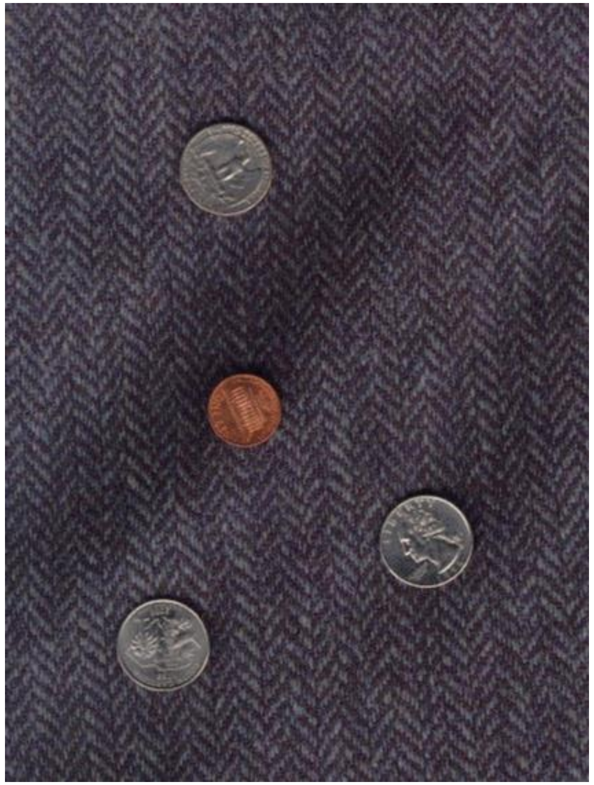


Note: a different Hough transform (with separate accumulators) was used for each circle radius (quarters vs. penny).

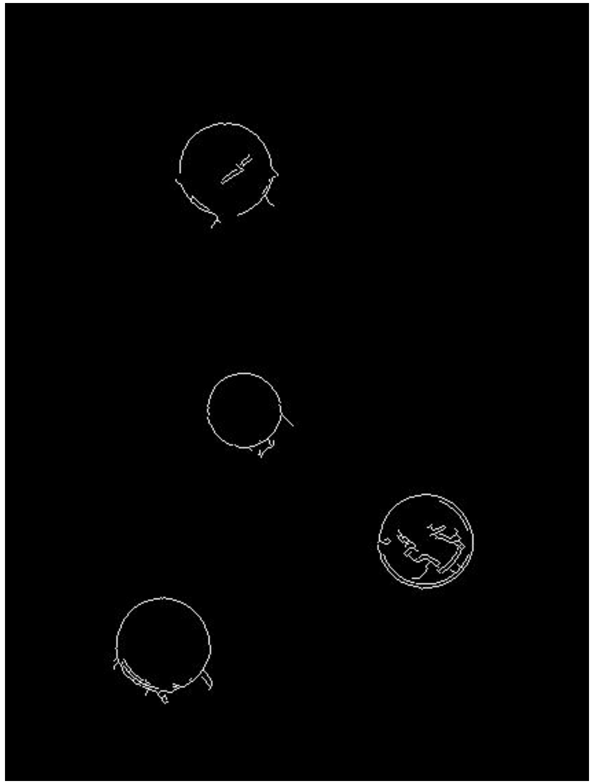
Slide credit: Kristen Grauman

Canvas Quiz: RANSAC

Original



Edges



Let's find circles of any radius from 6 to 55 pixels

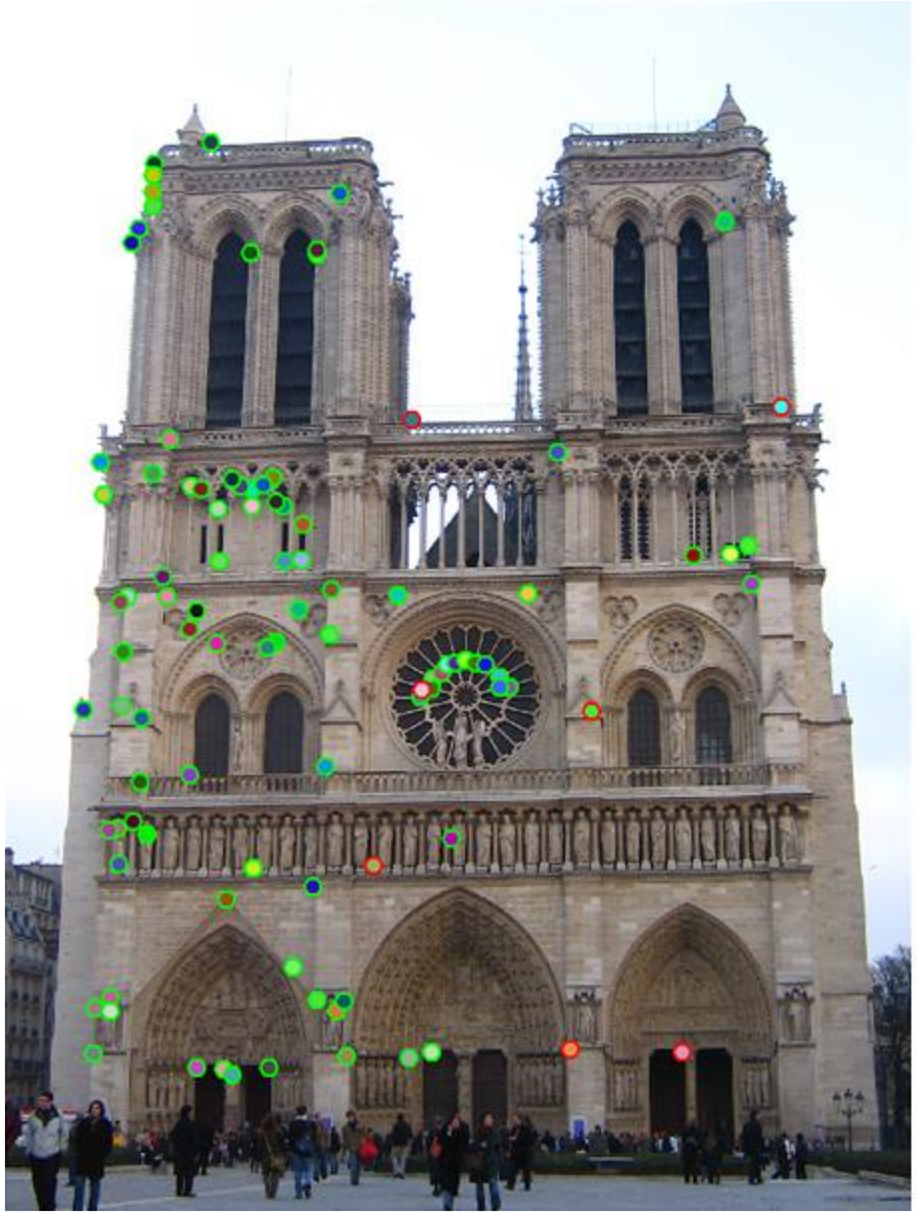
Let's assume that for a particular coin, 10% of the overall edge pixels are "inliers" (on the perimeter of that coin)

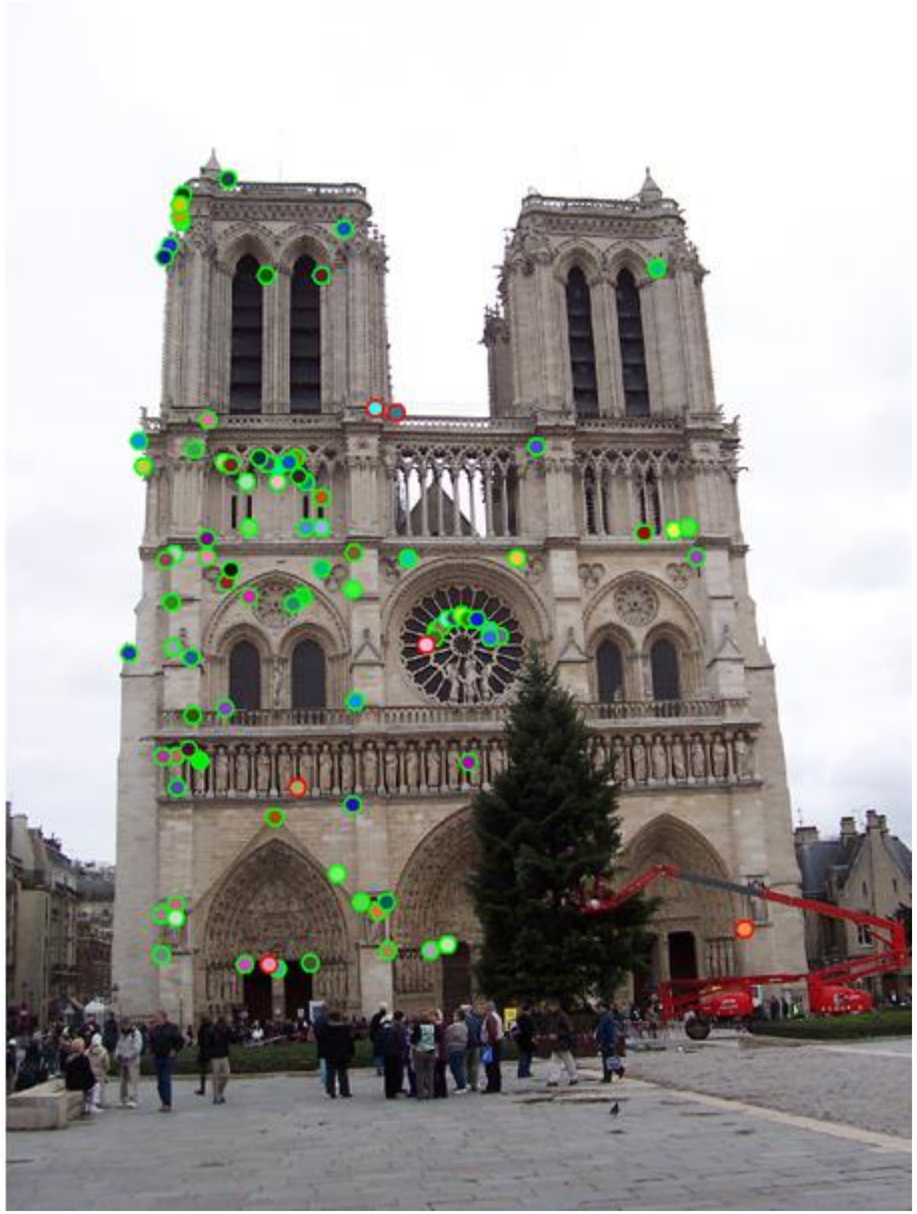
Recall this equation to estimate the number of RANSAC iterations needed, N

$$N = \log(1-p) / \log\left(1-(1-e)^s\right)$$

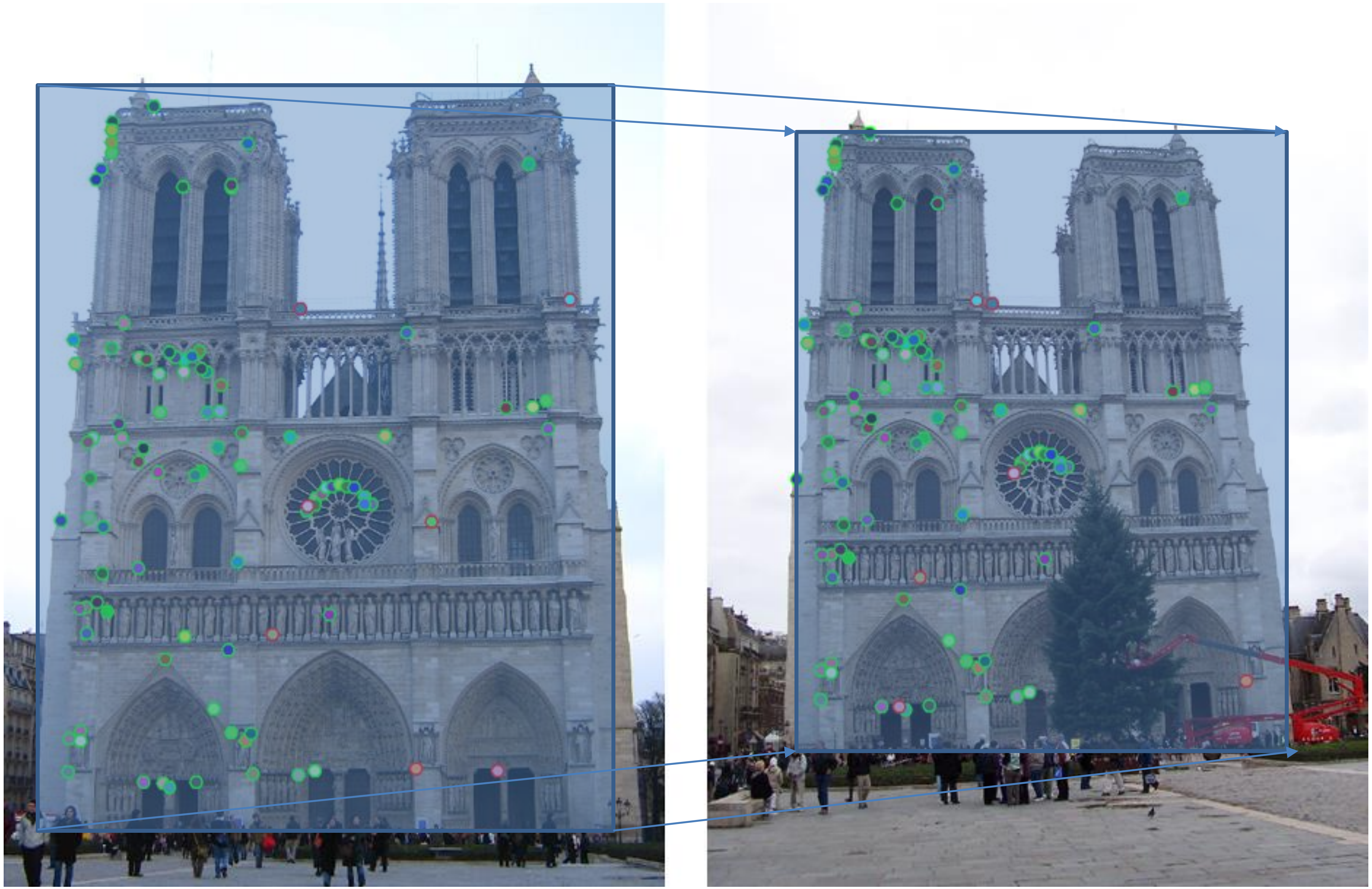
s = number of samples needed to fit a model
p = desired probability of finding an outlier free solution
e = proportion of outliers

How do we fit the best alignment?

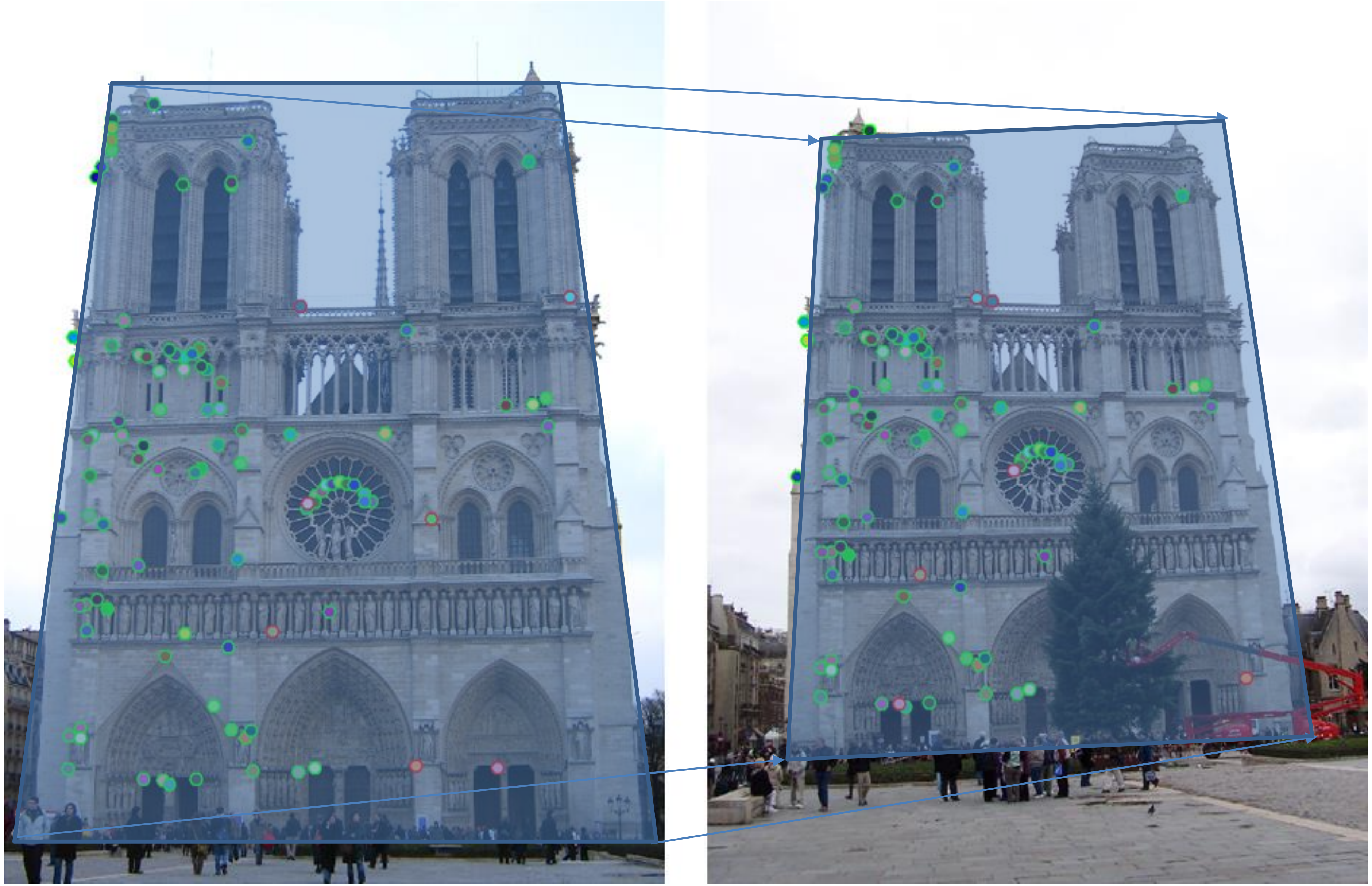




How do we fit the best alignment?



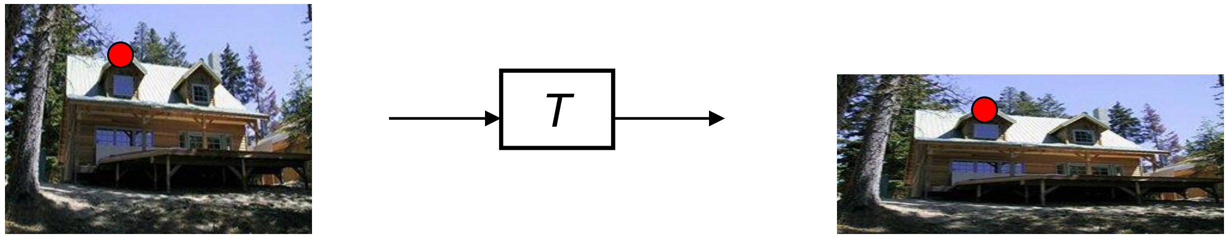
How do we fit the best alignment?



Alignment

- Alignment: find parameters of model that maps one set of points to another
- Typically want to solve for a global transformation that accounts for *most* true correspondences
- Difficulties
  - Noise (typically 1-3 pixels)
  - Outliers (often 50%)
  - Many-to-one matches or multiple objects

Parametric (global) warping



p = (x,y) **p’** = (x’,y’)

Transformation T is a coordinate-changing machine:

p’ = *T*(p)

What does it mean that *T* is global and parametric?

- Global: Is the same for any point p
- Parametric: can be described by just a few numbers

We’re going to focus on *linear* transformations, we can represent T as a matrix multiplication

p’ = **T**p

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{T} \begin{bmatrix} x \\ y \end{bmatrix}$$

Common transformations

original

Transformed

translation

rotation

aspect

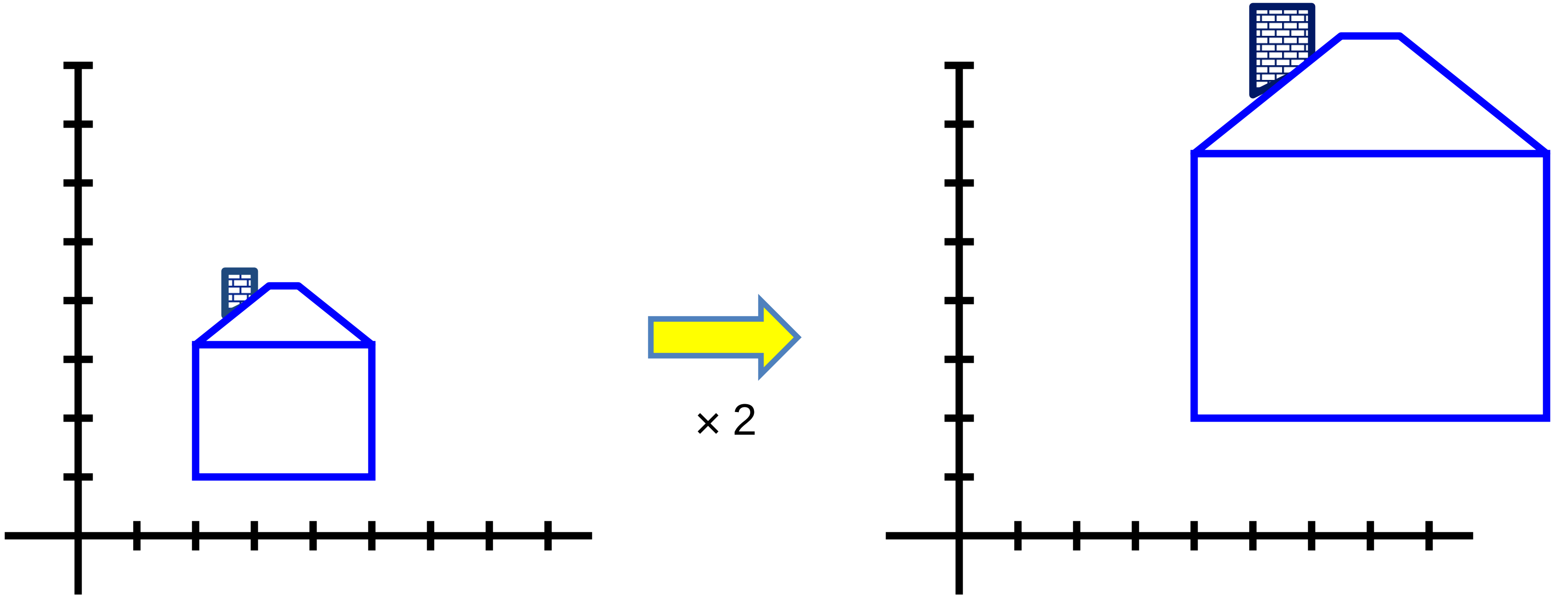
affine

perspective

Slide credit (next few slides):
A. Efros and/or S. Seitz

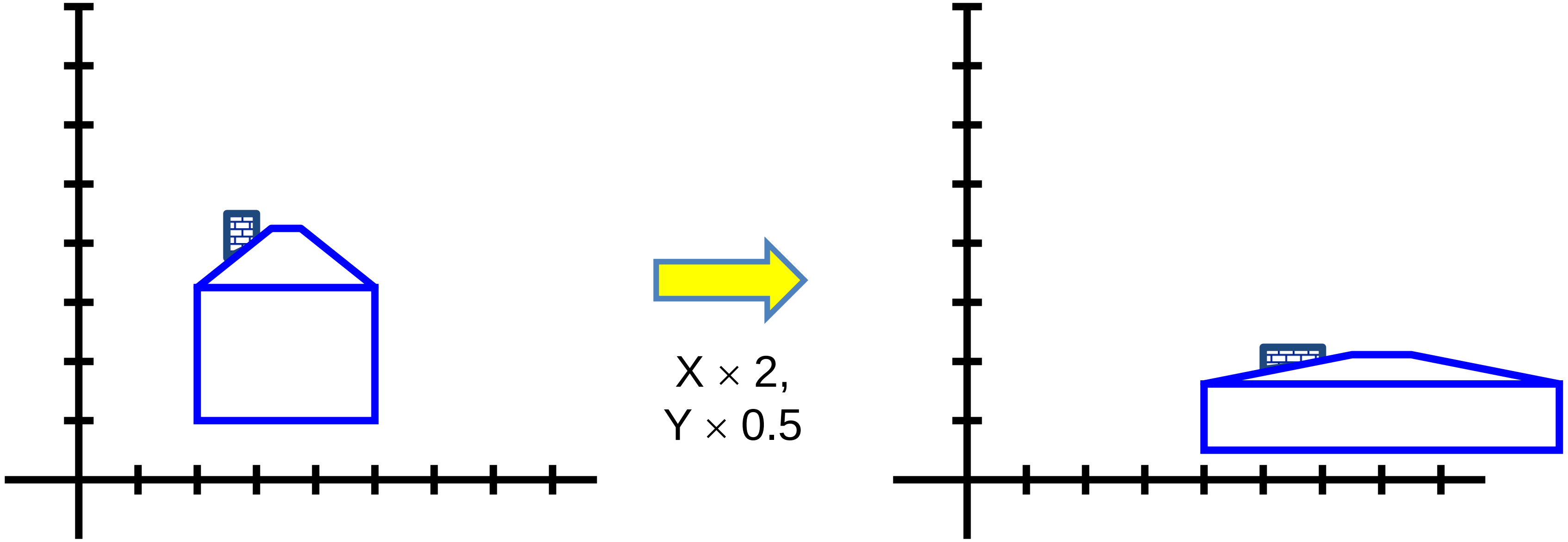
Scaling

- *Scaling* a coordinate means multiplying each of its components by a scalar
- *Uniform scaling* means this scalar is the same for all components:



Scaling

- *Non-uniform scaling*: different scalars per component:

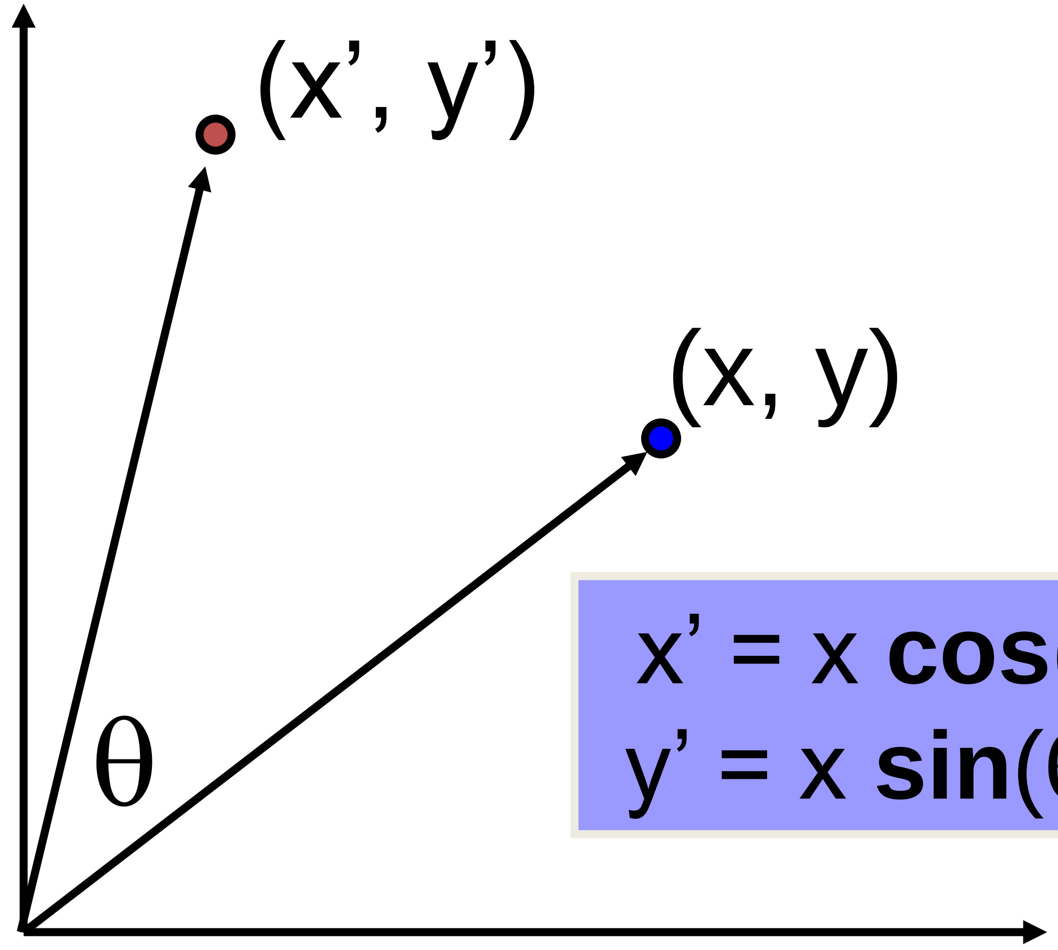


Scaling

- Scaling operation: $$x' = ax$$ $$y' = by$$

- Or, in matrix form: $$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}}_{\textit{scaling matrix } S} \begin{bmatrix} x \\ y \end{bmatrix}$$

2-D Rotation (around the origin)



$$x' = x\ \mathbf{cos}(\theta) - y\ \mathbf{sin}(\theta)$$
$$y' = x\ \mathbf{sin}(\theta) + y\ \mathbf{cos}(\theta)$$

2-D Rotation

This is easy to capture in matrix form:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}}_{\mathbf{R}} \begin{bmatrix} x \\ y \end{bmatrix}$$

Even though sin(θ) and cos(θ) are nonlinear functions of θ,

- ***For a particular*** θ, ***x' is a linear combination of x and y***
- ***For a particular*** θ, ***y' is a linear combination of x and y***

What is the inverse transformation?

- Rotation by $-\theta$
- For rotation matrices $\mathbf{R}^{-1} = \mathbf{R}^{T}$

Basic 2D transformations

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & \alpha_x \\ \alpha_y & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Shear

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta \\ \sin\Theta & \cos\Theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Rotate

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Affine

Affine is any combination of translation, scale, rotation, shear

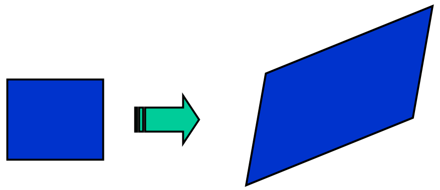
2D Affine Transformations

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Affine transformations are combinations of …

- Linear transformations, and
- Translations

Parallel lines remain parallel



Slide credit: Kristen Grauman

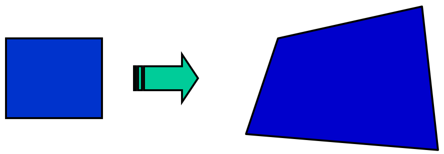
Projective Transformations

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Projective transformations:

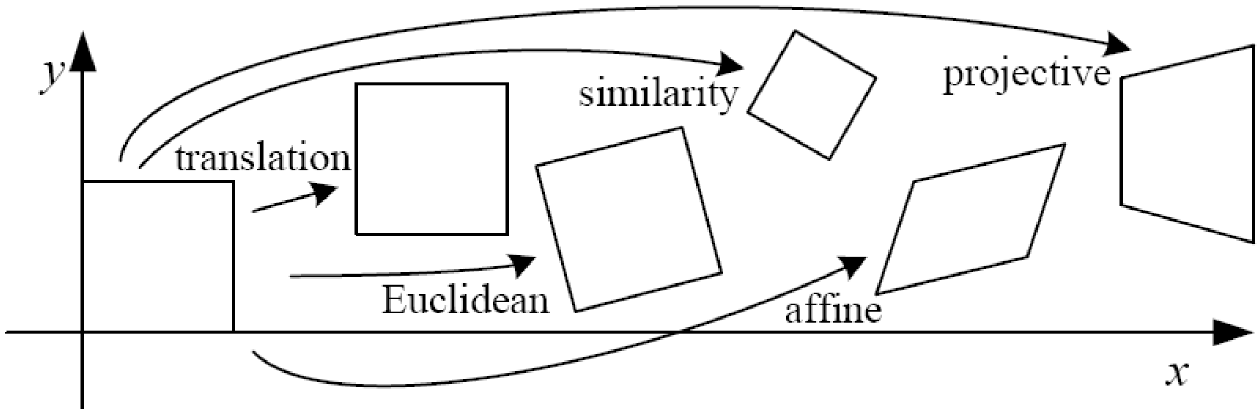
- Affine transformations, and
- Projective warps

Parallel lines do not necessarily remain parallel



Slide credit: Kristen Grauman

2D image transformations (reference table)



Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$\begin{bmatrix} \boldsymbol{I} & \boldsymbol{t} \end{bmatrix}_{2\times 3}$	2	orientation $+ \cdots$	
rigid (Euclidean)	$\begin{bmatrix} \boldsymbol{R} & \boldsymbol{t} \end{bmatrix}_{2\times 3}$	3	lengths $+ \cdots$	
similarity	$\begin{bmatrix} s\boldsymbol{R} & \boldsymbol{t} \end{bmatrix}_{2\times 3}$	4	angles $+ \cdots$	
affine	$\begin{bmatrix} \boldsymbol{A} \end{bmatrix}_{2\times 3}$	6	parallelism $+ \cdots$	
projective	$\begin{bmatrix} \tilde{\boldsymbol{H}} \end{bmatrix}_{3\times 3}$	8	straight lines	

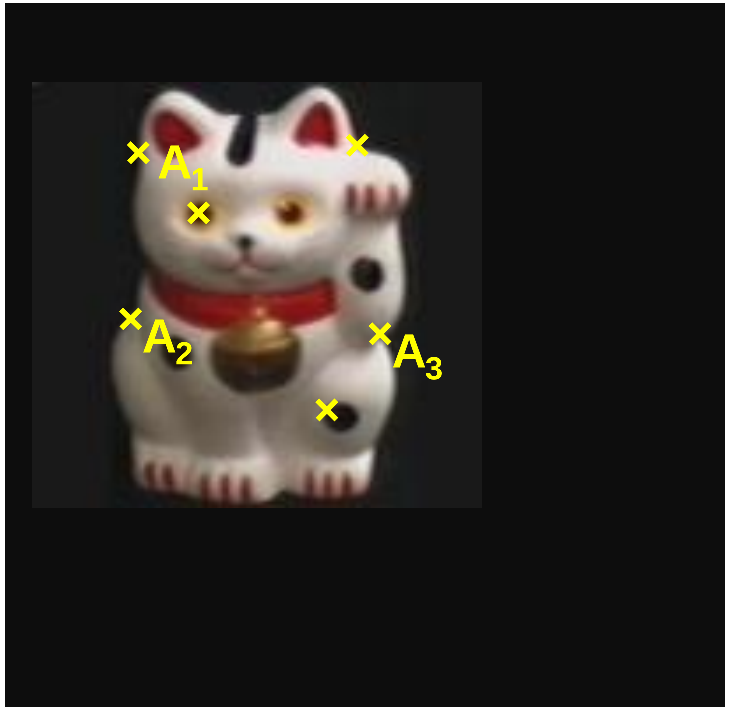
Szeliski 2.1

Example: solving for translation

Given matched points in {A} and {B}, estimate the translation of the object

$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Example: solving for translation



(t_x, t_y)

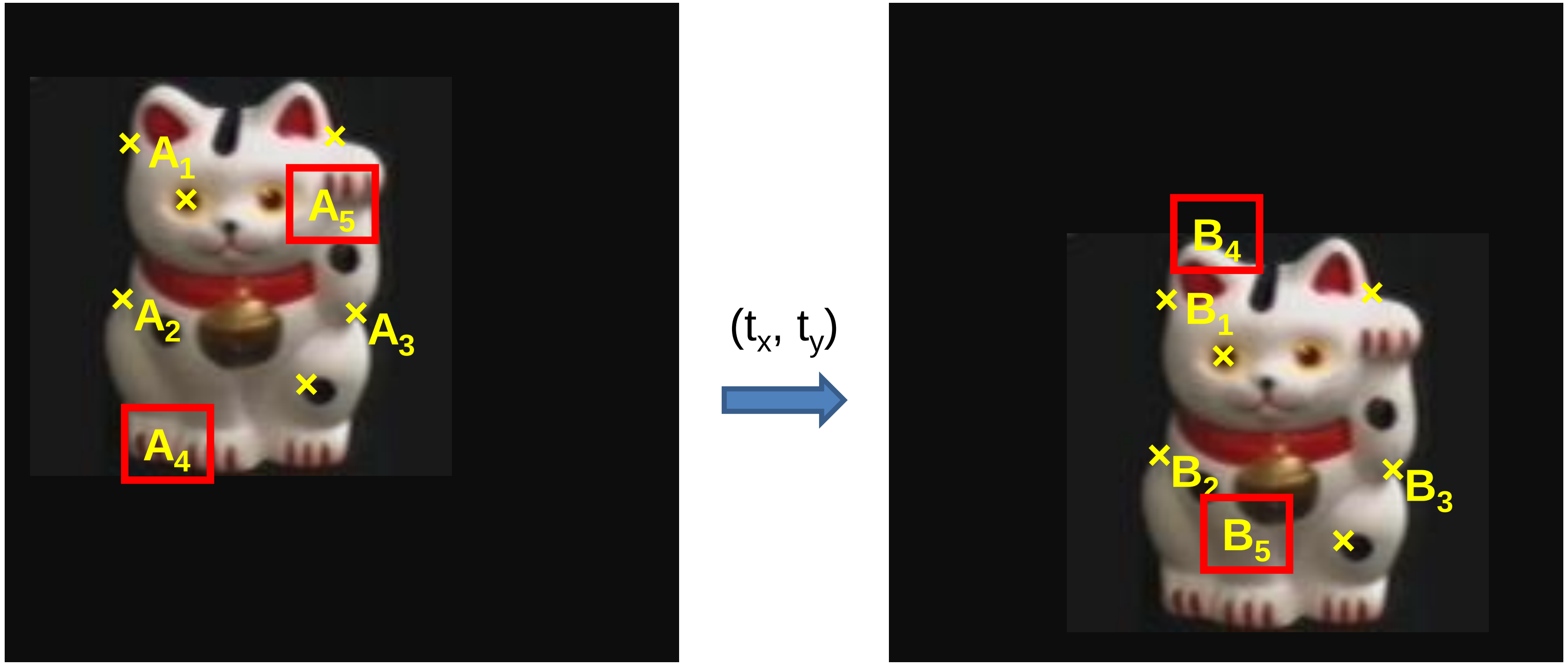
Least squares solution

1. Write down objective function
2. Derived solution
   a) Compute derivative
   b) Compute solution
3. Computational solution
   a) Write in form Ax=b
   b) Solve using pseudo-inverse or eigenvalue decomposition

$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \vdots & \vdots \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} t_x \\ t_y \end{bmatrix} = \begin{bmatrix} x_1^B - x_1^A \\ y_1^B - y_1^A \\ \vdots \\ x_n^B - x_n^A \\ y_n^B - y_n^A \end{bmatrix}$$

Example: solving for translation



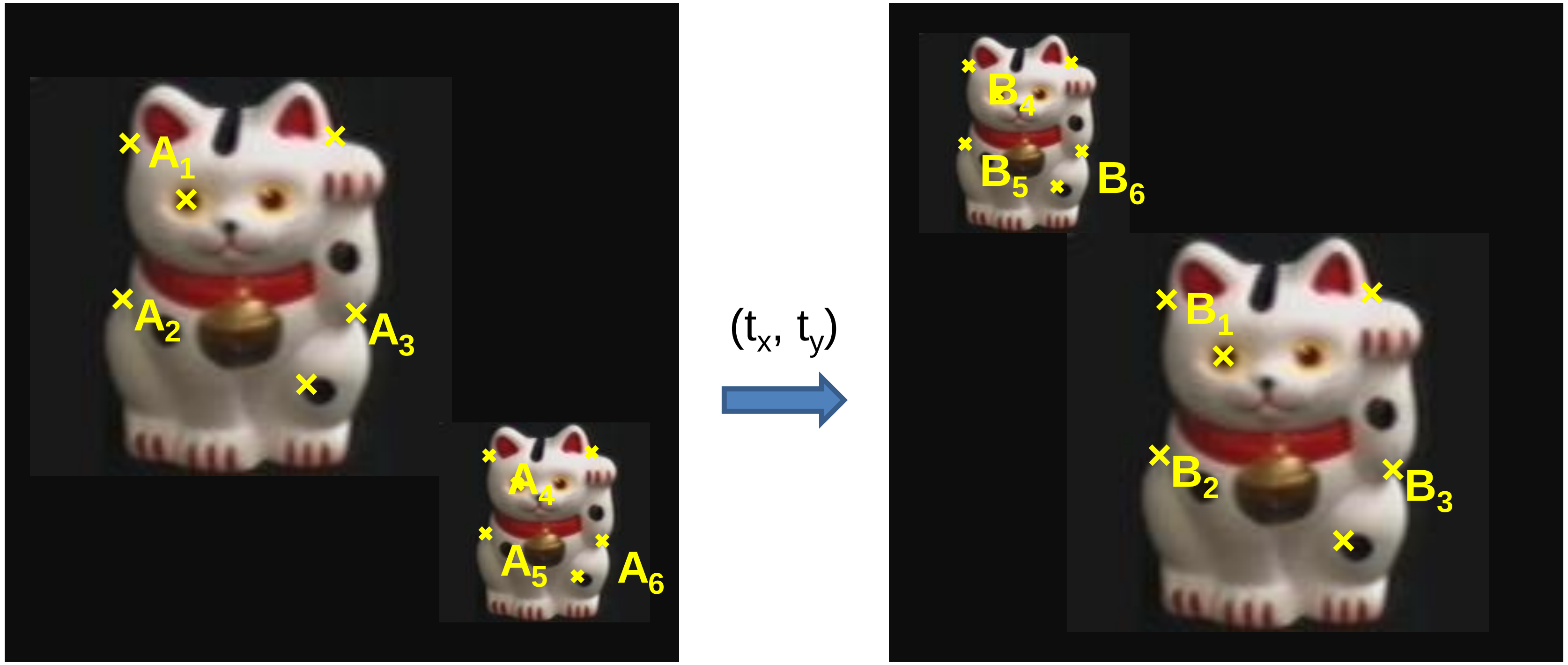
Problem: outliers

RANSAC solution

1. Sample a set of matching points (1 pair)
2. Solve for transformation parameters
3. Score parameters with number of inliers
4. Repeat steps 1-3 N times

$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Example: solving for translation



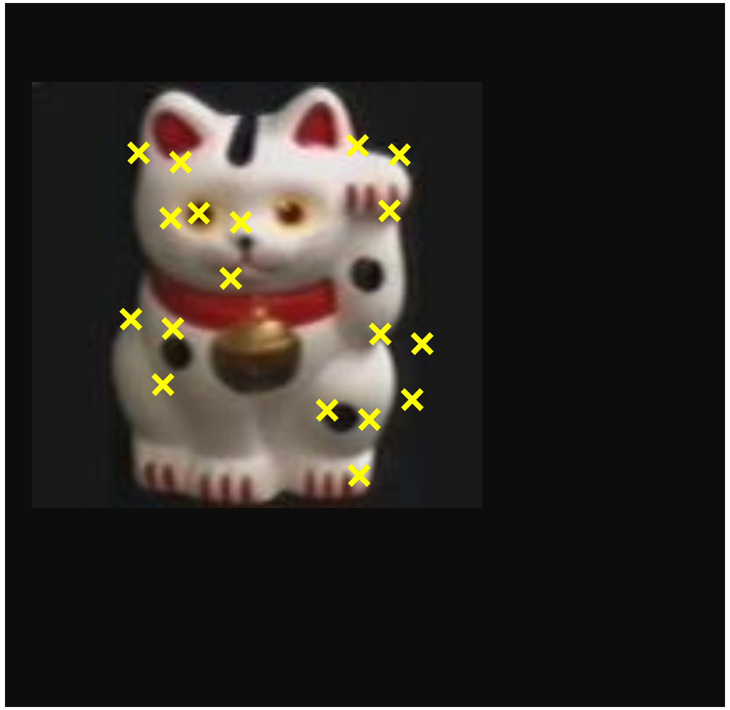
Problem: outliers, multiple objects, and/or many-to-one matches

Hough transform solution

1. Initialize a grid of parameter values
2. Each matched pair casts a vote for consistent values
3. Find the parameters with the most votes
4. Solve using least squares with inliers

$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Example: solving for translation



(t_x, t_y)

Problem: no initial guesses for correspondence

$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

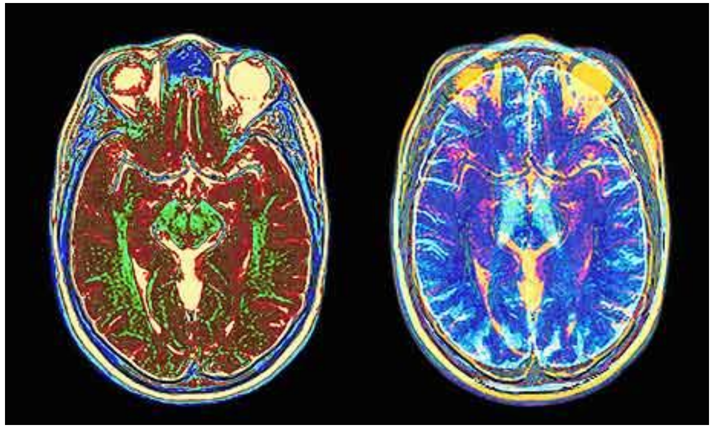
Fitting and Alignment: Methods

- ~~Global optimization / Search for parameters~~
  - ~~Least squares fit~~
  - ~~Robust least squares~~
  - ~~Other parameter search methods~~
- ~~Hypothesize and test~~
  - ~~Hough transform~~
  - ~~RANSAC~~
- Iterative Closest Points (ICP)

What if you want to align but have no prior matched pairs?

- Hough transform and RANSAC not applicable

- Important applications



Medical imaging: align brain scans or contours

Robotics: align point clouds

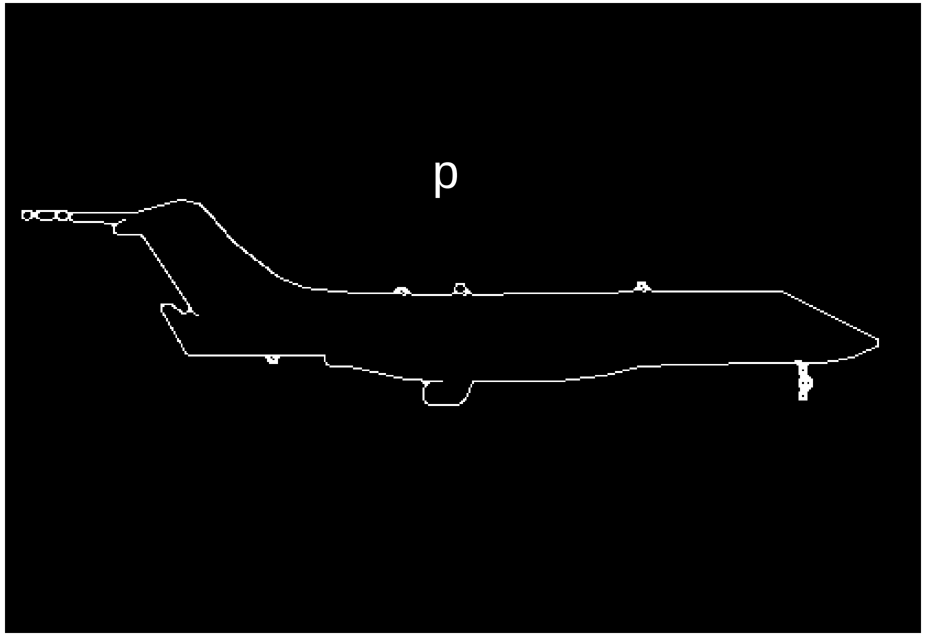
Iterative Closest Points (ICP) Algorithm

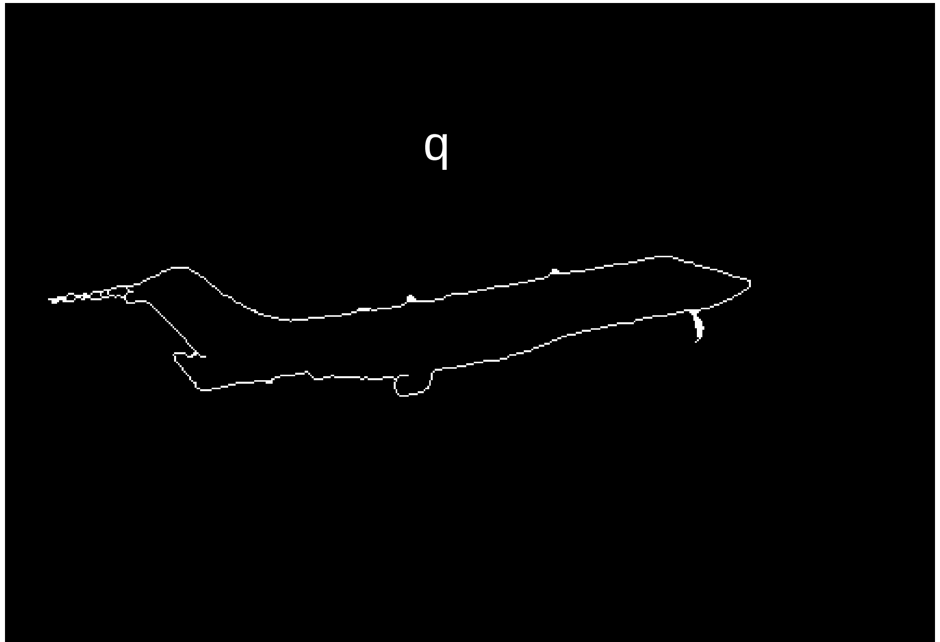
Goal: estimate transform between two dense sets of points

1. **Initialize** transformation (e.g., compute difference in means and scale)
2. **Assign** each point in {Set 1} to its nearest spatial neighbor in {Set 2}
3. **Estimate** transformation parameters
   - e.g., least squares or robust least squares
4. **Transform** the points in {Set 1} using estimated parameters
5. **Repeat** steps 2-4 until change is very small

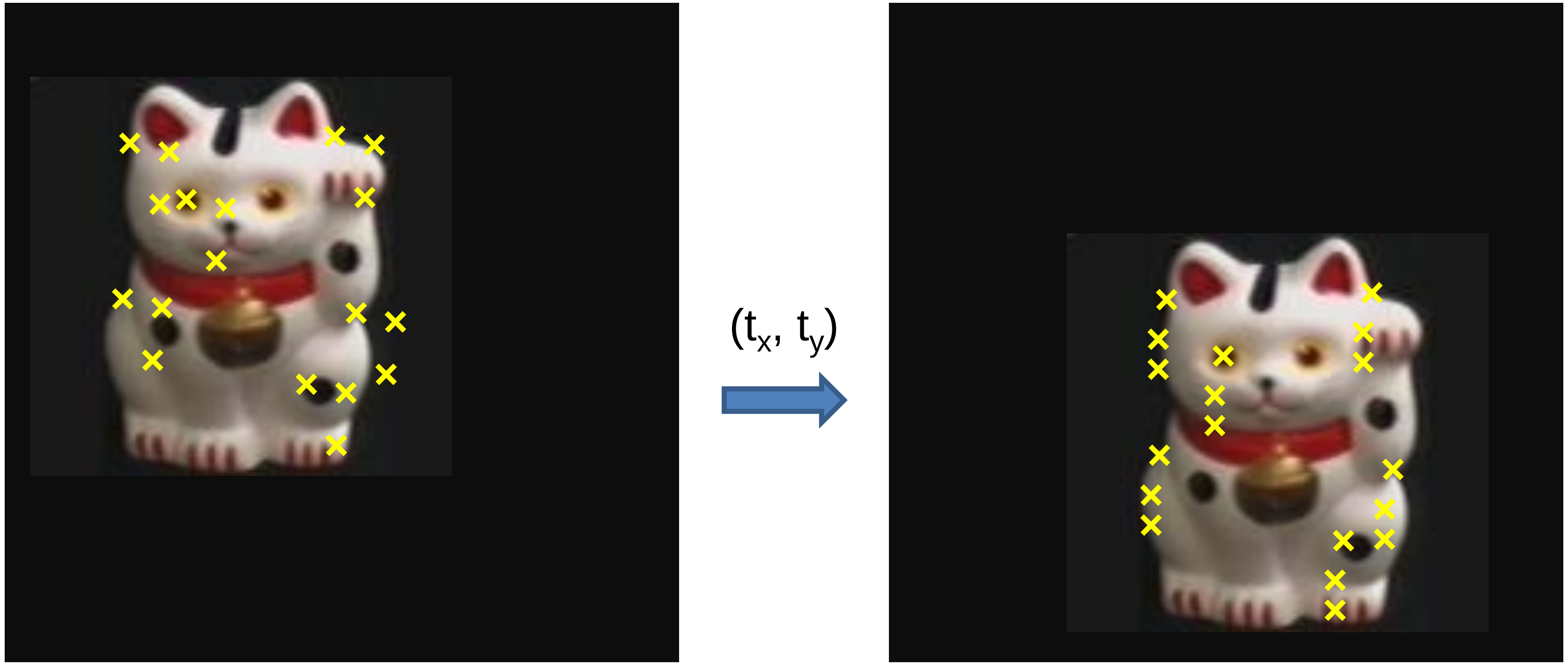
Example: aligning boundaries

1. Extract edge pixels $p_1 .. pn$ and $q_1 .. qm$
2. Compute initial transformation (e.g., compute translation and scaling by center of mass, variance within each image)
3. Get nearest neighbors: for each point p_i find corresponding $\mathrm{match(i)} = \underset{j}{\mathrm{argmin}}\, dist(pi, qj)$
4. Compute transformation ***T*** based on matches
5. Warp points ***p*** according to ***T***
6. Repeat 3-5 until convergence





Example: solving for translation



Problem: no initial guesses for correspondence

ICP solution

1. Find nearest neighbors for each point
2. Compute transform using matches
3. Move points using transform
4. Repeat steps 1-3 until convergence

$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

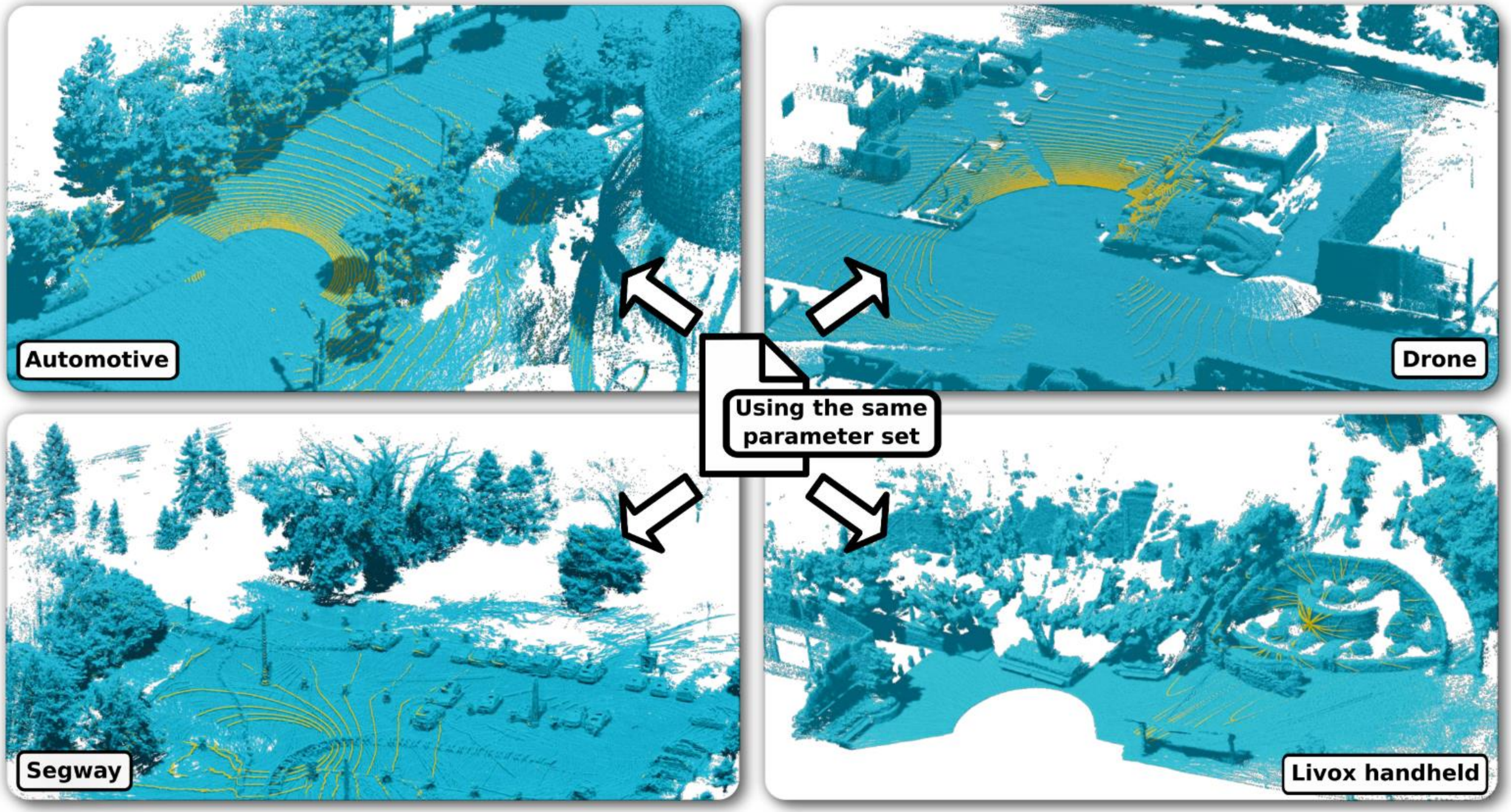
Sparse ICP

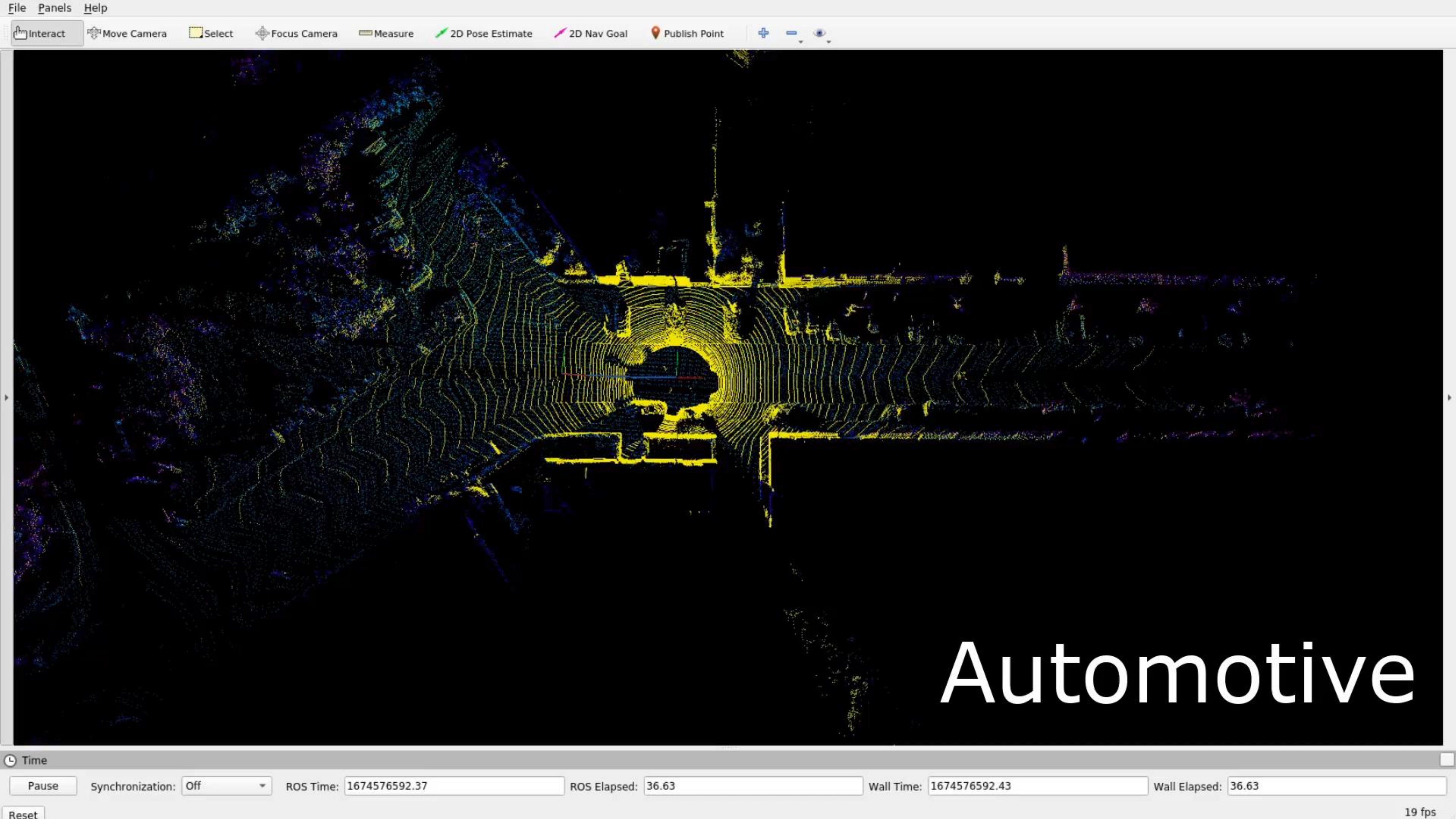
Sofien Bouaziz *Andrea Tagliasacchi* *Mark Pauly*

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

KISS-ICP: In Defense of Point-to-Point ICP – Simple, Accurate, and Robust Registration If Done the Right Way

Ignacio Vizzo   Tiziano Guadagnino   Benedikt Mersch   Louis Wiesmann   Jens Behley   Cyrill Stachniss




File Panels Help
Interact
Move Camera
Select
Focus Camera
Measure
2D Pose Estimate
2D Nav Goal
Publish Point
Automotive
Time
Pause
Synchronization: Off
ROS Time: 1674576592.37
ROS Elapsed: 36.63
Wall Time: 1674576592.43
Wall Elapsed: 36.63
Reset
19 fps

Algorithm Summaries

- Least Squares Fit
  - closed form solution
  - robust to noise
  - not robust to outliers
- Robust Least Squares
  - improves robustness to outliers
  - requires iterative optimization
- Hough transform
  - robust to noise and outliers
  - can fit multiple models
  - only works for a few parameters (1-4 typically)
- RANSAC
  - robust to noise and outliers
  - works with a moderate number of parameters (e.g, 1-8)
- Iterative Closest Point (ICP)
  - For local alignment only: does not require initial correspondences
  - Sensitive to initialization

Rough count of mentions in recent literature

- Keypoint 2,180 mentions
- SIFT 3,530 mentions
- “Least Squares” 2,290 mentions
- “Robust Least Squares” 4 mentions
- Hough: 901 mentions
- RANSAC: 1,690 mentions
- ICP: 895 mentions
- Affine 2,970
- ResNet: 8,510 mentions

Google search for site:https://openaccess.thecvf.com [term]
Seems to find results since 2013.