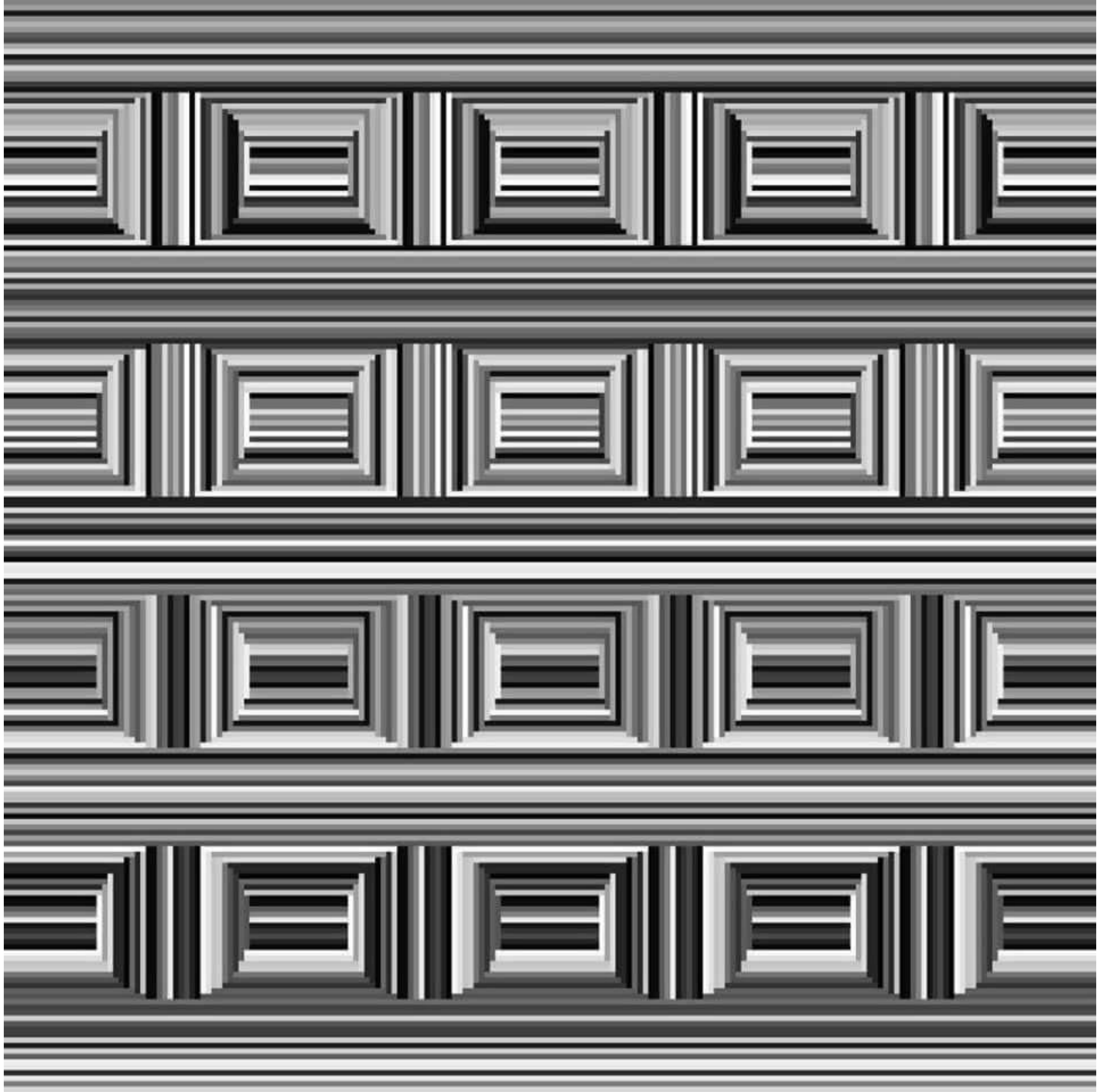
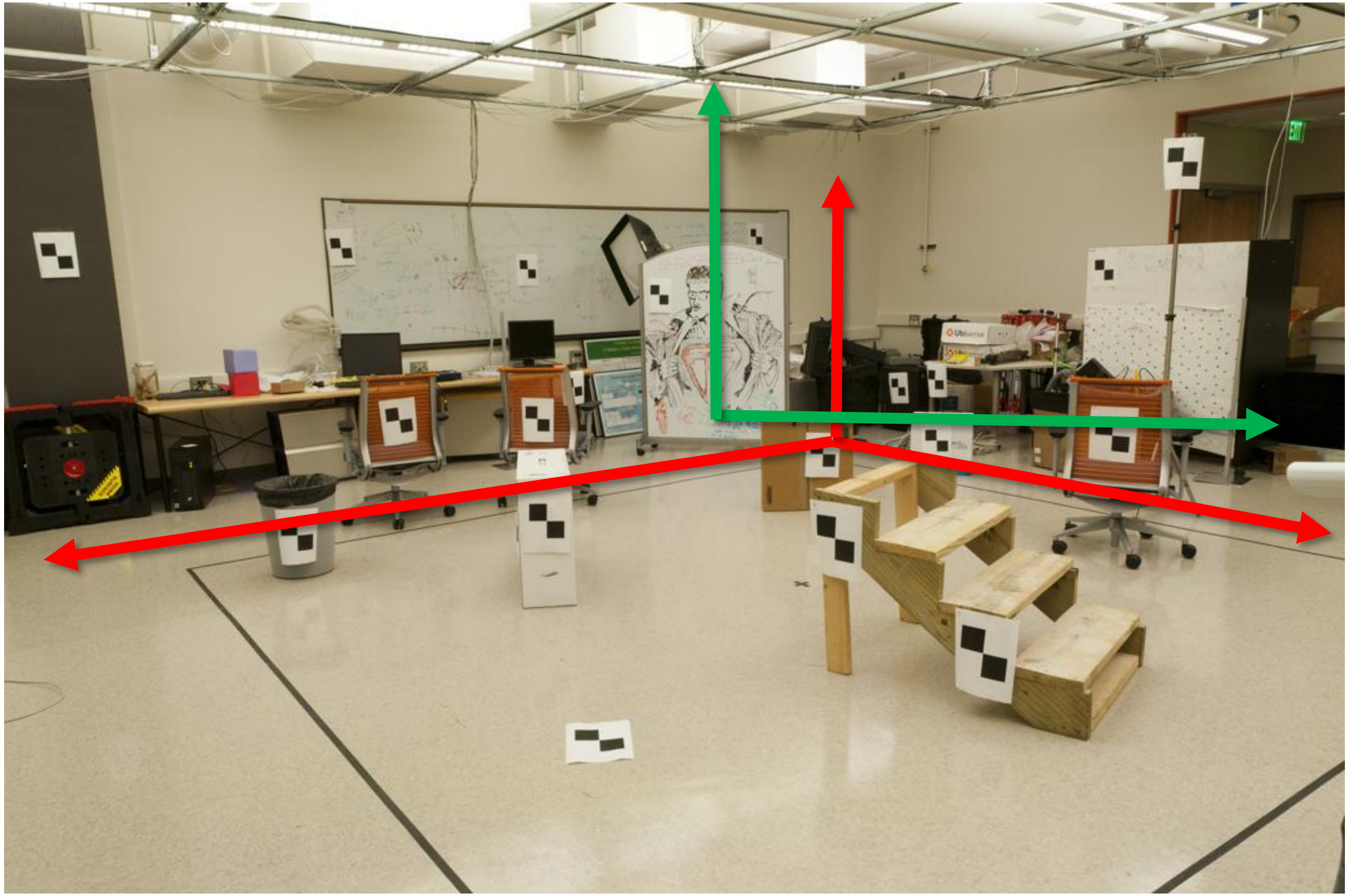



Erik Johansson

- Stereo and lidar can fall victim to reflections?
- Yes, there’s no easy way around that
- https://youtu.be/pBzU8TD1iks

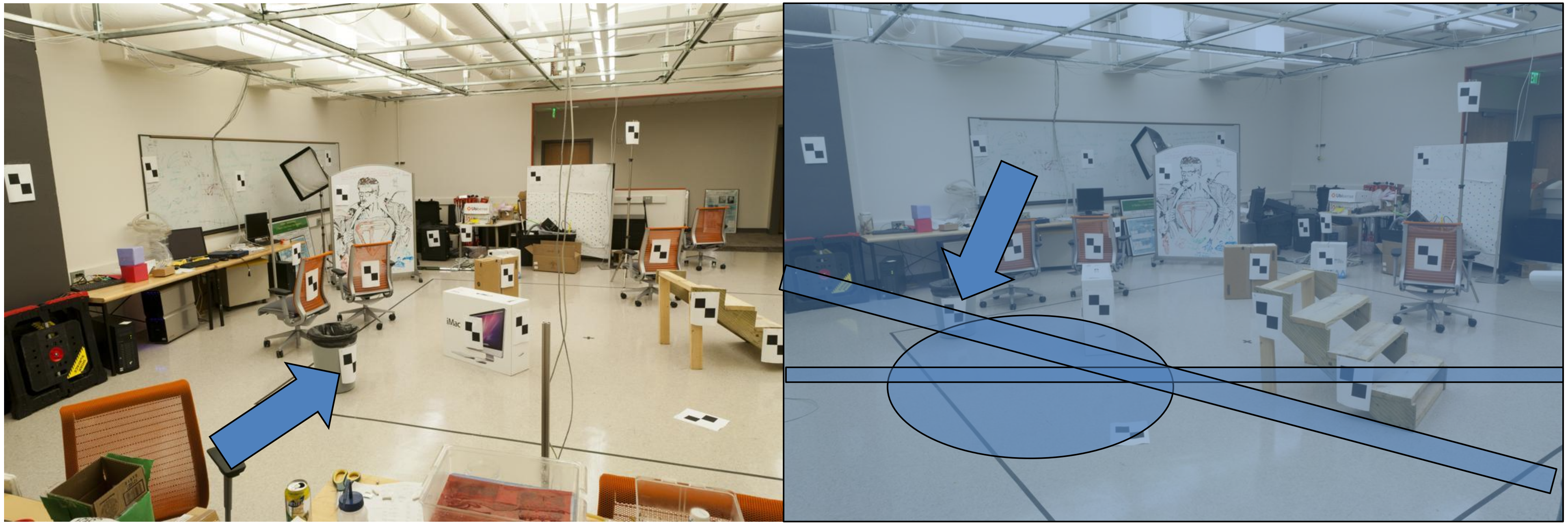
Last lecture: World vs Camera coordinates



Today's Outline

- Epipolar Geometry
  - Finding epipolar relationship between two images
  - Using epipolar geometry to rule out outliers
  - Finding dense correspondence along epipolar lines

Where do we need to search?



Epipolar Geometry and Stereo Vision

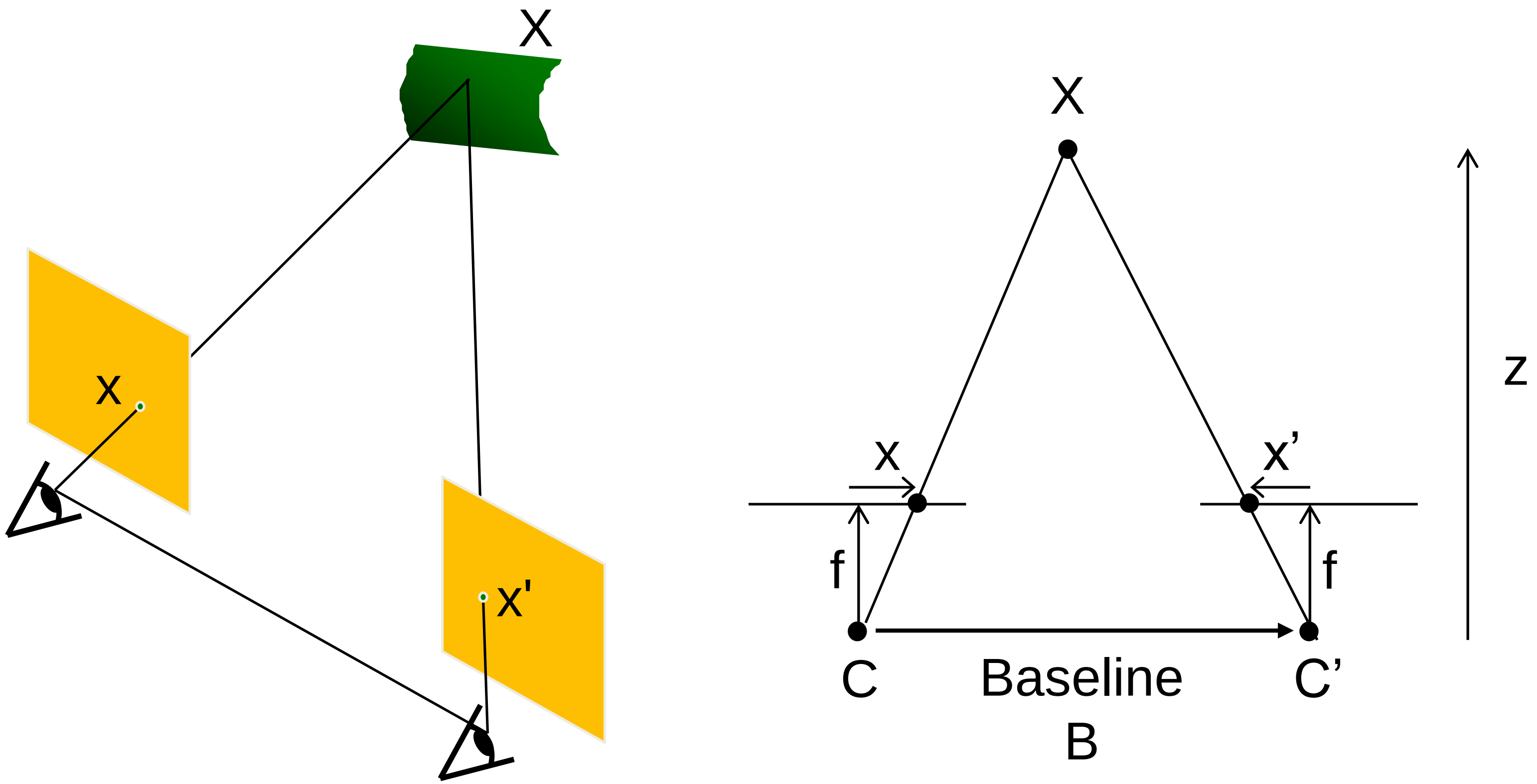
Chapter 11.3 in Szeliski

Many slides adapted from Derek Hoiem, Lana Lazebnik, Silvio Saverese, Steve Seitz, many figures from Hartley & Zisserman

- Epipolar geometry
  - Relates cameras from two positions

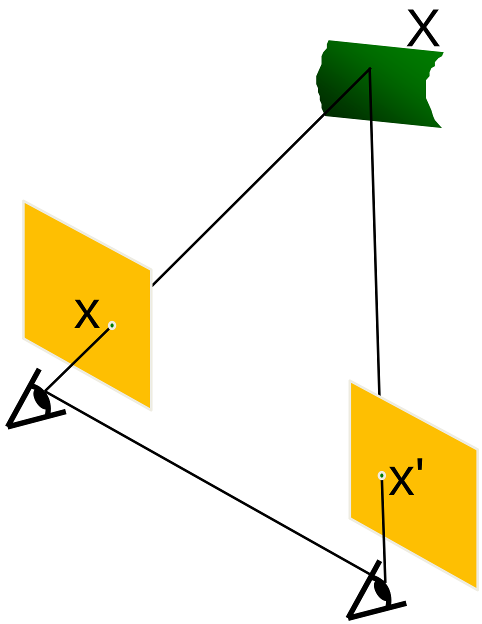
Depth from Stereo

- Goal: recover depth by finding image coordinate x’ that corresponds to x



Depth from Stereo

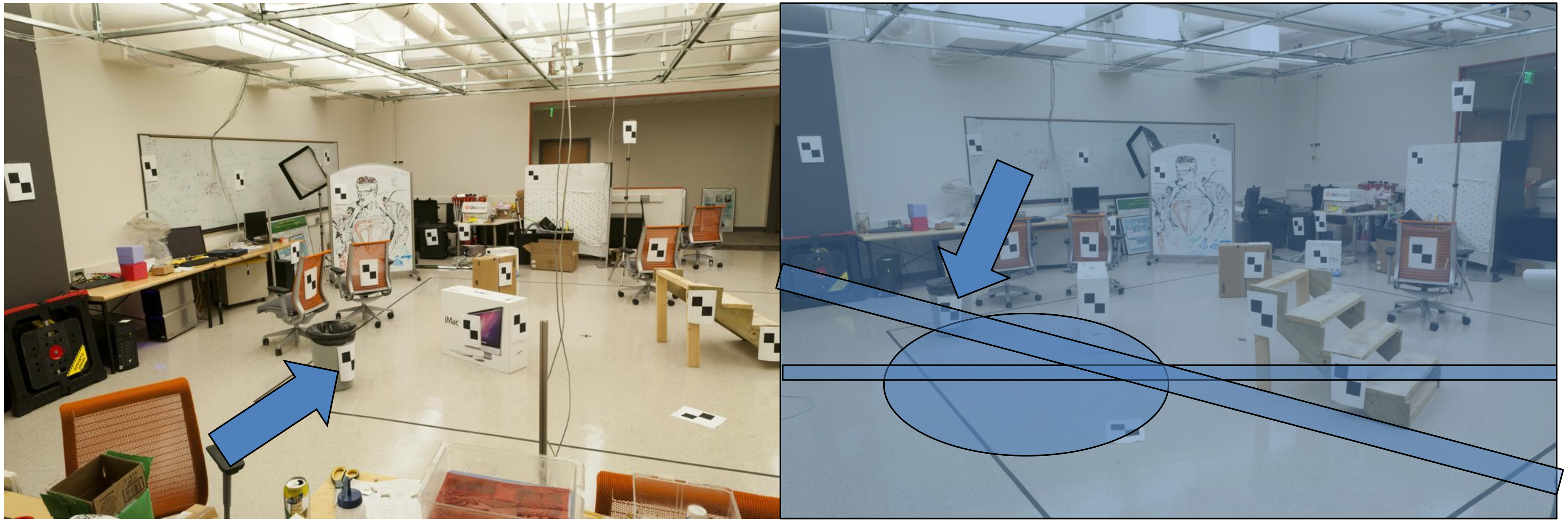
- Goal: recover depth by finding image coordinate x’ that corresponds to x
- Sub-Problems
  1. Calibration: How do we recover the relation of the cameras (if not already known)?
  2. Correspondence: How do we search for the matching point x’?



Correspondence Problem

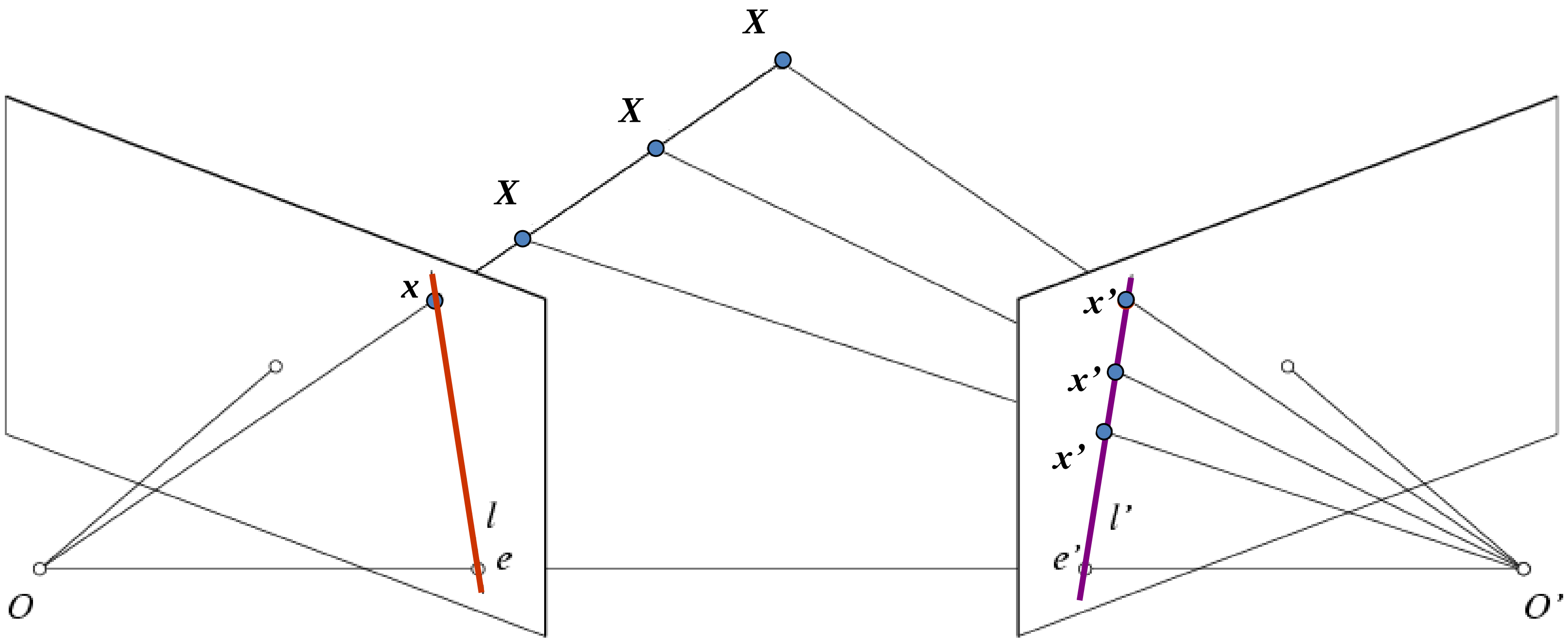
- We have two images taken from cameras with different intrinsic and extrinsic parameters
- How do we match a point in the first image to a point in the second? How can we constrain our search?

Where do we need to search?



Key idea: Epipolar constraint

Key idea: Epipolar constraint

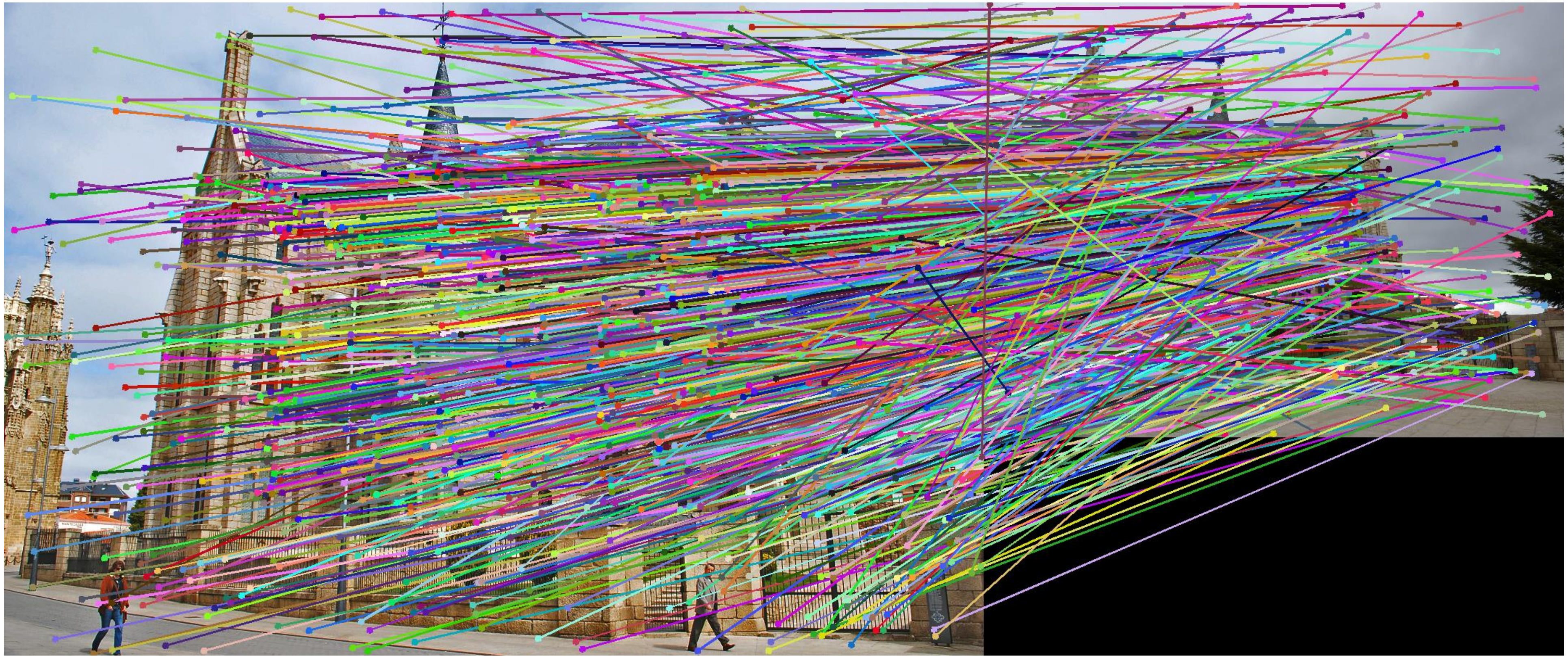


Potential matches for *x* have to lie on the corresponding line *l'*.

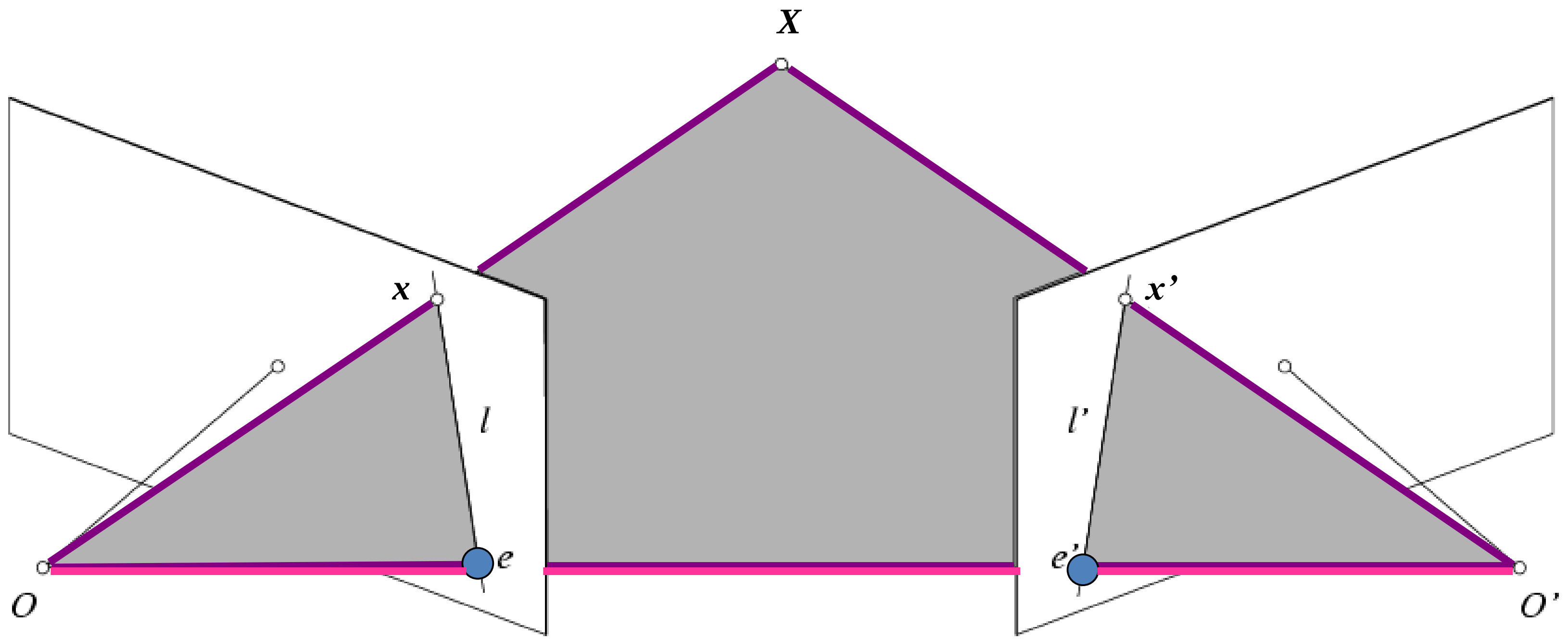
Potential matches for *x'* have to lie on the corresponding line *l*.

Wouldn’t it be nice to know where matches can live? To constrain our 2d search to 1d.

VLFeat's 800 most confident matches among 10,000+ local features.

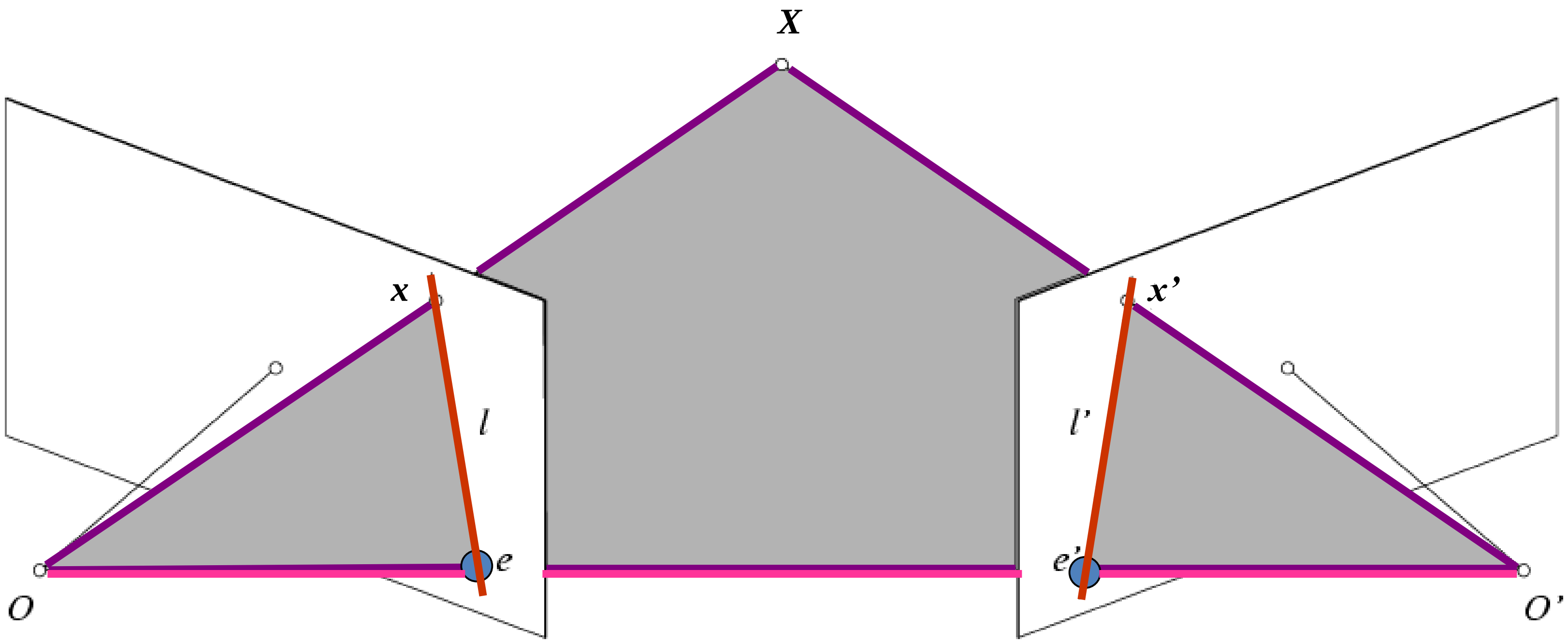


Epipolar geometry: notation



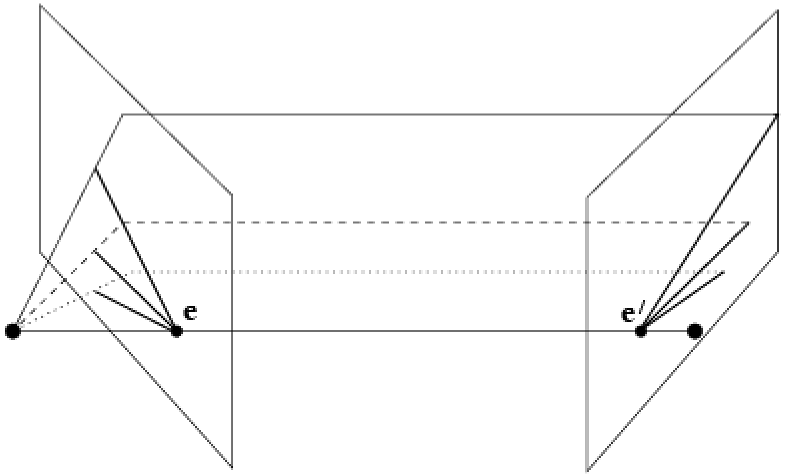
- **Baseline** – line connecting the two camera centers
- **Epipoles**
  = intersections of baseline with image planes
  = projections of the other camera center
- **Epipolar Plane** – plane containing baseline (1D family)

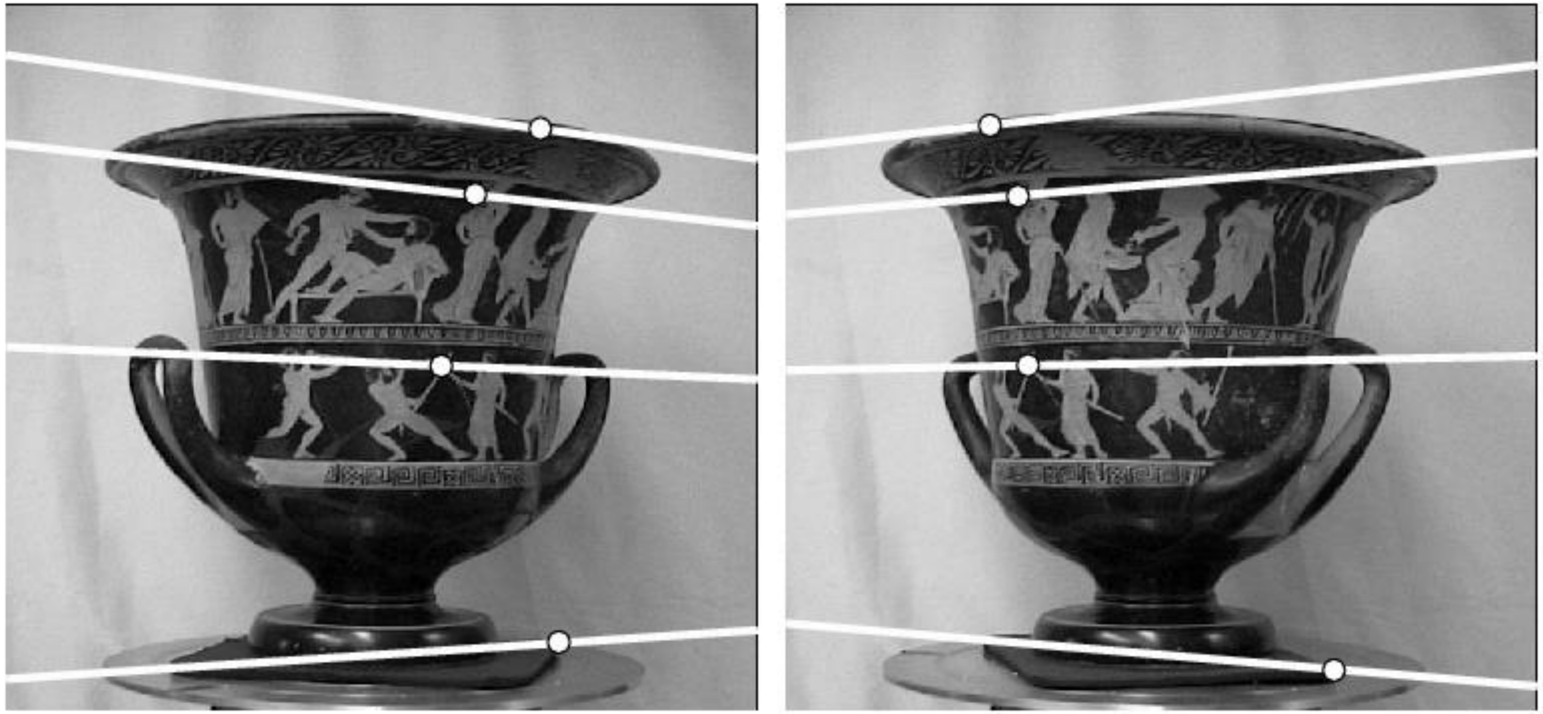
Epipolar geometry: notation



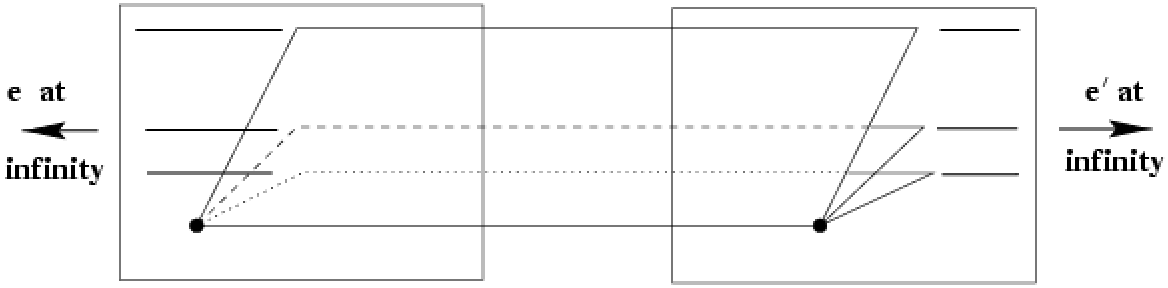
- **Baseline** – line connecting the two camera centers
- **Epipoles**
  = intersections of baseline with image planes
  = projections of the other camera center
- **Epipolar Plane** – plane containing baseline (1D family)
- **Epipolar Lines** - intersections of epipolar plane with image planes (always come in corresponding pairs)

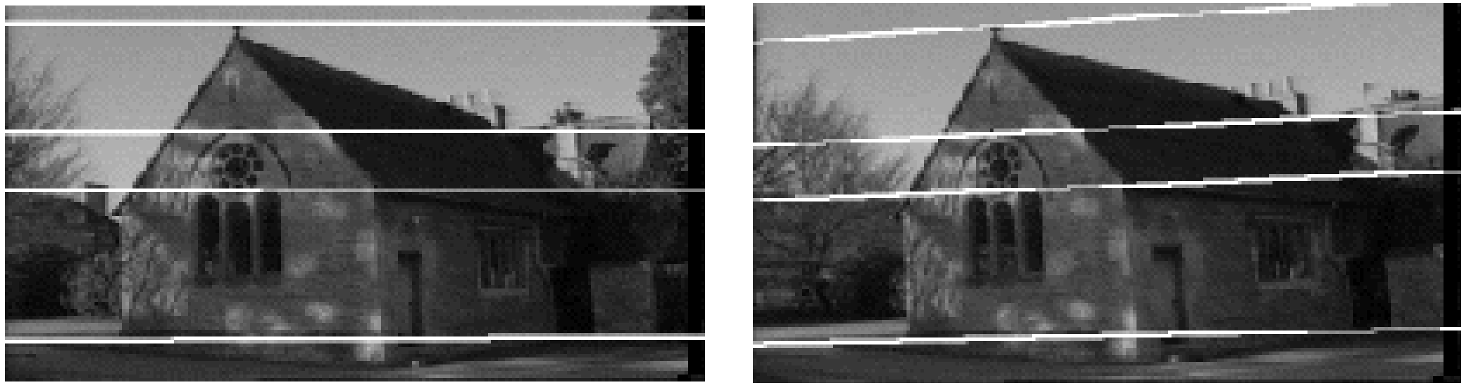
Example: Converging cameras





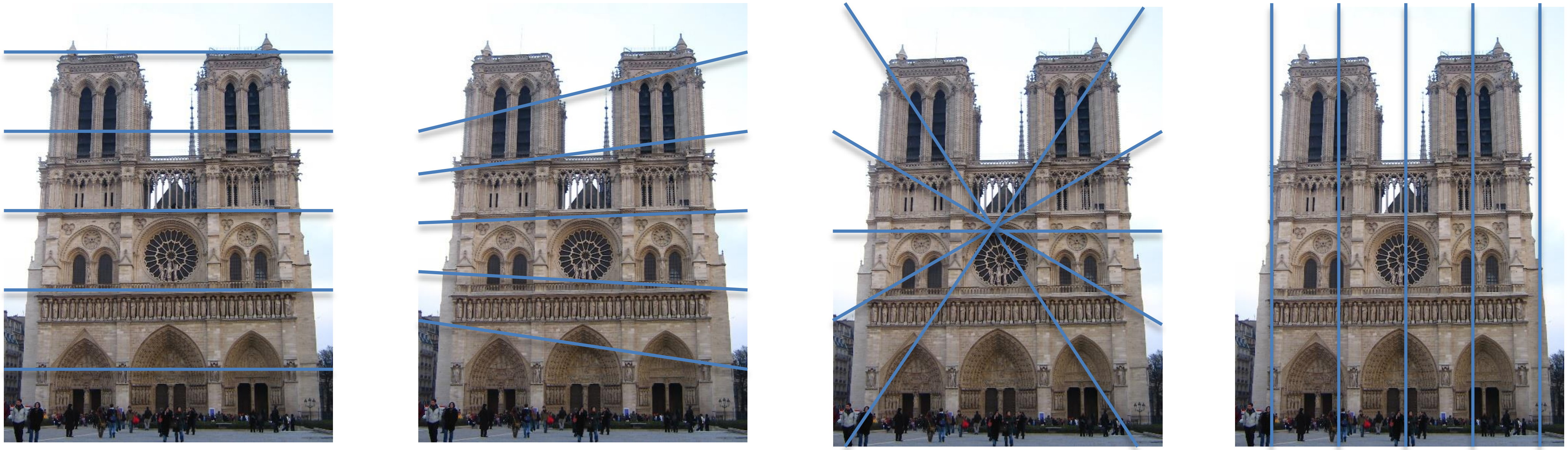
Example: Motion or displacement parallel to image plane





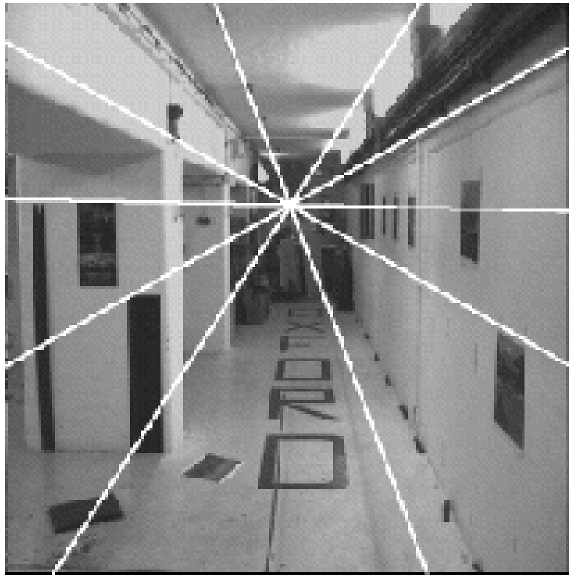
Example: Forward motion

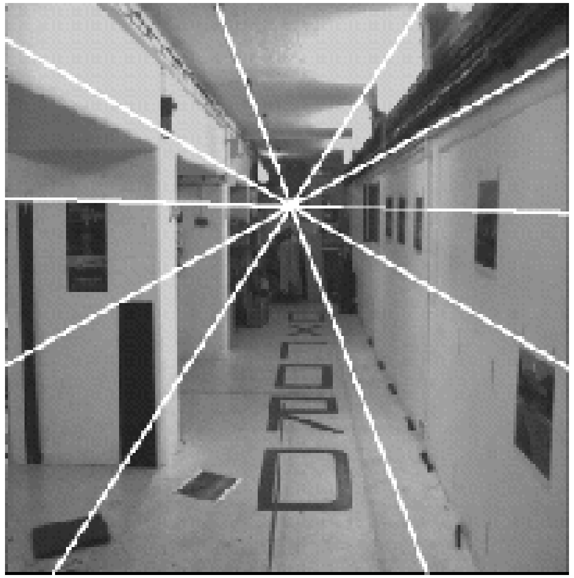
What would the epipolar lines look like if the camera moves directly forward?

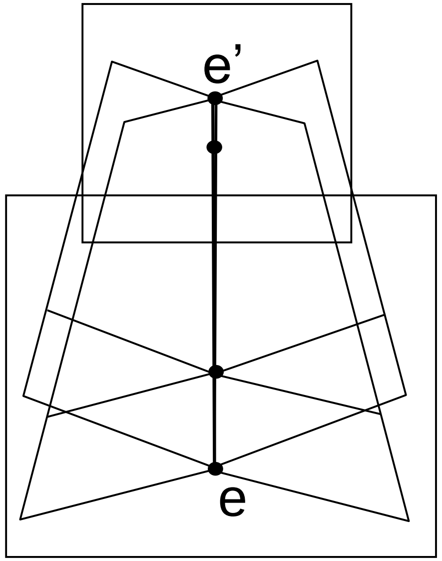


a b c d

Example: Forward motion

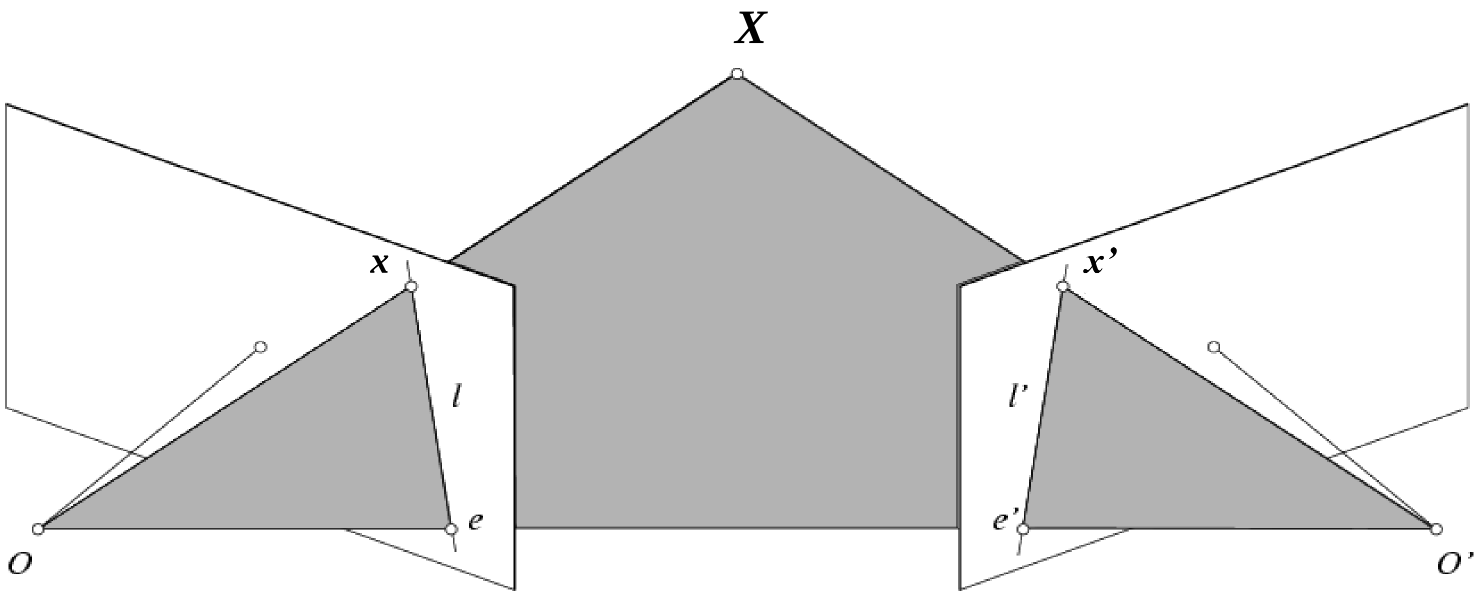






Epipole has same coordinates in both images.
Points move along lines radiating from e: “Focus of expansion”

Epipolar constraint: Calibrated case



Given the intrinsic parameters of the cameras:

1. Convert to normalized coordinates by pre-multiplying all points with the inverse of the calibration matrix; set first camera's coordinate system to world coordinates

$$\hat{x} = K^{-1} x = X$$

Homogeneous 2d point (3D ray towards X)

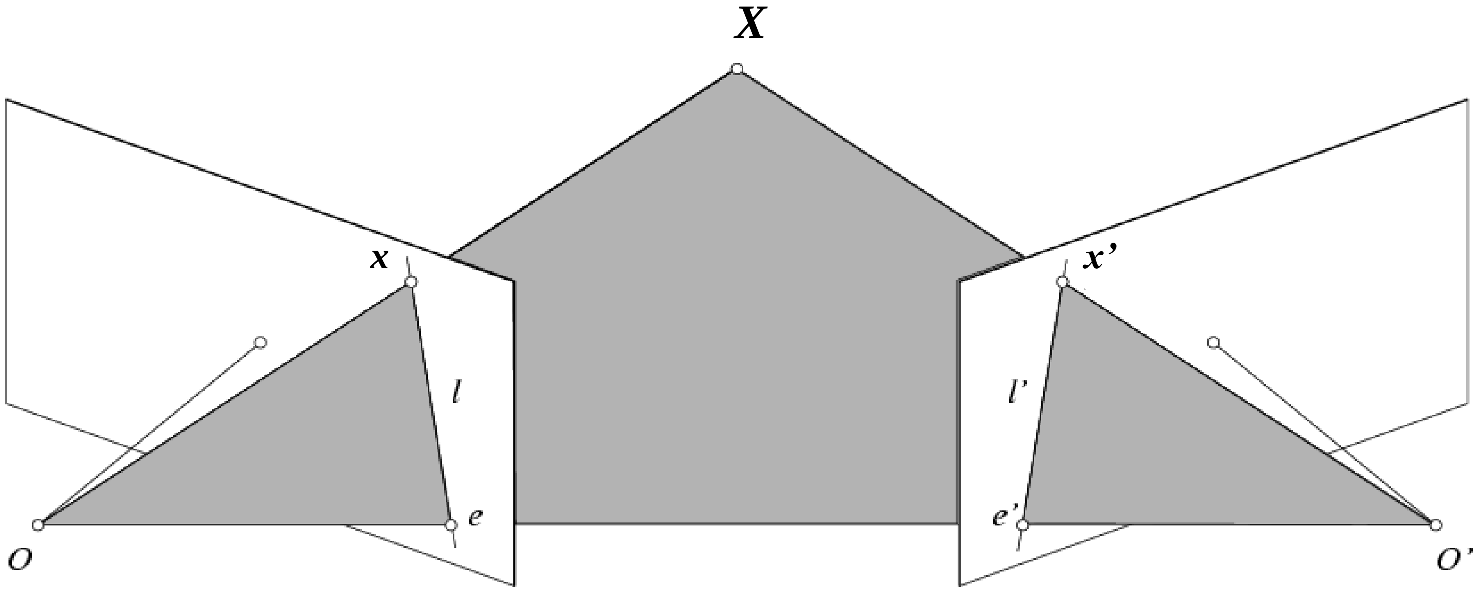
2D pixel coordinate (homogeneous)

3D scene point

$$\hat{x}' = K'^{-1} x' = X'$$

3D scene point in 2nd camera's 3D coordinates

Epipolar constraint: Calibrated case



Given the intrinsic parameters of the cameras:

1. Convert to normalized coordinates by pre-multiplying all points with the inverse of the calibration matrix; set first camera’s coordinate system to world coordinates
2. Define some *R* and *t* that relate X to X’ as below

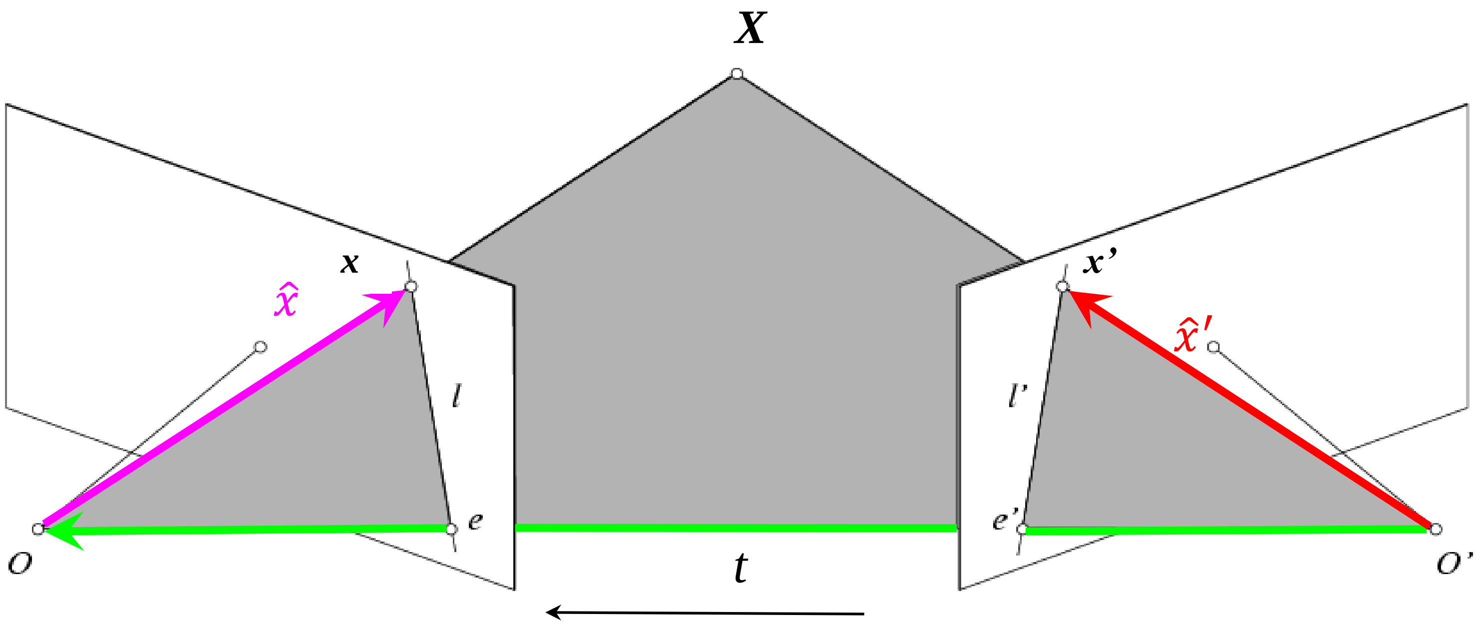
for some scale factor

$$\hat{x} = K^{-1}x = X$$

$$\hat{x}' = K'^{-1}x' = X'$$

$$\hat{x} = R\hat{x}' + t$$

Epipolar constraint: Calibrated case

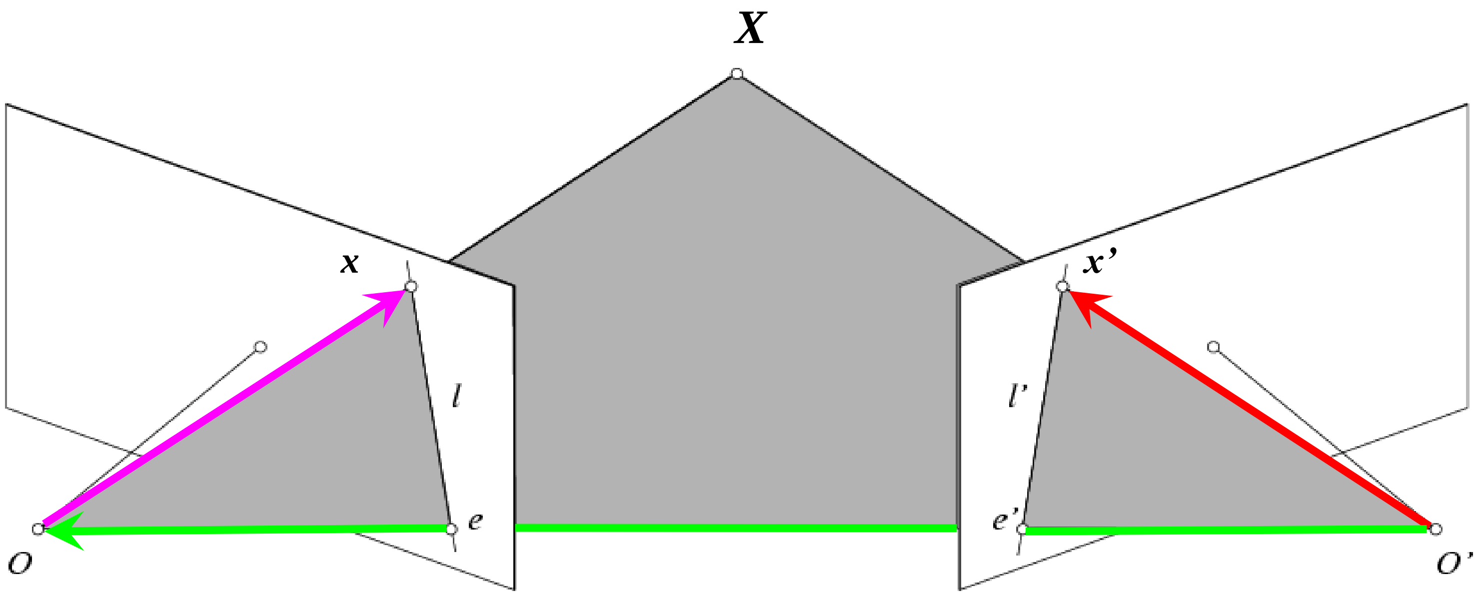


$$\hat{x} = K^{-1}x = X \qquad \hat{x}' = K'^{-1}x' = X'$$

$$\hat{x} = R\hat{x}' + t \quad \Rightarrow \quad \hat{x} \cdot [t \times (R\hat{x}')] = 0$$

(because $\hat{x}$, $R\hat{x}'$, and t are co-planar)

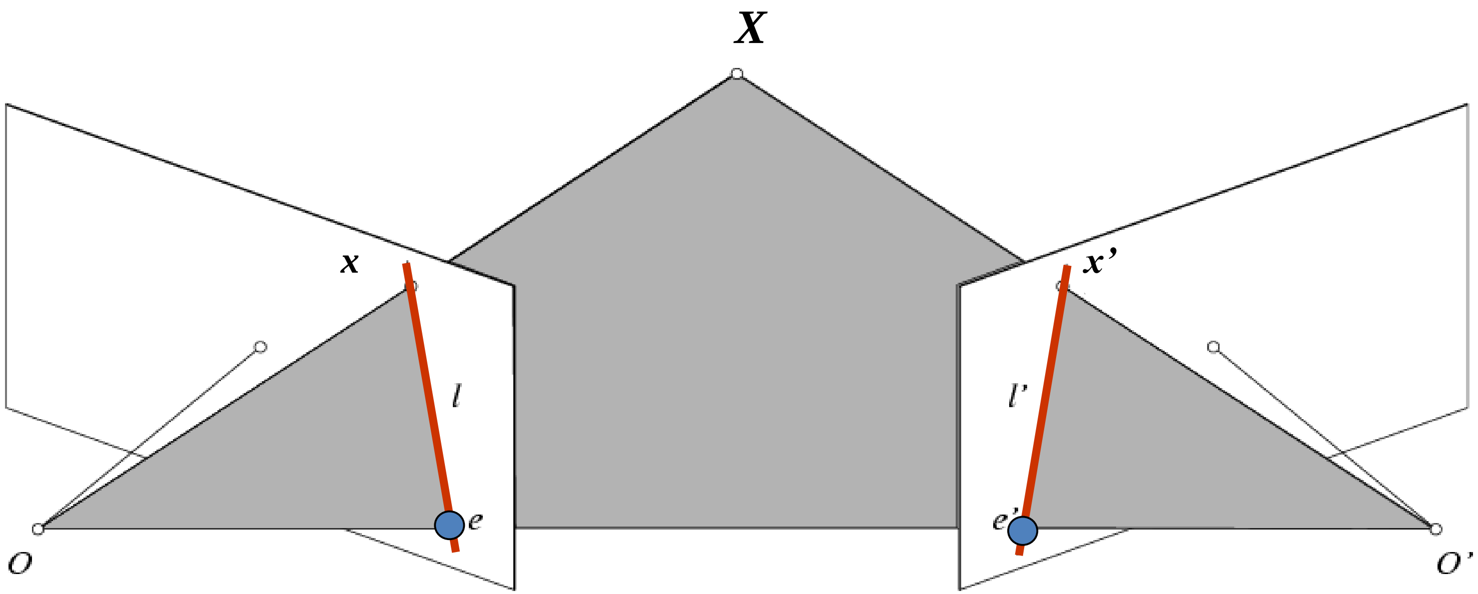
Essential matrix



$$\hat{x} \cdot [t \times (R\hat{x}')] = 0 \quad \Rightarrow \quad \hat{x}^T E \hat{x}' = 0 \quad \text{with} \quad E = [t]_\times R$$

Essential Matrix
(Longuet-Higgins, 1981)

Properties of the Essential matrix



$$\hat{x} \cdot [t \times (R\hat{x}')] = 0 \quad \Rightarrow \quad \hat{x}^T E \hat{x}' = 0 \quad \text{with} \quad E = [t]_{\times} R$$

Drop ^ below to simplify notation

Skew-symmetric matrix

- *E x'* is the epipolar line associated with *x'* (*l = E x'*)
- $E^T x$ is the epipolar line associated with *x* ($l' = E^T x$)
- *E e'* = 0 and $E^T e = 0$
- *E* is singular (rank two)
- *E* has five degrees of freedom
  - (3 for R, 2 for t because it's up to a scale)

The Fundamental Matrix

Without knowing K and K', we can define a similar relation using *unknown* normalized coordinates

$$\hat{x}^T E \hat{x}' = 0$$

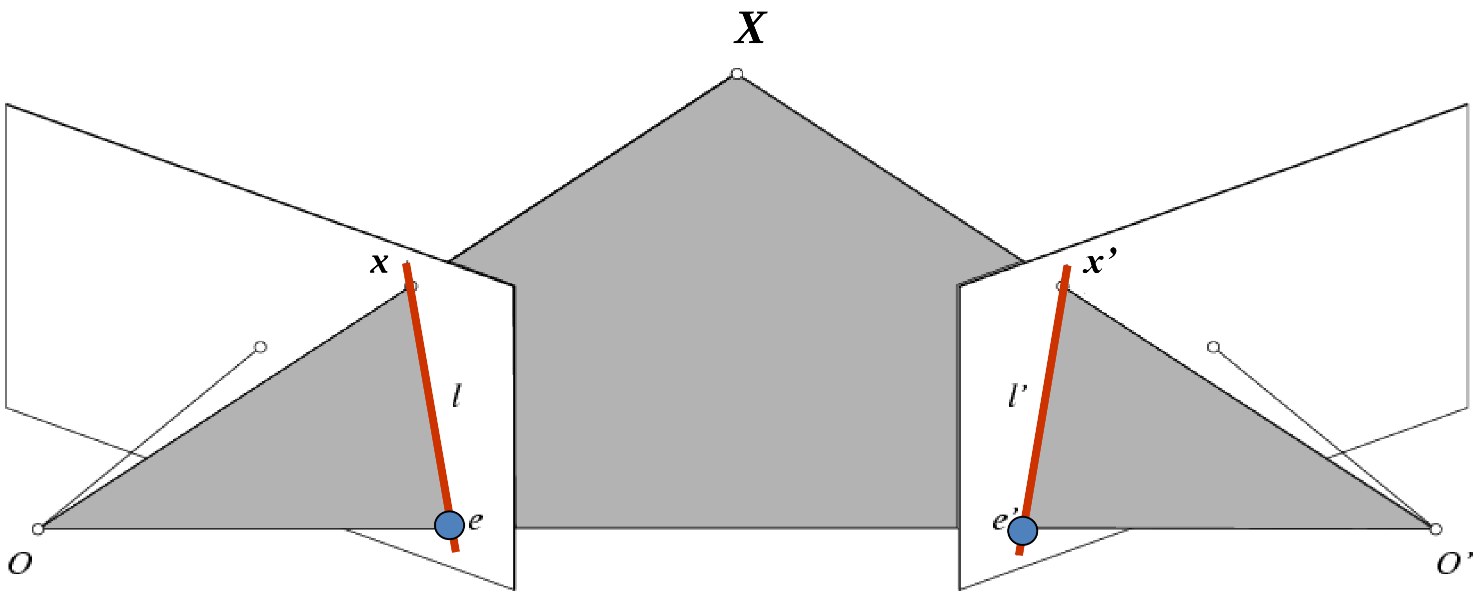
$$\hat{x} = K^{-1} x$$

$$\hat{x}' = K'^{-1} x'$$

$$x^T F x' = 0 \quad \text{with} \quad F = K^{-T} E K'^{-1}$$

Fundamental Matrix
(Faugeras and Luong, 1992)

Properties of the Fundamental matrix



$$x^T F x' = 0 \quad \text{with} \quad F = K^{-T} E K'^{-1}$$

- $F x' = 0$ is the epipolar line associated with x'
- $F^T x = 0$ is the epipolar line associated with x
- $F e' = 0$ and $F^T e = 0$
- F is singular (rank two): det(F)=0
- F has seven degrees of freedom: 9 entries but defined up to scale, det(F)=0

Estimating the Fundamental Matrix

- 8-point algorithm
  - Least squares solution using SVD on equations from 8 pairs of correspondences
  - Enforce det(F)=0 constraint using SVD on F
- 7-point algorithm
  - Use least squares to solve for null space (two vectors) using SVD and 7 pairs of correspondences
  - Solve for linear combination of null space vectors that satisfies det(F)=0
- Minimize reprojection error
  - Non-linear least squares

Note: estimation of F (or E) is degenerate for a planar scene.

8-point algorithm

1. Solve a system of homogeneous linear equations
    a. Write down the system of equations

$$\mathbf{x}^T F \mathbf{x}' = 0$$

$$uu'f_{11} + uv'f_{12} + uf_{13} + vu'f_{21} + vv'f_{22} + vf_{23} + u'f_{31} + v'f_{32} + f_{33} = 0$$

$$\mathrm{A}\boldsymbol{f} = \begin{bmatrix} u_1u_1' & u_1v_1' & u_1 & v_1u_1' & v_1v_1' & v_1 & u_1' & v_1' & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_nu_v' & u_nv_n' & u_n & v_nu_n' & v_nv_n' & v_n & u_n' & v_n' & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ \vdots \\ f_{33} \end{bmatrix} = \mathbf{0}$$

8-point algorithm

1. Solve a system of homogeneous linear equations
   a. Write down the system of equations
   b. Solve **f** from A**f**=**0** using SVD

Matlab:

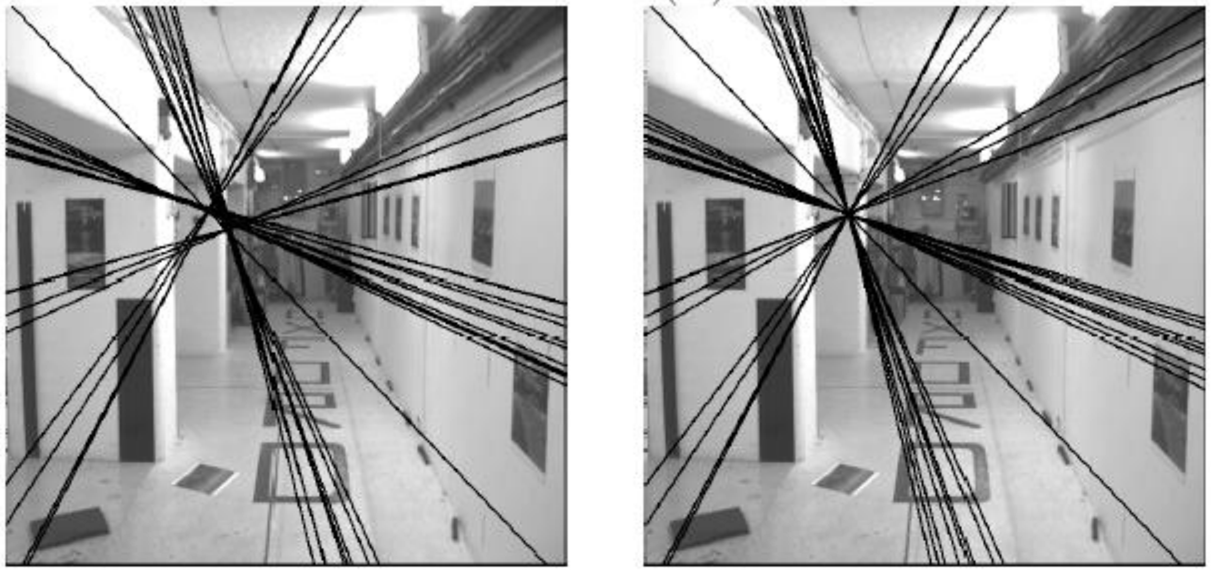
```
[U, S, V] = svd(A);
f = V(:, end);
F = reshape(f, [3 3])';
```

For python, see

numpy.linalg.svd

Need to enforce singularity constraint

Fundamental matrix has rank 2 : $\det(\mathbf{F}) = 0$.



Left : Uncorrected F – epipolar lines are not coincident.

Right : Epipolar lines from corrected F.

8-point algorithm

1. Solve a system of homogeneous linear equations
    a. Write down the system of equations
    b. Solve **f** from A**f**=**0** using SVD

Matlab:
```
[U, S, V] = svd(A);
f = V(:, end);
F = reshape(f, [3 3])';
```

2. Resolve det(F) = 0 constraint using SVD

Matlab:
```
[U, S, V] = svd(F);
S(3,3) = 0;
F = U*S*V';
```

For python, see

numpy.linalg.svd

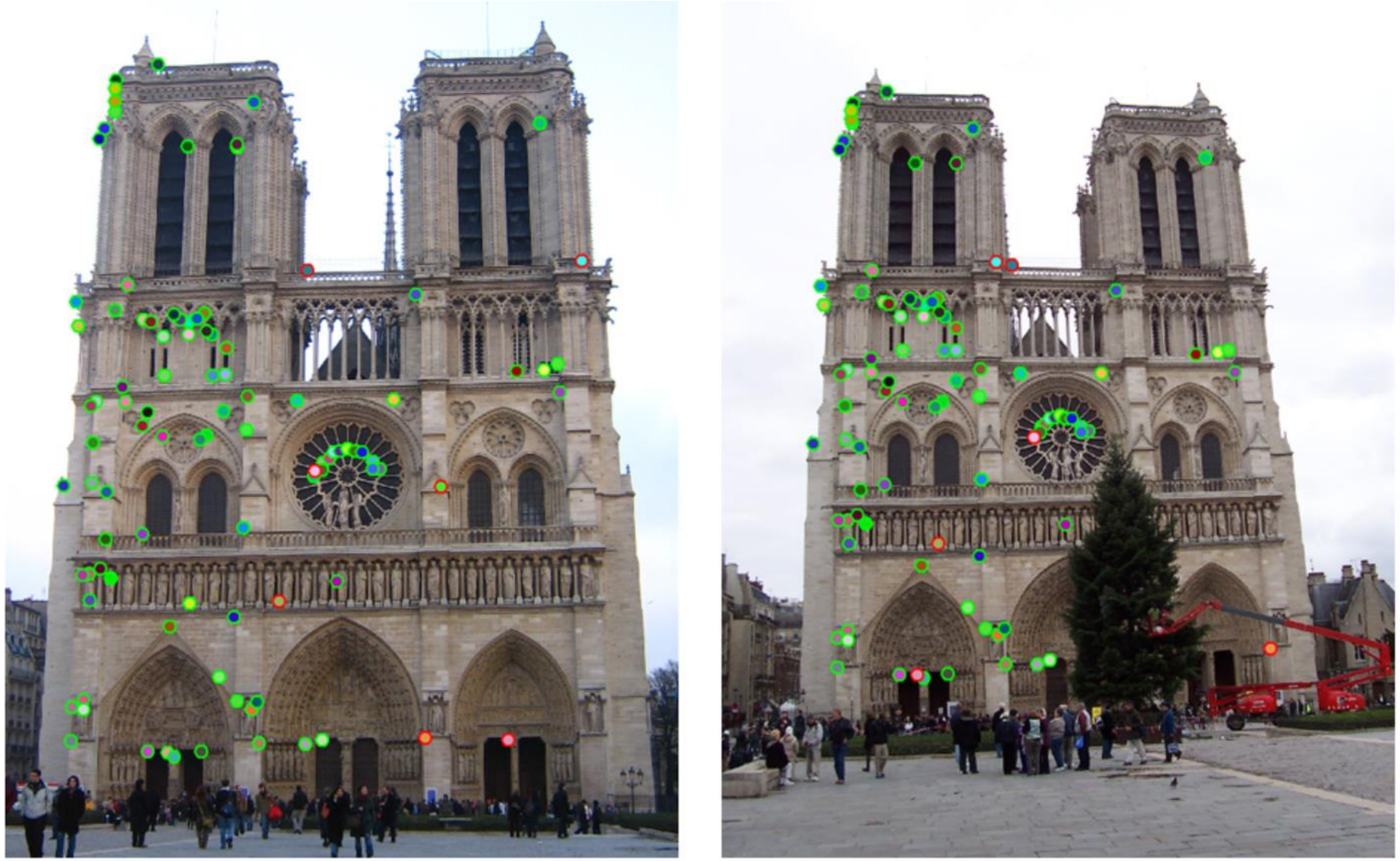
8-point algorithm

1. Solve a system of homogeneous linear equations
   a. Write down the system of equations
   b. Solve **f** from A**f**=**0** using SVD
2. Resolve det(F) = 0 constraint by SVD

Notes:

- Use RANSAC to deal with outliers (sample 8 points)
  – How to test for outliers?

How to test for outliers?



The top 100 most confident local feature matches from a baseline implementation of project 2. In this case, 93 were correct (highlighted in green) and 7 were incorrect (highlighted in red).

Project 2: Local Feature Matching

Problem with eight-point algorithm

$$\begin{bmatrix} u'u & u'v & u' & v'u & v'v & v' & u & v \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \end{bmatrix} = -1$$

Problem with eight-point algorithm

250906.36	183269.57	921.81	200931.10	146766.13	738.21	272.19	198.81
2692.28	131633.03	176.27	6196.73	302975.59	405.71	15.27	746.79
416374.23	871684.30	935.47	408110.89	854384.92	916.90	445.10	931.81
191183.60	171759.40	410.27	416435.62	374125.90	893.65	465.99	418.65
48988.86	30401.76	57.89	298604.57	185309.58	352.87	846.22	525.15
164786.04	546559.67	813.17	1998.37	6628.15	9.86	202.65	672.14
116407.01	2727.75	138.89	169941.27	3982.21	202.77	838.12	19.64
135384.58	75411.13	198.72	411350.03	229127.78	603.79	681.28	379.48

$$\begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \end{bmatrix} = -1$$

Poor numerical conditioning

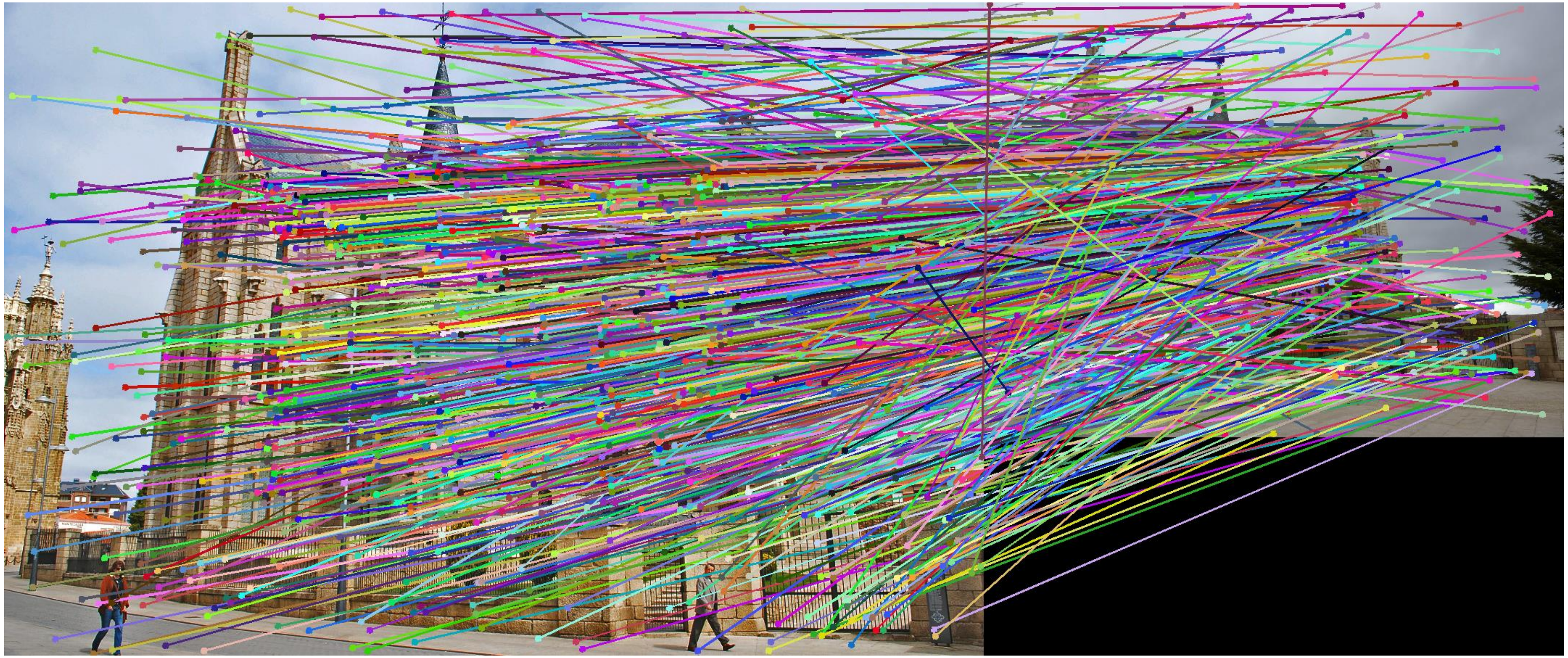
Can be fixed by rescaling the data

The normalized eight-point algorithm

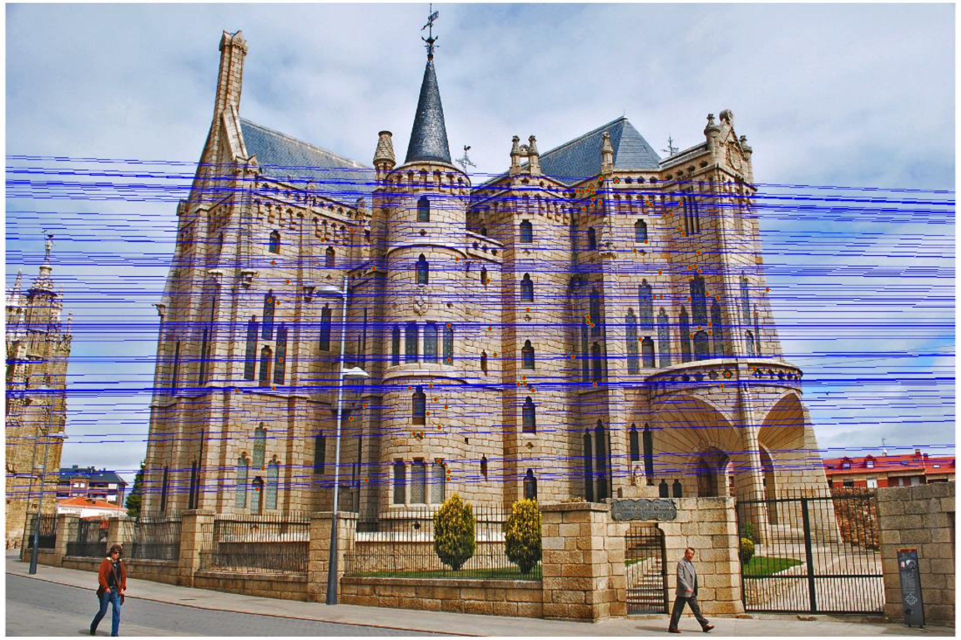
(Hartley, 1995)

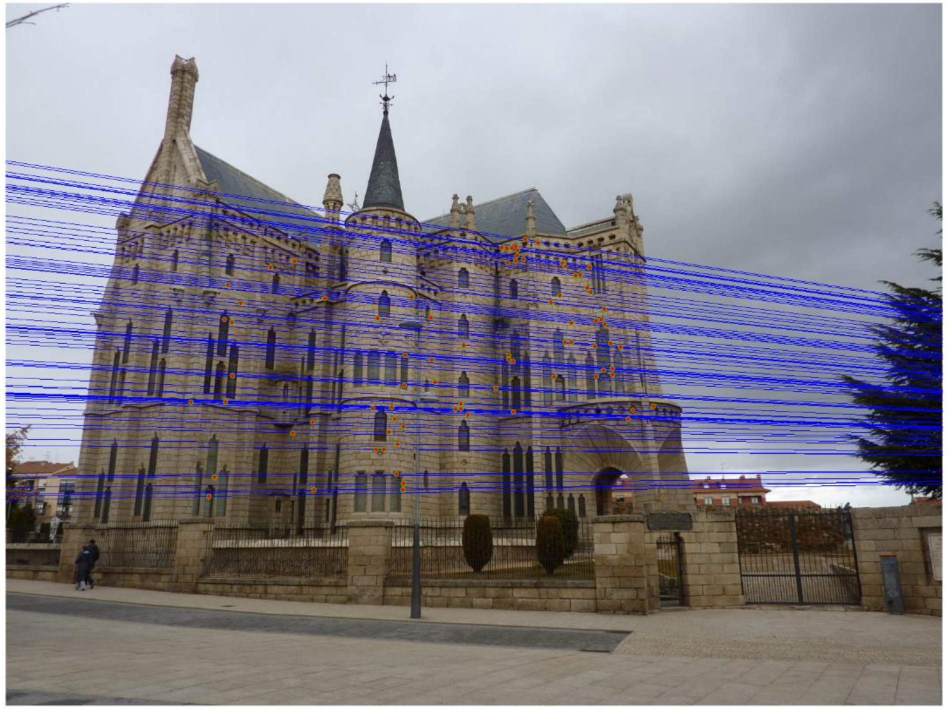
- Center the image data at the origin, and scale it so the mean squared distance between the origin and the data points is 2 pixels
- Use the eight-point algorithm to compute ***F*** from the normalized points
- Enforce the rank-2 constraint (for example, take SVD of ***F*** and throw out the smallest singular value)
- Transform fundamental matrix back to original units: if ***T*** and ***T'*** are the normalizing transformations in the two images, than the fundamental matrix in original coordinates is $T'^T F T$

VLFeat's 800 most confident matches among 10,000+ local features.

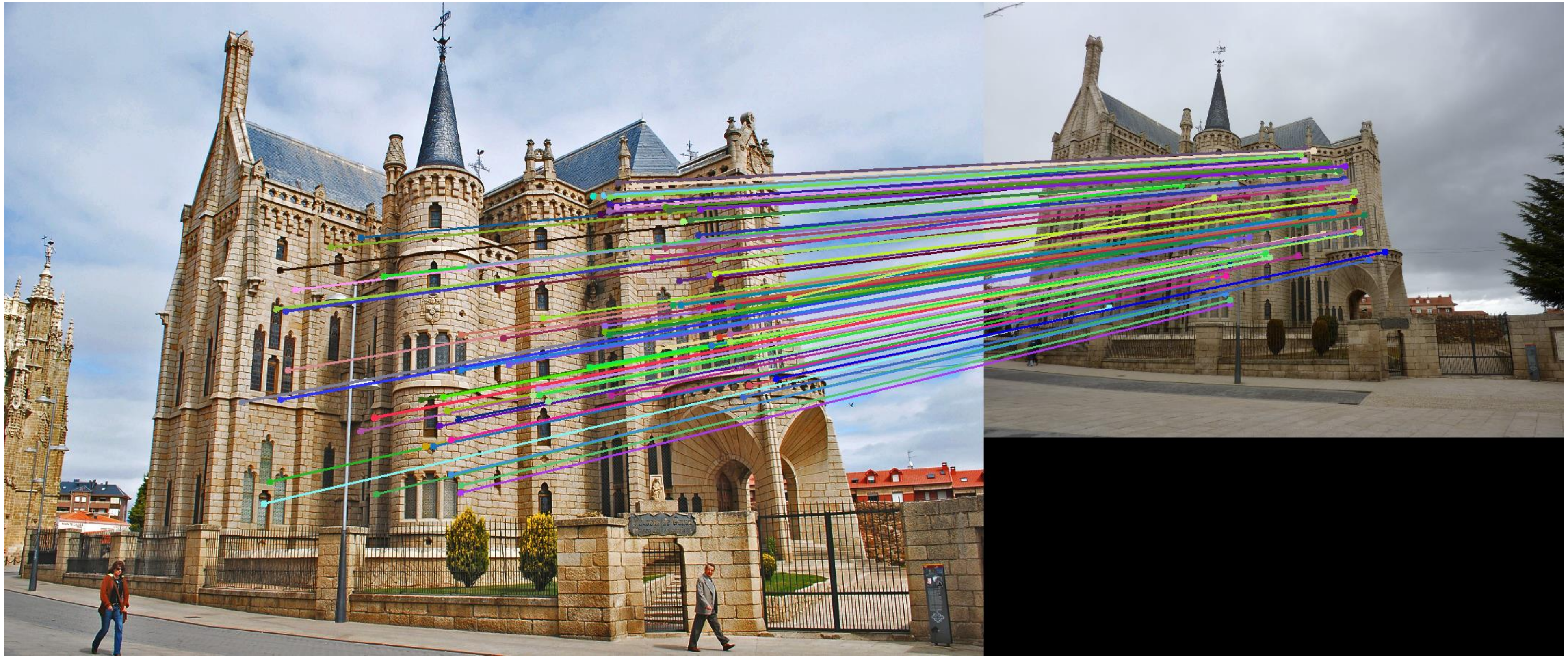


Epipolar lines





Keep only the matches at are “inliers” with respect to the “best” fundamental matrix



The scale of algorithm name quality

better

RANSAC

SIFT

Deep Learning

Optical Flow

Hough Transform

Neural Networks

Essential and Fundamental Matrix

Dynamic Programming

worse

In class written Quiz format

- 15 to 20 short answer or multiple choice questions
- Typically can be done in half an hour
- No calculators needed
- Closed book
- Only covers material discussed in class, not book. But the book is still a useful resource
- Covers all material through the quiz date