Subspace Methods for Visual Learning and Recognition

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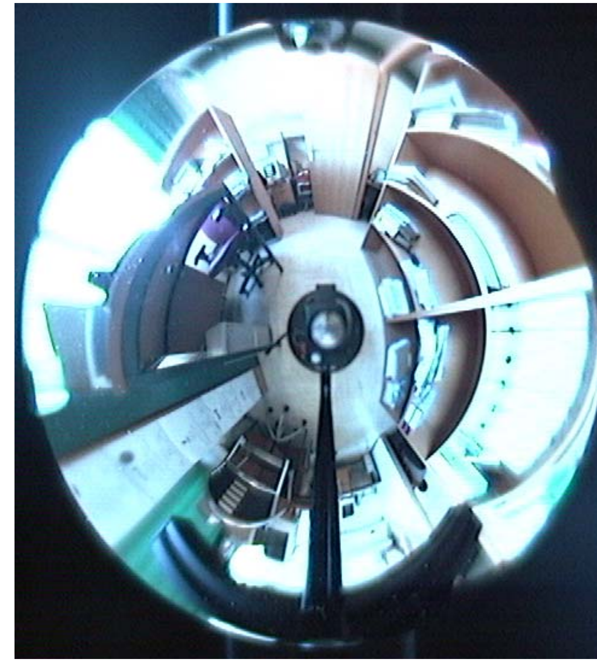
Outline Part 1

- Motivation
- Appearance based learning and recognition
- Subspace methods for visual object recognition
- Principal Components Analysis (PCA)
- Linear Discriminant Analysis (LDA)
- Canonical Correlation Analysis (CCA)
- Independent Component Analysis (ICA)
- Non-negative Matrix Factorization (NMF)
- Kernel methods for non-linear subspaces

Outline Part 2

- Robot localization
- Robust representations and recognition
- Robust PCA recognition
- Scale invariant recognition using PCA
- Illumination insensitive recognition
- Representations for panoramic images
- Incremental building of eigenspaces
- Multiple eigenspaces for efficient representation
- Robust building of eigenspaces
- Research issues

The name of the game



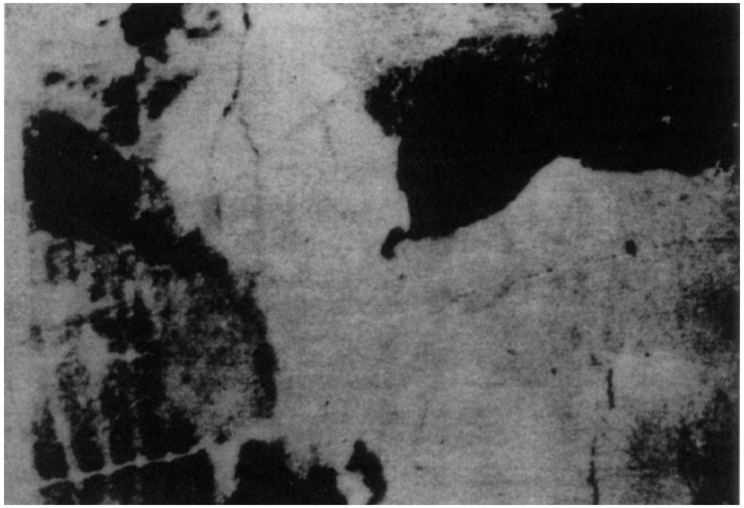
- complex objects/scenes
- varying pose (3D rotation, scale)
- cluttered background/foreground
- occlusions (noise)
- varying illumination

Object Representation

- High-Level Shape Models (e.g., Generalized Cylinders)
  - Idealized Images
  - Texture Less
- Mid-Level Shape Models (e.g. CAD models, Superquadrics)
  - More Complex
  - Well-defined geometry
- Low-level Appearance Based Models (e.g. Eigenspaces)
  - Most complex
  - Complicated shapes

Problems

Segmentation:



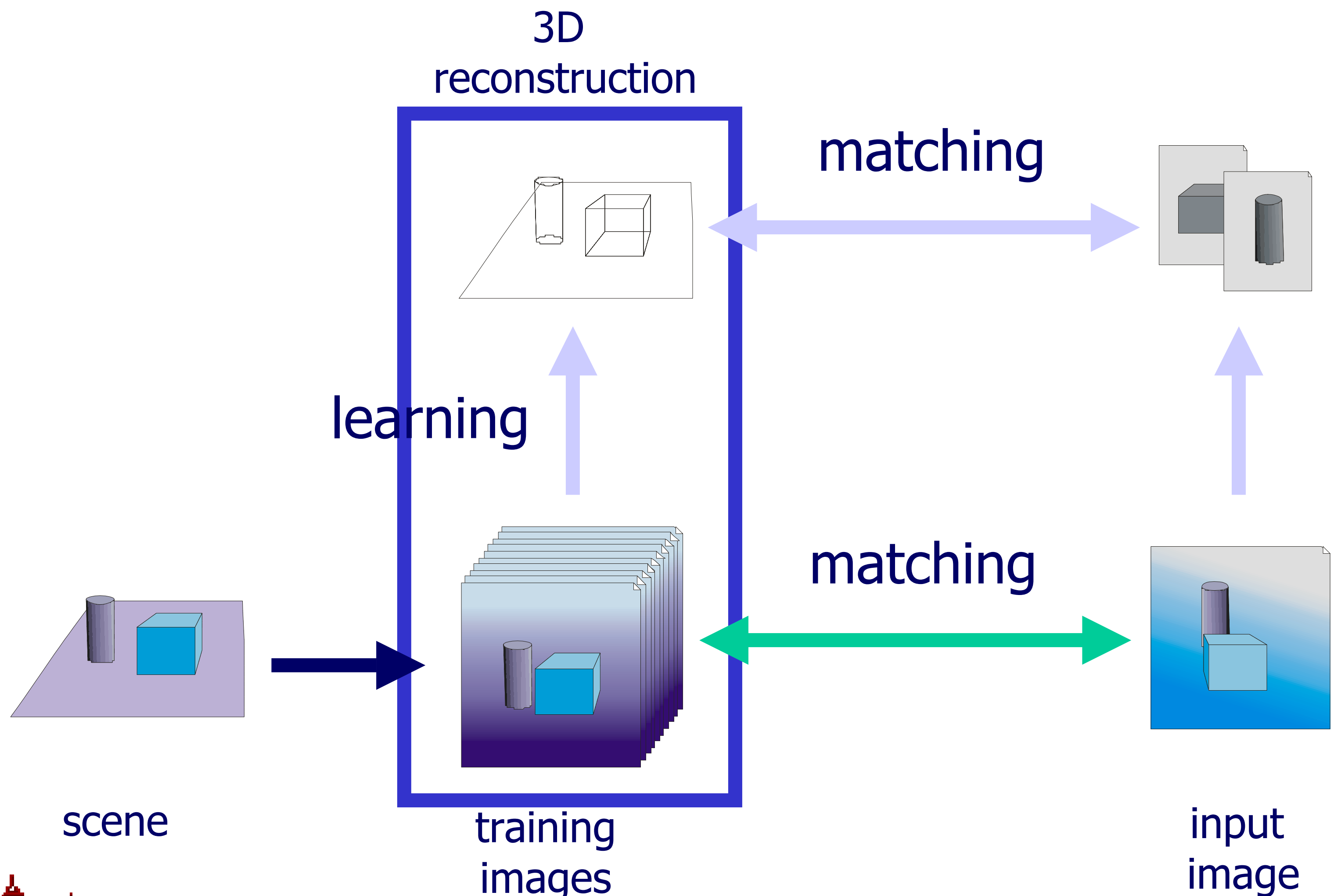
Pose/Shape:

a

Illumination

Example

Learning and recognition



Appearance-based approaches

A renewed attention in the appearance-based approaches

Encompass combined effects of:

- shape,
- reflectance properties,
- pose in the scene,
- illumination conditions.

Acquired through an automatic learning phase.

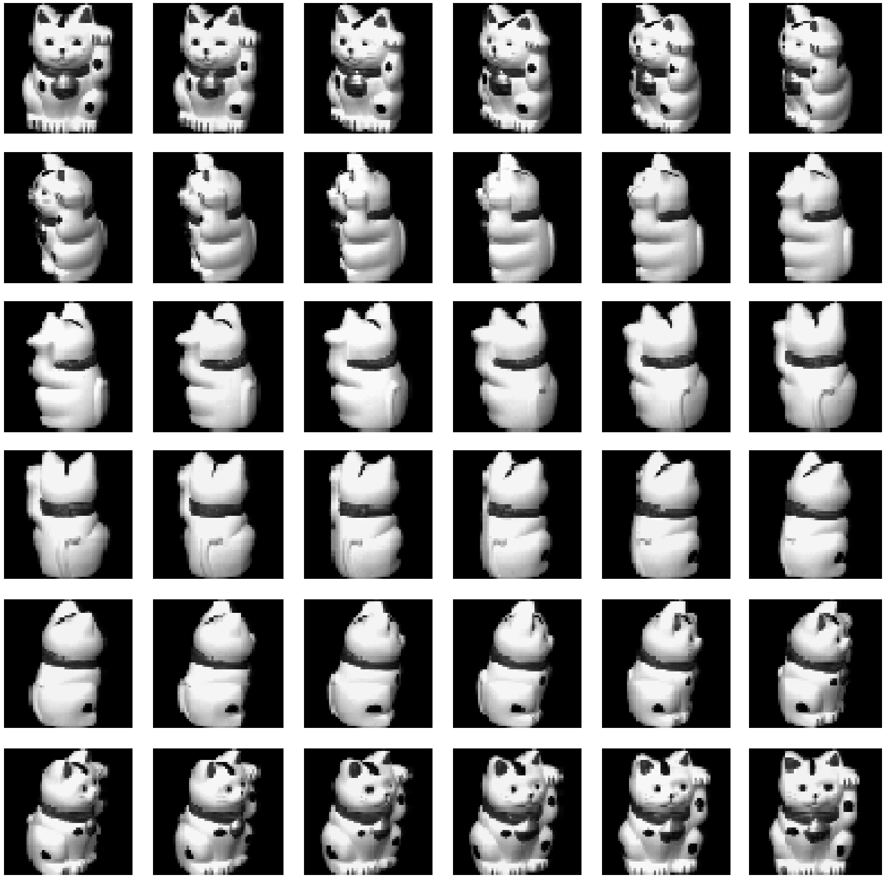
Appearance-based approaches

A variety of successful applications:

- Human face recognition e.g. [Beymer & Poggio, Turk & Pentland]
- Visual inspection e.g. [Yoshimura & Kanade]
- Visual positioning and tracking of robot manipulators, e.g. [Nayar & Murase]
- Tracking e.g., [Black & Jepson]
- Illumination planning e.g., [Murase & Nayar]
- Image spotting e.g., [Murase & Nayar]
- Mobile robot localization e.g., [Jogan & Leonardis]
- Background modeling e.g., [Oliver, Rosario & Pentland]

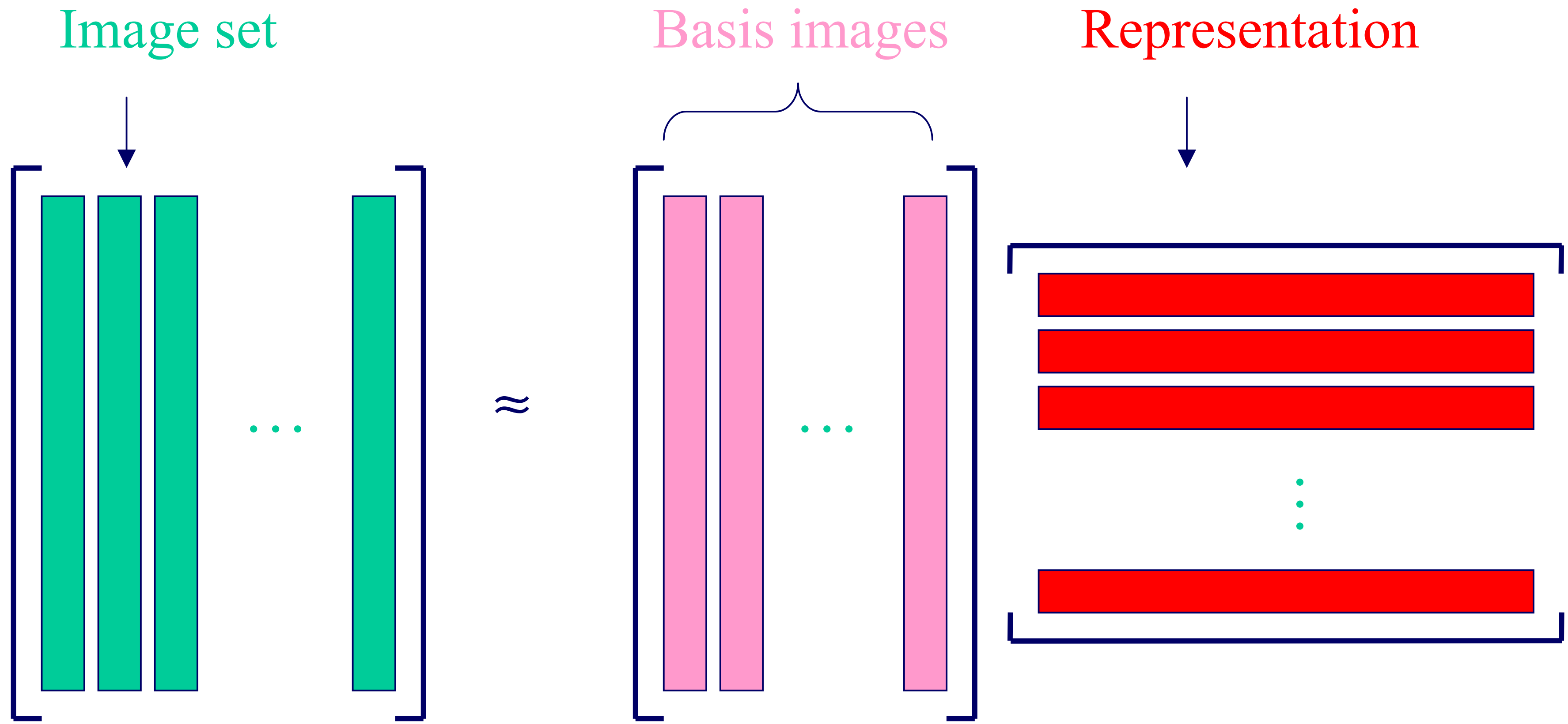
Appearance-based approaches

Objects are represented by a large number of views:



Subspace Methods

- Images are represented as points in the N-dimensional vector space
- Set of images populate only a small fraction of the space
- Characterize subspace spanned by images

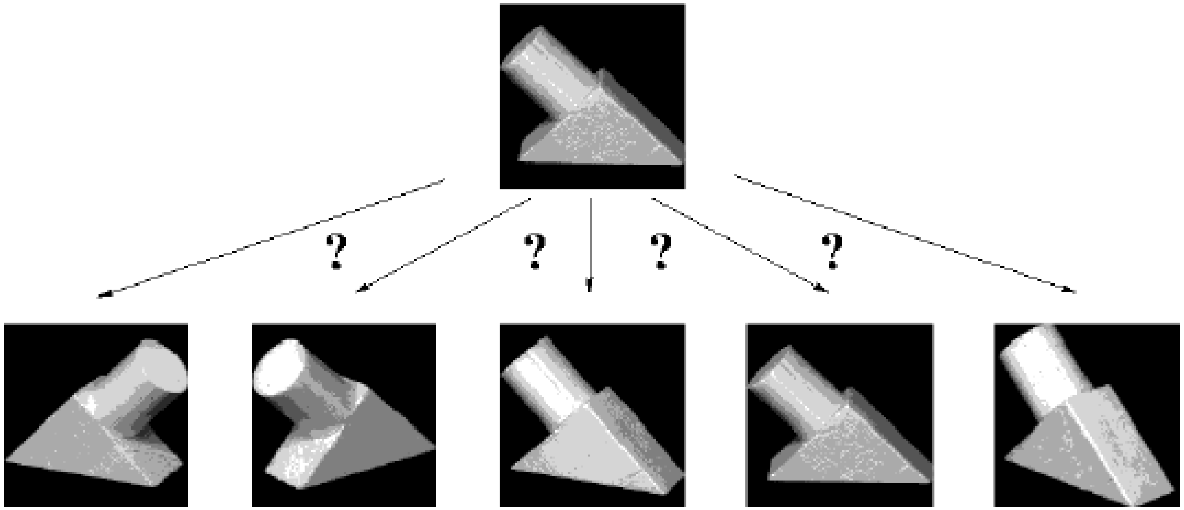


Subspace Methods

Properties of the representation:

- Optimal Reconstruction $\Rightarrow$ PCA
- Optimal Separation $\Rightarrow$ LDA
- Optimal Correlation $\Rightarrow$ CCA
- Independent Factors $\Rightarrow$ ICA
- Non-negative Factors $\Rightarrow$ NMF
- Non-linear Extension $\Rightarrow$ Kernel Methods

Image Matching



$$\rho = \frac{\mathbf{x}^T \mathbf{y}}{\|\mathbf{x}\| \|\mathbf{y}\|} > \Theta$$

Normalized images $\quad \|\mathbf{x} - \mathbf{y}\|^2 < \Psi$

$\Rightarrow$ Compress images

Eigenspace representation

- Image set (normalised, zero-mean)

$$X = [\ \mathbf{x}_0 \quad \mathbf{x}_1 \quad \ldots \quad \mathbf{x}_{n-1}\ ];\ \ X \in \mathbb{R}^{m \times n}$$

- We are looking for orthonormal basis functions:

$$U = [\ \mathbf{u}_0 \quad \mathbf{u}_1 \quad \ldots \quad \mathbf{u}_k\ ];\ \ k << n$$

- Individual image is a linear combination of basis functions

$$\mathbf{x}_i \approx \tilde{\mathbf{x}}_i = \sum_{j=0}^{p} q_j(\mathbf{x}_i)\mathbf{u}_j$$

$$\| \mathbf{x} - \mathbf{y} \|^2 \approx \| \sum_{j=1}^{k} q_j(\mathbf{x})\mathbf{u}_j - \sum_{j=1}^{k} q_j(\mathbf{y})\mathbf{u}_j \|^2 =$$

$$\| \sum_{j=1}^{k} (q_j(\mathbf{x}) - q_j(\mathbf{y}))\mathbf{u}_j \|^2 = \| q_j(\mathbf{x}) - q_j(\mathbf{y}) \|^2$$

Best basis functions ν?

- **Optimisation problem**

$$\sum_{i=0}^{n-1} ||\mathbf{x}_i - \sum_{j=0}^{k} q_j(\mathbf{x}_i)\mathbf{u}_j||^2 \rightarrow \min$$

- **Taking the *k* eigenvectors with the largest eigenvalues of**

$$C = XX^T = \begin{bmatrix} \mathbf{x}_0 & \mathbf{x}_1 & \dots & \mathbf{x}_{n-1} \end{bmatrix} \begin{bmatrix} \mathbf{x}_0^\top \\ \mathbf{x}_1^\top \\ \dots \\ \mathbf{x}_{n-1}^\top \end{bmatrix}$$

- **PCA or Karhunen-Loéve Transform (KLT)**

$$C\mathbf{u}_i = \lambda_i \mathbf{u}_i$$

Efficient eigenspace computation

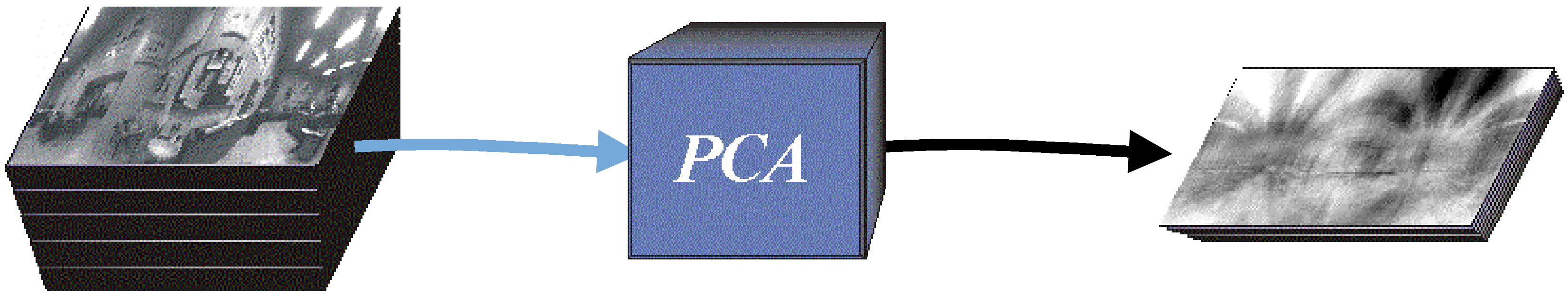
- *n* << *m*
- Compute the eigenvectors u'_i, $i = 0,\dots,n-1$, of the inner product matrix

$$Q = X^\top X = \begin{bmatrix} \mathbf{x}_0^\top \\ \mathbf{x}_1^\top \\ \dots \\ \mathbf{x}_{n-1}^\top \end{bmatrix} \begin{bmatrix} \mathbf{x}_0 & \mathbf{x}_1 & \dots & \mathbf{x}_{n-1} \end{bmatrix}; \quad Q \in \mathbb{R}^{n \times n}$$

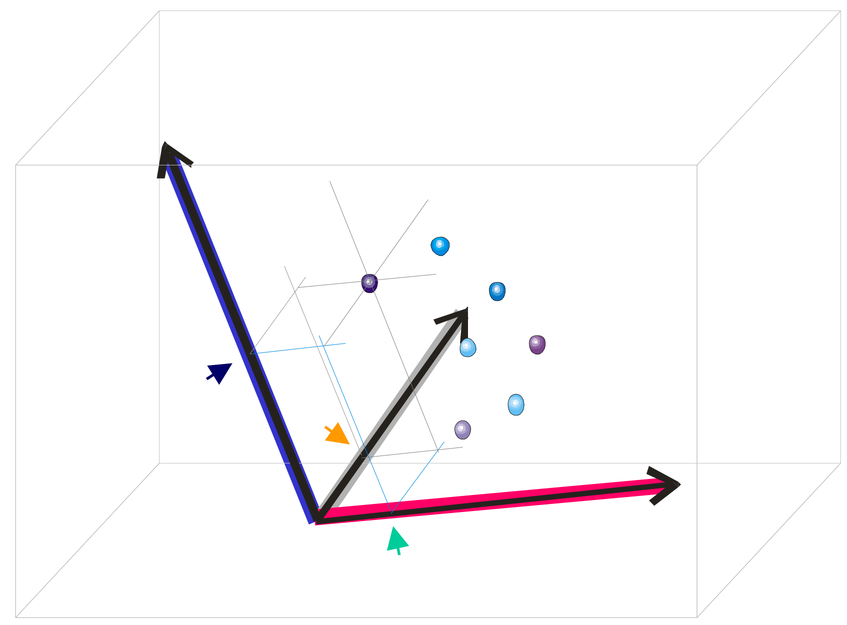
- The eigenvectors of $XX^\top$ can be obtained by using $XX^\top X v_i' = \lambda'_i X v_i'$:

$$\mathbf{u}_i = \frac{1}{\sqrt{\lambda'_i}} X \mathbf{u}'_i$$

Principal Component Analysis



Principal Component Analysis



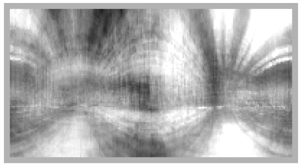
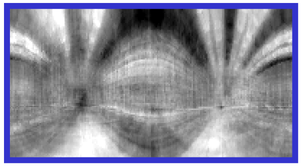
 $=q_1\cdot$  $+\,q_2\cdot$  $+\,q_3\cdot$  $+\ldots$

Image representation with PCA

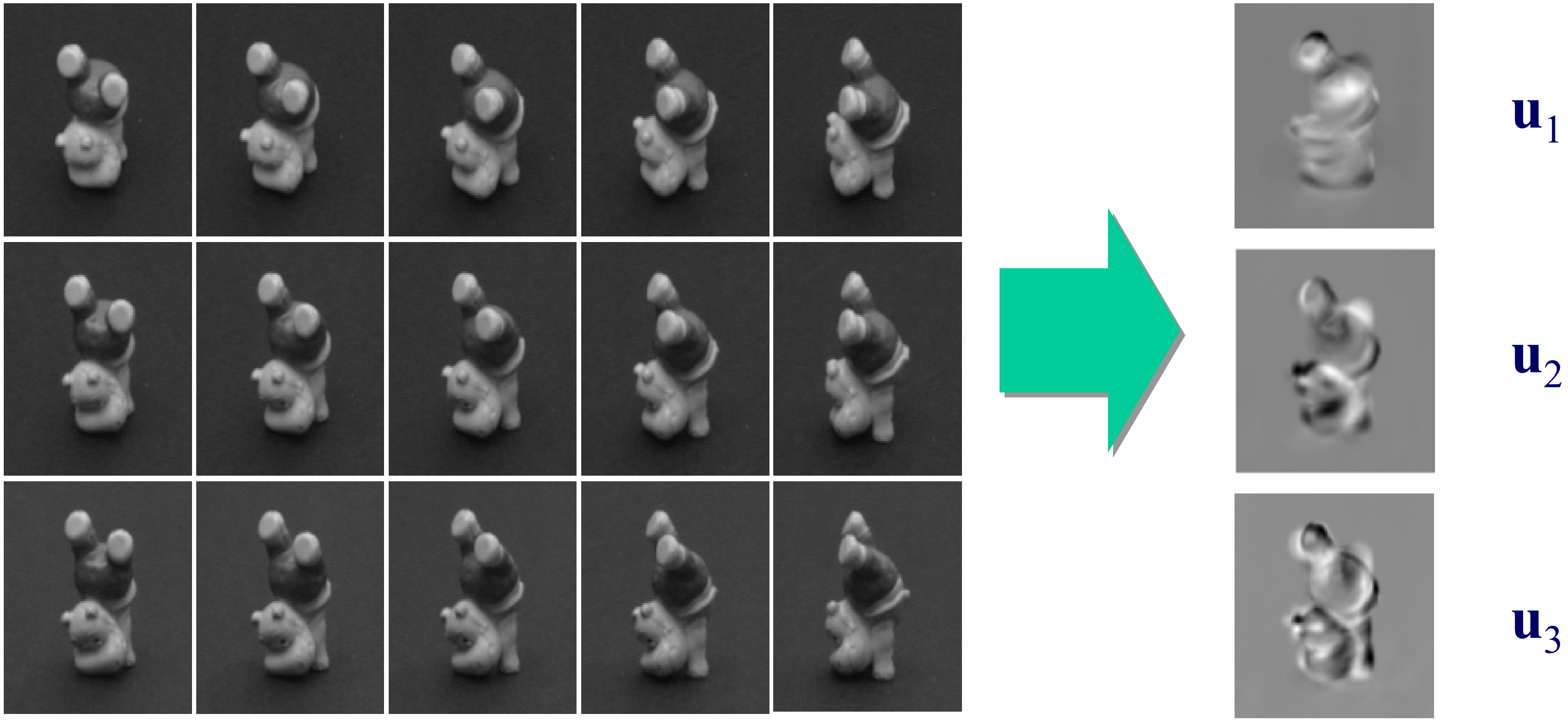
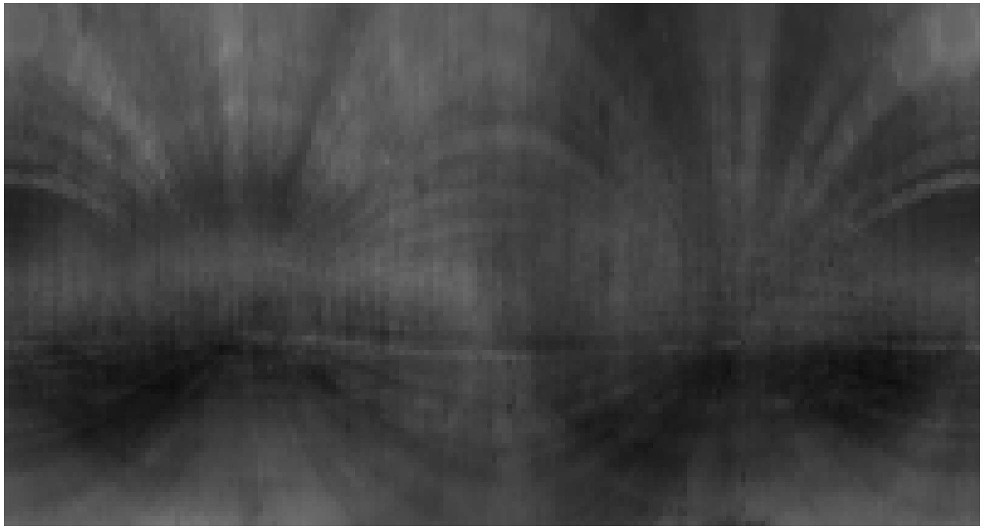


Image presentation with PCA



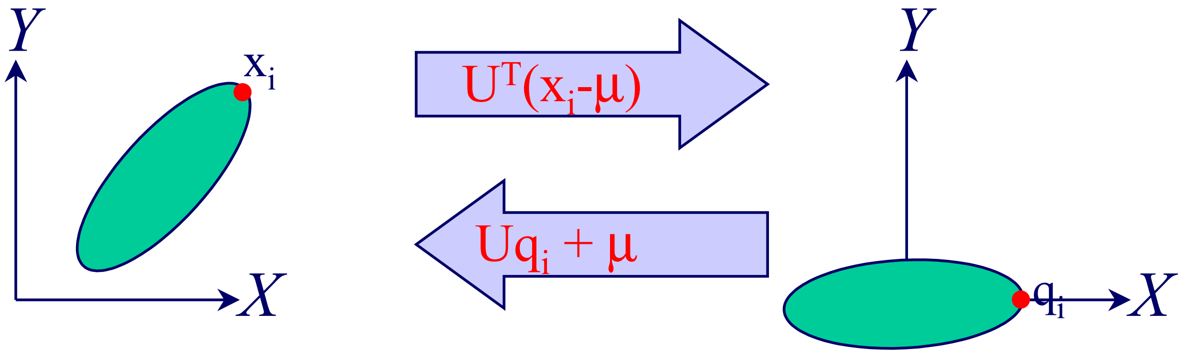
Properties PCA

- Any point x_i can be projected to an appropriate point q_i by :

$$q_i = U^T(x_i - \mu)$$

- and conversely (since $U^{-1} = U^T$)

$$Uq_i + \mu = x_i$$



Properties PCA

- It can be shown that the mean square error between x_i and its reconstruction using only m principle eigenvectors is given by the expression :

$$\boxed{\sum_{j=1}^{N} \lambda_j - \sum_{j=1}^{m} \lambda_j = \sum_{j=m+1}^{N} \lambda_j}$$

- PCA minimizes reconstruction error

- PCA maximizes variance of projection

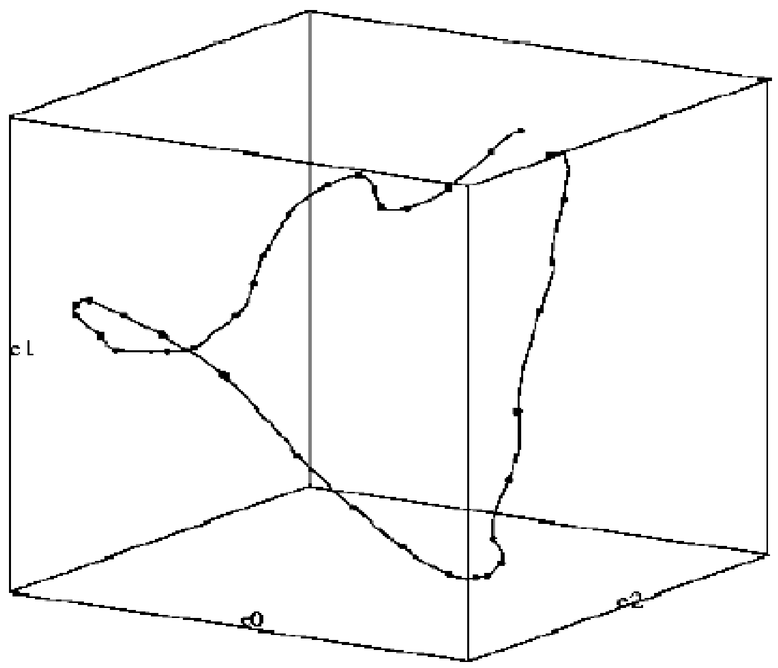
- Finds a more “natural” coordinate system for the sample data.

PCA for visual recognition and pose estimation

Objects are represented as coordinates in an n-dimensional eigenspace.

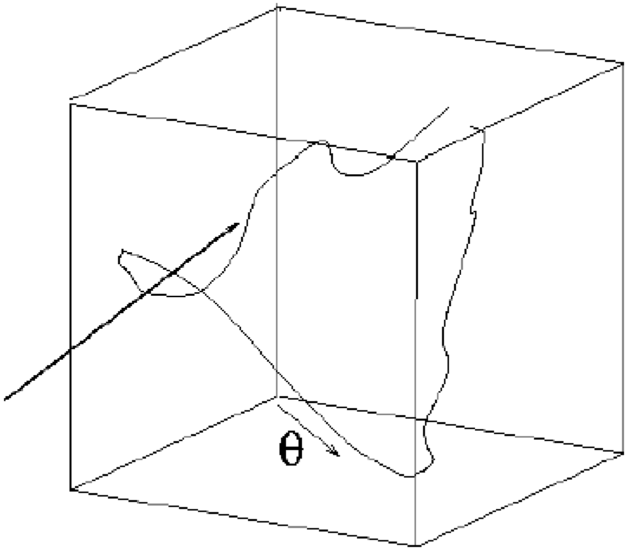
An example:

> 3-D space with points representing individual objects or a manifold representing **parametric eigenspace** (e.g., orientation, pose, illumination).



PCA for visual recognition and pose estimation

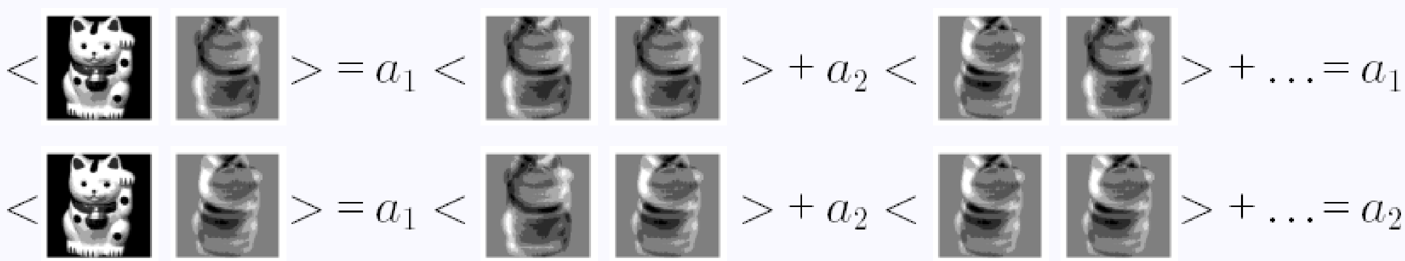
- Calculate coefficients
- Search for the nearest point (individual or on the curve)
- Point determines object and/or pose



Calculation of coefficients

To recover a_i the image is projected onto the eigenspace

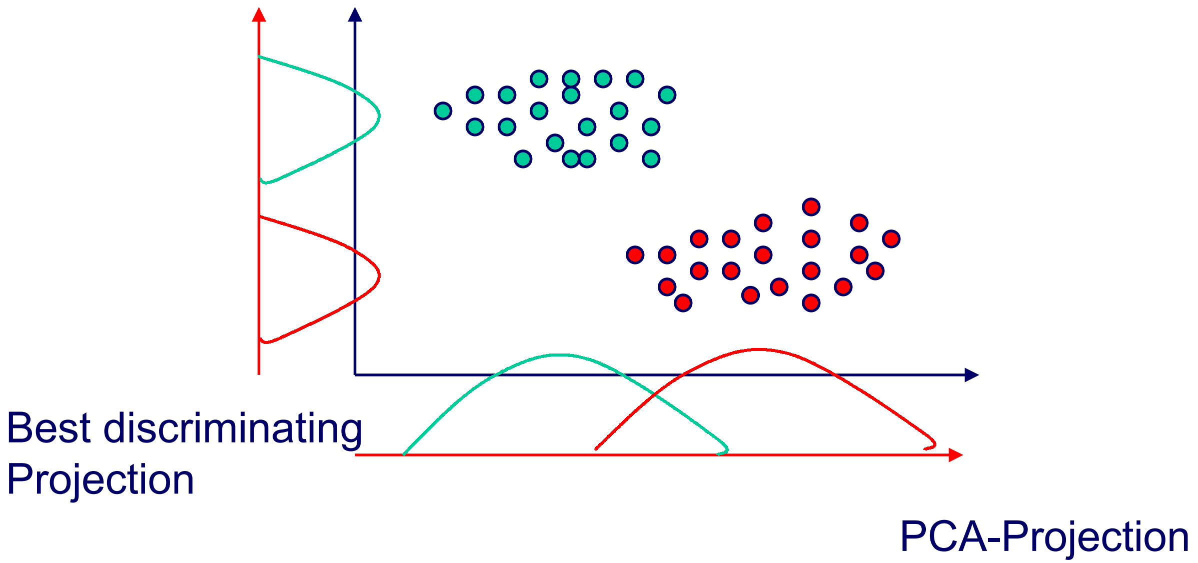
$$a_i(\mathbf{x}) = < \mathbf{x}, \mathbf{e}_i > = \sum_{j=1}^{m} x_j e_{i_j} \qquad 1 \leq i \leq p$$



- Complete image x_i is required to calculate a_i.
- Corresponds to Least-Squares Solution

Linear Discriminant Analysis (LDA)

- **PCA minimizes projection error**



- **PCA is „unsupervised“ no information on classes is used**
- **Discriminating information might be lost**

- **Linear Discriminance Analysis (LDA)**

  – Maximize distance between classes
  – Minimize distance within a class

⇨ Fisher Linear Discriminance

$$\rho(\mathbf{w}) = \frac{\mathbf{w}^T \mathbf{S_B} \mathbf{w}}{\mathbf{w}^T \mathbf{S_W} \mathbf{w}}$$

LDA: Problem formulation

- **n Sample images:** $\{x_1, \cdots, x_n\}$
- **c classes:** $\{\chi_1, \cdots, \chi_c\}$
- **Average of each class:**
$$\mu_i = \frac{1}{n_i} \sum_{x_k \in \chi_i} x_k$$
- **Total average:**
$$\mu = \frac{1}{n} \sum_{k=1}^{N} x_k$$

LDA: Practice

- **Scatter of class i:**

$$S_i = \sum_{x_k \in \chi_i} (x_k - \mu_i)(x_k - \mu_i)^T$$

- **Within class scatter:**

$$S_W = \sum_{i=1}^{c} S_i$$

- **Between class scatter:**

$$S_B = \sum_{i=1}^{c} |\chi_i| (\mu_i - \mu)(\mu_i - \mu)^T$$

- **Total scatter:**

$$S_T = S_W + S_B$$

LDA: Practice

- **After projection:**

$$y_k = W^T x_k$$

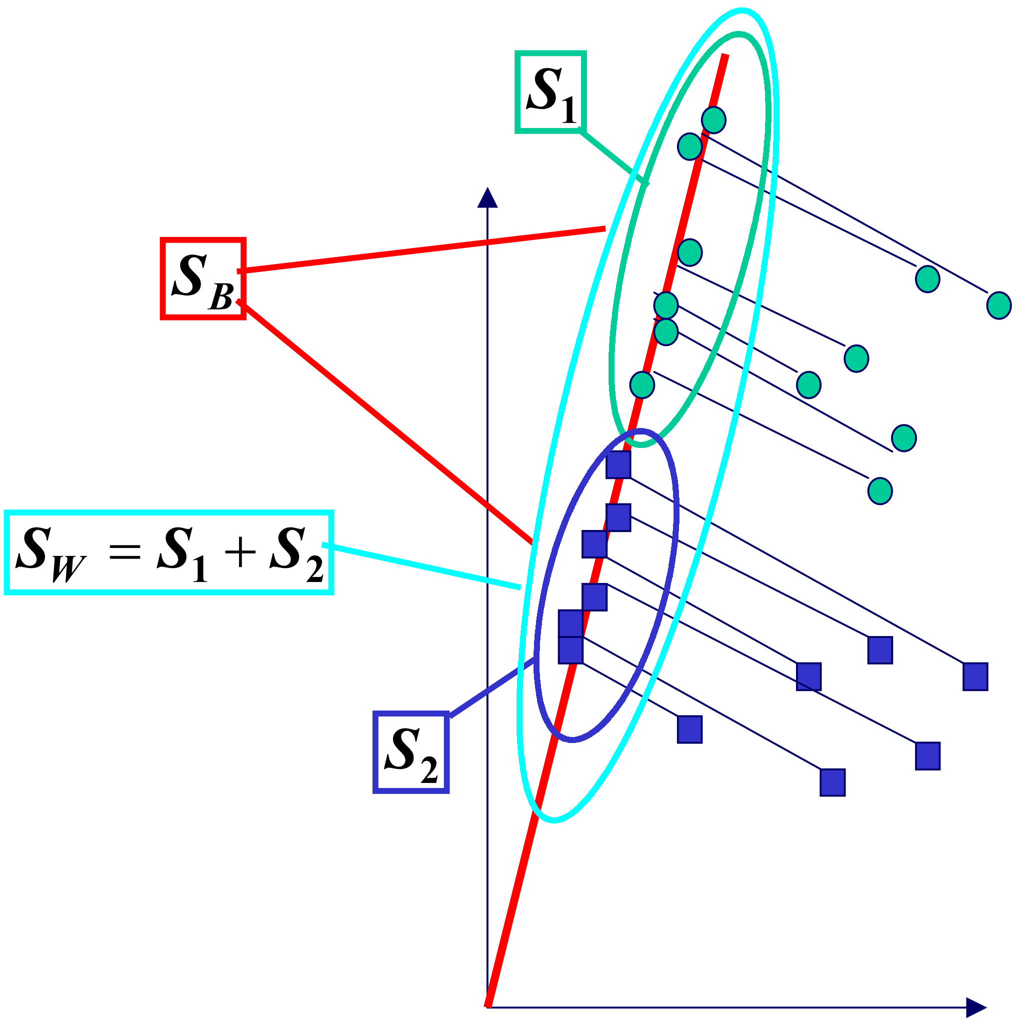
  - Between class scatter (of y's):

$$\tilde{S}_B = W^T S_B W$$

  - Within class scatter (of y's):

$$\tilde{S}_W = W^T S_W W$$

LDA



Good separation

- Maximization of

$$\rho(\mathbf{w}) = \frac{\mathbf{w}^T \mathbf{S_B} \mathbf{w}}{\mathbf{w}^T \mathbf{S_W} \mathbf{w}}$$

- is given by solution of generalized eigenvalue problem

$$\mathbf{S_B}\mathbf{w} = \lambda \mathbf{S_w}\mathbf{w}$$

- For the c-class case we obtain (at most) c-1 projections as the largest eigenvalues of

$$\mathbf{S_B}\mathbf{w}_i = \lambda \mathbf{S_w}\mathbf{w}_i$$

LDA

- **How to calculate LDA for high-dimensional images?**

- **Problem: S_W is always singular**
  - Number of pixels in each image is larger than the number of images in the training set

1. **Fischerfaces → Reduce dimension by PCA and then perform LDA**

2. **Simultaneous diagonalization of S_W and S_B**

LDA

- Fischerfaces (Belhumeur et.al. 1997)

- Reduce dimensionality to n-c with PCA

$$\mathbf{U}_{pca} = \arg\max_{U} |\mathbf{U}^{\mathbf{T}}\mathbf{Q}\mathbf{U}| = [\mathbf{u}_1\ \mathbf{u}_2\ \ldots\ \mathbf{u}_{\mathbf{n-c}}]$$

- Further reduce to c-1 with FLD

$$\mathbf{W}_{fld} = \arg\max_{\mathbf{W}} \frac{|\mathbf{W}^{\mathbf{T}}\mathbf{W}_{\mathbf{pca}}^{\mathbf{T}}\mathbf{S}_{\mathbf{B}}\mathbf{W}_{\mathbf{pca}}\mathbf{W}|}{|\mathbf{W}^{\mathbf{T}}\mathbf{W}_{\mathbf{pca}}^{\mathbf{T}}\mathbf{S}_{\mathbf{W}}\mathbf{W}_{\mathbf{pca}}\mathbf{W}|} = [\mathbf{w}_1\ \mathbf{w}_2\ \ldots\ \mathbf{w}_{\mathbf{c-1}}]$$

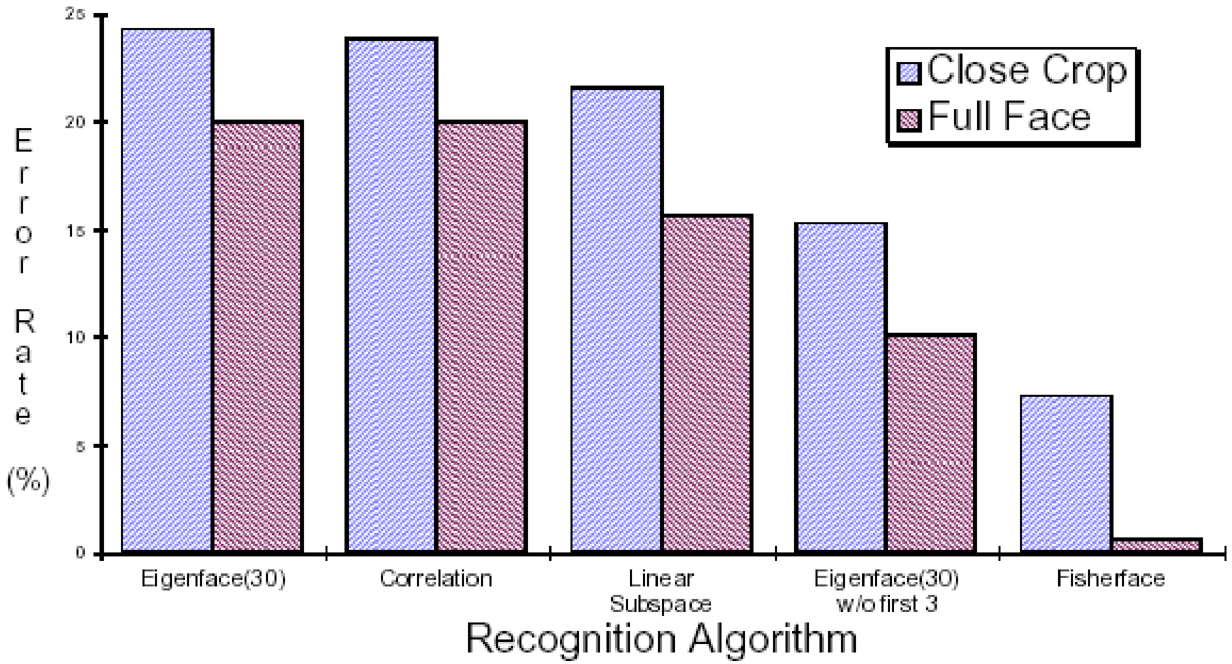
- The optimal projection becomes

$$\mathbf{W}_{opt} = \mathbf{W}_{fld}^{\mathbf{T}}\mathbf{U}^{\mathbf{T}}$$

LDA

- Example Fisherface of recognition Glasses/NoGlasses (Belhumeur et.al. 1997)

- Example comparison for face recognition (Belhumeur et.al. 1997)



- Superior performance than PCA for face recognition
- Noise sensitive
- Requires larger training set, more sensitive to different training data [Martinez&Kak2001]

Canonical Correlation Analysis (CCA)

- **Also „supervised“ method but motivated by regression tasks, e.g. pose estimation.**

- **Canonical Correlation Analysis relates two sets of observations by determining pairs of directions that yield maximum correlation between these sets.**

- **Find a pair of directions (canonical factors)** $w_x \in \Re^p$, $w_y \in \Re^q$, **so that the correlation of the projections c =** $w_x^T x$ **and d =** $w_y^T y$ **becomes maximal.**

What is CCA?

Canonical Correlation
$0 \leq r \leq 1$

$$\rho = \frac{E[cd]}{\sqrt{E[c^2]E[d^2]}} =$$

$$\frac{E[\mathbf{w}_x^T \mathbf{x}\mathbf{y}^T \mathbf{w}_y]}{\sqrt{E[\mathbf{w}_x^T \mathbf{x}\mathbf{x}^T \mathbf{w}_x]E[\mathbf{w}_y^T \mathbf{y}\mathbf{y}^T \mathbf{w}_y]}} =$$

Between Set
Covariance
Matrix

$$\frac{\mathbf{w}_x^T \mathbf{C}_{xy} \mathbf{w}_y}{\sqrt{\mathbf{w}_x^T \mathbf{C}_{xx} \mathbf{w}_x \mathbf{w}_y^T \mathbf{C}_{yy} \mathbf{w}_y}}$$

What is CCA?

- Finding solutions

$$\mathbf{w} = \begin{pmatrix} \mathbf{w}_x \\ \mathbf{w}_y \end{pmatrix} \qquad \mathbf{A} = \begin{pmatrix} 0 & \mathbf{C}_{xy} \\ \mathbf{C}_{yx} & 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} \mathbf{C}_{xx} & 0 \\ 0 & \mathbf{C}_{yy} \end{pmatrix}$$

Rayleigh Quotient

$$r = \frac{\mathbf{w}^T \mathbf{A} \mathbf{w}}{\mathbf{w}^T \mathbf{B} \mathbf{w}}$$

Generalized Eigenproblem

$$\mathbf{A}\mathbf{w} = \mu \mathbf{B}\mathbf{w}$$

CCA for images

- Same problem as for LDA
- Computationally efficient algorithm based on SVD

$$\mathbf{A} = \mathbf{C}_{xx}^{-\frac{1}{2}} \mathbf{C}_{xy} \mathbf{C}_{yy}^{-\frac{1}{2}}$$

$$\mathbf{A} = \mathbf{U}\mathbf{D}\mathbf{V}^{T}$$

$$\mathbf{w}_{xi} = \mathbf{C}_{xx}^{-\frac{1}{2}} \mathbf{u}_{i}$$

$$\mathbf{w}_{yi} = \mathbf{C}_{yy}^{-\frac{1}{2}} \mathbf{v}_{i}$$

Properties of CCA

- At most min(p,q,n) CCA factors
- Invariance w.r.t. affine transformations
- Orthogonality of the Canonical factors

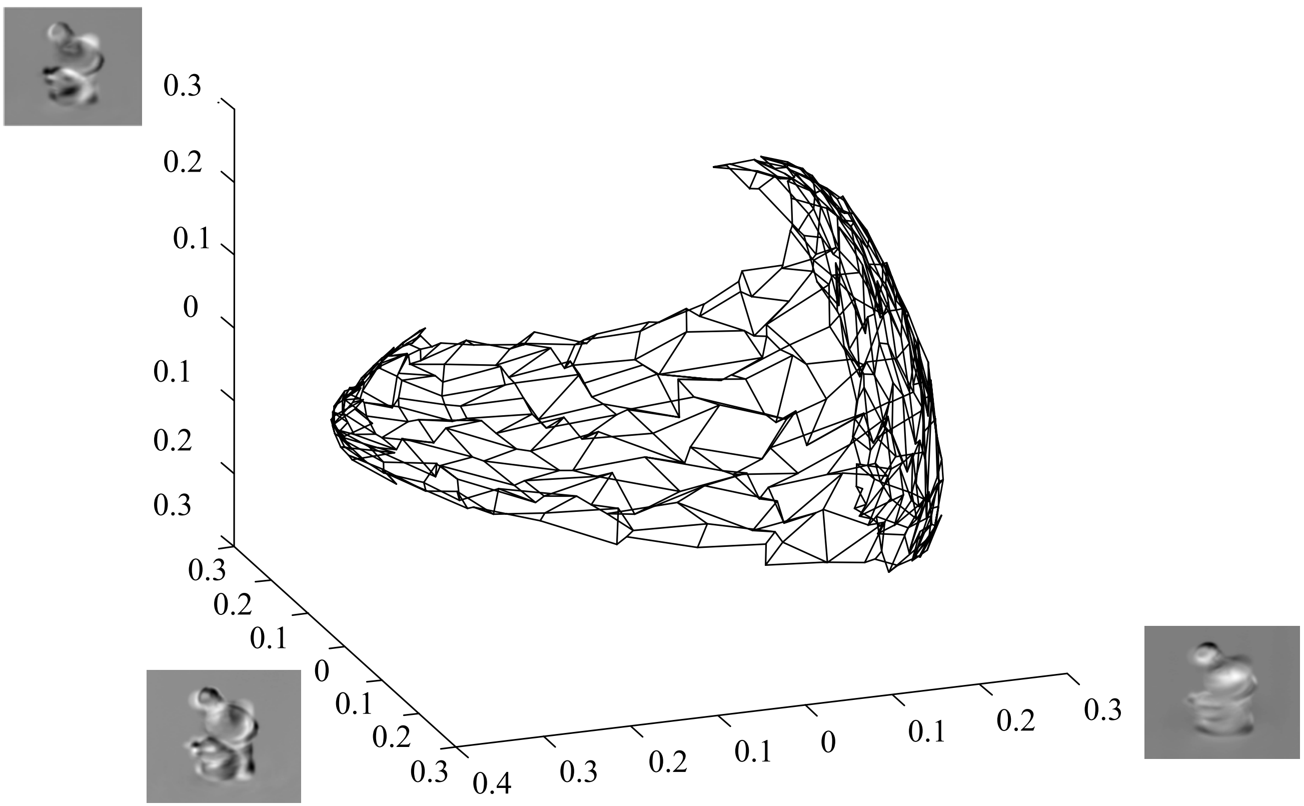
$$\mathbf{w}_{xi}^{T}\mathbf{C}_{xx}\mathbf{w}_{xj} = \mathbf{0}$$

$$\mathbf{w}_{yi}^{T}\mathbf{C}_{yy}\mathbf{w}_{yj} = \mathbf{0}$$

$$\mathbf{w}_{xi}^{T}\mathbf{C}_{xy}\mathbf{w}_{yj} = \mathbf{0}$$

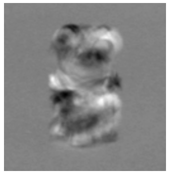
CCA Example

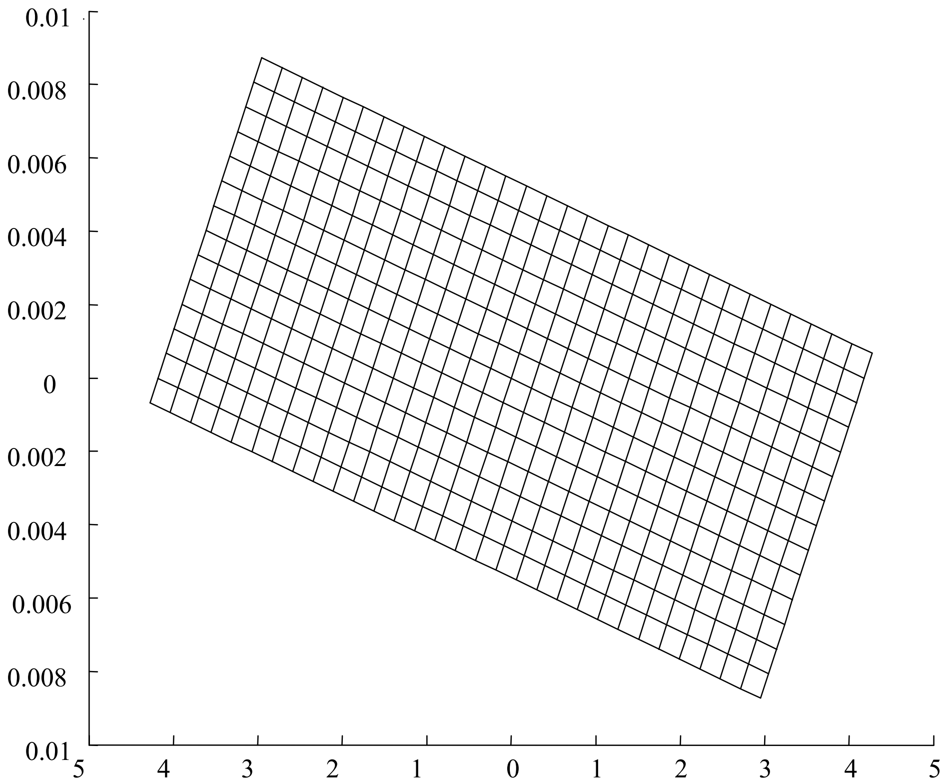
Parametric eigenspace obtained by PCA for 2DoF in pose

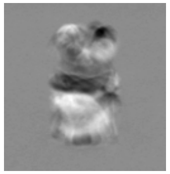


CCA representation

(projections of training images onto $\mathbf{w}_{x1}$, $\mathbf{w}_{x2}$)







Independent Component Analysis (ICA)

- ICA is a powerful technique from signal processing (Blind Source Separation)
- Can be seen as an extension of PCA
- PCA takes into account only statistics up to 2nd order
- ICA finds components that are statistically independent (or as independent as possible)

→ Local descriptors, sparse coding

Independent Component Analysis (ICA)

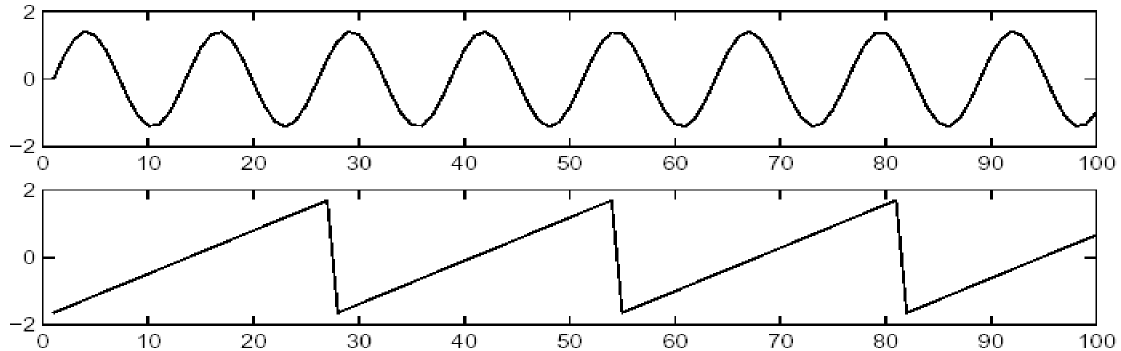
- m scalar variables $X=(x_1 \ldots x_m)^T$
- They are assumed to be obtained as linear mixtures of n sources $S=(s_1 \ldots s_n)^T$

$$\mathbf{X} = \mathbf{AS}$$

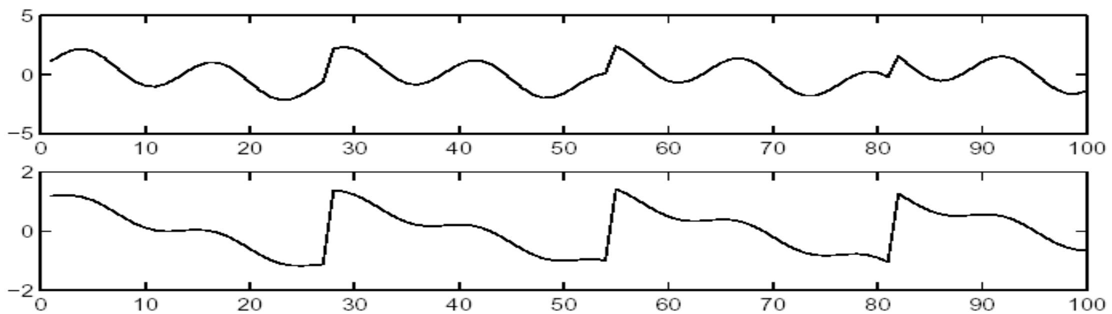
- Task: Given X find A, S (under the assumption that S are independent)

ICA Example

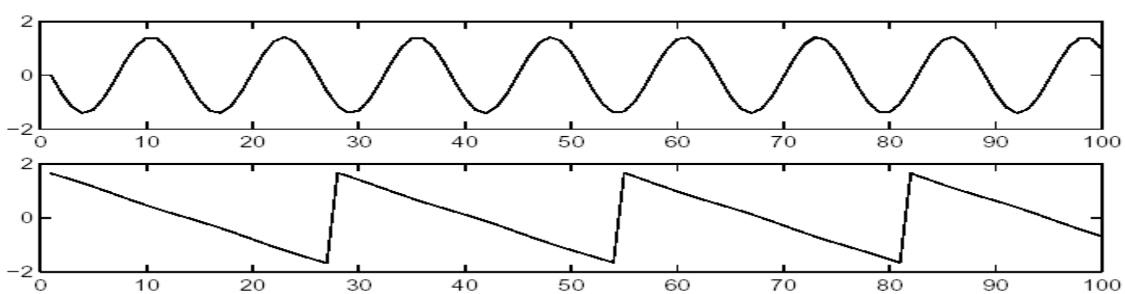
Original Sources



Mixtures

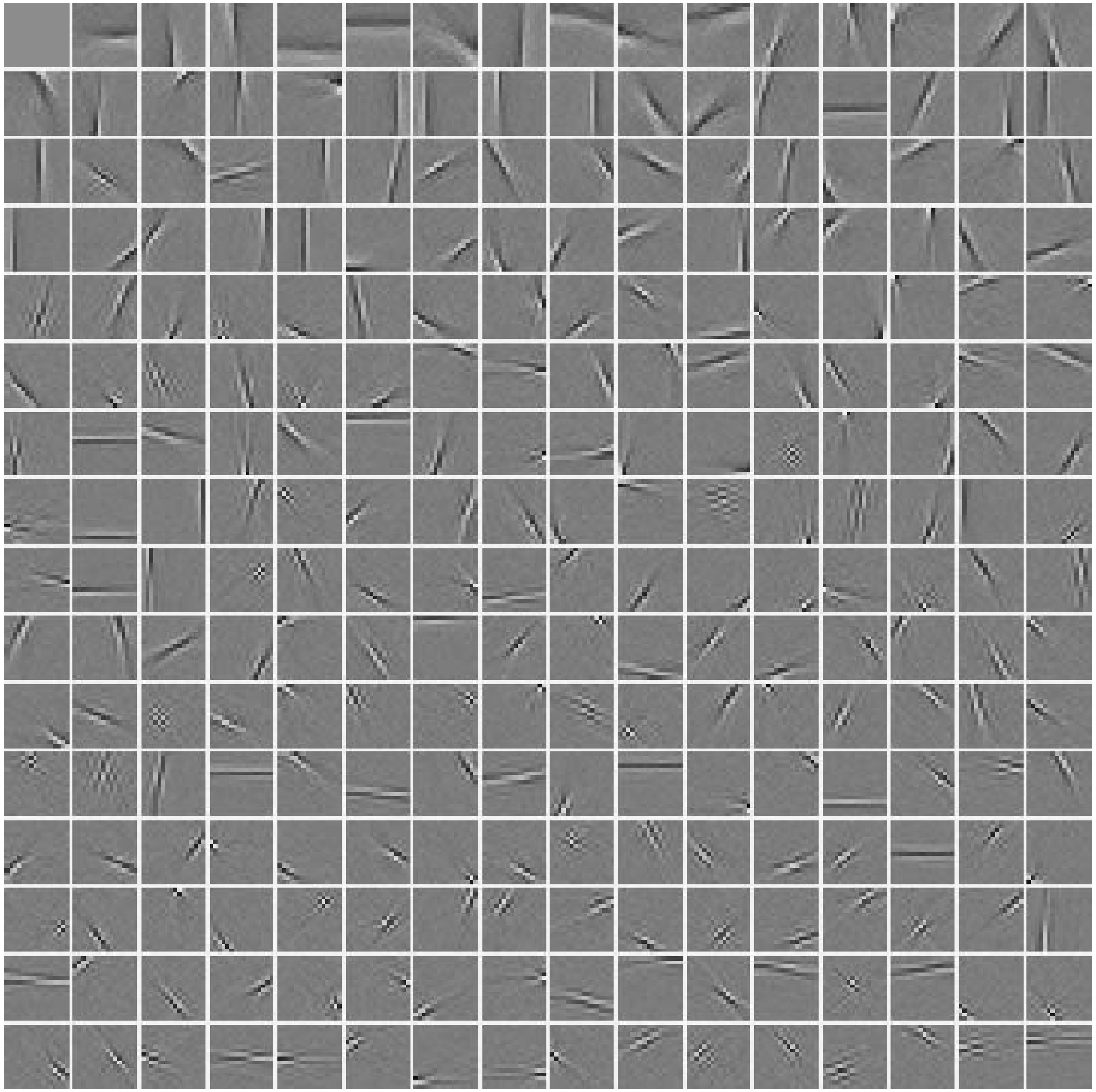


Recovered Sources



ICA Example

ICA basis obtained from 16x16 patches of natural images (Bell&Sejnowski 96)



ICA Algorithms

1. Minimize (Maximize) function
   - Complex matrix (tensor) functions

2. Adaptive Algorithms based on stochastic gradient
   - Measure of independence
   - Non-Gaussian, e.g. Kurtosis, Negentropy

   - Fast ICA Algorithm (Hyvärinen)

$$1. \qquad \mathbf{W} = \mathbf{W} / \sqrt{\| \mathbf{W}\mathbf{W}^T \|}$$

Repeat until convergence

$$2. \qquad \mathbf{W} = \frac{3}{2}\mathbf{W} - \frac{1}{2}\mathbf{W}\mathbf{W}^T\mathbf{W}$$

ICA Properties

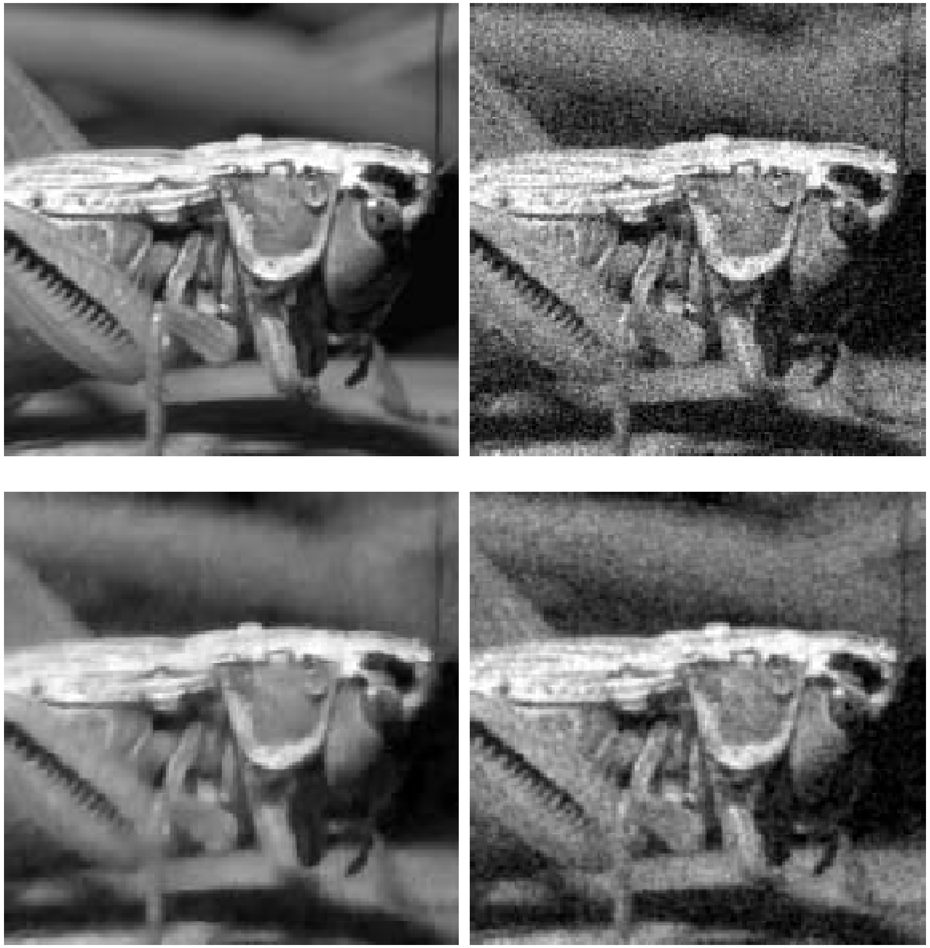
- ICA works only for Non-Gaussian Sources
- Usually centering and Whiteing of data is performed
- We can not measure the variance of the components

$$\mathbf{X} = \mathbf{A}\mathbf{P}^{-1}\mathbf{P}\mathbf{S}$$

- ICA does not provide ordering
- ICA components are not orthogonal

ICA for Noise Suppression

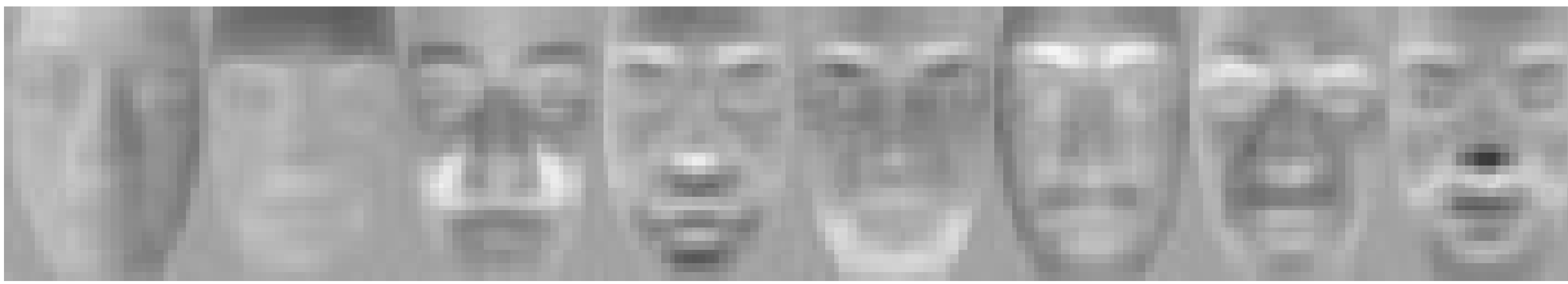
- Sparse Code Shrinkage (similar to Wavelet Shrinkage Hyvärinen 99)

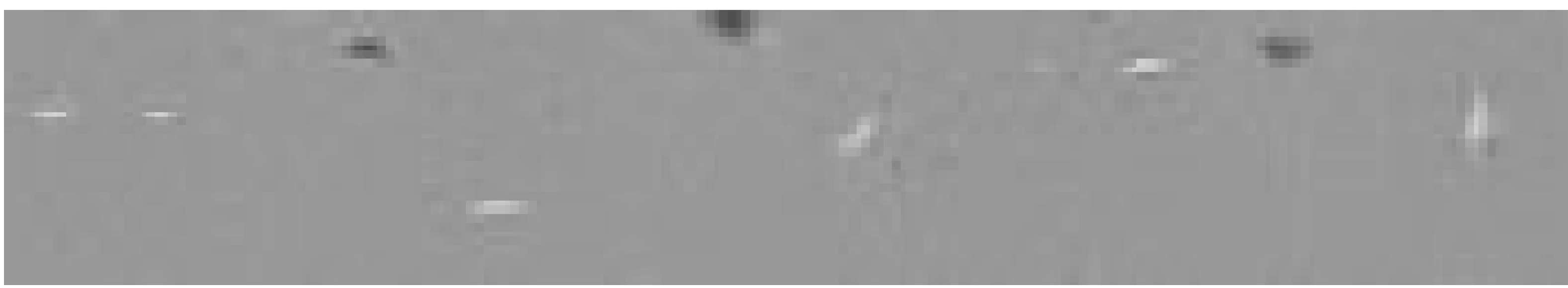


Face Recognition using ICA

- PCA vs. ICA on Ferret DB (Baek et.al. 02)

PCA





ICA

Non-Negative Matrix Factorization (NMF)

- How can we obtain part-based representation?
- Local representation where parts are added
- E.g. learn from a set of faces the parts a face consists of, i.e. eyes, nose, mouth, etc.
- Non-Negative Matrix Factorization (Lee & Seung 1999) lead to part based representation

$V \approx WH$

- **PCA** : *W* are orthonormal basis vectors

$$W = [\overrightarrow{w}_1, \overrightarrow{w}_2, \cdots, \overrightarrow{w}_n], \qquad \overrightarrow{w}_i \cdot \overrightarrow{w}_j = \delta_{ij}$$

- **VQ** : *H* are unity vectors

$$H = [\vec{h}_1, \vec{h}_2, \cdots, \vec{h}_n], \qquad \vec{h}_j^T = [0,0,1,0,\cdots,0]$$

- **NMF** : *V,W,H* are non-negative

$$V_{ij}, W_{ij}, H_{ij} \geq 0 \qquad \forall i, j$$

NMF - Cost functions

- **Euclidean distance between *A* and *B***

$$\left\| A - B \right\|^2 = \sum_{ij} (A_{ij} - B_{ij})^2$$

- **Divergence of *A* from *B* (Relative entropy)**

$$D(A \| B) = \sum_{ij} \left(A_{ij} \log \frac{A_{ij}}{B_{ij}} - A_{ij} + B_{ij} \right)$$

NMF - update rules

- $\|V - WH\|^2$ **is non-increasing under**

$$H_{a\mu} \leftarrow H_{a\mu} \frac{(W^T V)_{a\mu}}{(W^T WH)_{a\mu}} \qquad W_{i\mu} \leftarrow W_{i\mu} \frac{(VH^T)_{i\mu}}{(WHH^T)_{i\mu}}$$

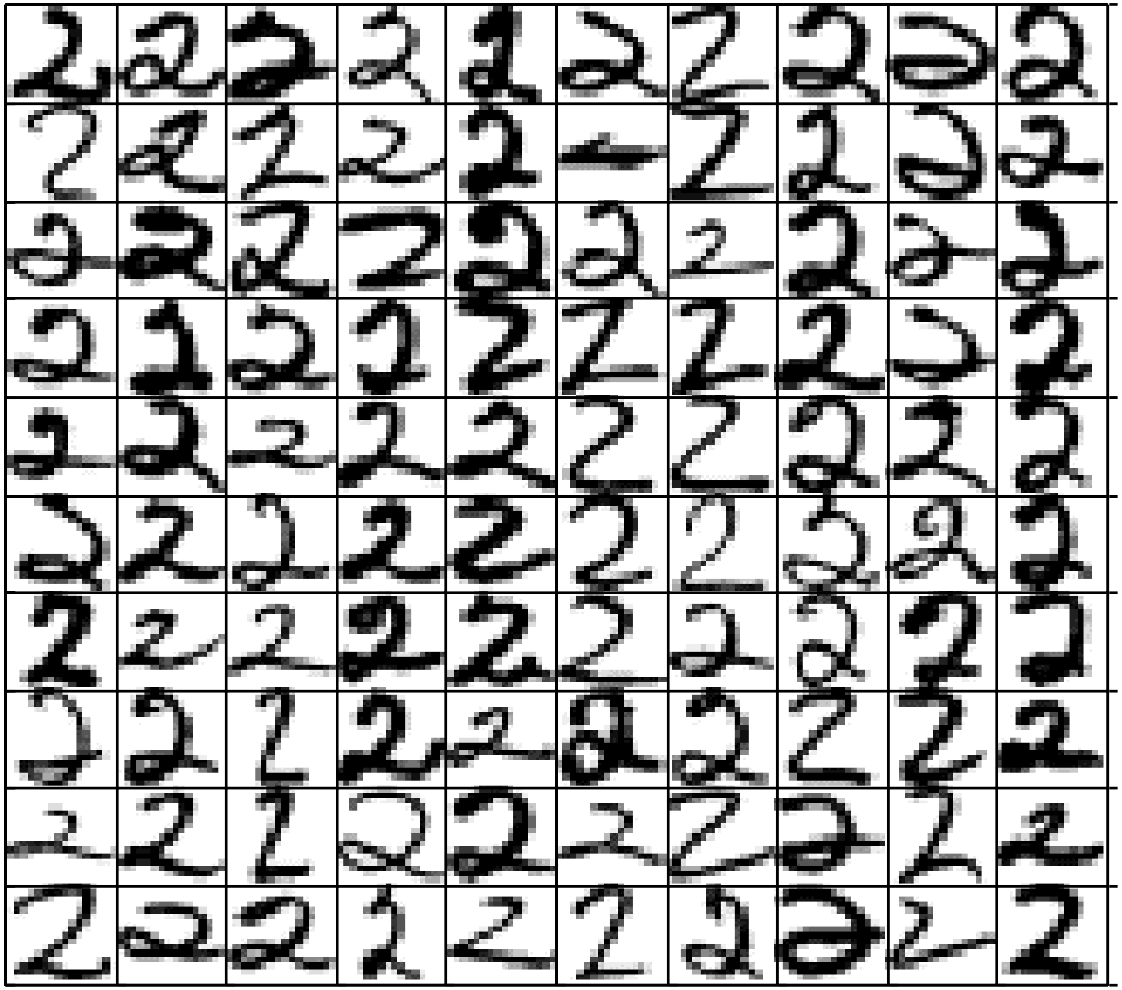
- $D(V\|WH)$ **is non-increasing under**

$$H_{a\mu} \leftarrow H_{a\mu} \frac{\sum_i W_{ia} V_{i\mu} / (WH)_{i\mu}}{\sum_k W_{ka}} \qquad W_{ia} \leftarrow W_{ia} \frac{\sum_\mu H_{a\mu} V_{i\mu} / (WH)_{i\mu}}{\sum_\nu W_{a\nu}}$$

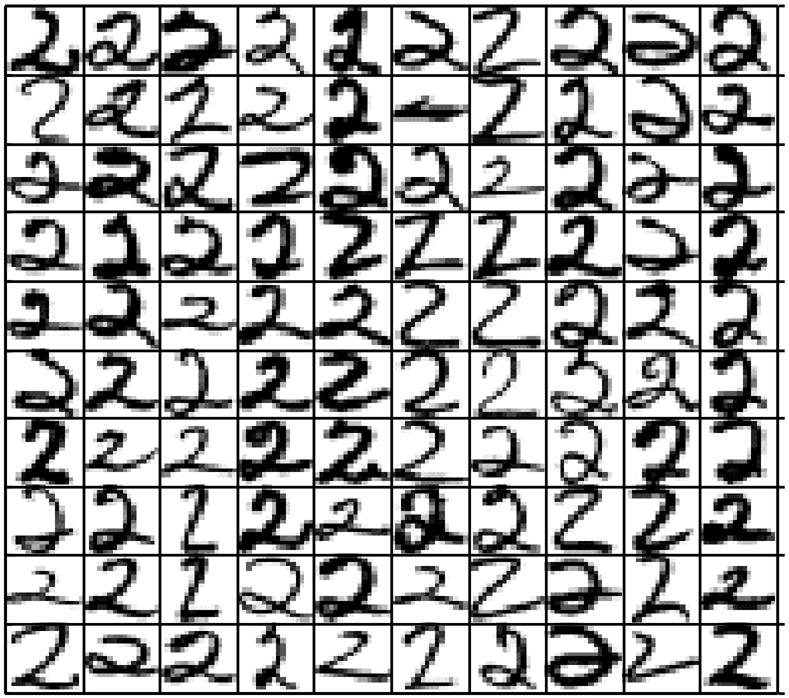
- **We can start with random matrices for W and H and update each matrix iteratively until W and H are at a stationary point - the cost functions are invariant at this point.**

Concrete example – Handwritten Digits

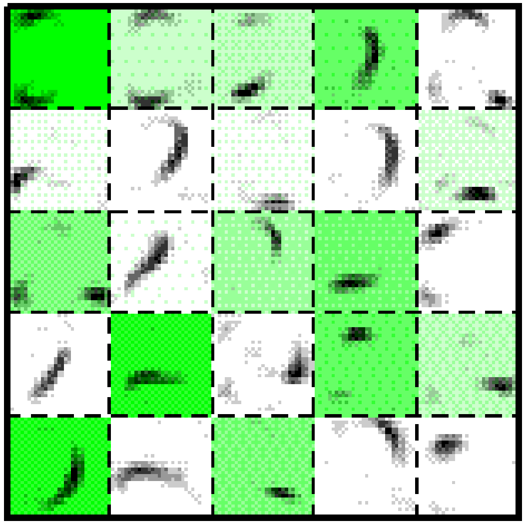
- **Training data set for learning process**



Find basis images from the training set

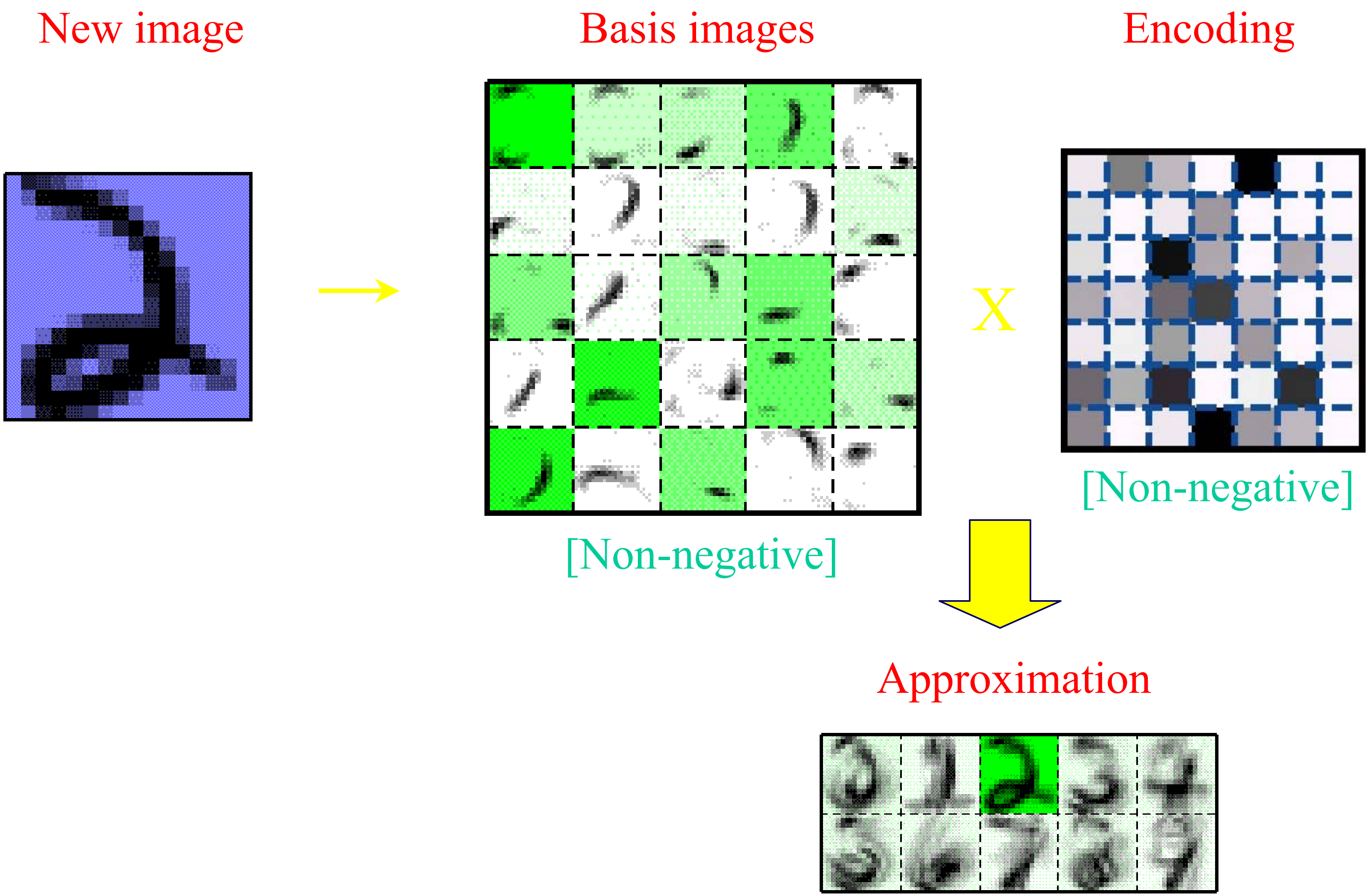


Training images

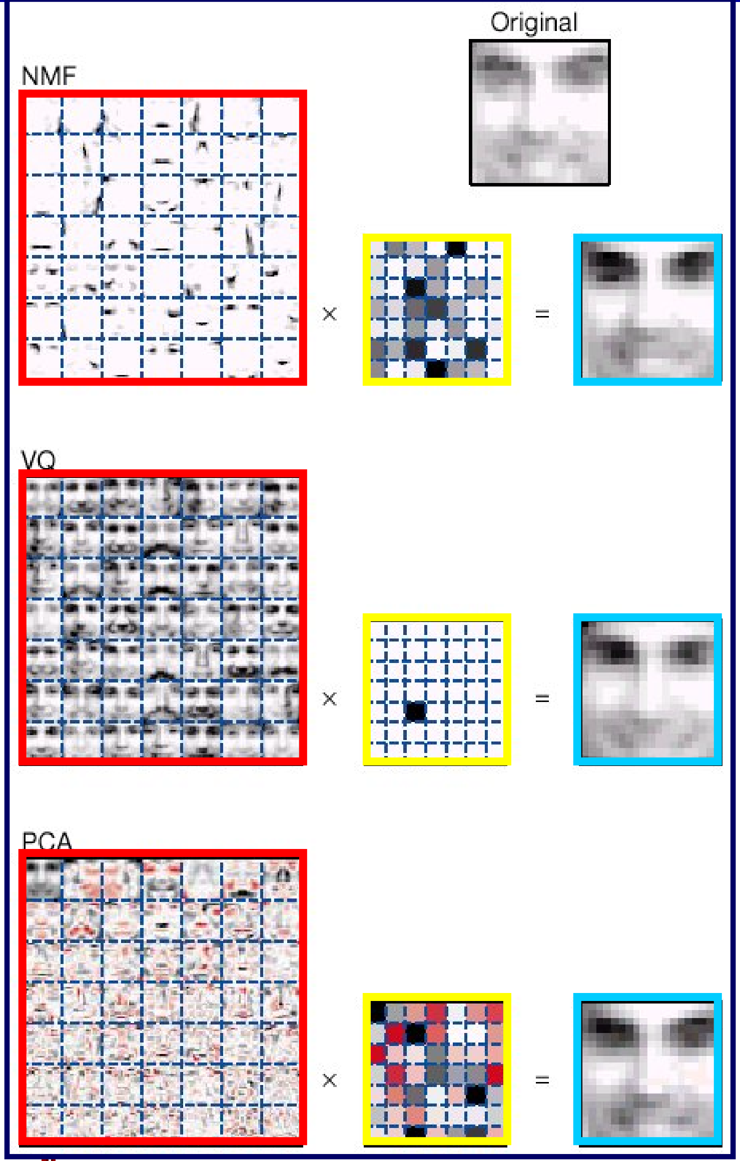


Basis images

Reconstruction



Face features



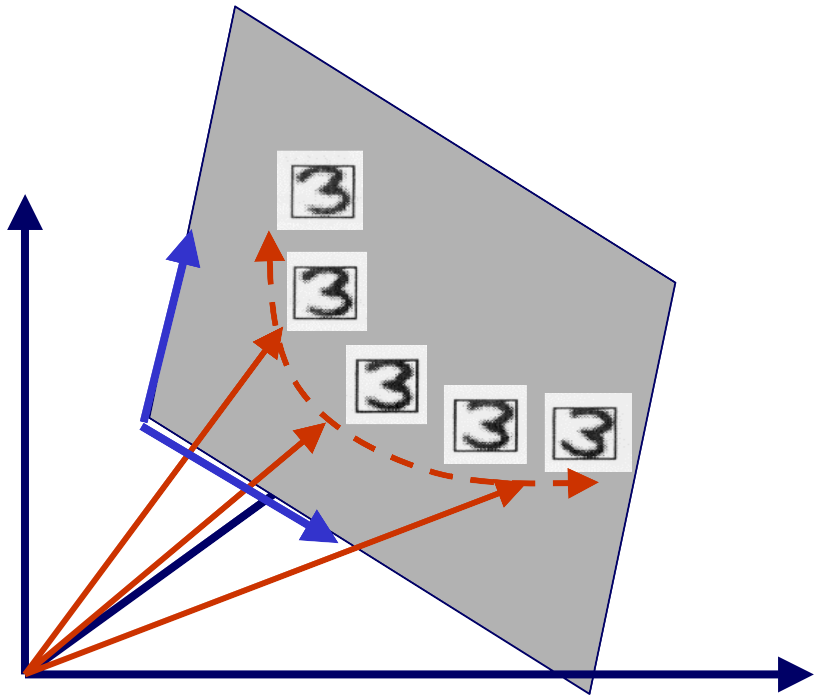
Basis images

Encoding (Coefficients)

Reconstructed image

Kernel Methods

- **All presented methods are linear**



- **Can we generalize to non-linear methods in a computational efficient manner?**

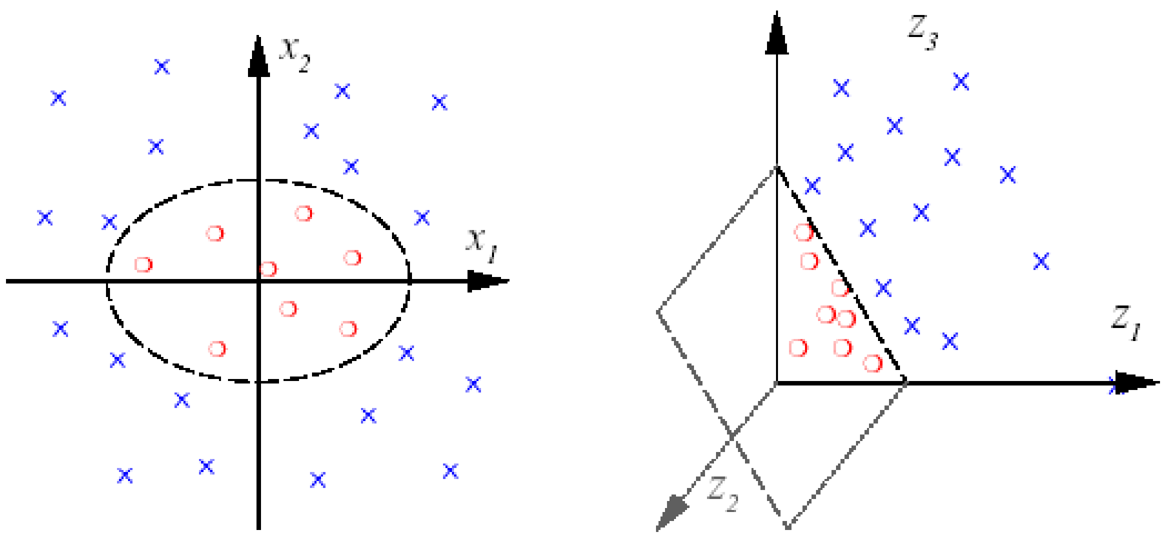
Kernel Methods

- Kernel Methods are powerful methods (introduced with Support Vector Machines) to generalize linear methods

BASIC IDEA:

1. Non-linear mapping of data in high dimensional space
2. Perform linear method in high-dimensional space

➡ Non-linear method in original space



Kernel Trick

- **Problem: High-dimensional spaces:**
  - **E.g. N=16x16 polynomial of degree 5 $\Rightarrow 10^{10}$**

- **Can we avoid computing the non-linear mapping directly?**
  - **E.g. polynomial and inner products**

$$\Phi(\mathbf{x})\Phi(\mathbf{y}) = (x_1^2, \sqrt{2}x_1x_2, x_2^2)(y_1^2, \sqrt{2}y_1y_2, y_2^2) = (\mathbf{xy})^2 = k(\mathbf{x}, \mathbf{y})$$

- **If algorithm can be specified in terms of dot products and non-linearity satisfies Mercers condition we can apply the kernel trick**

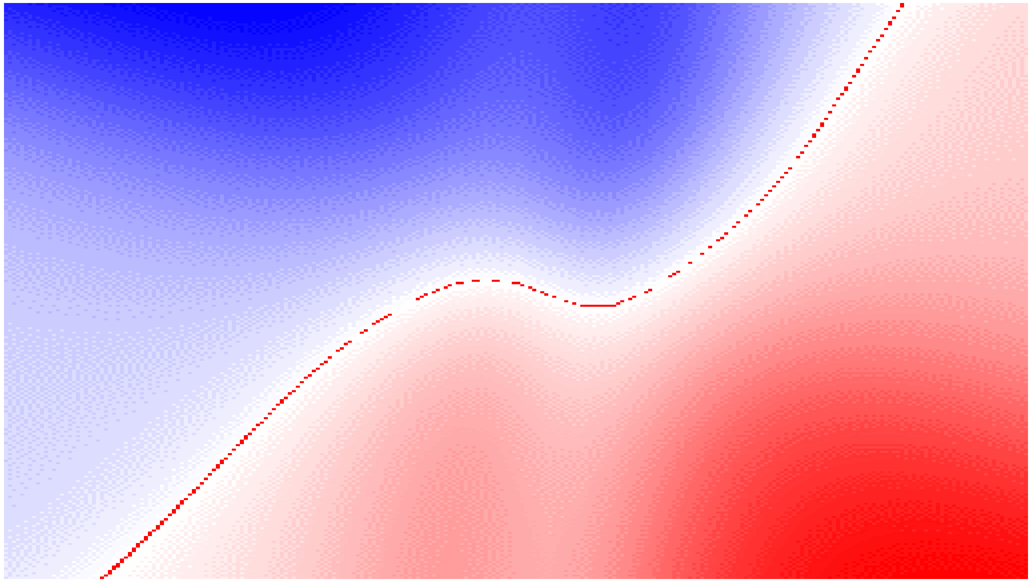
Example kernels

Gaussian: $$k(\mathrm{x},\mathrm{y}) = \exp\left(-\frac{\|\mathrm{x}-\mathrm{y}\|^2}{2\sigma^2}\right)$$

Polynomial: $$k(\mathbf{x},\mathbf{y}) = (\mathbf{x}\cdot\mathbf{y}+c)^d, c \geq 0$$

Sigmoid: $$k(\mathrm{x},\mathrm{y}) = \sigma(\kappa\cdot(\mathrm{x}\cdot\mathrm{y})+\Theta), \kappa, \Theta \in R$$

Nonlinear separation can be achieved.



Kernel Principal Component Analysis

KPCA carries out a linear PCA in the feature space *F*

The extracted features take the nonlinear form

$$f_k(\mathrm{x}) = \sum_{i=1}^{l} \alpha_i^k k(\mathrm{x}_i \mathrm{x}),$$

The α_i^k are the components of the *k*-th eigenvector of the matrix

$$(k(\mathrm{x}_i \mathrm{x}_j))_{ij}$$

KPCA and Dot Products

Find eigenvectors **V** and eigenvalues λ of the covariance matrix

$$C = \frac{1}{l}\sum_{i=1}^{l} \Phi(\mathbf{x}_i) \cdot \Phi(\mathbf{x}_i)^{\mathbf{T}}.$$

Again, replace

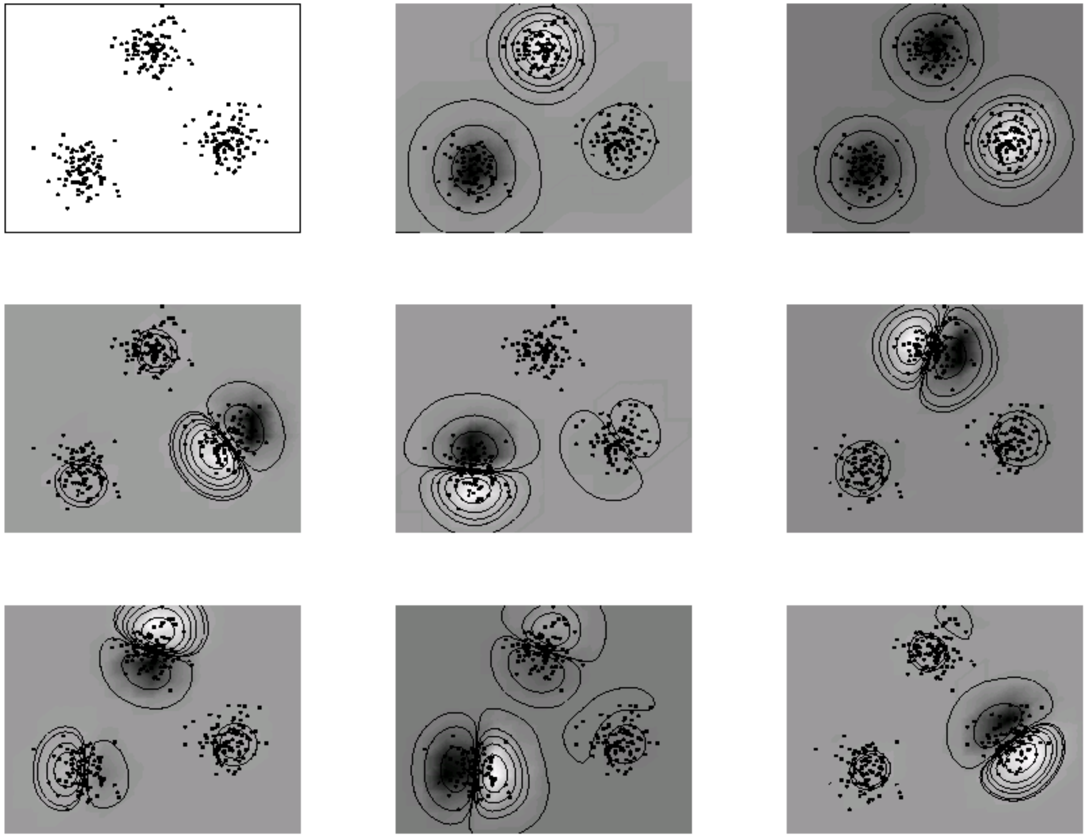
$$\Phi(\mathrm{x}) \cdot \Phi(\mathrm{y}).$$

with

$$k(\mathbf{x},\mathbf{y}).$$

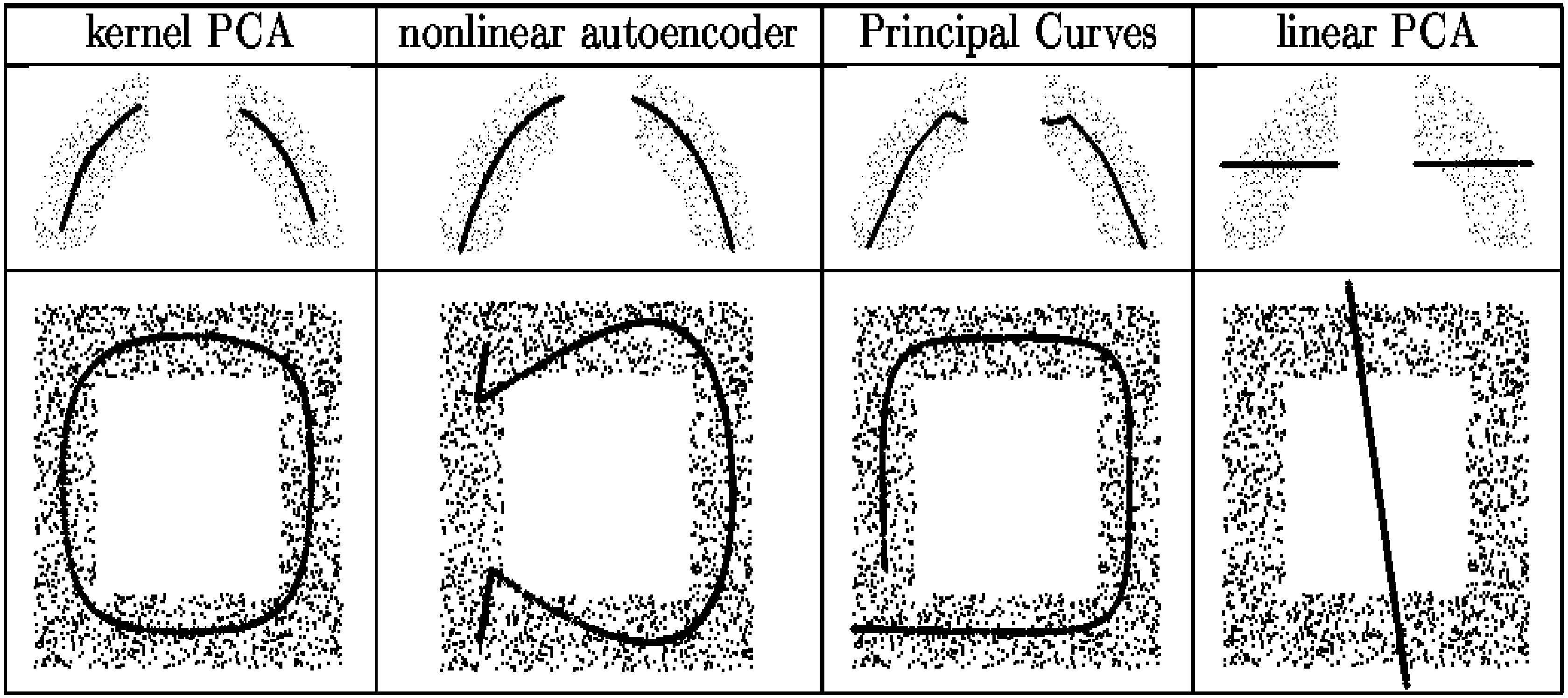
Kernel PCA Toy Example

Artificial data set from three point sources, 100 point each.



De-noising in 2-dimensions

- A half circle and a square in the plane
- De-noised versions are the solid lines



Kernel-CCA

- **Reformulation of CCA for finite sample size n**

X	…	**p $\times$ n matrix of training images**
Y	…	**q $\times$ n matrix of pose parameters**

$$\hat{\mathbf{A}} = \frac{1}{n-1}\begin{pmatrix} 0 & \mathbf{XY}^T \\ \mathbf{XY}^T & 0 \end{pmatrix}, \hat{\mathbf{B}} = \frac{1}{n-1}\begin{pmatrix} \mathbf{XX}^T & 0 \\ 0 & \mathbf{YY}^T \end{pmatrix}$$

- **Theorem**

The component vectors w_x^*, w_y^* of the extremum points w^* of

$$r = \frac{\mathbf{w}^T \hat{\mathbf{A}} \mathbf{w}}{\mathbf{w}^T \hat{\mathbf{B}} \mathbf{w}}$$

lie in the span of the training data **X**, **Y**, i.e.,

$$\exists \mathbf{f}, \mathbf{g} : \mathbf{w}_x^* = \mathbf{X}\mathbf{f}, \mathbf{w}_y^* = \mathbf{Y}\mathbf{g}$$

Kernel-CCA III

$$\mathbf{K} = \mathbf{X}^T \mathbf{X}$$

$$\mathbf{L} = \mathbf{Y}^T \mathbf{Y}$$

n × n Inner Product (Gram) Matrix

$$r = \frac{\begin{pmatrix} \mathbf{f}^T & \mathbf{g}^T \end{pmatrix} \begin{pmatrix} \mathbf{0} & \mathbf{KL} \\ \mathbf{LK} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{f} \\ \mathbf{g} \end{pmatrix}}{\begin{pmatrix} \mathbf{f}^T & \mathbf{g}^T \end{pmatrix} \begin{pmatrix} \mathbf{KK} & \mathbf{0} \\ \mathbf{0} & \mathbf{LL} \end{pmatrix} \begin{pmatrix} \mathbf{f} \\ \mathbf{g} \end{pmatrix}}$$

Kernel-CCA

- **Apply non-linear transformations $\phi()$, $\theta()$ to the data**

$$\phi(X) = < \phi(x_1), \ldots, \phi(x_n) > \qquad \theta(Y) = < \theta(y_1), \ldots, \theta(y_n) >$$

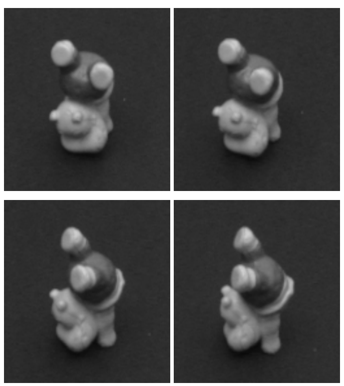
- **The Kernel Trick:**

$$K_{ij} = \phi(x_i)^T \phi(x_j) = \mathbf{k}_\phi(x_i, x_j) \qquad L_{ij} = \theta(y_i)^T \theta(y_j) = \mathbf{k}_\theta(y_i, y_j)$$

Inner Product in Feature Space

Kernel Evaluation in Input Space

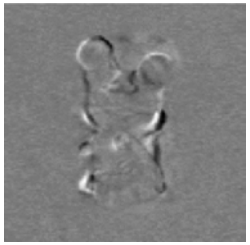
Hippo:
Rotated through 360^o (1 DOF) in 2^o steps.



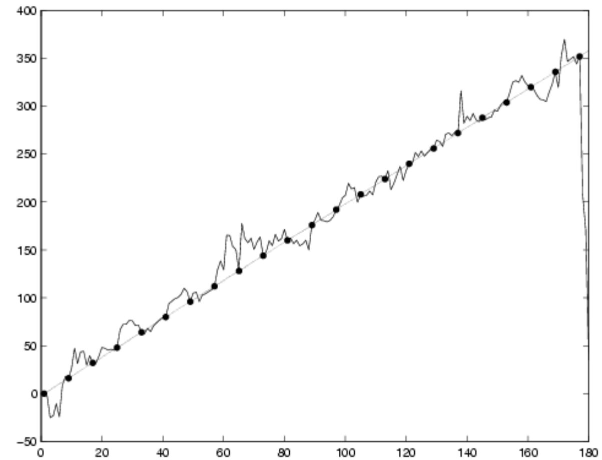
Linear Y-Encoding: y_i = turntable position α_i in degrees.

CCA-Factor

$\mathbf{w}_{x1}$



Estimates for y_i

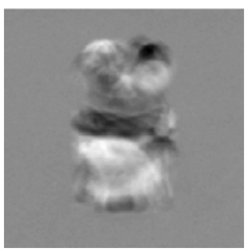


Trigonometric Y-Encoding:

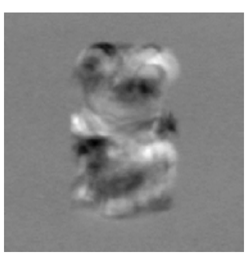
$y_i = <\sin(\alpha_i),\cos(\alpha_i)>$

CCA-Factors

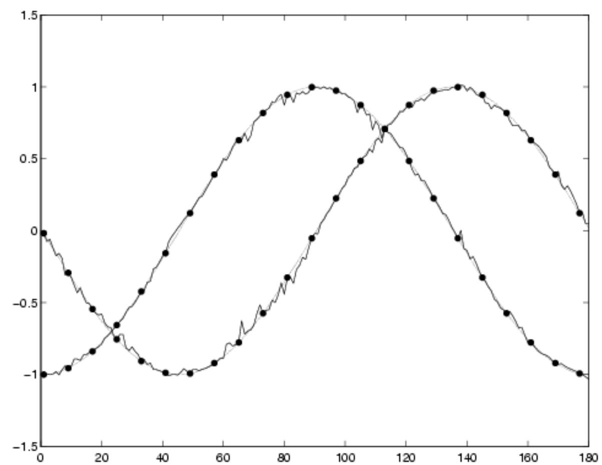
$\mathbf{w}_{x1}$



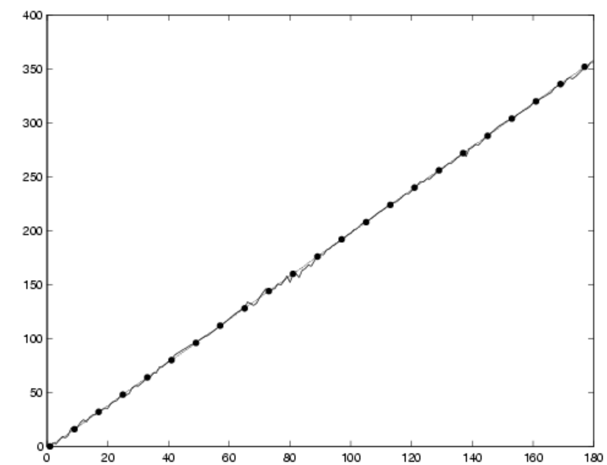
$\mathbf{w}_{x2}$



Estimates for y_{i1}, y_{i2}

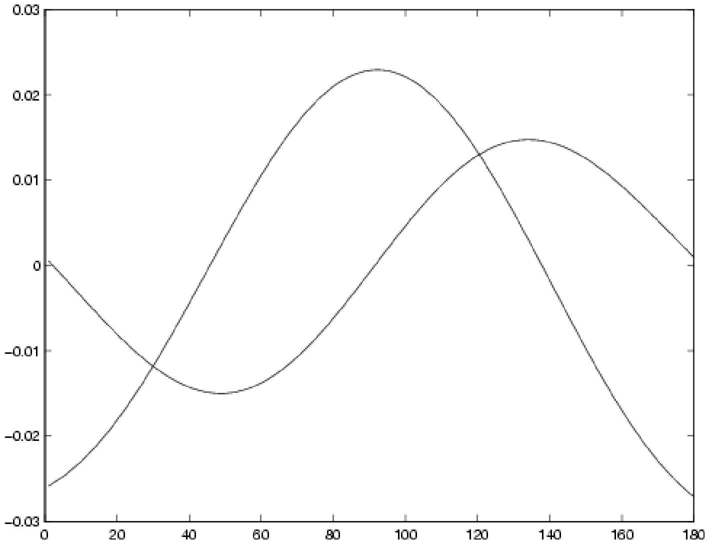


atan2



Experiments

- Using Kernel-CCA, optimal output features can be found automatically



Application of a RBF-kernel to the scalar output parameters α_i yielded two factors pairs with a canonical correlation of 1.

Outline Part 1

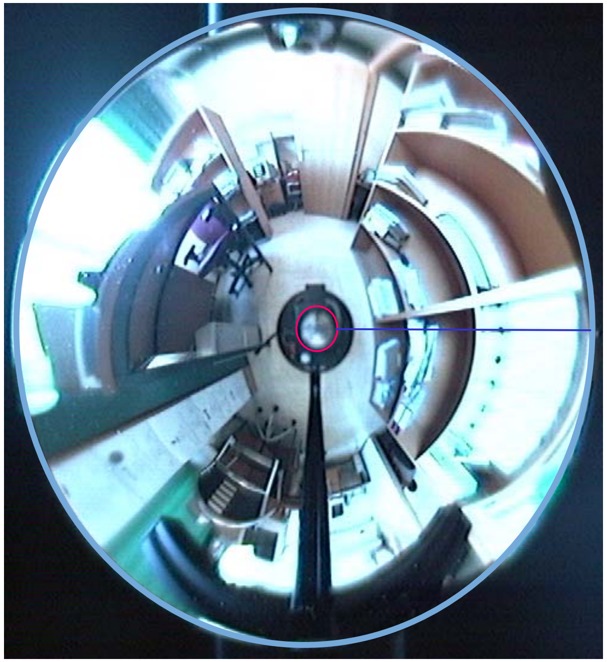
- Motivation
- Appearance based learning and recognition
- Subspace methods for visual object recognition
- Principal Components Analysis (PCA)
- Linear Discriminant Analysis (LDA)
- Canonical Correlation Analysis (CCA)
- Independent Component Analysis (ICA)
- Non-negative Matrix Factorization (NMF)
- Kernel methods for non-linear subspaces

Outline Part 2

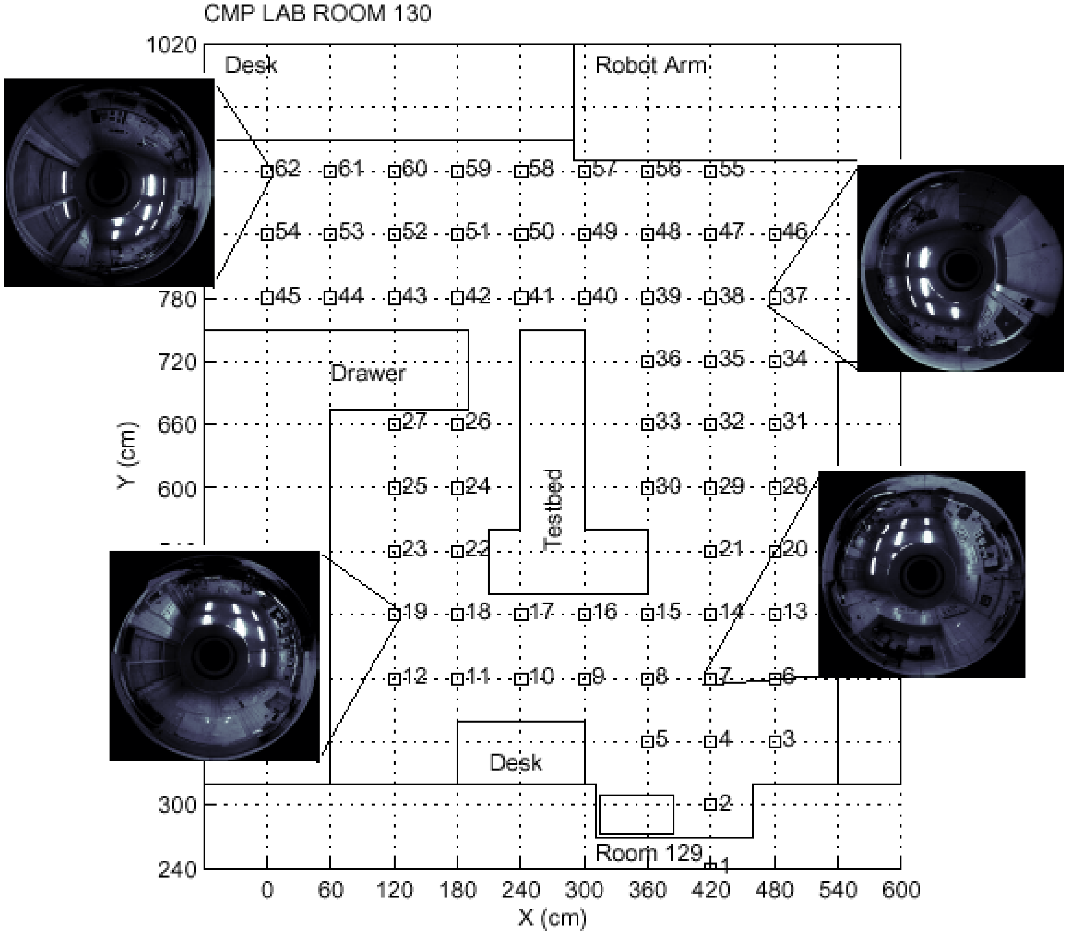
- Robot localization
- Robust representations and recognition
- Robust recognition using PCA
- Scale invariant recognition using PCA
- Illumination insensitive recognition
- Representations for panoramic images
- Incremental building of eigenspaces
- Multiple eigenspaces for efficient representation
- Robust building of eigenspaces
- Research issues

Mobile Robot

Panoramic image



Environment map



- environments are represented by a large number of views
- localisation = recognition

Compression with PCA

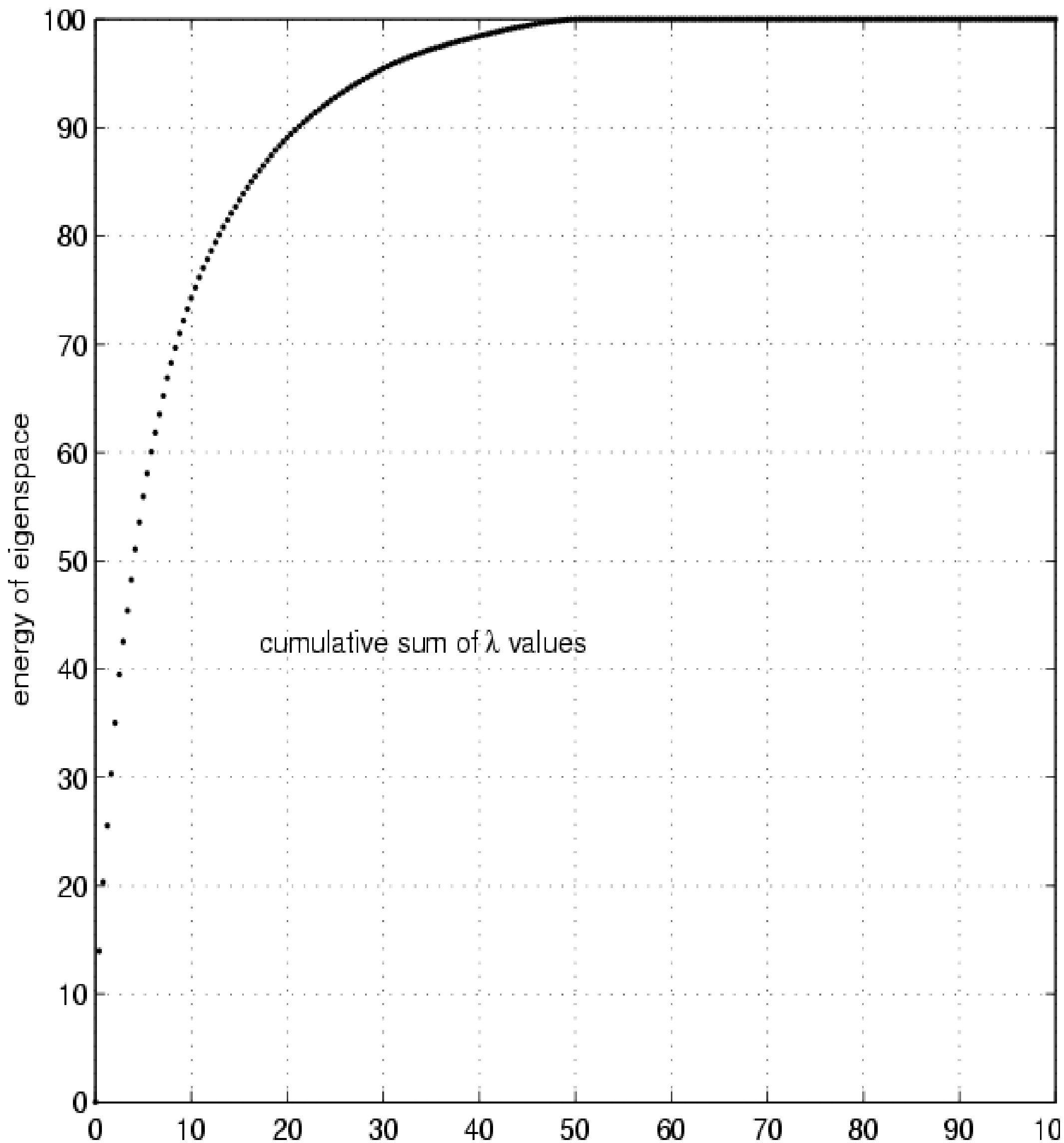
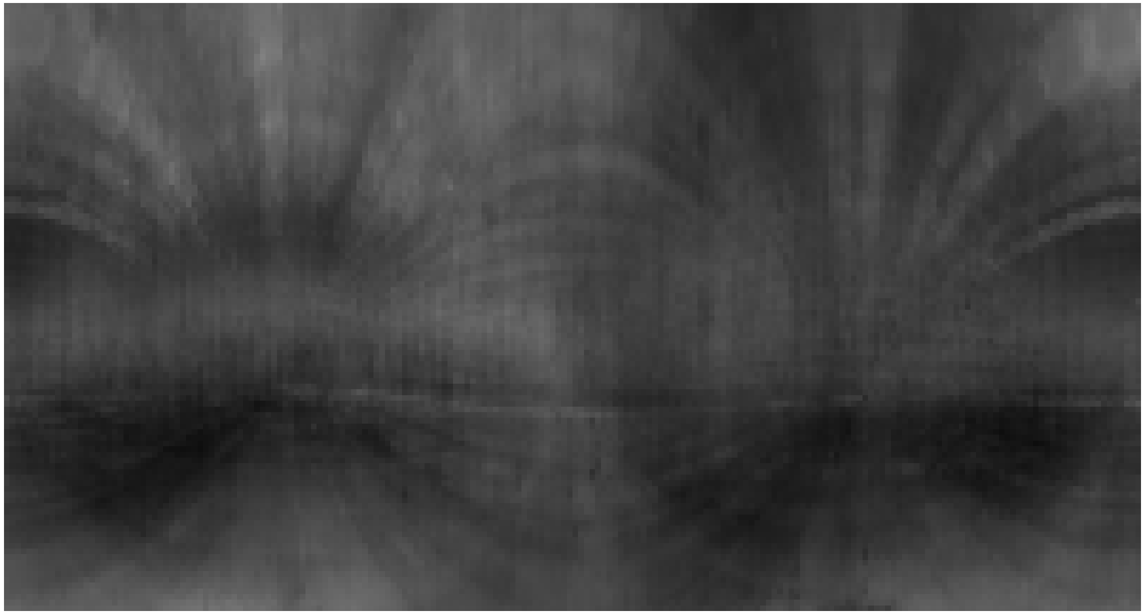
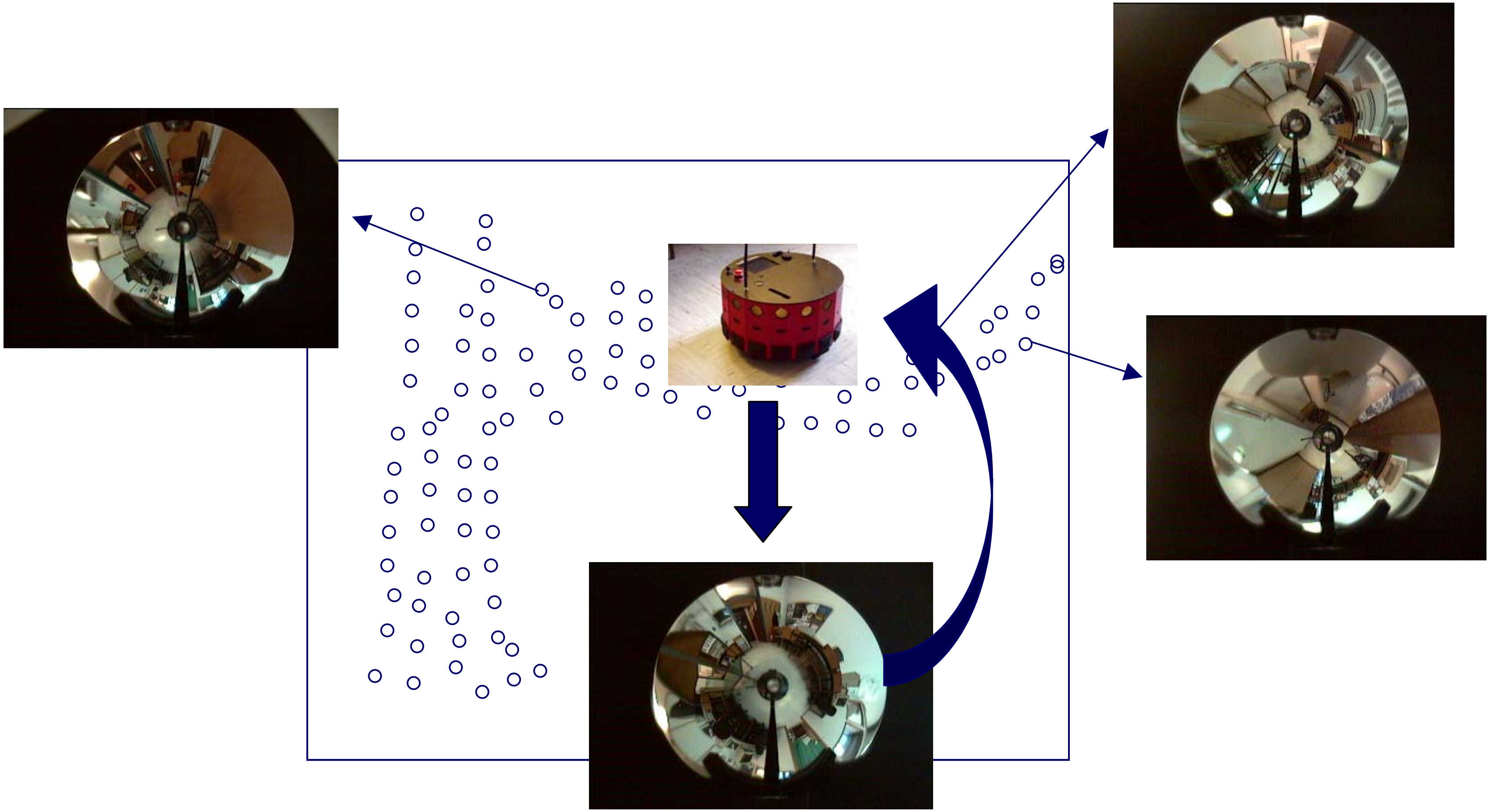


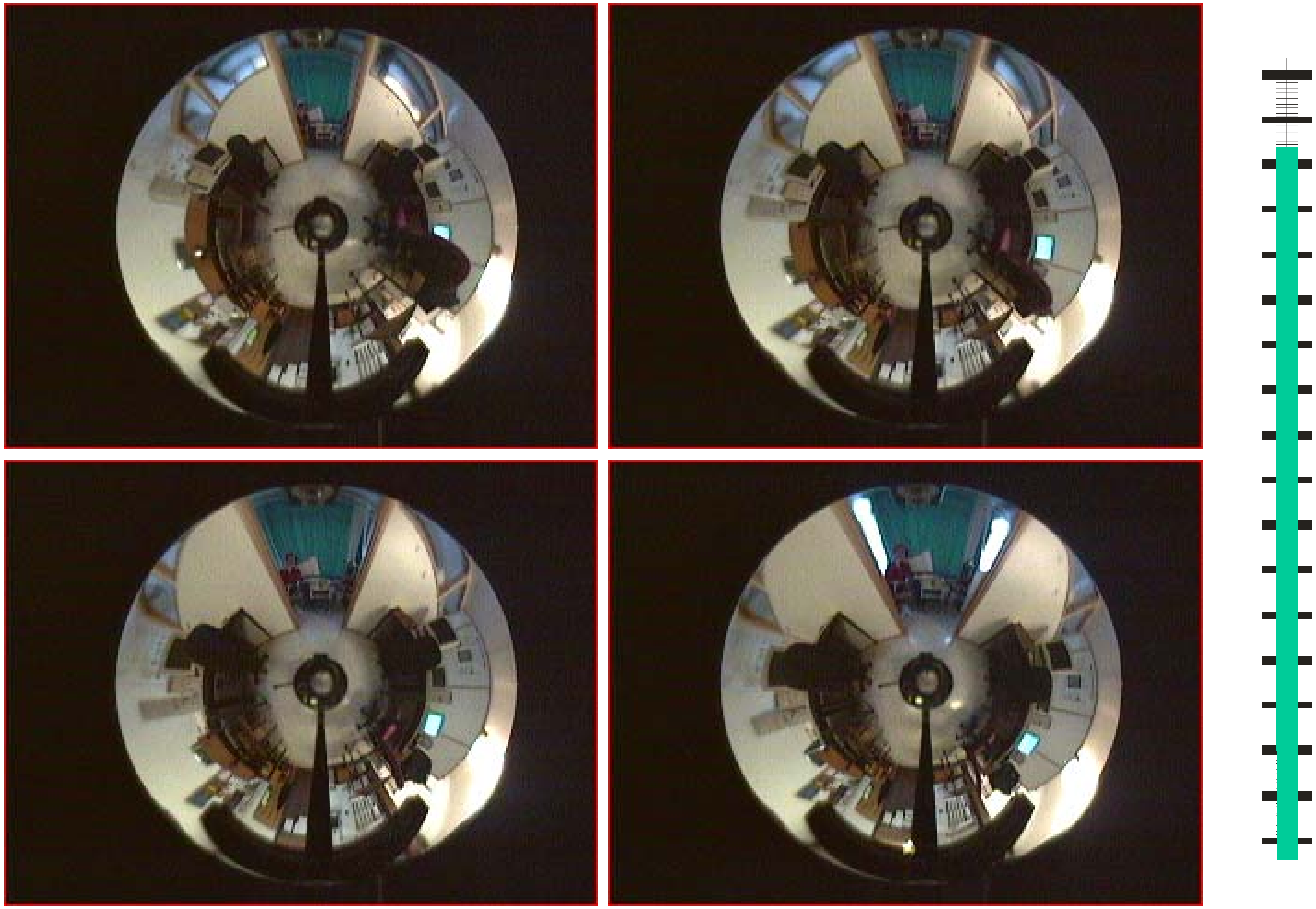
Image representation with PCA



Localisation



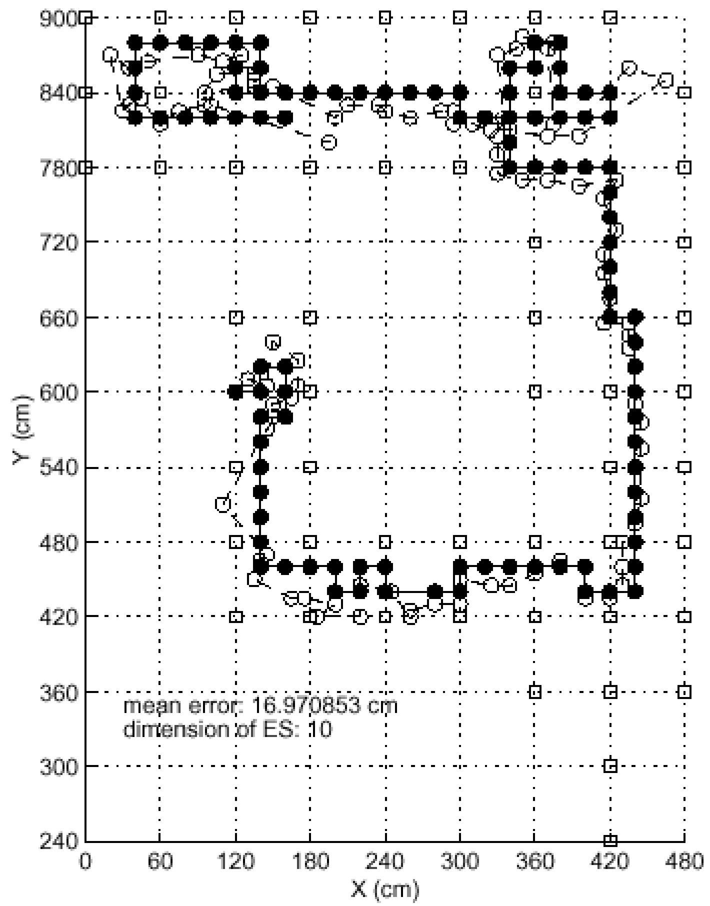
Distance vs. similarity



Robot localisation

- Interpolated hyper-surface represents the memorized environment.
- The parameters to be retrieved are related to position and orientation.
- Parameters of an input image are obtained by scalar product.

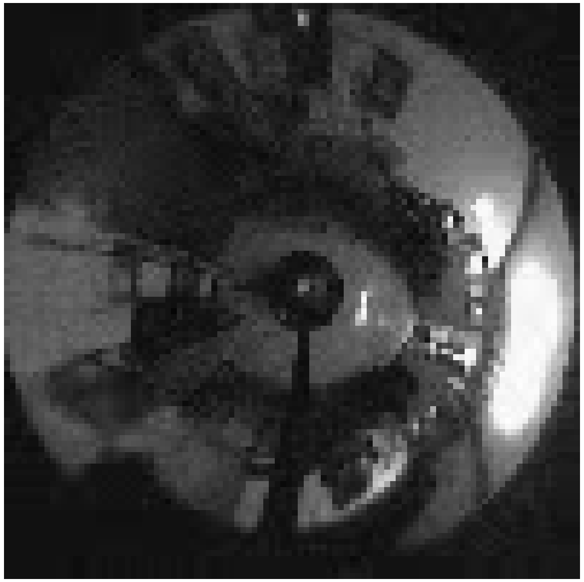
Localisation



Enhancing recognition and representations

- **Occlusions, varying background, outliers**
  - Robust recognition using PCA
- **Scale variance**
  - Multiresolution coefficient estimation
  - Scale invariant recognition using PCA
- **Illumination variations**
  - Illumination insensitive recognition
- **Rotated panoramic images**
  - Spinning eigenimages
- **Incremental building of eigenspaces**
- **Multiple eigenspaces for efficient representations**
- **Robust building of eigenspaces**

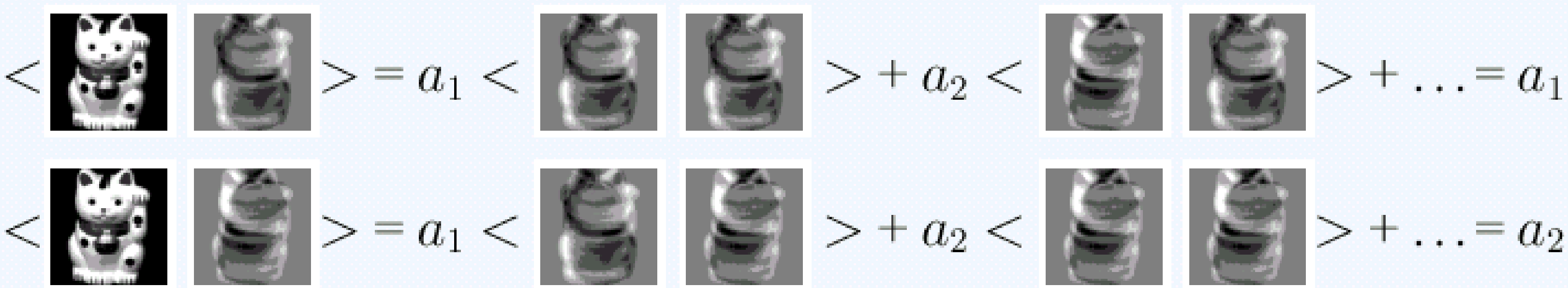
Occlusions



Standard recovery of coefficients

To recover a_i the image is projected onto the eigenspace

$$a_i(\mathbf{x}) = <\mathbf{x}, \mathbf{e}_i> = \sum_{j=1}^{m} x_j e_{i_j} \qquad 1 \leq i \leq p$$

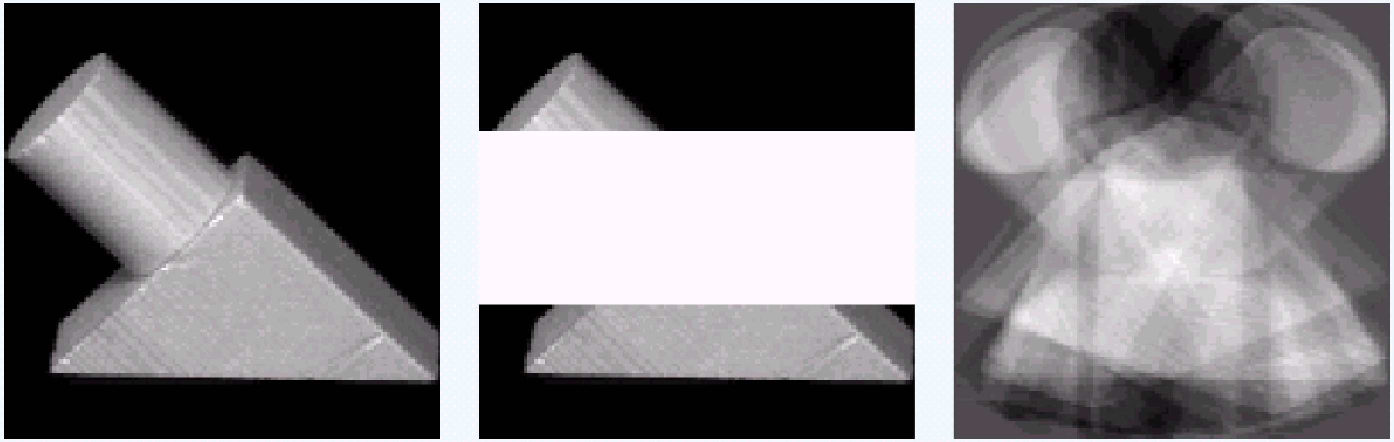


Complete image x_i is required to calculate a_i.
Corresponds to Least-Squares Solution

Non-robustness

Drawbacks: Prone to errors caused by

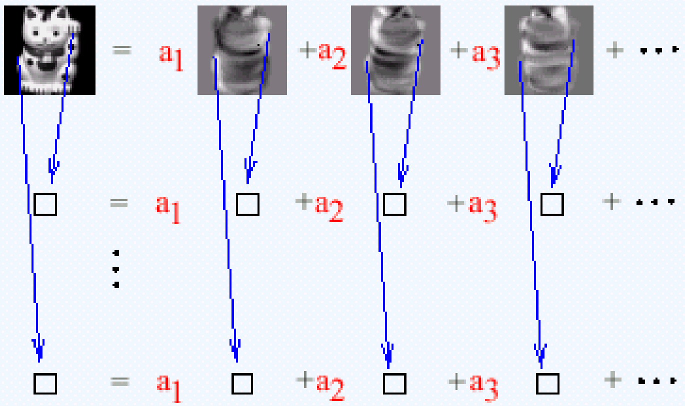
- occlusions (outliers)
- cluttered background



Original Occluded Reconstruction

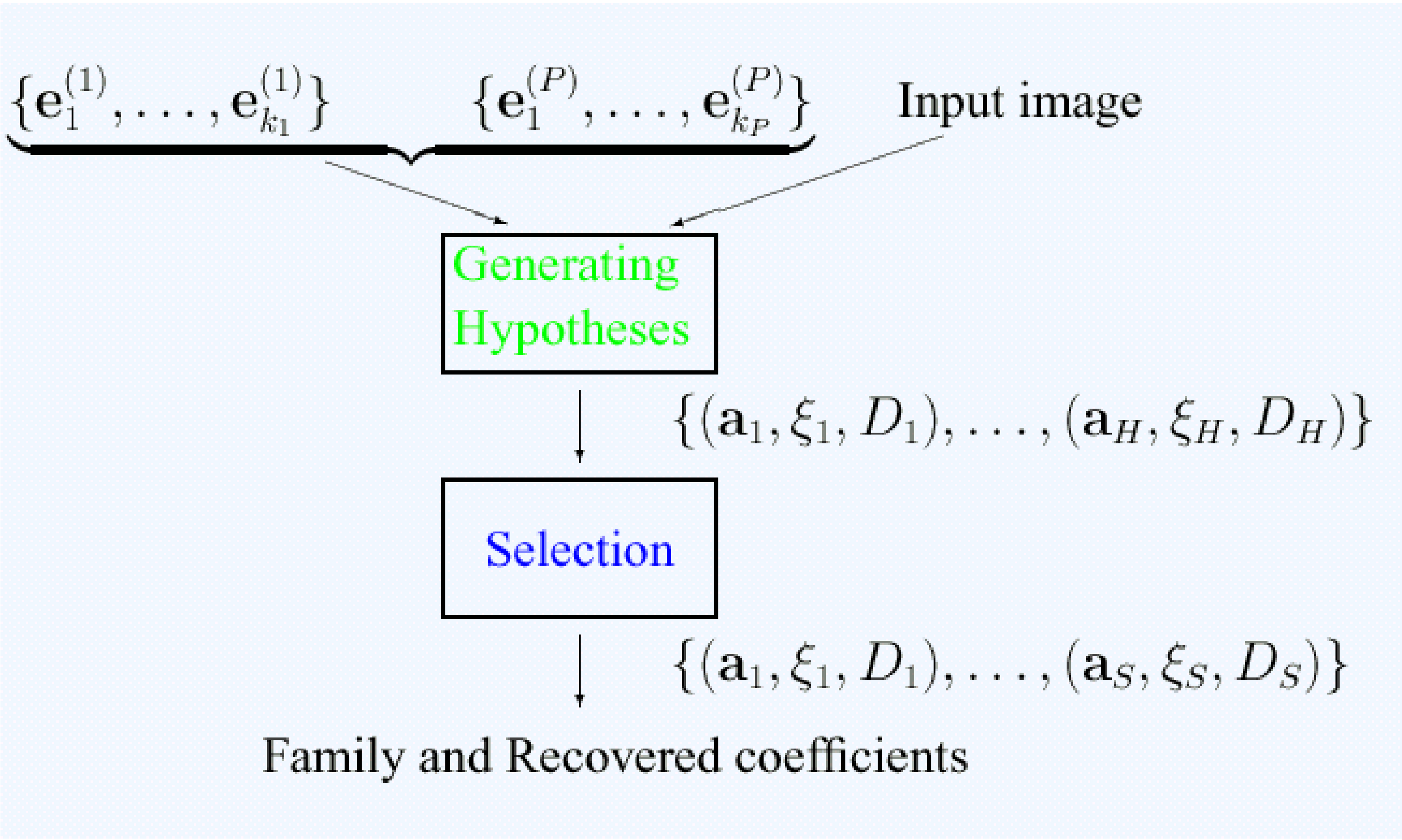
Robust method

- **Major idea:** Instead of using the standard approach we:
  - **subset of data points** $\longrightarrow$ linear system of equations
  - **Robust** solution of this system of equations
  - Perform **multiple hypotheses**



- Hypothesize-and-test paradigm
- Competing hypotheses are subject to a **selection** procedure based on the MDL principle.

Robust algorithm



Selection

Three cases:

1. **One object**: Select best match (c_{ii})
2. Multiple **non-overlapping** objects: Select local maximum (c_{ii})
3. Multiple **overlapping** objects: MDL-criterion:

The objective function:

$$F(\mathbf{h}) = \mathbf{h}^T \mathbf{C} \mathbf{h}$$

$\mathbf{h}^T = [h_1, h_2, \ldots, h_R]$ — set of hypotheses

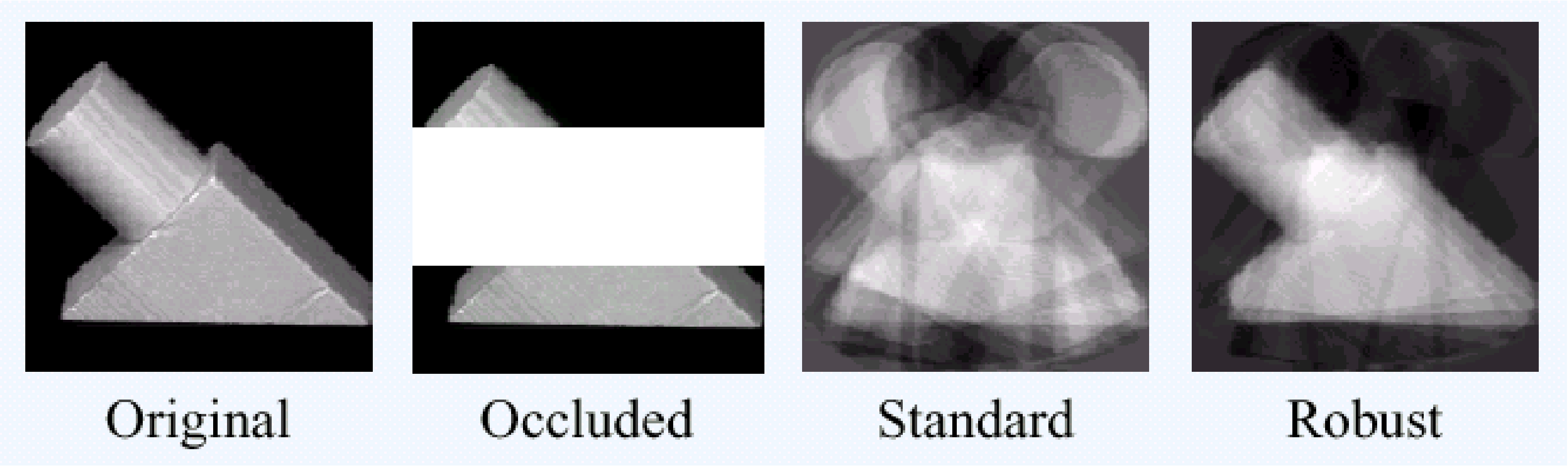
Diagonal terms of $\mathbf{C}$ express the cost-benefit value for hypothesis i

$$c_{ii} = \mathrm{K}_1 |D_i| - \mathrm{K}_2 ||\vec{\xi}_i||_{D_i} - \mathrm{K}_3 N_i$$

Off-diagonal terms handle overlapping hypotheses

$$c_{ij} = \frac{-\mathrm{K}_1 |D_i \cap D_j| + \mathrm{K}_2 \xi_{ij}}{2}$$

Robust recovery of coefficients



Robustness – Experimental results

Experimental testing on a standard database COIL of 1440 images (20 objects under 72 orientations).

Recognition and pose estimation

Pose estimation :

Method	Salt & Pepper [%]				Gaussian Noise [σ]				Occlusions [%]			
	0	25	50	75	75	150	225	300	15	30	45	60
Standard	2	3	3	48	3	3	4	24	3	25	31	45
Robust	2	3	3	4	4	5	6	10	3	3	16	29

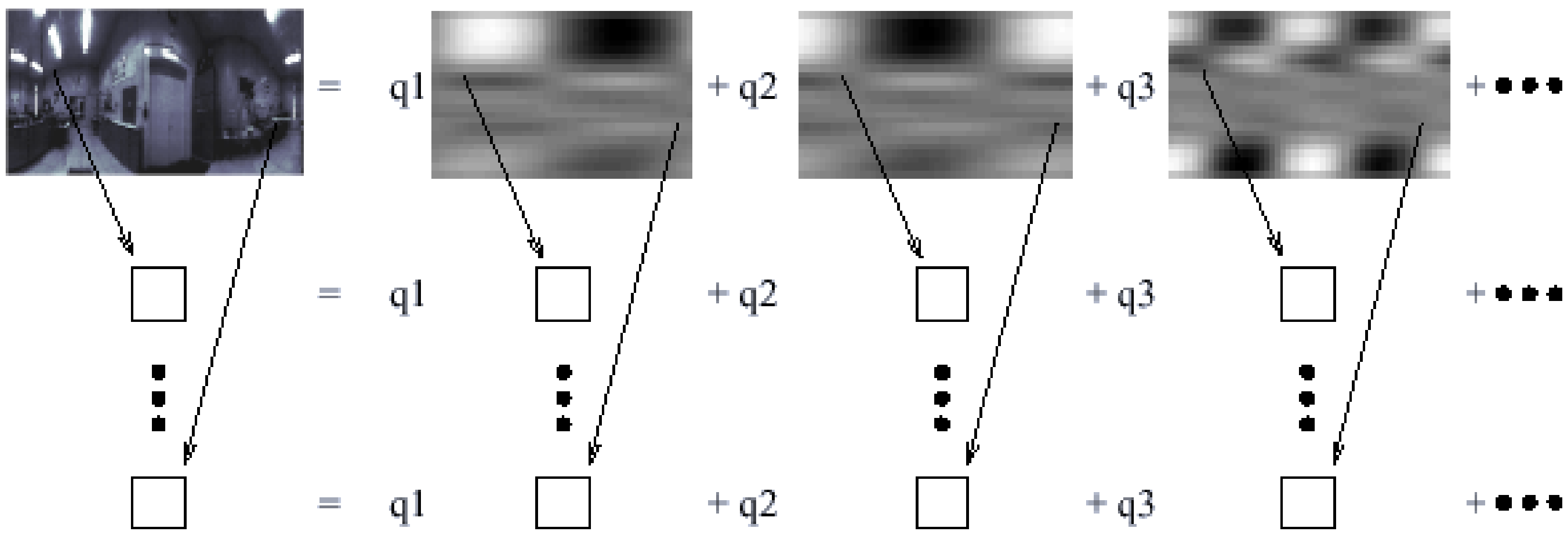
Recognition (50 % salt & pepper noise):

Method	Recognition Rate	Mean absolute orientation error
Standard	46 %	22°
Robust	75 %	6°

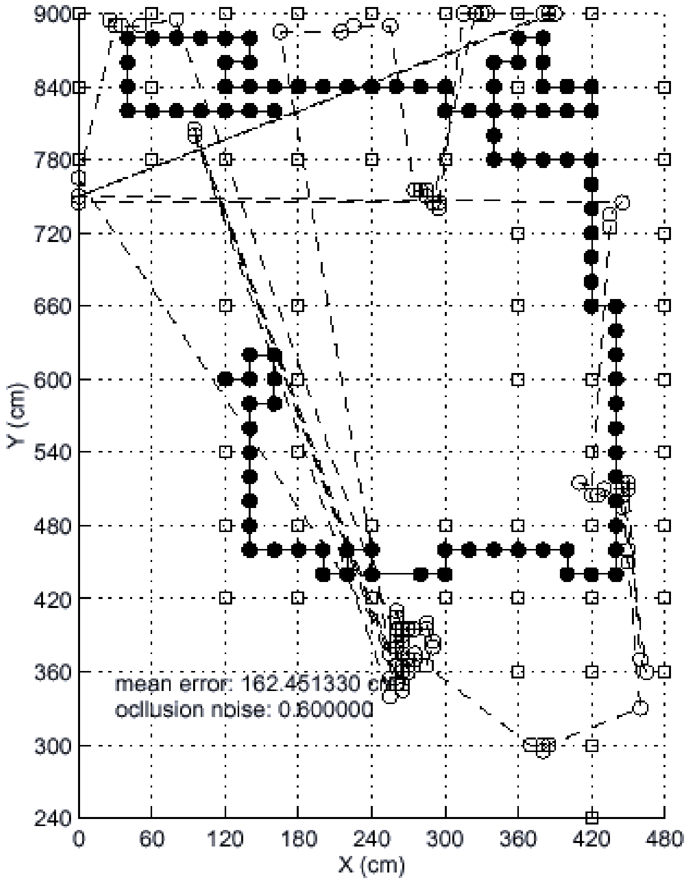
Recognition (50 % occlusion):

Method	Recognition Rate	Mean absolute orientation error
Standard	12 %	57°
Robust	66 %	29°

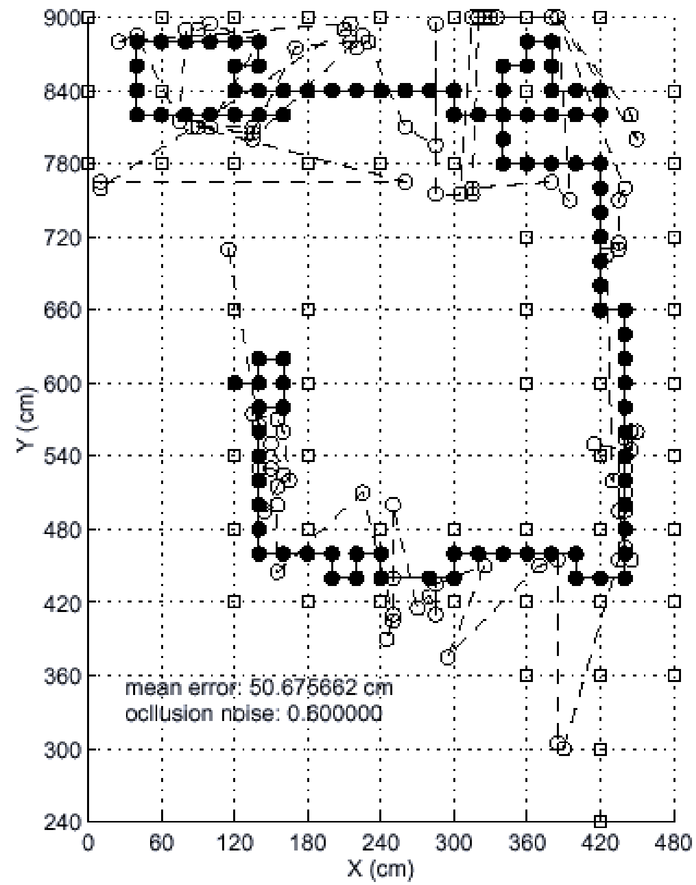
Robust localisation under occlusions



Robust localisation at 60% occlusion



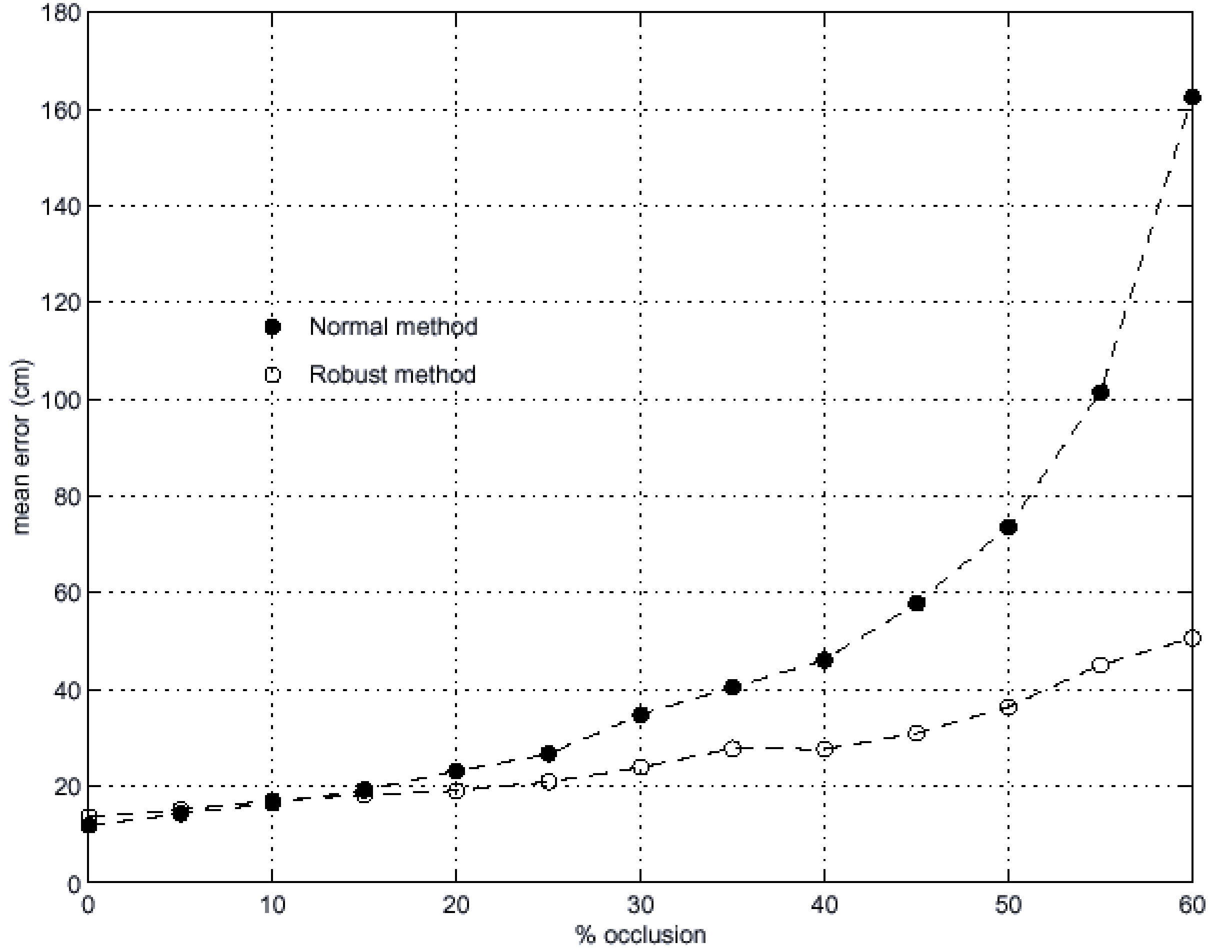
Standard approach



Robust approach

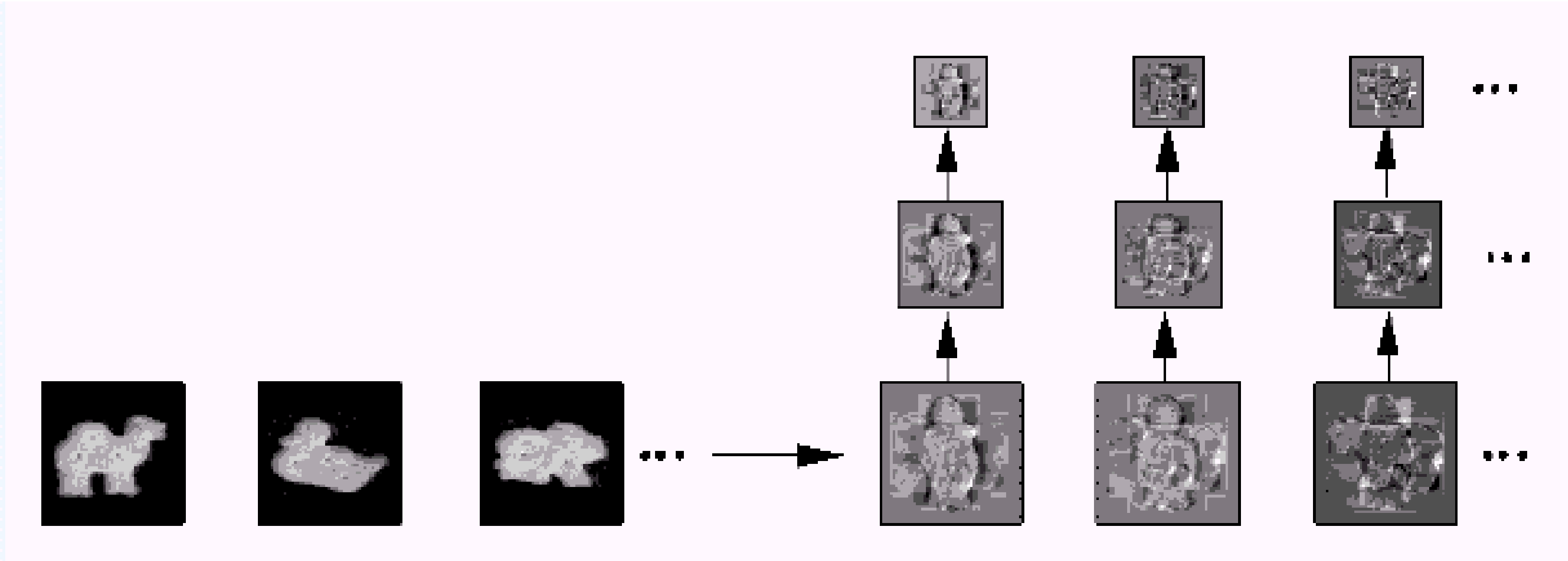
Mean error of localisation

- **Mean error of localisation with respect to % of occlusion**



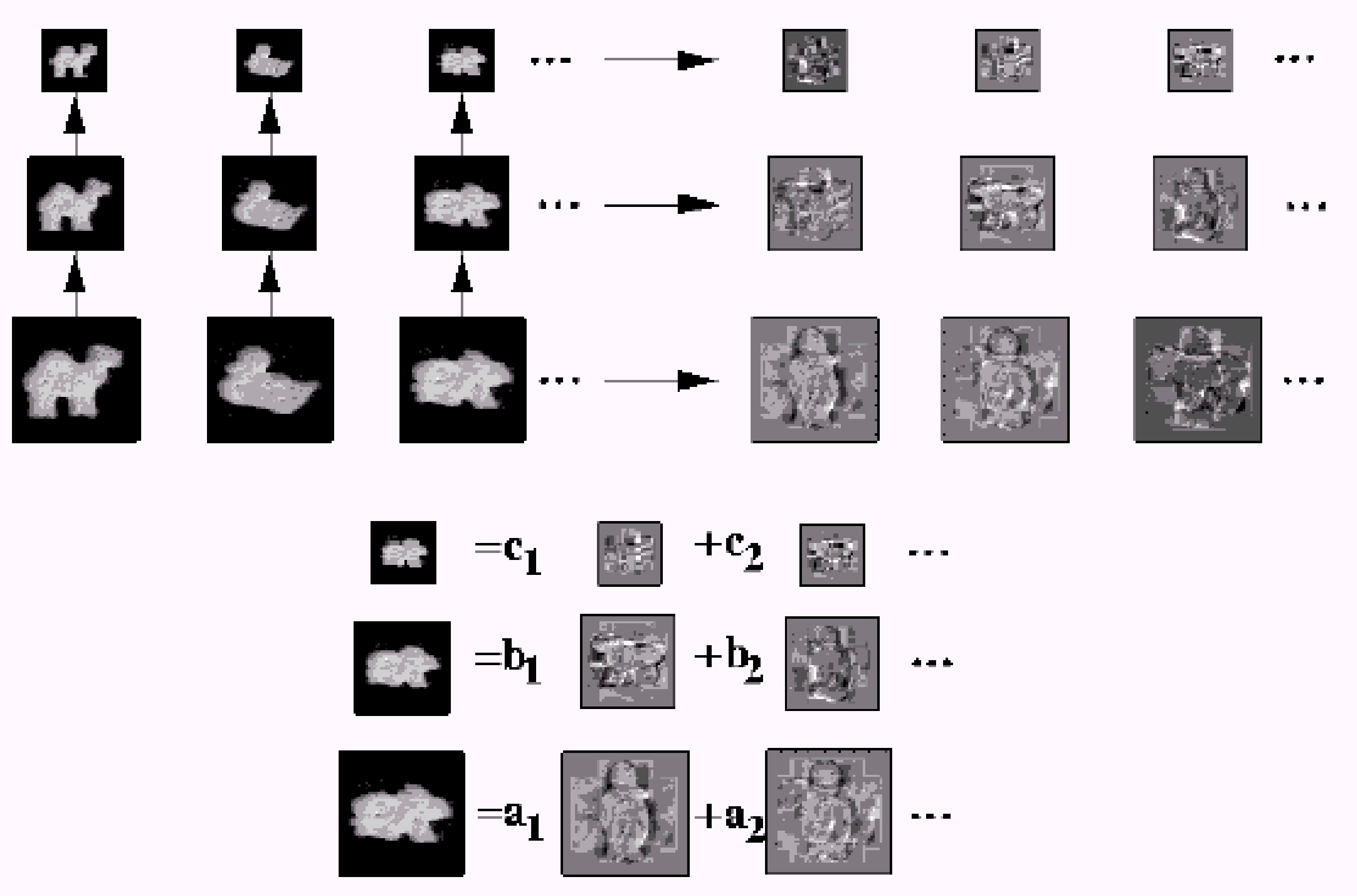
Multiresolution coefficient estimation

- **Multiresolution**
  - a well-known technique to reduce computational complexity
  - a search for the solution at the coarsest level and then a refinement through finer scales
- **Standard eigenspace method cannot be applied in an ordinary multiresolution way — it relies on the orthogonality of eigenimages.**



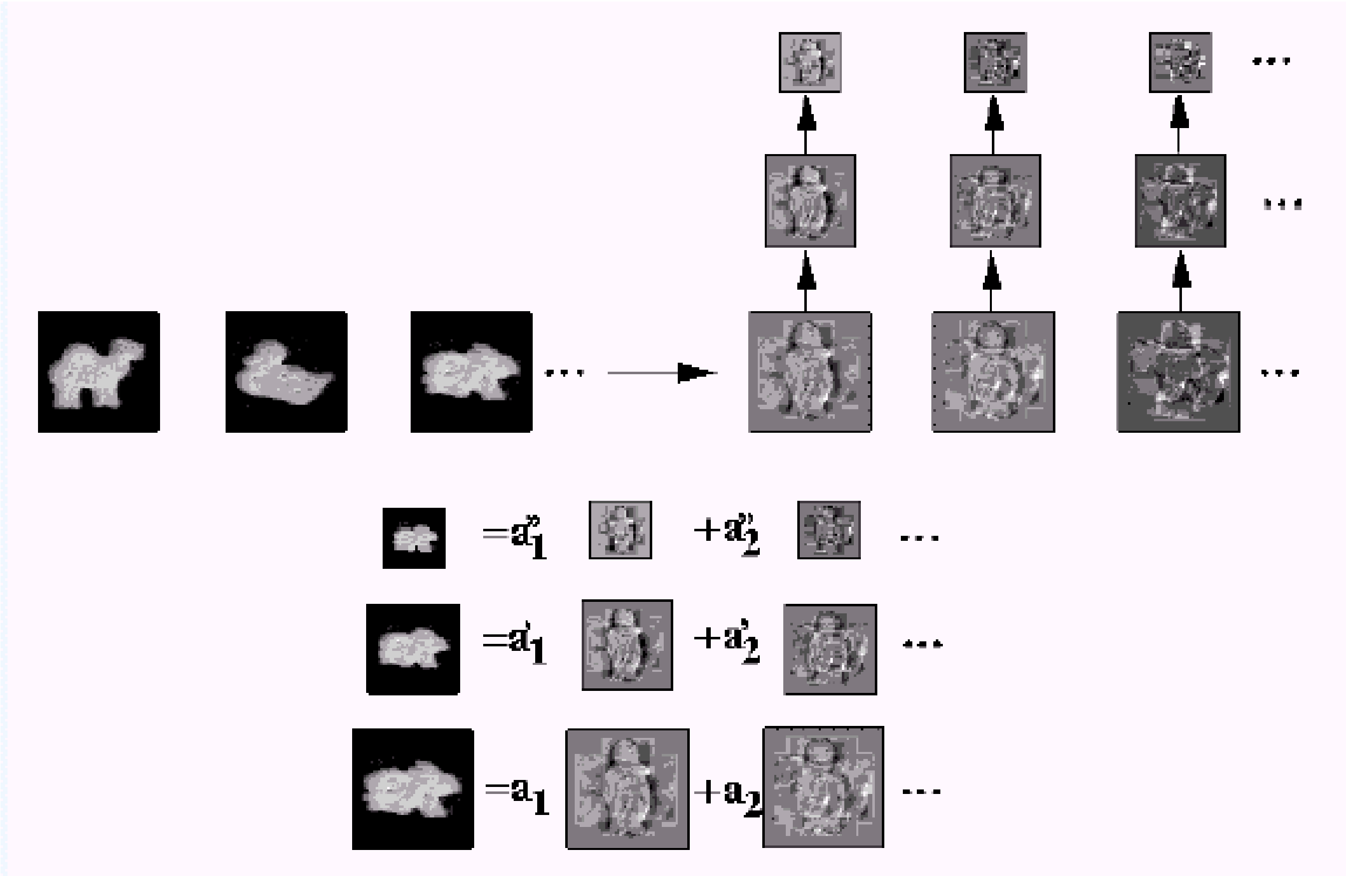
Standard multiresolution coefficient estimation

- Eigenimages in each resolution layer are computed from a set of templates in that layer
- Computationally costly and requires additional storage space



Robust multiresolution coefficient estimation

- Robust method requires only a single set of eigenimages obtained on the finest resolution.
- Linear system of equations: does not require orthogonality.



Multiresolution coefficient estimation

Linear System of Equations:

$$\tilde{x}(\mathbf{r}_j) = \sum_{i=1}^{p} a_i e_i(\mathbf{r}_j) \quad ,$$

Convolution:

$$(f * \tilde{x})(\mathbf{r}_j) = \sum_{i=1}^{p} a_i (f * e_i)(\mathbf{r}_j) \quad ,$$

Sub-sampling:

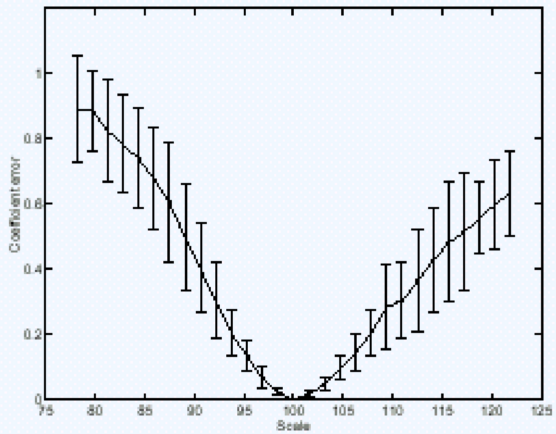
$$\tilde{x}_{\downarrow}(\mathbf{r}_j) = \sum_{i=1}^{p} a_i e_{i\downarrow}(\mathbf{r}_j) \quad ,$$

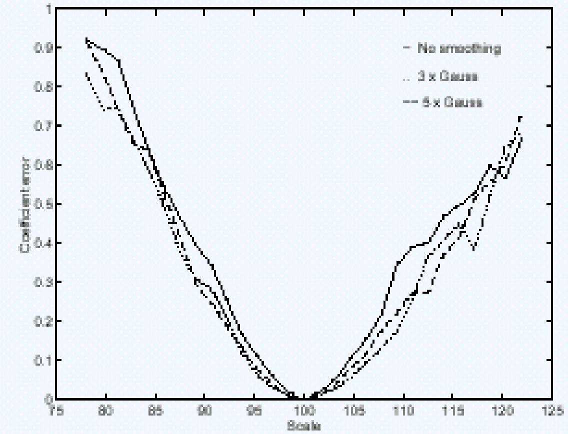
Convolution & Sub-sampling:

$$(f * \tilde{x})_{\downarrow}(\mathbf{r}_j) = \sum_{i=1}^{p} a_i (f * e_i)_{\downarrow}(\mathbf{r}_j) \quad ,$$

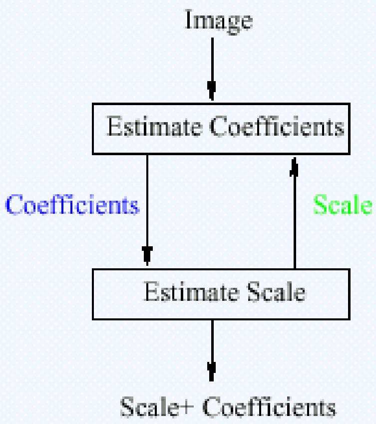
Same coefficients on convolved and sub-sampled eigenimages

Scale Sensitivity:





1. Generate multiple hypotheses at different scales.
2. **Estimate scale & coefficients simultaneously**.



Scale estimation

Minimize:

$$E_s(\alpha) = (\mathbf{s}(\mathbf{x}, \alpha) - \sum_{i=1}^{p} a_i \mathbf{e}_i)^2$$

$\mathbf{s}(\mathbf{x}, \alpha)$: image scaled by α.

- **Gradient descent**[Black]:
  Taylor series expansion of $\mathbf{s}(\mathbf{x}, \alpha)$
  - Small scale changes

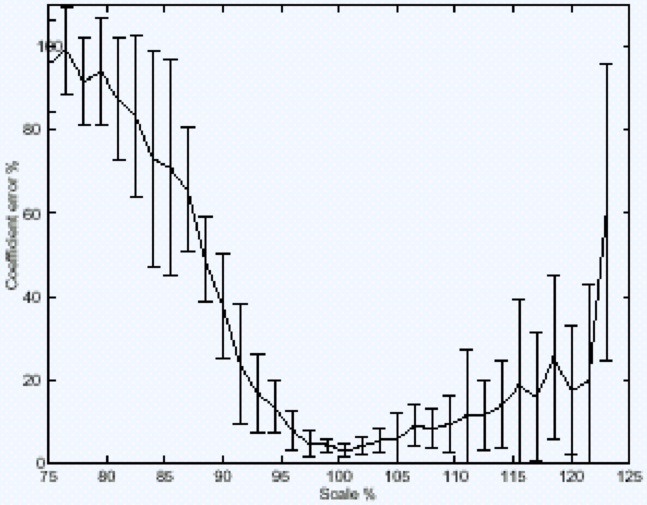
  $\longrightarrow$ High resolution

- **Coarse exhaustive search**:
  - Computationally costly

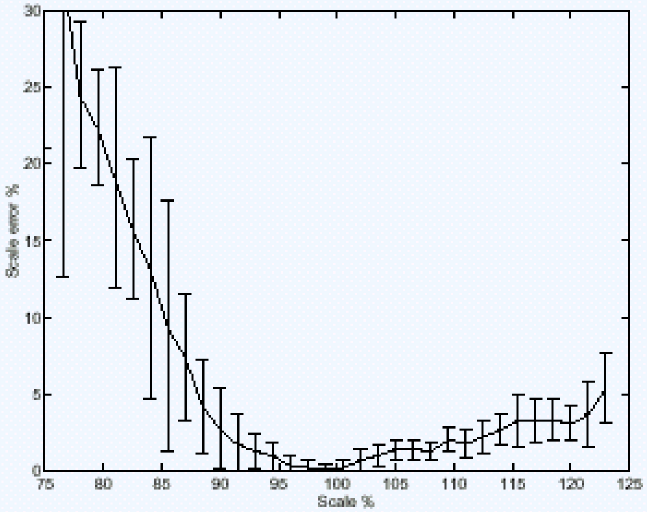
  $\longrightarrow$ Low resolution

Numerical demonstration

Coefficient Error:

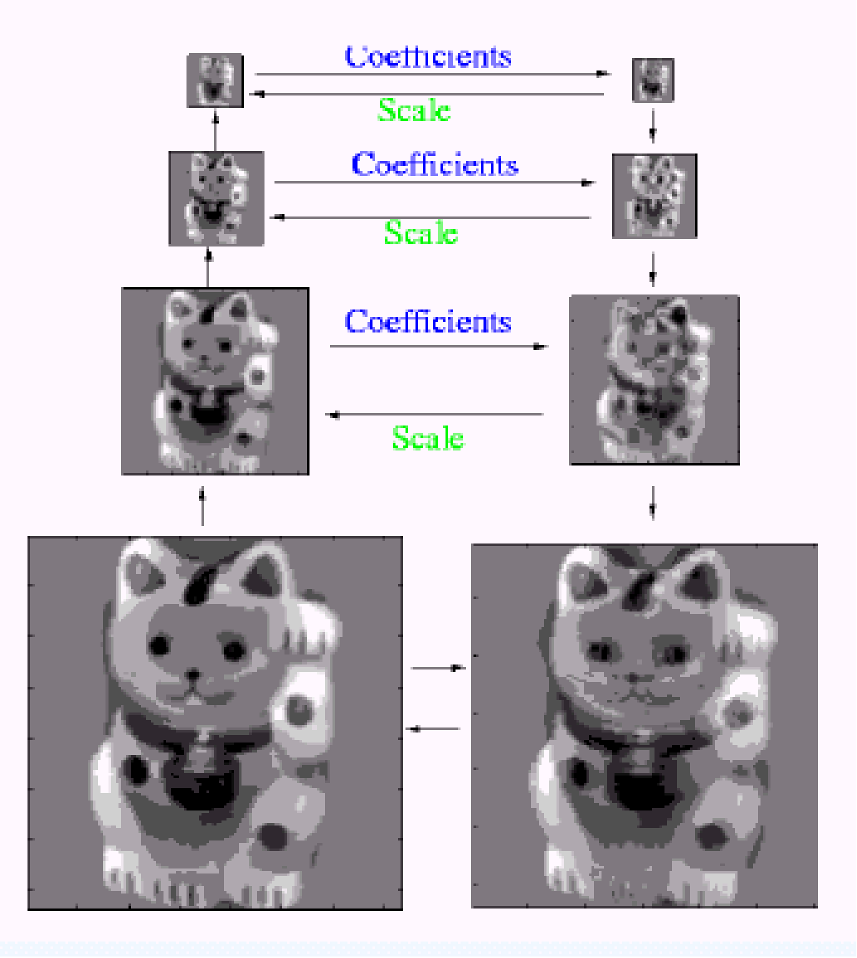


Scale Error:

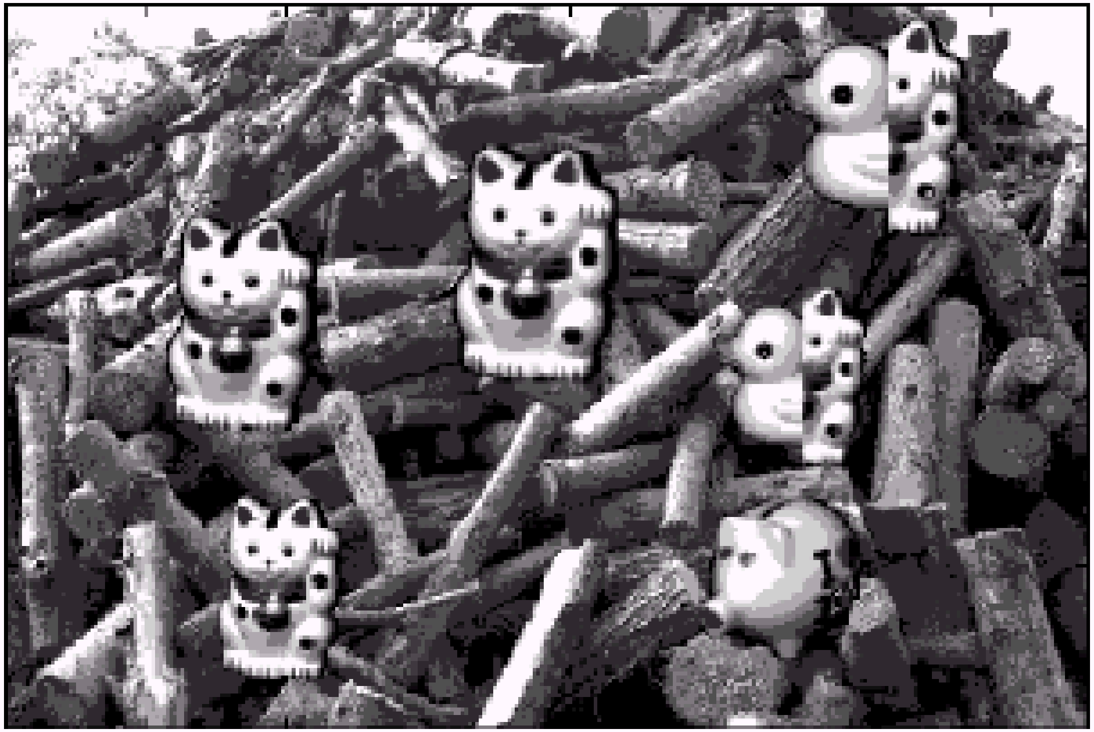


Multiresolution approach

- Estimate scale & coefficients simultaneously in the pyramid
- Efficient search structure



Experimental results – test image

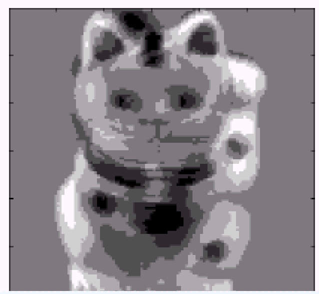


Experimental results

Cat

120% Scaled cat

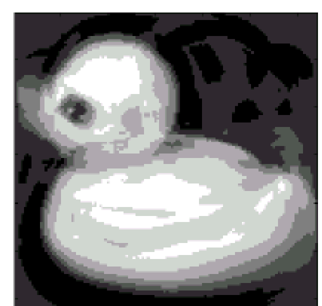
Occluded cat



120% Scaled occluded cat

Occluded duck

120% Scaled occluded duck



Illumination insensitive recognition

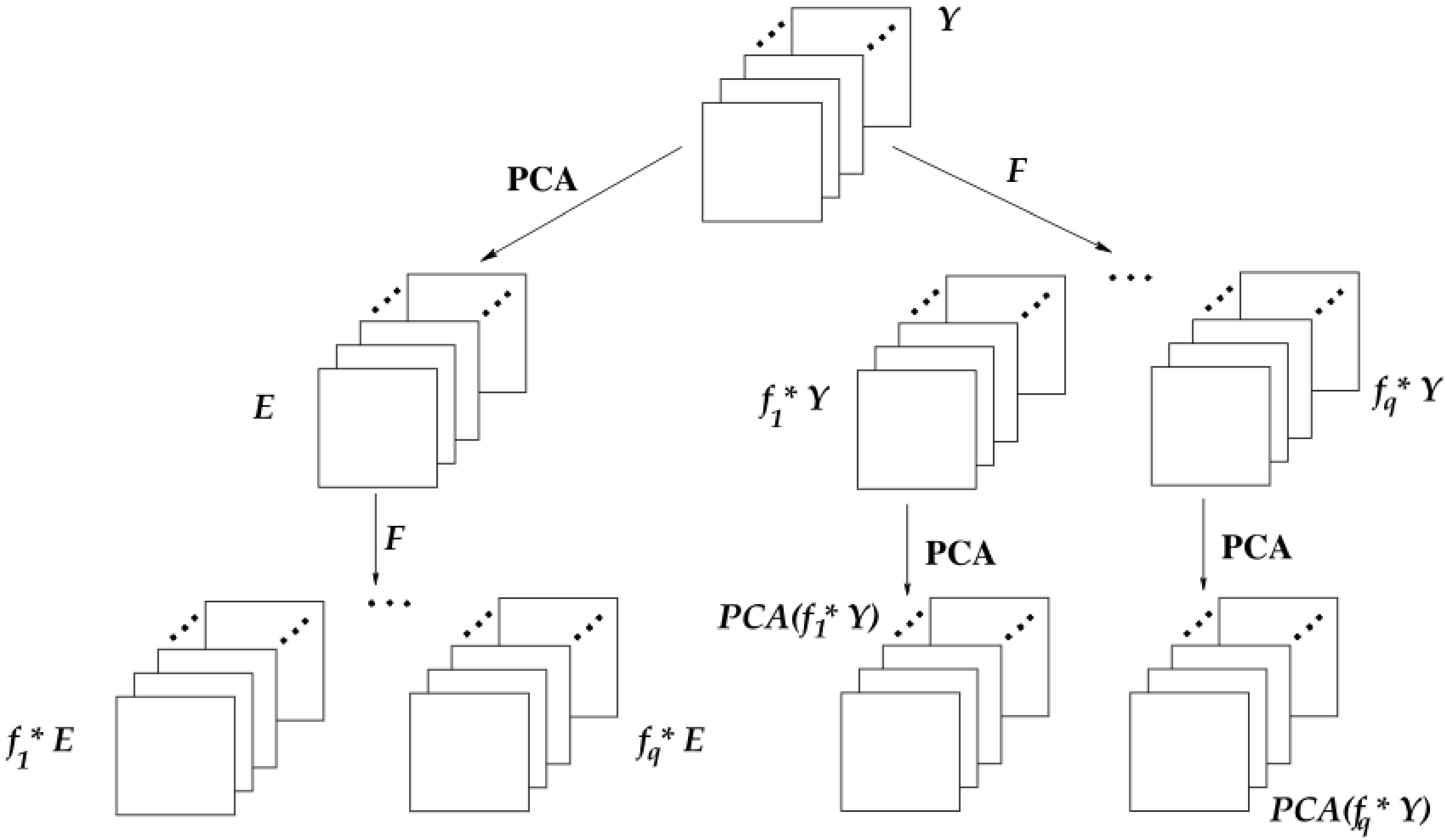
- Recognition of objects under varying illumination
  - global illumination changes
  - highlights
  - shadows
- Dramatic effects of illumination on objects appearance
- Training set under a single ambient illumination

Illumination insensitive recognition

Our Approach

- *Global* eigenspace representation
- *Local* gradient based filters
- Efficient combination of global and local representations
- Robust coefficient recovery in eigenspaces

Eigenspaces and filtering



Filtered eigenspaces

$$y_{r_i} = \sum_{j=1}^{n} q_j e_{jr_i} \quad 1 \leq i \leq k$$

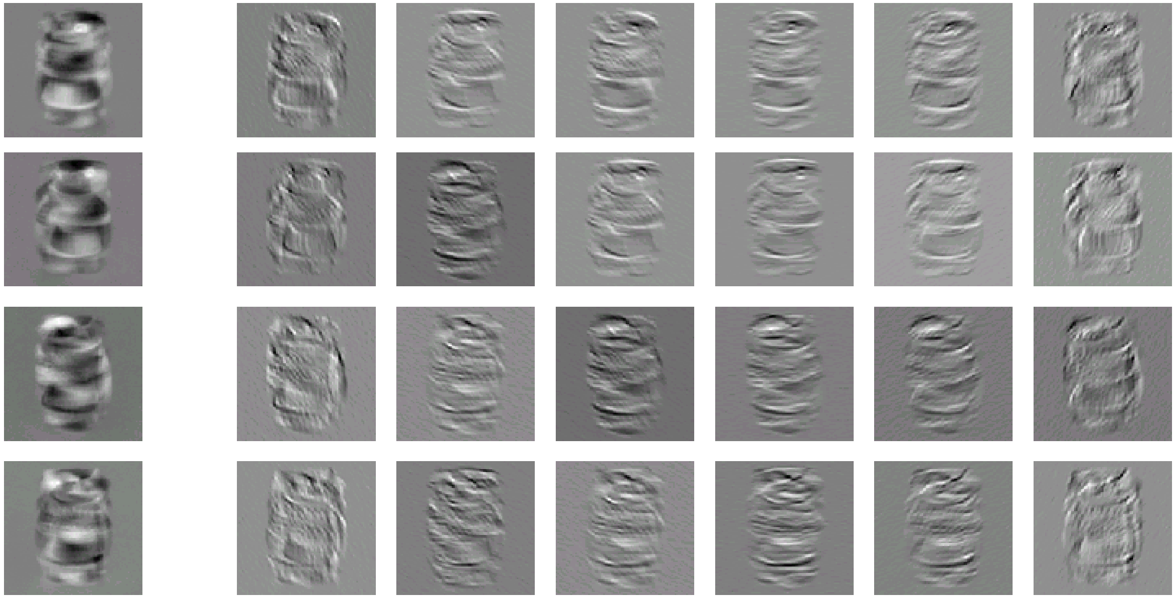
$$(f * x)(r) = \sum_{i=1}^{p} q_j (f * e_i)(r)$$

Gradient-based filters

Global illumination

Gradient-based filters

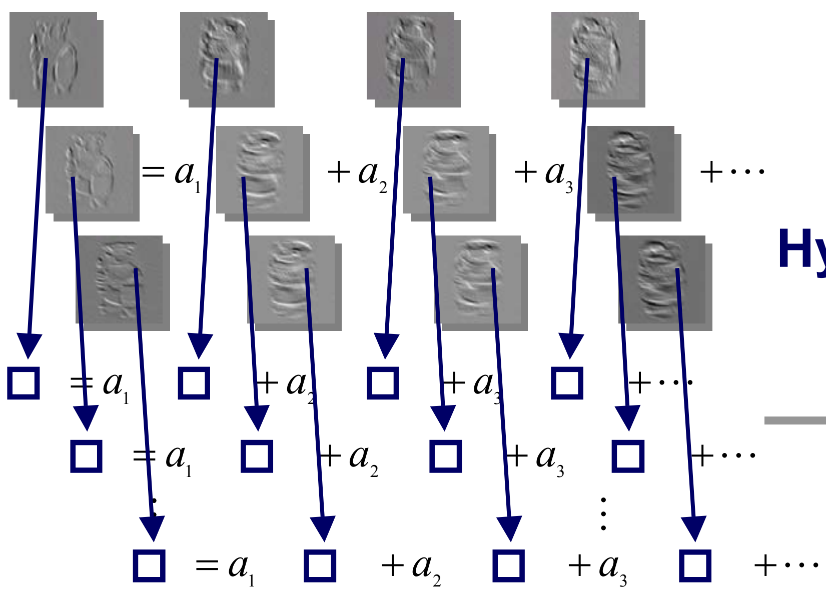
Steerable filters [Simoncelli]



Highlights and shadows

Robust coefficient recovery

Robust solution of linear equations



Hypothesize
&
Select

Experimental results

Test images

Our approach

Standard method

→ Demo

Experimental results

Robust filtered method - all eigenvectors used

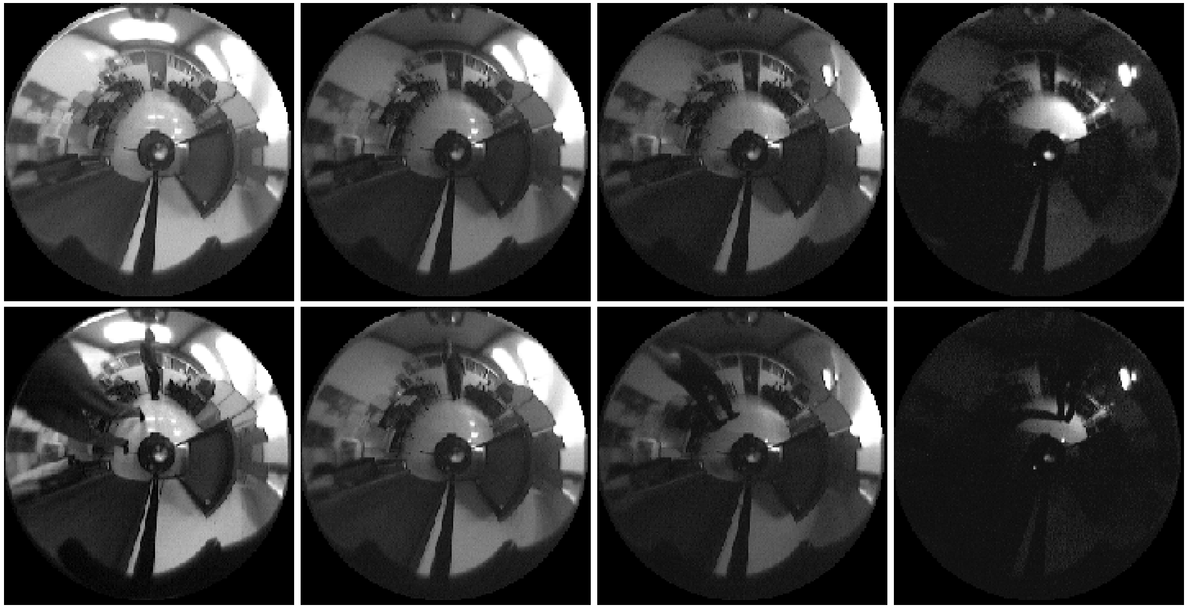
obj.	1	2	3	4	5	%	ang.
1	360	0	0	0	0	100.0	5.25
2	0	308	16	0	0	95.1	10.55
3	0	0	504	0	0	100.0	1.05
4	19	4	3	332	2	92.2	3.37
5	15	2	17	0	578	94.4	3.34
avg.						96.4	4.19

Standard method - all eigenvectors used

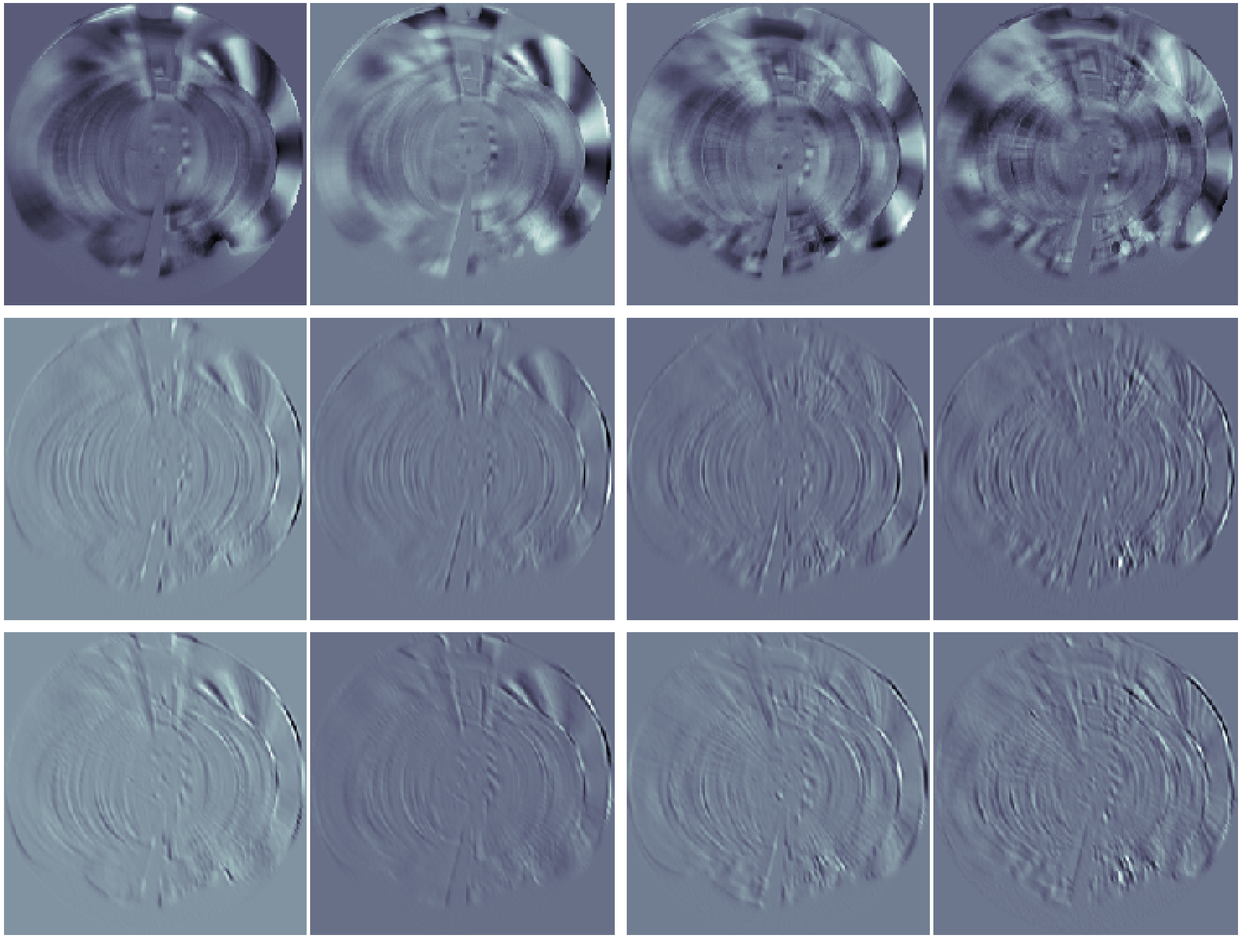
obj.	1	2	3	4	5	%	ang.
1	141	0	14	26	179	39.2	10.50
2	0	254	62	5	3	78.4	18.90
3	0	4	317	0	183	62.9	3.47
4	23	6	38	249	44	69.2	7.11
5	3	1	51	0	557	91.0	6.82
avg.						70.3	8.53

Illumination invariant localisation

- **Illumination variations and occlusions**

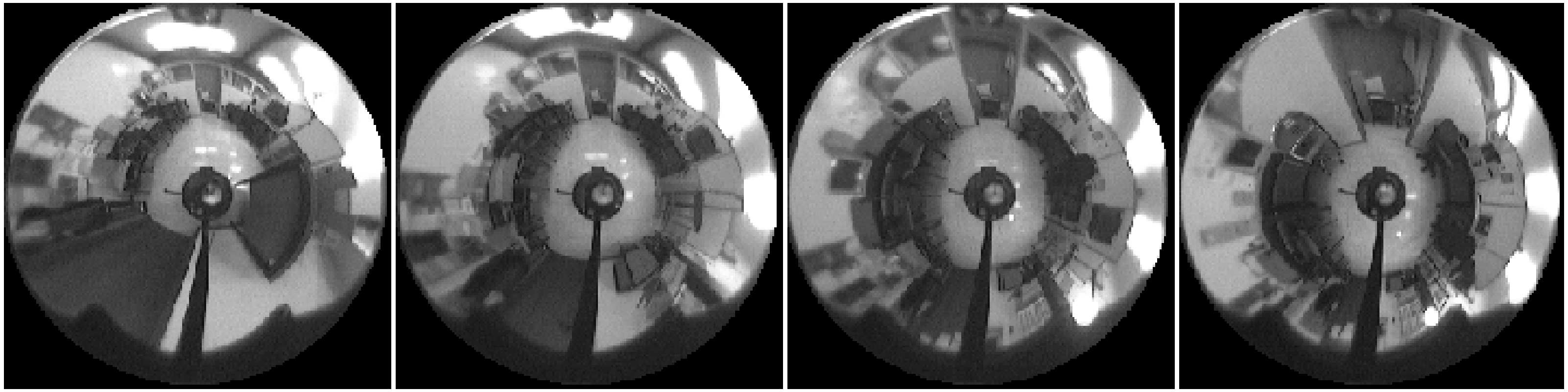


Filtered eigenvectors



Experimental results

- **Training set: straight path, uniform illumination**



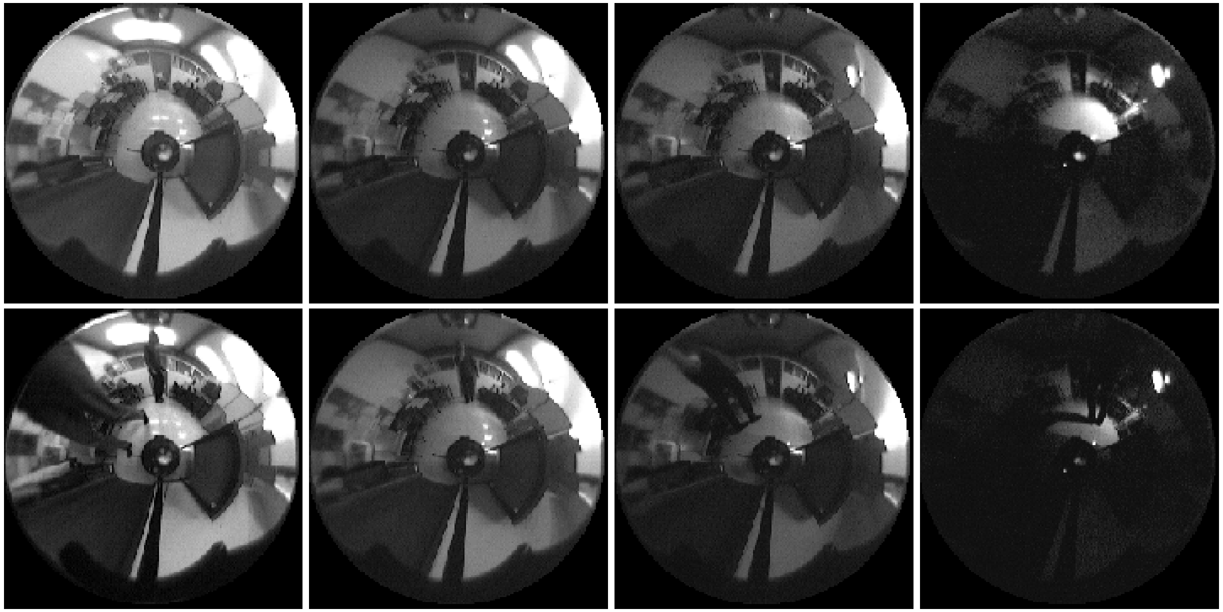
Experimental results

Test sets T/1/2/3

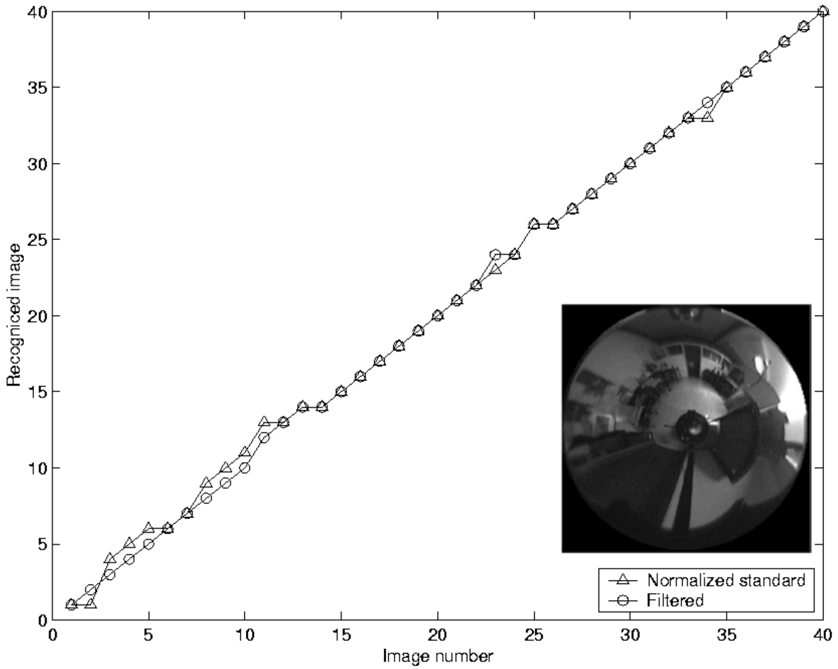
without occlusion

Test sets 4/5/6/7

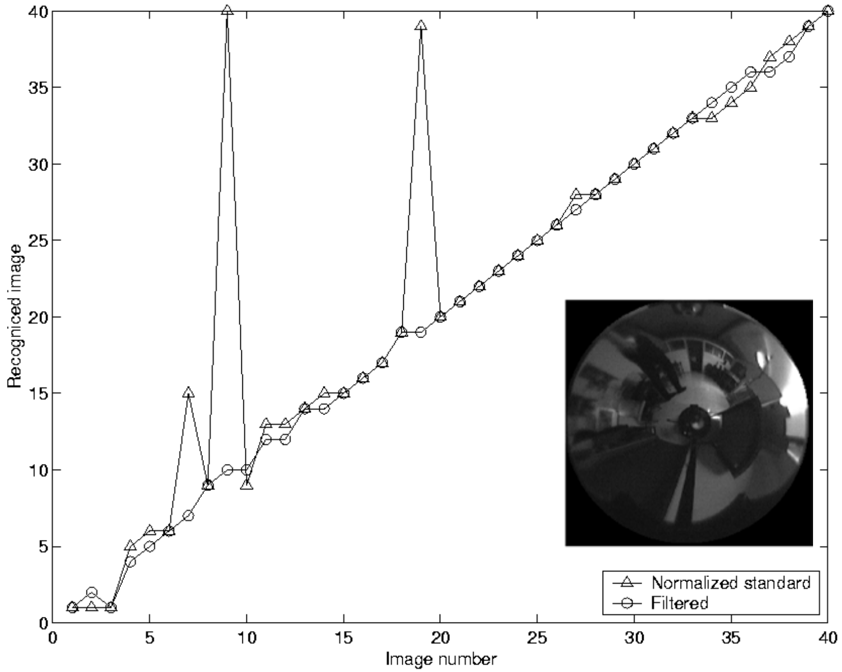
with occlusion



- ## Comparison with standard method

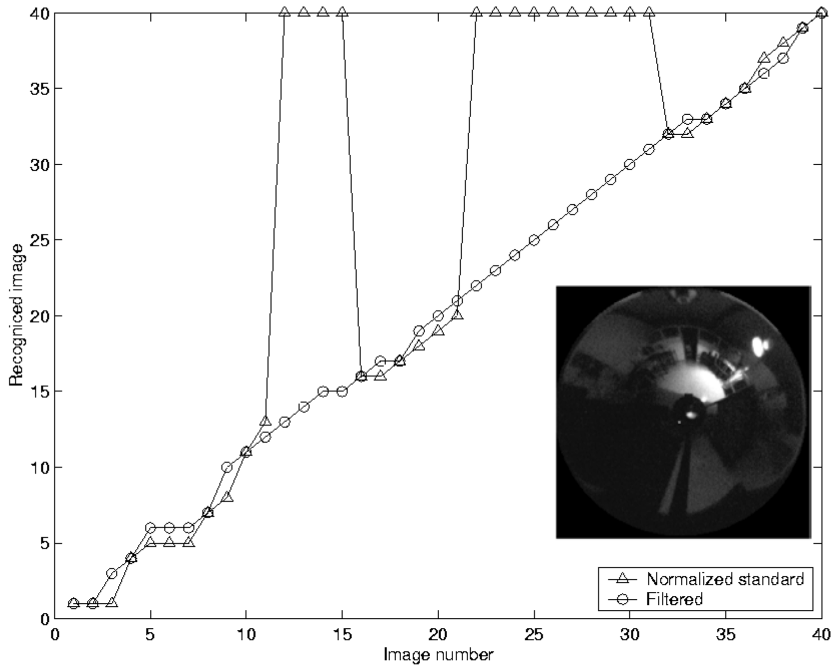


Test set 2

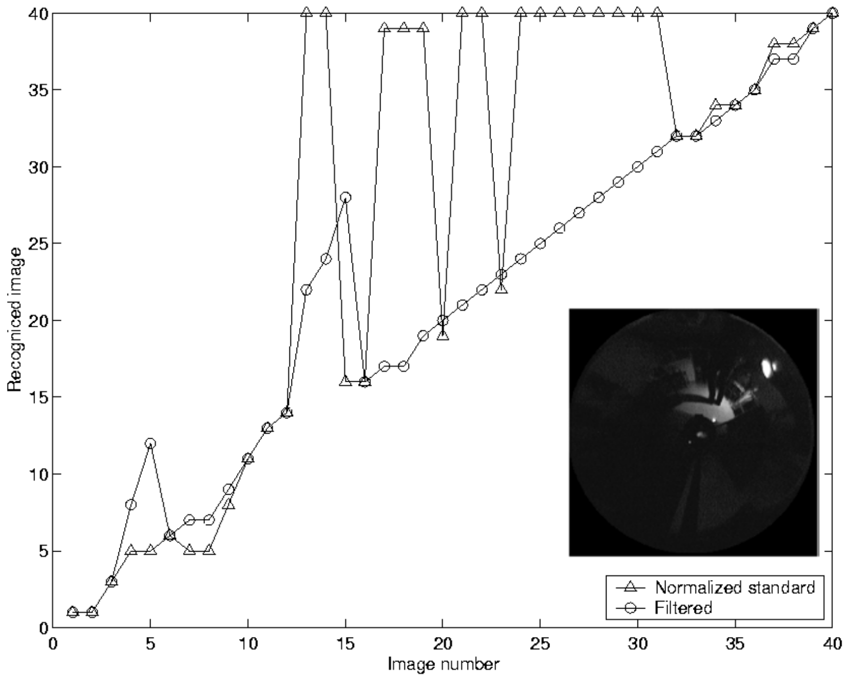


Test set 6

- **Comparison with standard method**



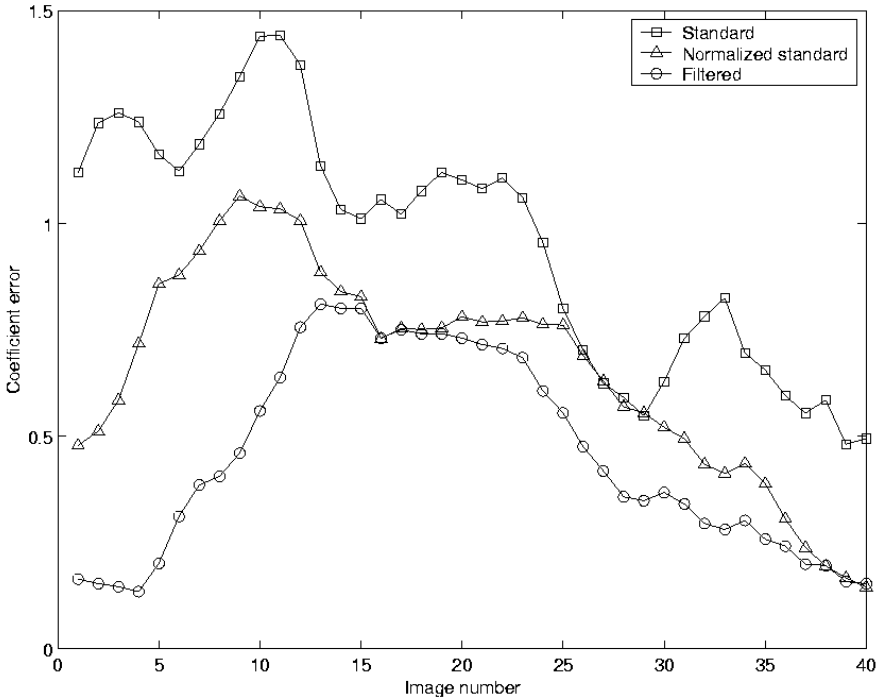
Test set 3



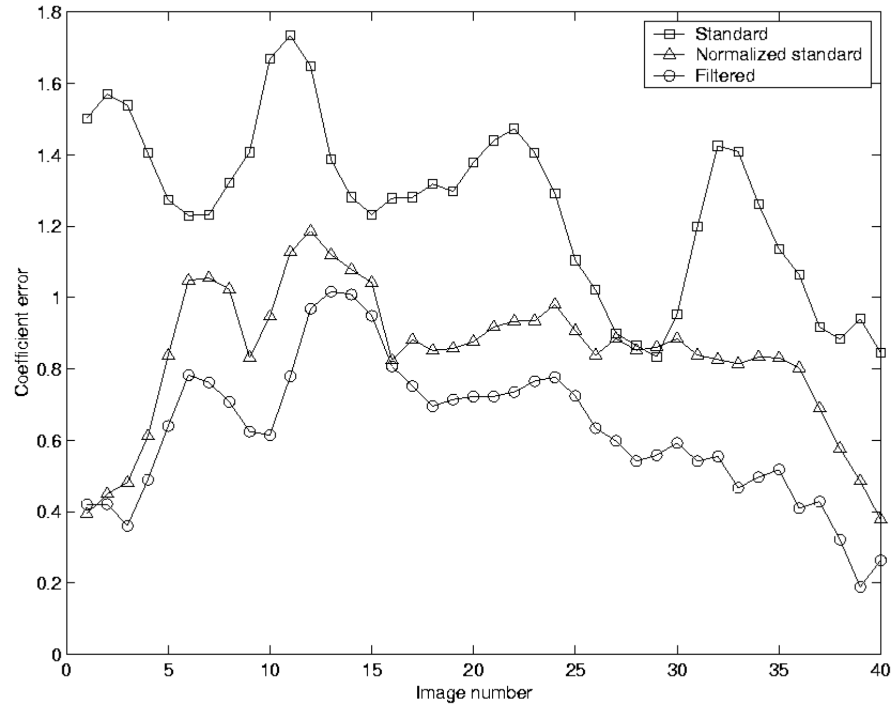
Test set 7

- **Comparison with standard method**

  Coefficient error, test sets 2 and 6



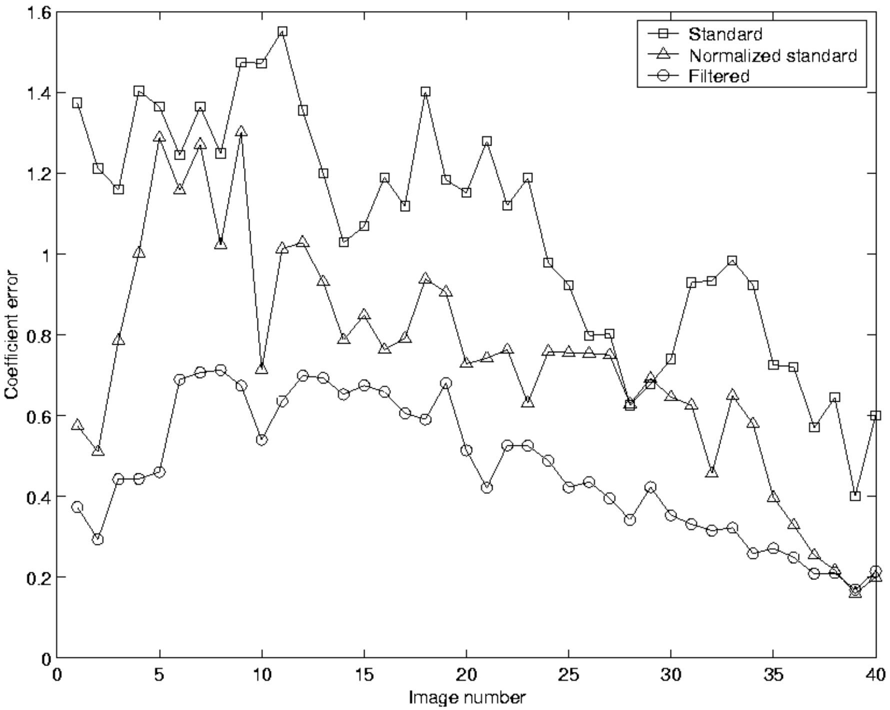
Test set 2



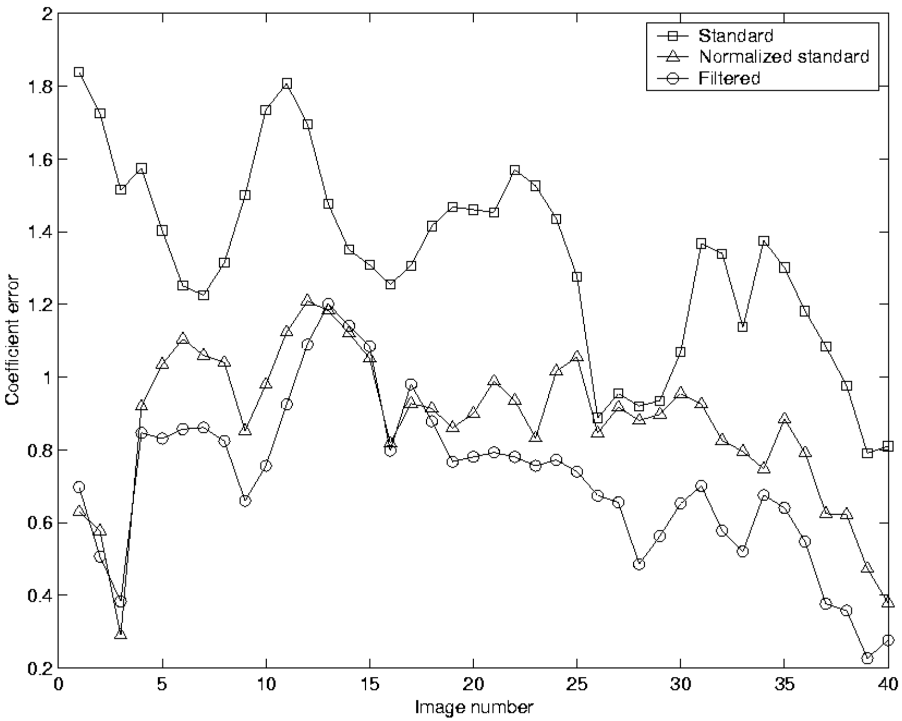
Test set 6

- **Comparison with standard method**

  Coefficient error, test sets 3 and 7



Test set 3



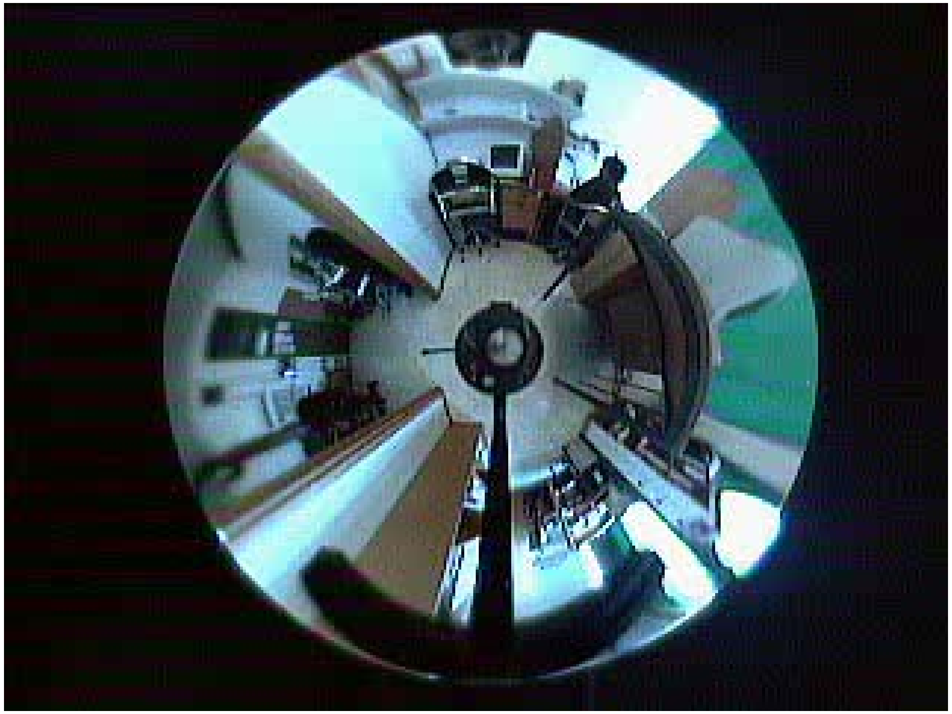
Test set 7

Experimental results

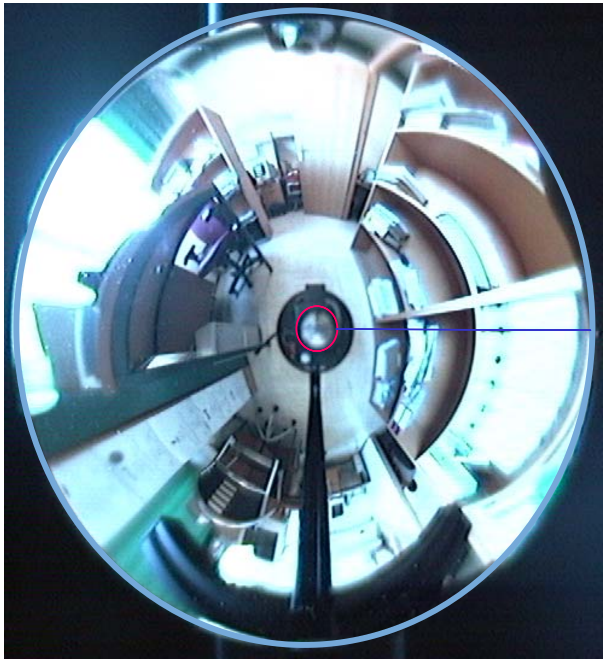
- **Average localisation error (in cm).**

Set	1	2	3	4	5	6	7
Standard	7	48.7	73.8	2.5	13.5	57.8	108.0
Normalized	1.5	3.3	65.0	0.8	3.3	19.0	68.3
Filtered.	0	1.3	4.0	0.5	1.8	2.3	14.0

Rotated panoramic images



Unwrapping



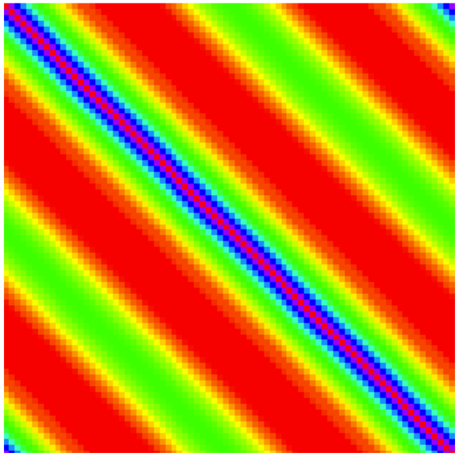
A rotated panoramic image

- **rotated/shifted *n* times**

$$X^{mn} = \begin{bmatrix} \mathbf{x}_0 & \mathbf{x}_1 & \dots & \mathbf{x}_{n-1} \end{bmatrix}$$

- **Inner product matrix $Q=X^TX$**
- **symmetric, Toeplitz, circulant**

$$Q = \begin{bmatrix} q_0 & q_1 & \dots & q_{n-2} & q_{n-1} \\ q_{n-1} & q_0 & q_1 & \dots & q_{n-2} \\ q_{n-2} & q_{n-1} & q_0 & q_1 & \dots \\ \dots & \dots & \dots & \dots & \dots \\ q_1 & \dots & q_{n-2} & q_{n-1} & q_0 \end{bmatrix}$$



Eigenvectors of a circulant matrix

- **Shift theorem: the eigenvectors of a general circulant matrix are the *N* basis vectors from the Fourier matrix *F*=[u_0', u_1', ... , u_{n-1}'], where**

$$\mathbf{u}'_i = \left[1, \omega^i, \omega^{2i}, \ldots, \omega^{(n-1)i}\right]^\top, \quad i = 0, \ldots, n-1$$

- **The eigenvalues can be calculated simply by retrieving the magnitude of the DFT of one row of *Q***

$$\omega = e^{-2\pi j/n}, \quad j = \sqrt{-1}$$

$$\lambda_i = \sum_{l=0}^{n-1} q_l \, \omega^{i\,l}$$

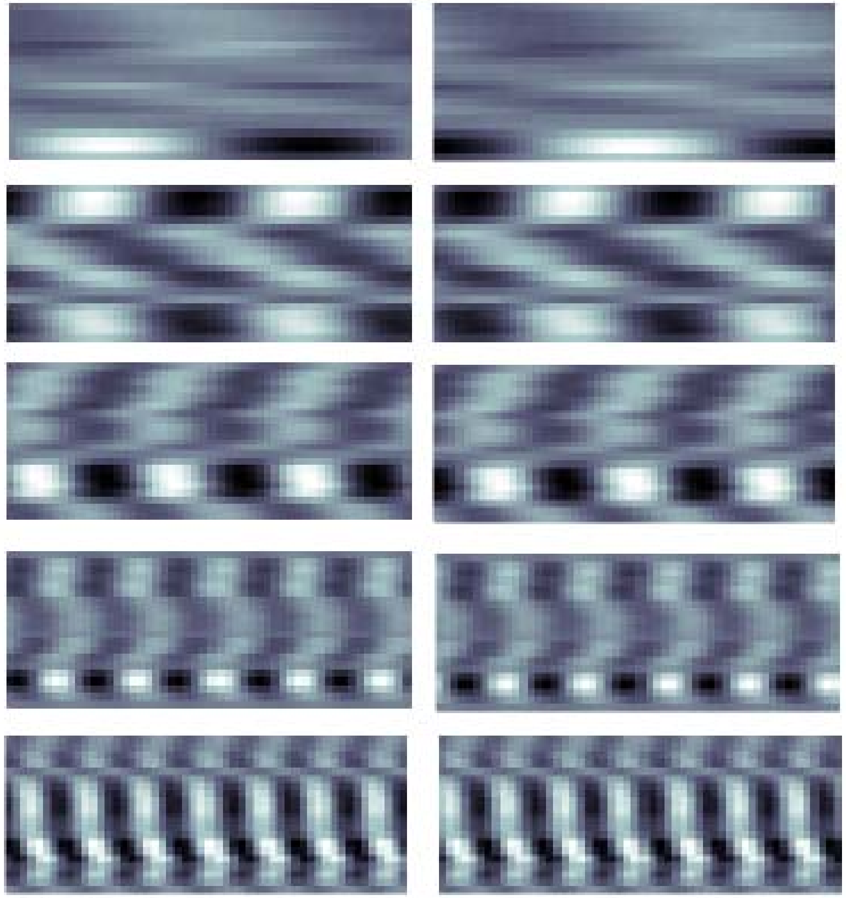
From u_i' to u_i

- **The eigenvectors of XX^T can be obtained by using $XX^T X u_i' = \lambda_i' X u_i'$:**

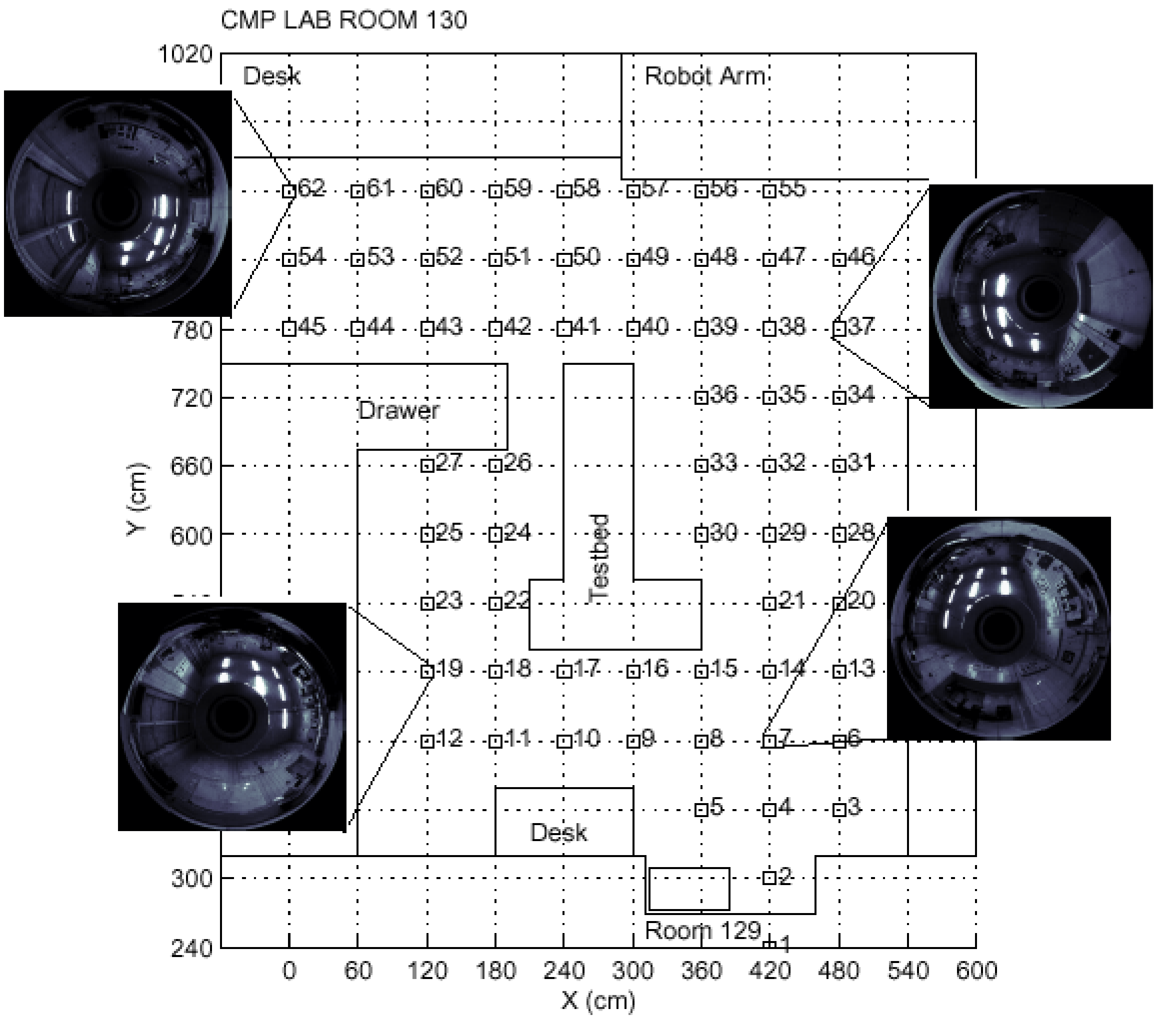
$$\mathbf{u}_i = \frac{1}{\sqrt{\lambda_i'}} X \mathbf{u}_i'$$

- **eigenvectors** u_i
  - same frequency as $\mathbf{u}_i'$,
  - phase and amplitude may change

Eigenvectors



Generalisation to several locations



A set of rotated images

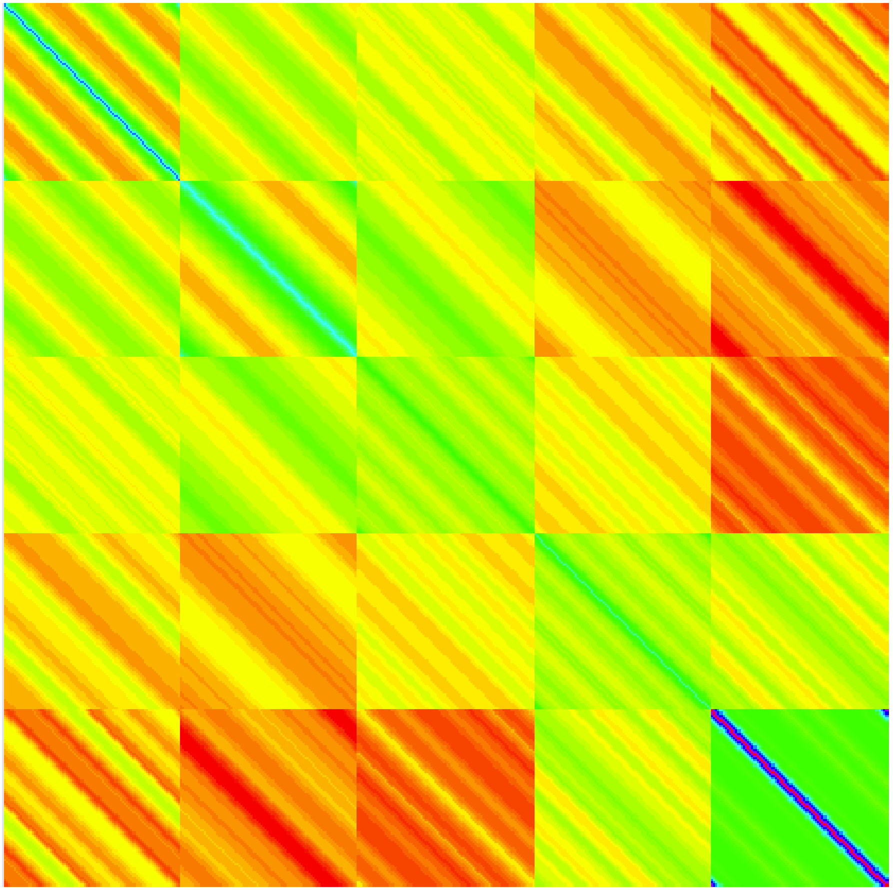
- ***P*** **different locations, each shifted** ***n*** **times**

$$A = X^T X = \begin{bmatrix} Q_{00} & Q_{01} & \dots & Q_{0,P-1} \\ Q_{10} & Q_{11} & \dots & Q_{1,P-1} \\ \dots & \dots & \dots & \dots \\ Q_{P-1,0} & Q_{P-1,1} & \dots & Q_{P-1,P-1} \end{bmatrix}$$

- **Every Q_{ij} is circulant (but in general not symmetric!)**
- **Is it possible to exploit these properties?**

It is still possible to compute the eigenvectors without performing the SVD decomposition of A.

Rotated panoramic images



Eigenvalue problem

- Solution of the problem

$$A\mathbf{w}' = \mu\mathbf{w}'$$

- matrix blocks Q_{jl} of A are circulant matrices
- every circulant matrix can be diagonalised in the same basis by Fourier matrix F
- all the submatrices Q_{jk} have the same set of eigenvectors

$$\mathbf{u}'_i, \quad i = 0, \ldots, n-1$$

$$\mathbf{w}'_i = \left[\alpha_{i0}\mathbf{u}'^T_i, \alpha_{i1}\mathbf{u}'^T_i, \ldots, \alpha_{i,P-1}\mathbf{u}'^T_i\right]^T$$

- $A\mathrm{w}' = \mu\mathrm{w}'$ **written blockwise:**

$$\sum_{l=0}^{P-1} Q_{jl}(\alpha_{il}\mathbf{u}'_i) = \mu\alpha_{ij}\mathbf{u}'_i, \quad j = 0, \ldots, P-1$$

- **Since** u_i' **is an eigenvector of every** Q_{jl},

$$\sum_{l=0}^{P-1} \alpha_{il}\lambda^i_{jl}\mathbf{u}'_i = \mu\alpha_{ij}\mathbf{u}'_i, \quad j = 0, \ldots, P-1,$$

- λ^i_{jk} **is an eigenvalue of** Q_{jl} **corresponding to** u_i'.

Derivations

- **This implies a new eigenvalue problem**

$$\Lambda \alpha_i = \mu \alpha_\iota,$$

- **where**

$$\Lambda = \begin{bmatrix} \lambda_{00}^i & \lambda_{01}^i & \cdots & \lambda_{0,P-1}^i \\ \lambda_{10}^i & \lambda_{11}^i & \cdots & \lambda_{1,P-1}^i \\ \cdots & \cdots & \cdots & \cdots \\ \lambda_{P-1,0}^i & \lambda_{P-1,1}^i & \cdots & \lambda_{P-1,P-1}^i \end{bmatrix}$$

- **and**

$$\boldsymbol{\alpha}_i = \left[\alpha_{i0}, \alpha_{i1}, \ldots, \alpha_{i,P-1}\right]^\top$$

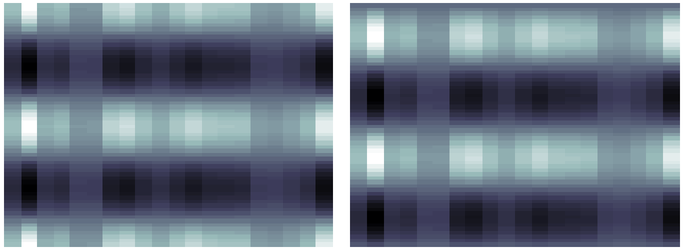
Computational complexity

- Since $Q_{jk}=Q^{T}_{kj}$, it can be proved that Λ is Hermitian and
- we have *P* linearly independent eigenvectors α_i,
- which provide *P* linearly independent eigenvectors w'_i.
- Since the same procedure can be performed for every v'_i,
- we can obtain $N \cdot P$ linearly independent eigenvectors of *A*.

It is therefore possible to solve the eigen-problem using *N* decompositions of order *P* (as opposed to decomposition of $P \cdot N$).

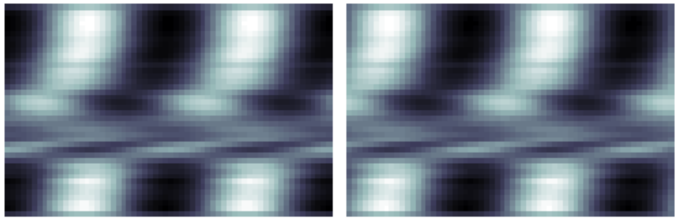
Complex eigenspace of spinning images

- **Real and imaginary part of one of the vectors:**

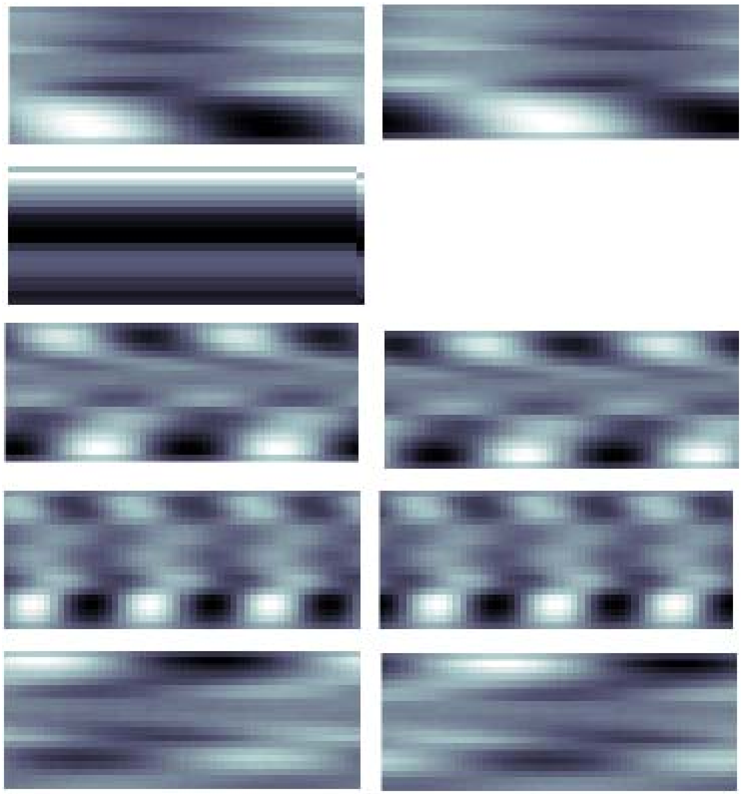


- **Real and imaginary part of one of the w vectors:**

$$\mathbf{w}_i = \frac{1}{\sqrt{\mu_i}} X \mathbf{w}'_i$$



Eigenvectors

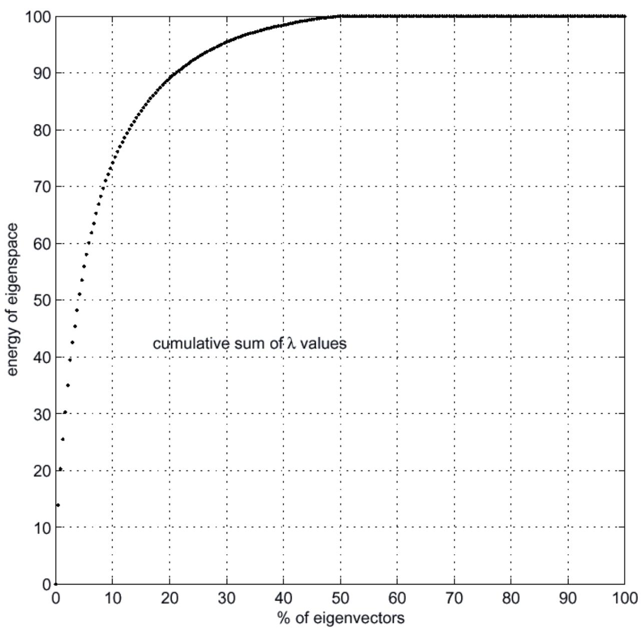


Energy distribution

- compressing efficiency of the eigenspace

$$\frac{\sum_{i=0}^{K-1} \lambda_i}{\sum_{j=0}^{S-1} \lambda_j} \cdot 100\%$$

- 62 images, each in 50 different orientations



Timings for building the eigenspace

- Images of dimensions 40x68, each image was rotated/shifted 68 times, i.e., for 40 locations we got 2720 images.
- This is also the number of image elements (the border case when the covariance matrix is of the same size as the inner product matrix, and the complexity of the SVD method reaches its upper bound).

locations (P)	XX^T	X^TX	CPLX
10	2507.3	55.8	16.1
20	2569.6	429.2	105.3
30	2634.8	1400.3	312.4
40	3007.7	3252.3	853.2

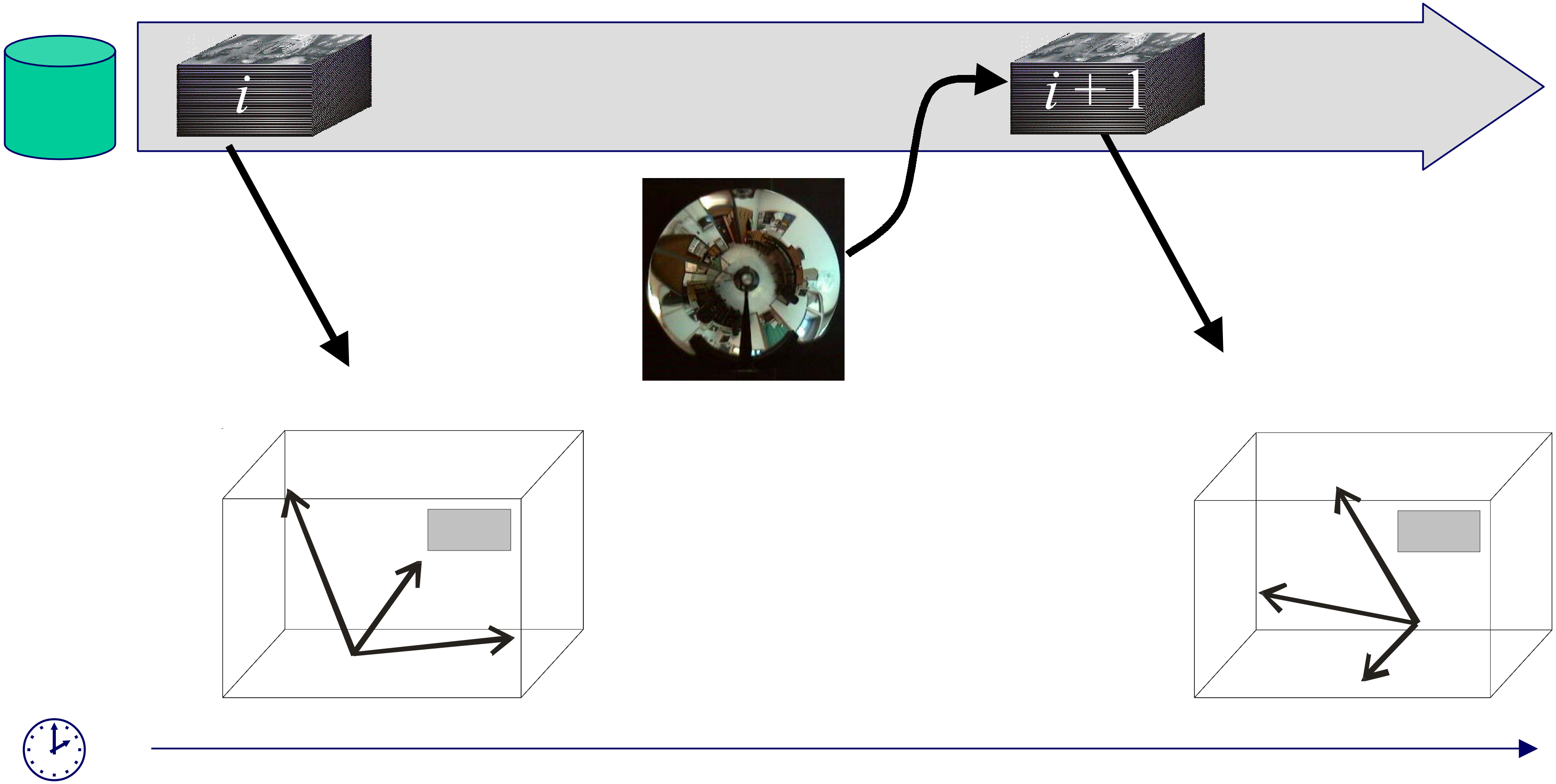
Eigenspace of spinning images

- K-L expansion of a set of rotated panoramic images
- SVD on the complete covariance matrix is not necessary
- Instead, we solve a set of smaller eigen-problems
- The final eigenvectors are composed of locally varying harmonic functions (analytic functions!)

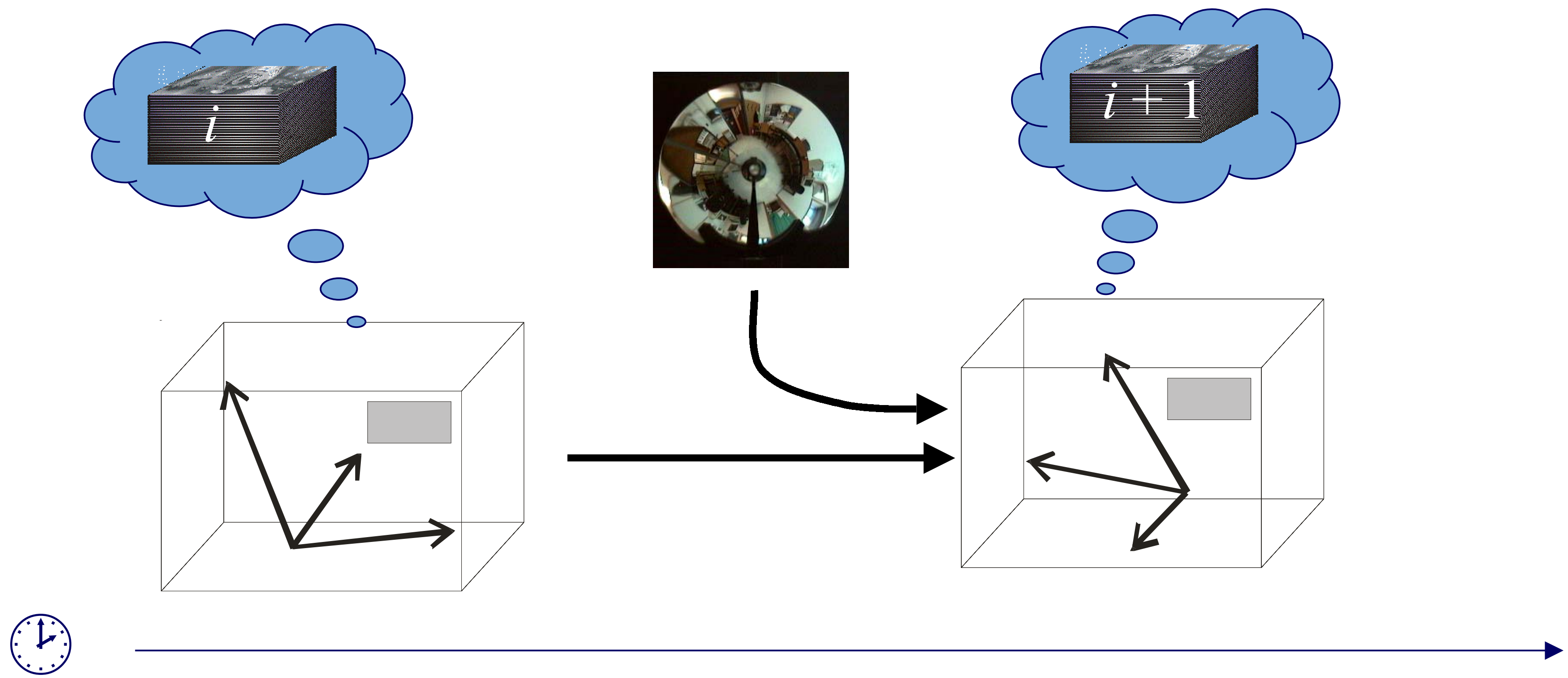
Other approaches

- Images stored in arbitrary orientation
- Images stored in a reference orientation (e.g. gyrocompass)
- Autocorrelation
- FFT power spectra
- Zero Phase Representation
- Eigenspace of spinning-images

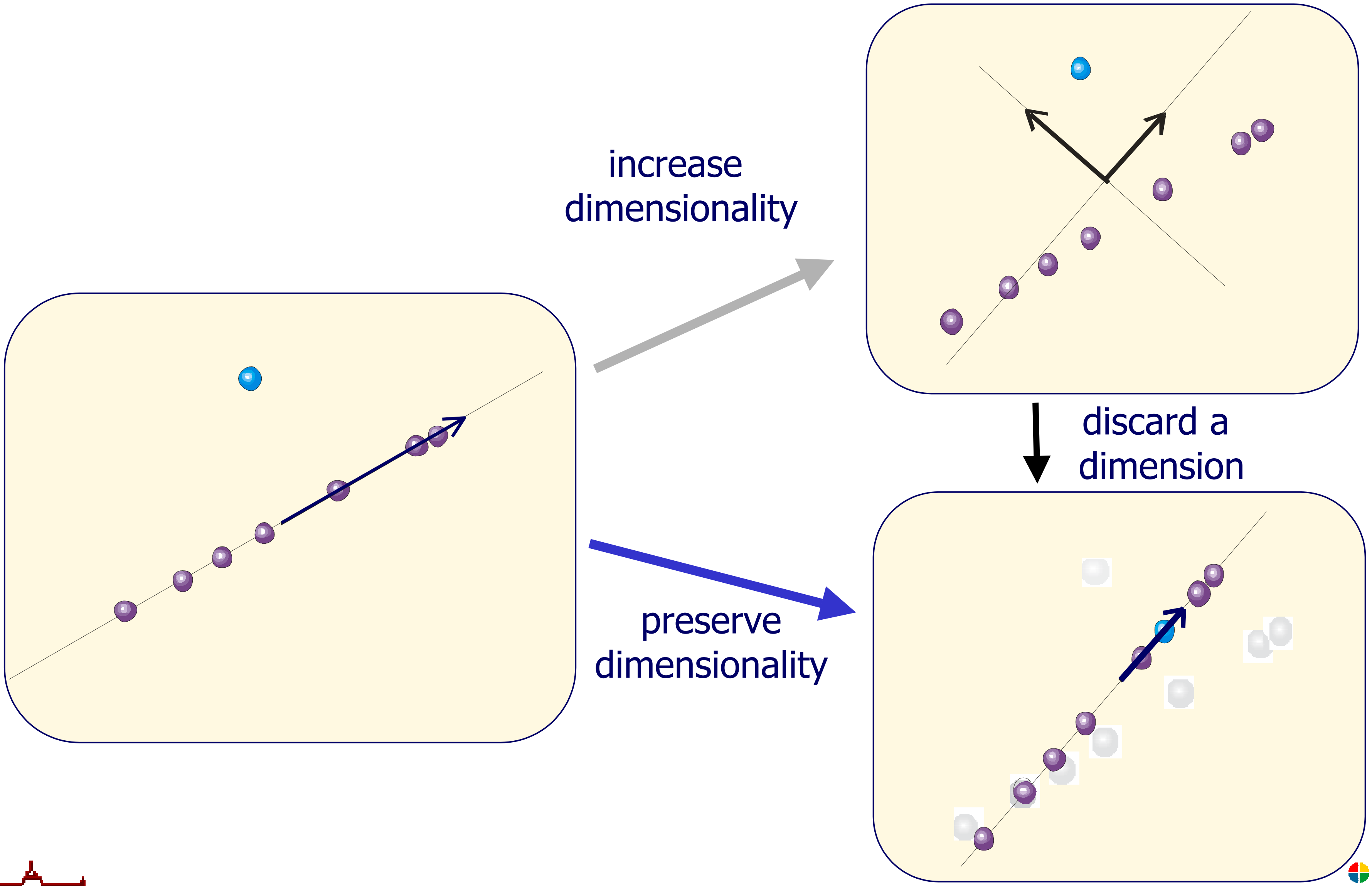
Batch computation of PCA



Incremental computation of PCA

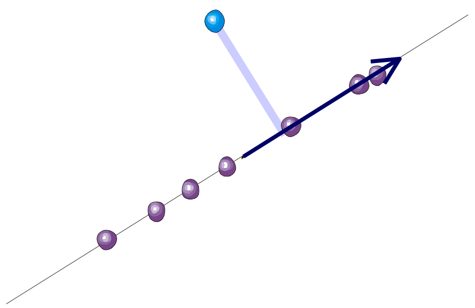


Incremental computation of PCA – Algorithm

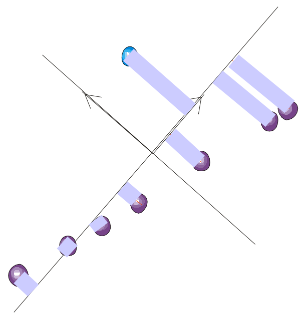


Determining the number of eigenvectors

- **Increase the number of dimensions by one if the distance between the last image and its projection is high.**



- Increase the number of dimensions by one if the cumulative distance between the images and their projections is high.



Incremental PCA in detail

- **Extend *U* with a residual $\mathbf{h}_n$ of the new image $\mathbf{y}$ and rotate by *R***
- **Rotation matrix *R* is a result of the eigenproblem**

$$U' = [U \ \mathbf{h}_n] \, R \qquad ; \ R \in \mathbb{R}^{(k+1)\times(k+1)}$$

- **Λ' is the new eigenvalue matrix**

$$D \ R = R \ \Lambda'$$

Incremental PCA in detail

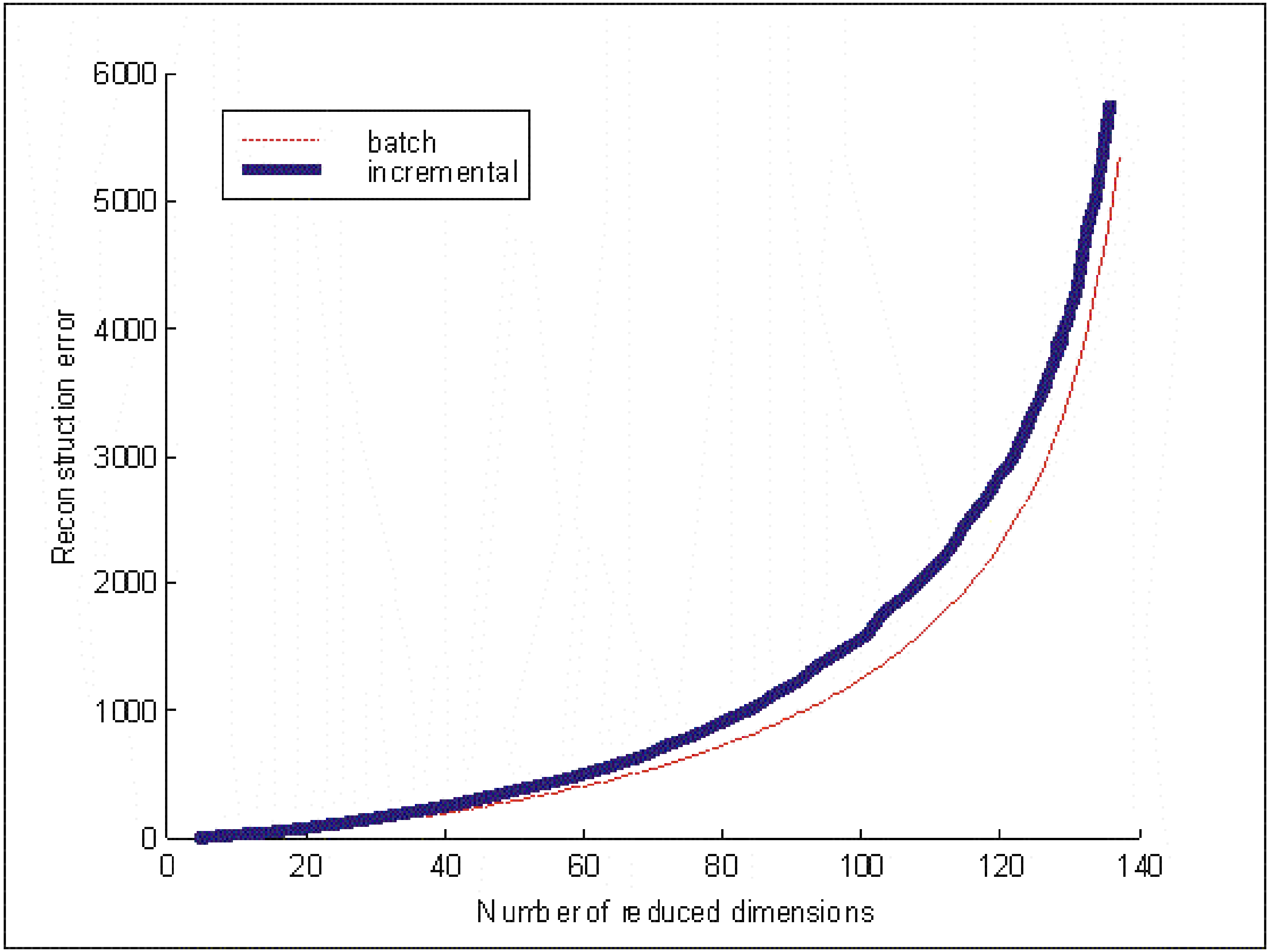
- *D* is assembled from the current eigenvalues Λ, the new image y and its corresponding coefficients a

$$D\ R = R\ \Lambda'$$

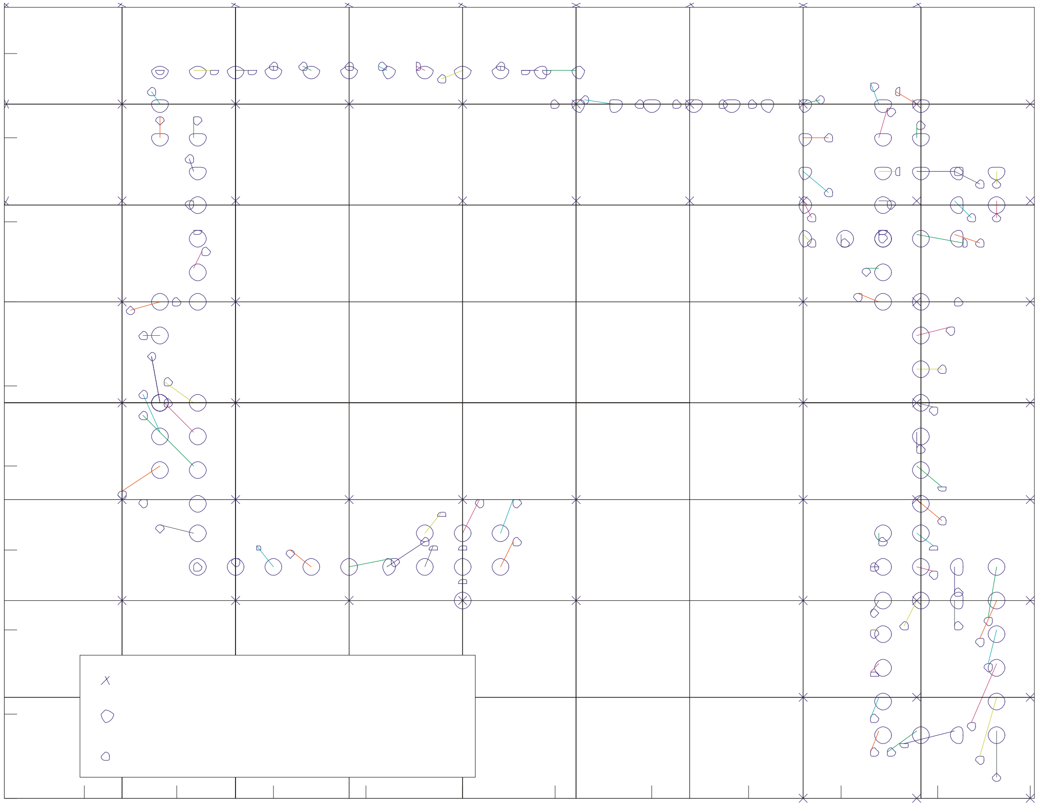
$$D = \frac{n}{n+1}\left[\begin{array}{cc} \Lambda & \mathbf{0} \\ \mathbf{0}^\top & 0 \end{array}\right] + \frac{n}{(n+1)^2}\left[\begin{array}{cc} \mathbf{a}\mathbf{a}^\top & \gamma\mathbf{a} \\ \gamma\mathbf{a}^\top & \gamma^2 \end{array}\right]$$

$$\gamma = \mathbf{h}_n(\mathbf{y} - \overline{\mathbf{x}})$$

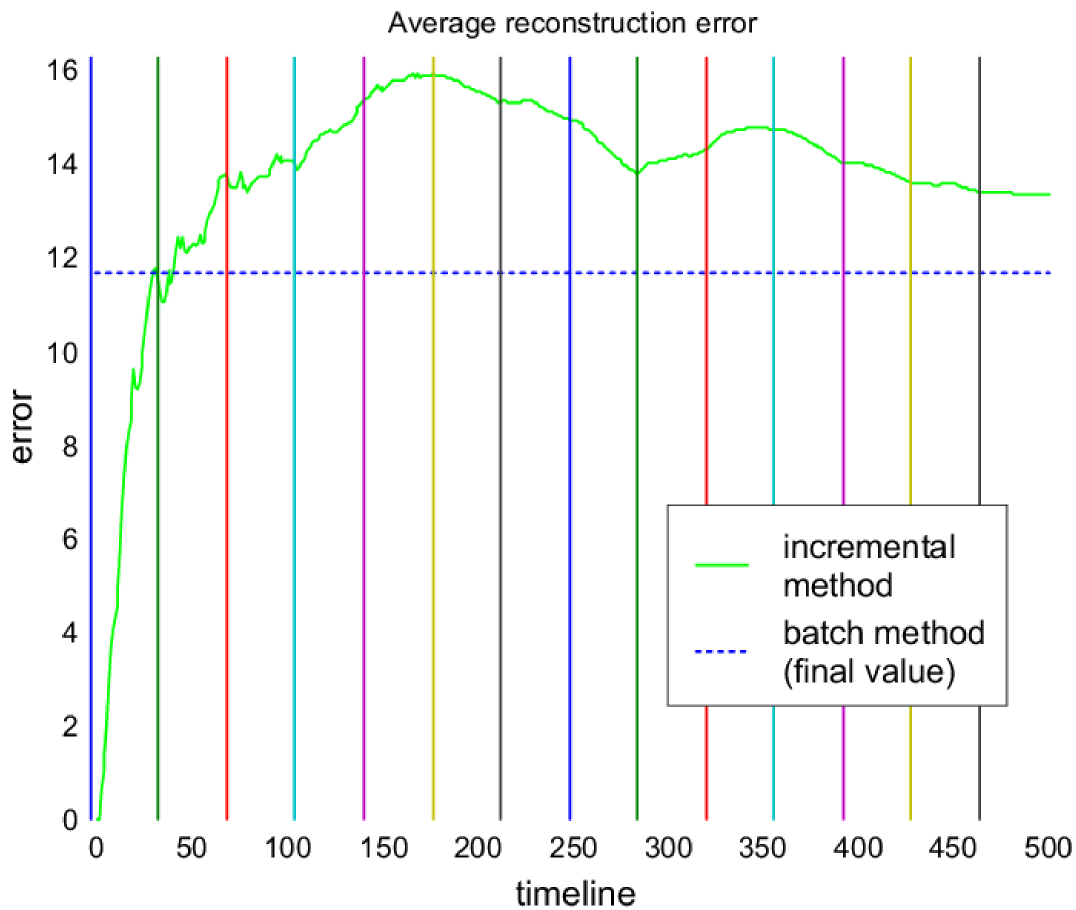
Comparison with batch method

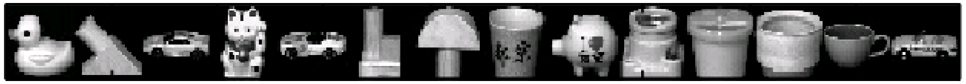


Localization with incremental method



Reconstruction error through time





Multiple Eigenspaces - Motivation

- **A single eigenspace**
  - high dimensionality
  - low-dimensional structure of data is ignored
  - poor generalisation
- **Ad-hoc partitioning of the image set is not efficient**

Multiple eigenspaces – our goal

- **Systematically construct multiple low-dimensional eigenspaces from a set of training images**

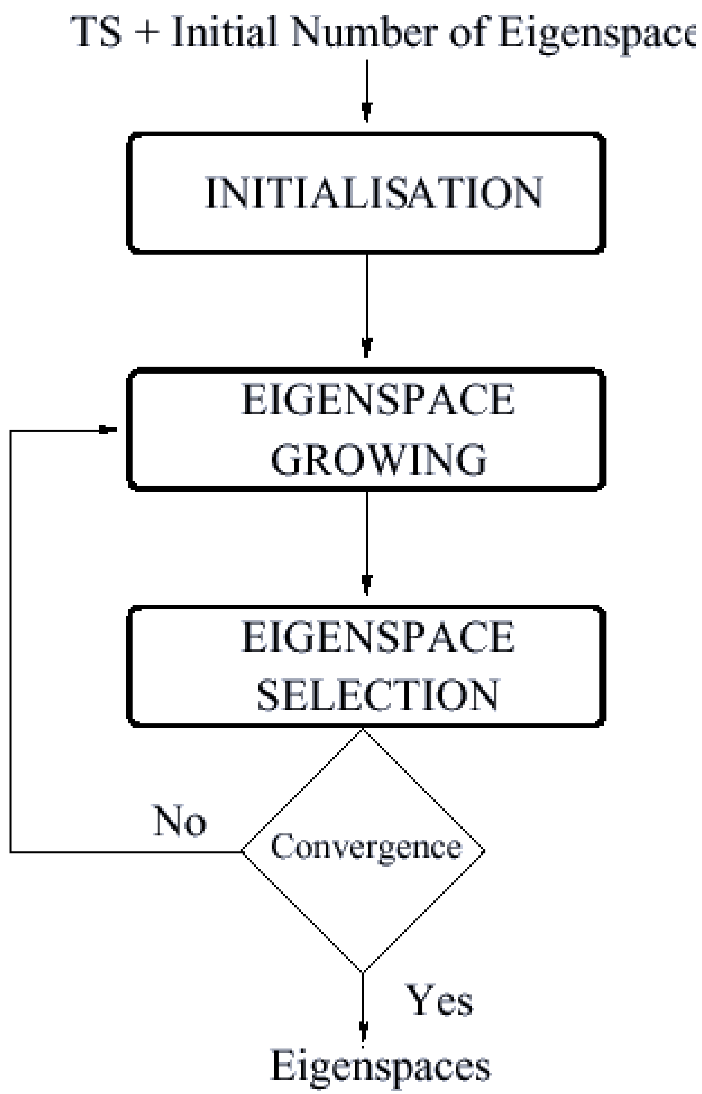
$$\mathcal{X} = \{\mathbf{x}_1, \mathbf{x}_2, \ldots \mathbf{x}_n | \mathbf{x}_i \in \mathbb{R}^n\}$$

- **Each image is described as a linear combination**

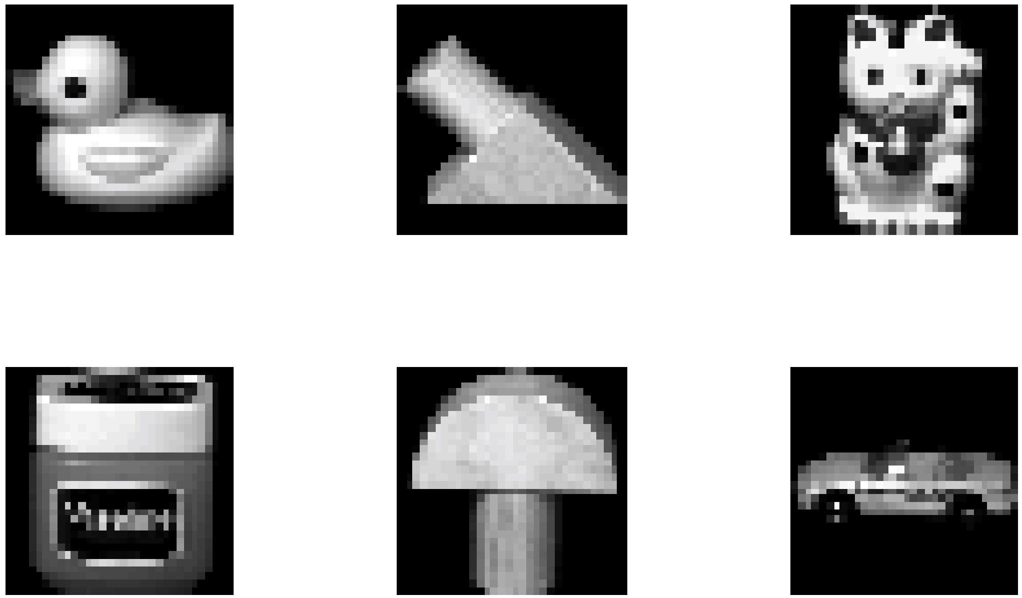
$$\mathbf{x}_i = \sum_{j=1}^{m} \mathcal{I}_j{}^{(i)} \sum_{l=1}^{d_j} c_{jl}^{(i)} \mathbf{u}_{jl}$$

- **Design a numerically feasible and** robust **procedure**

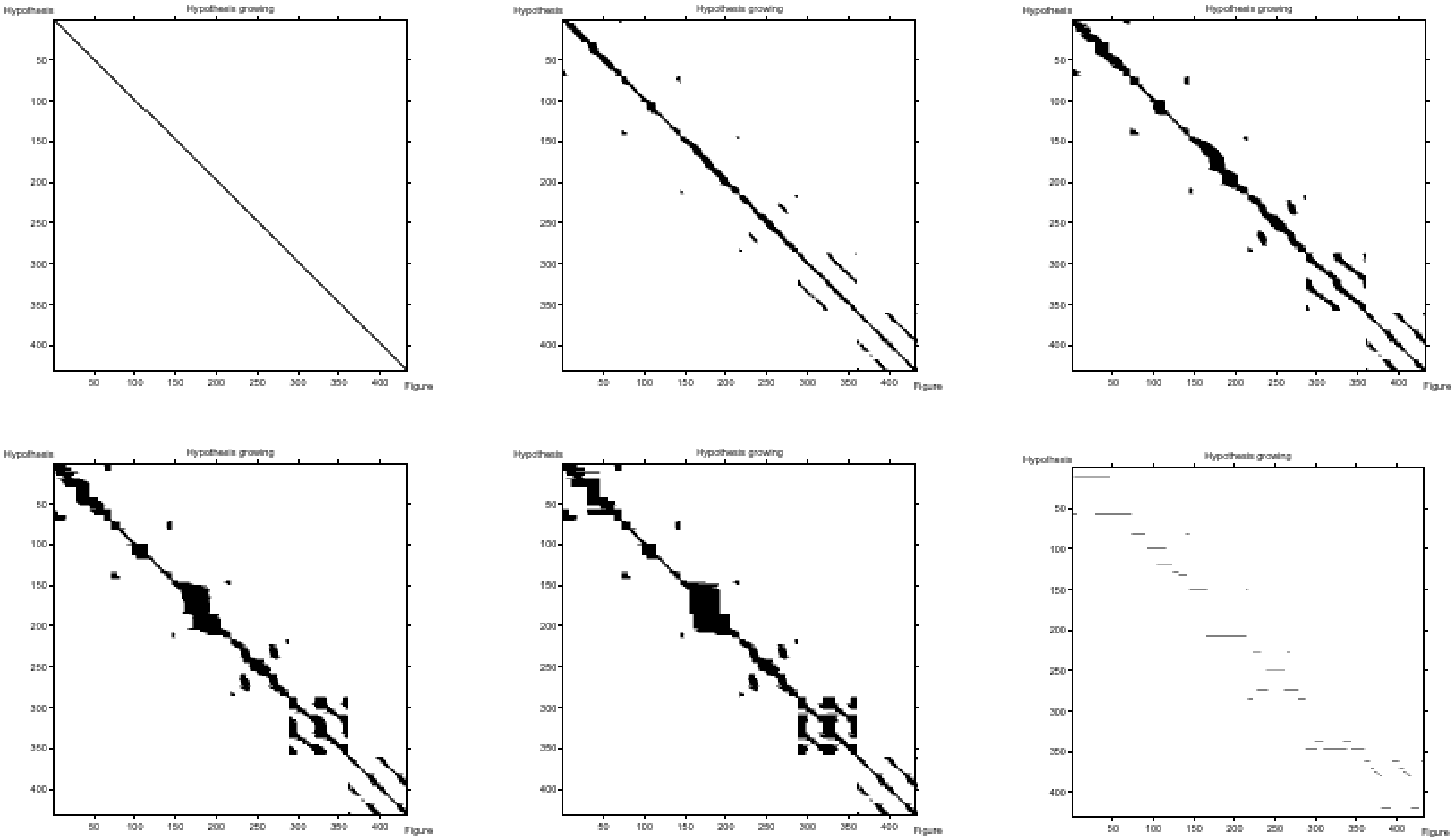
Eigenspace growing and selection

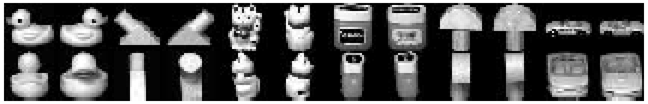


Multiple eigenspaces - experiments

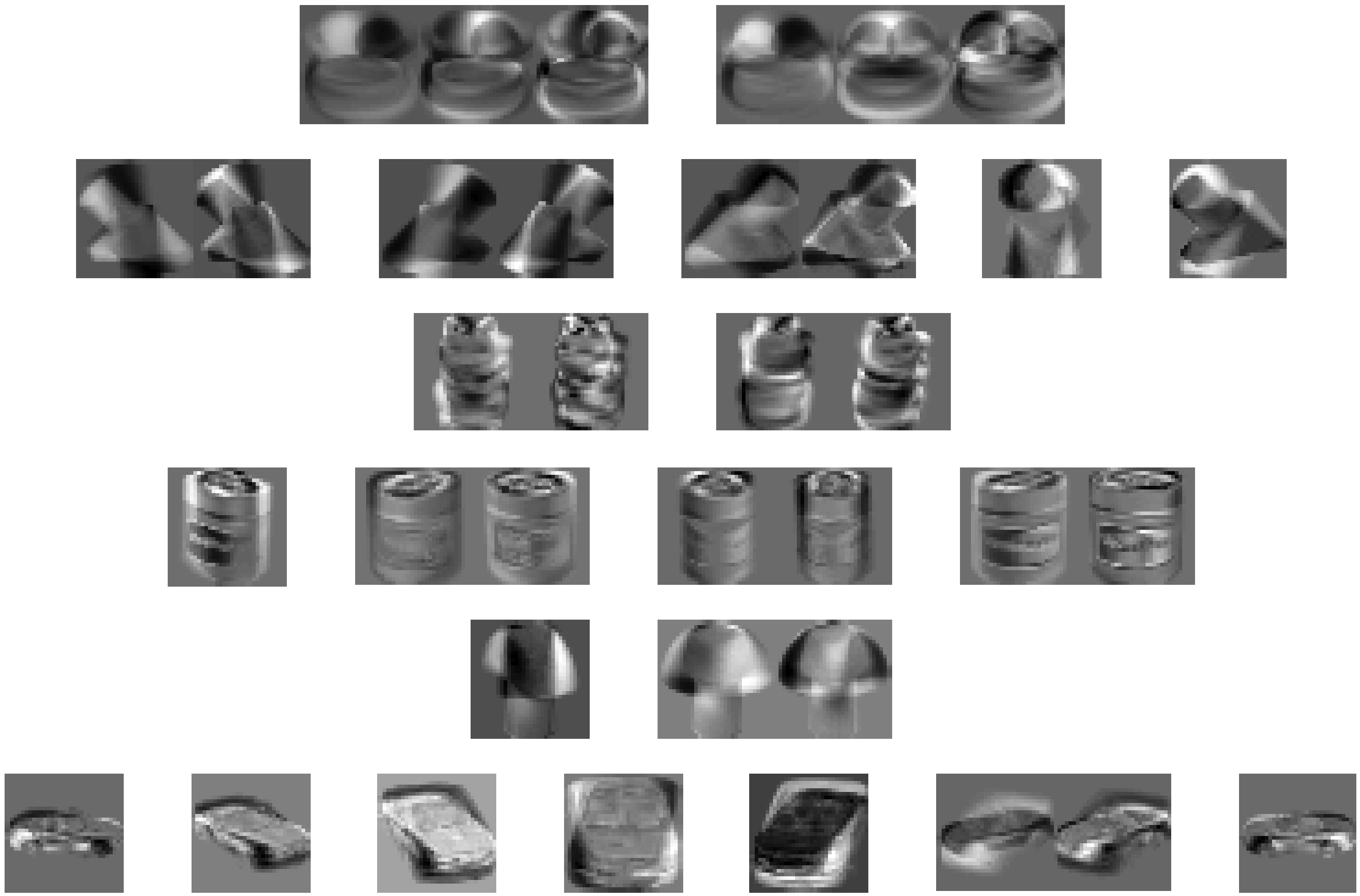


Eigenspace growing and selection

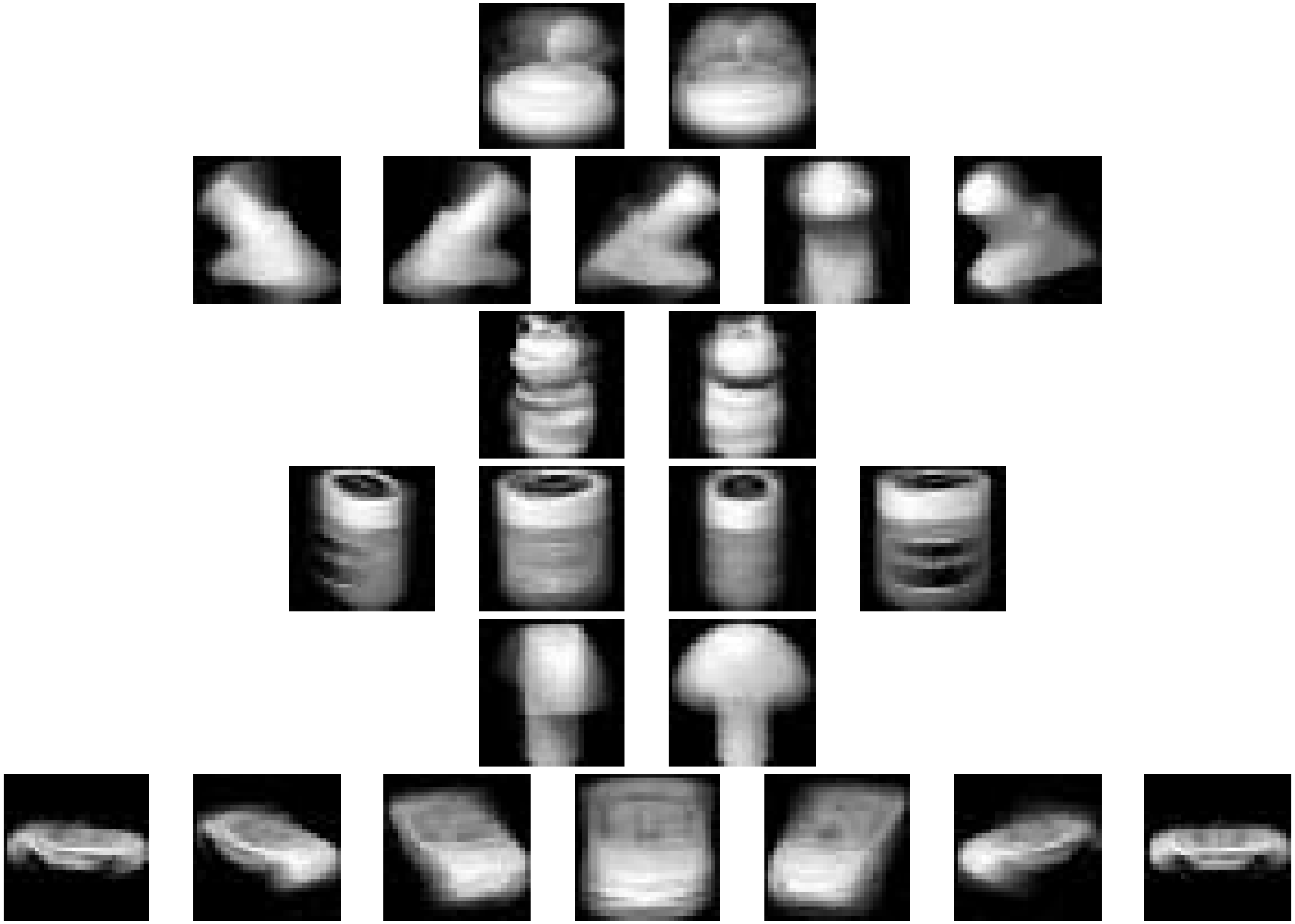




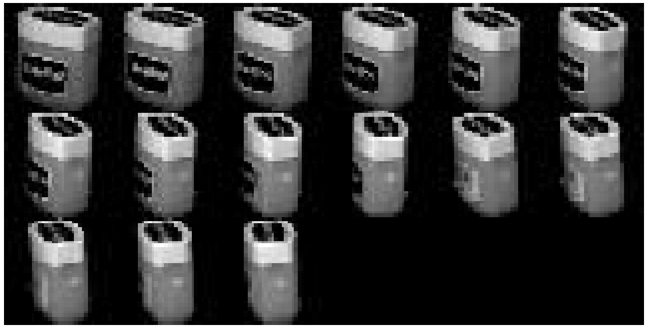
Eigenimages of individual eigenspaces

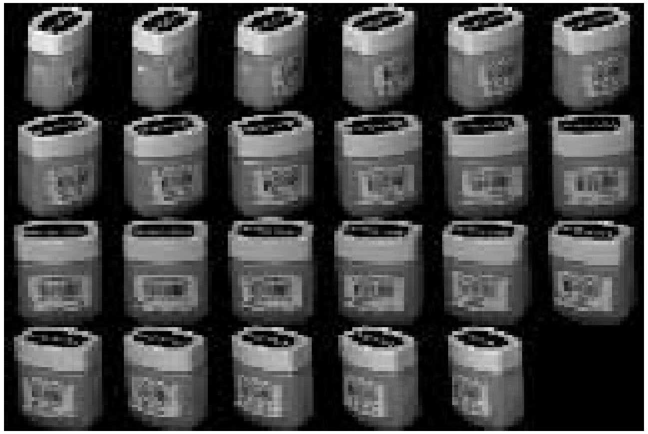


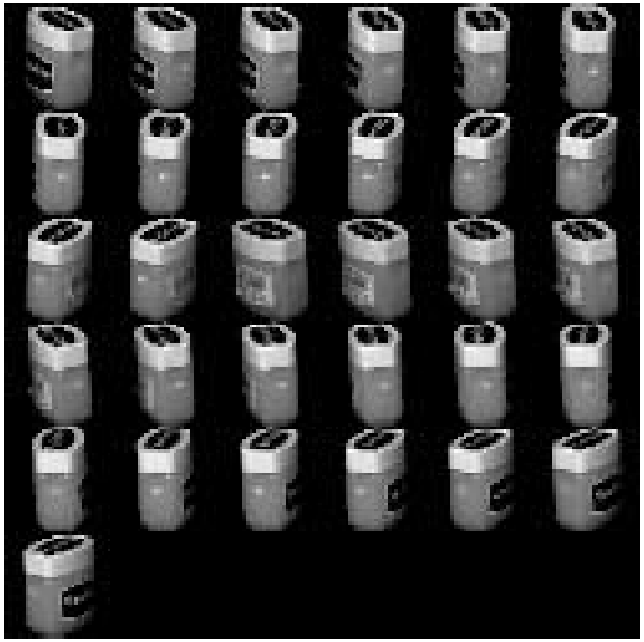
Mean images of individual eigenspaces



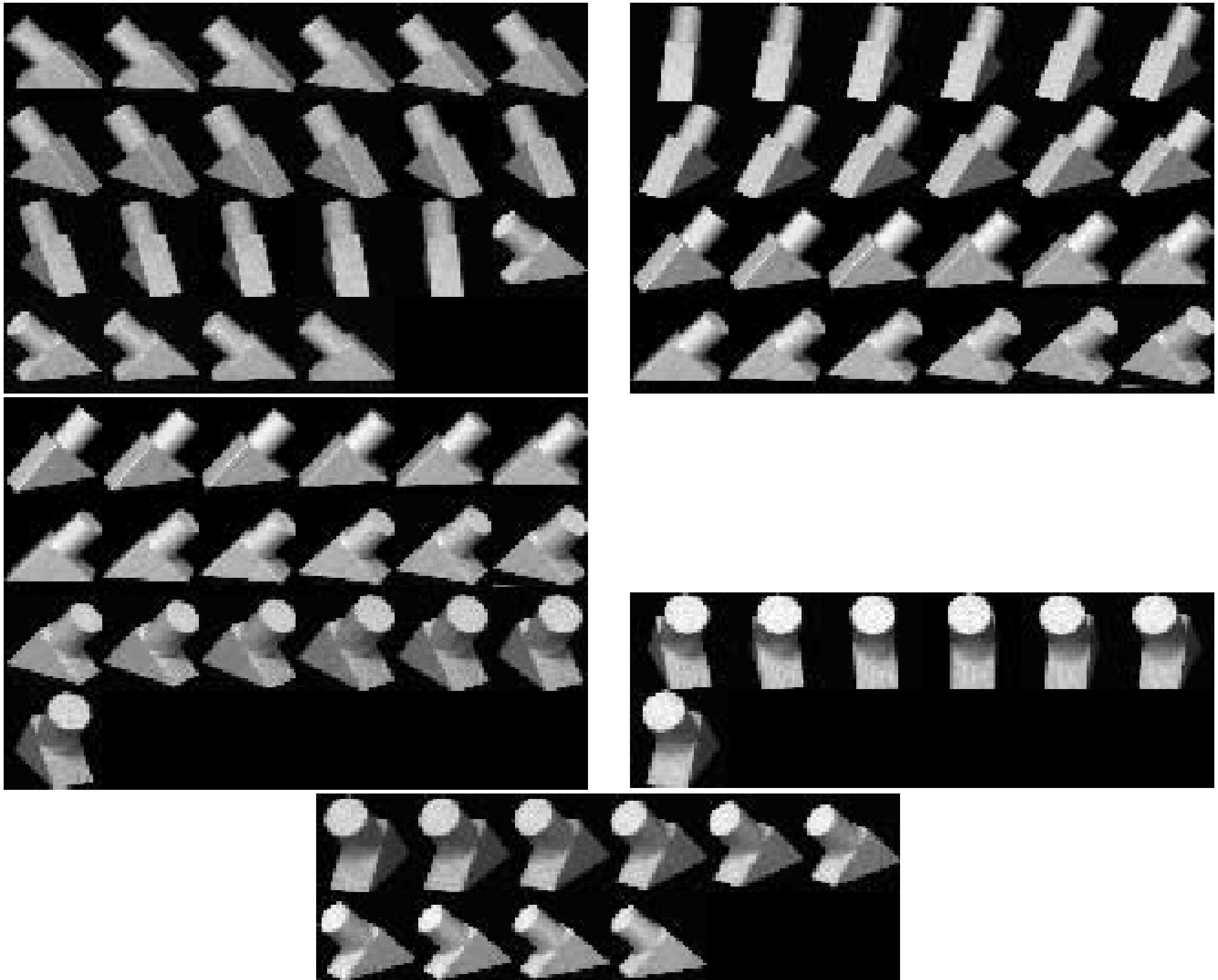
"Box" images in four eigenspaces



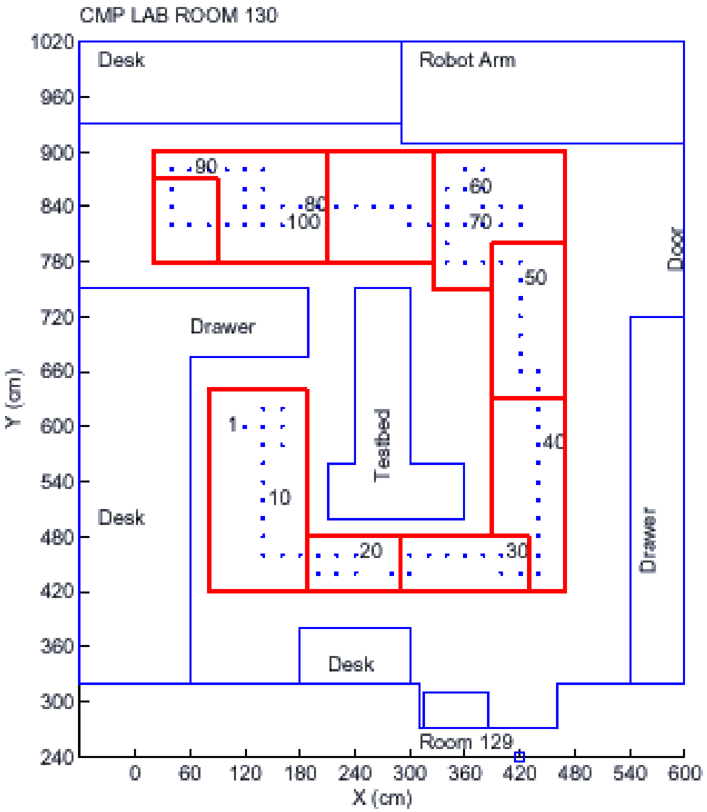




"Block" images in five eigenspaces

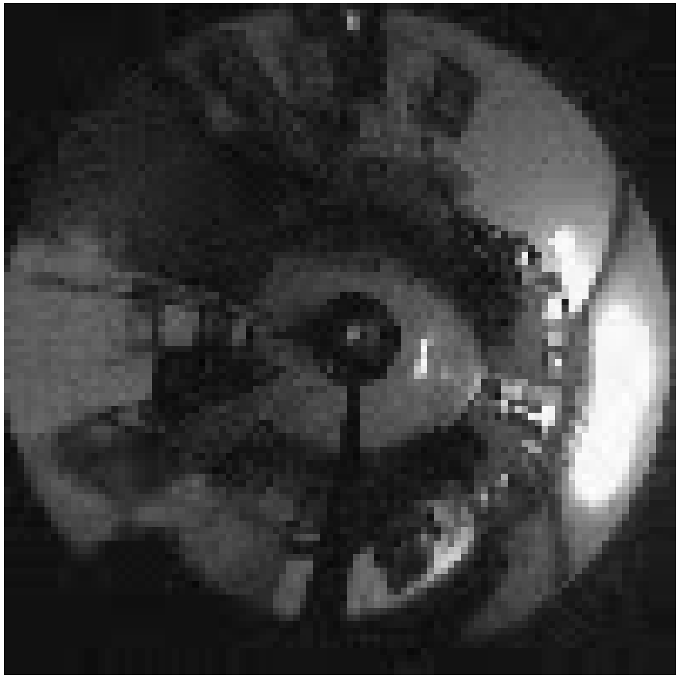


Multiple eigenspaces



Robust Subspace Learning

- **Subspace learning from data containing outliers:**
  - Detect outliers
  - Learn using only inliers.



[D. Skočaj, A. Leonardis, H. Bischof: A robust PCA algorithm for building representations from panoramic images, ECCV 2002]

EM algorithm for learning

Solving systems of linear equations

Only in non-missing pixels

E-step: $$\forall j: \quad x_{ij} = \sum_{p=1}^{k} u_{ip} a_{pj} \quad , \quad i = 1 \ldots m \mid x_{ij} \notin \mathcal{M}$$

M-step: $$\forall i: \quad x_{ij} = \sum_{p=1}^{k} u_{ip} a_{pj} \quad , \quad j = 1 \ldots n \mid x_{ij} \notin \mathcal{M}$$

$$0 = \alpha \sum_{p=1}^{k} u_{ip} (a_{p,j-1} - 2a_{pj} + a_{p,j+1})$$

$$j = 1 \ldots n \mid x_{ij} \in \mathcal{M}$$

Smoothing in missing pixels

Energy function

$$\mathcal{E} = \sum_{j=1}^{n} \sum_{i \in \mathcal{G}_j} \left(x_{ij} - \sum_{p=1}^{k} u_{ip} a_{pj} \right)^2 + \alpha \sum_{j=1}^{n} \sum_{i \in \mathcal{B}_j} \left(\sum_{p=1}^{k} u_{ip} a''_{pj} \right)^2$$

- **Minimization of reconstruction error in non-missing pixels** (the main property of PCA).
- **Smoothing reconstructed values in missing pixels** (additional constraint to prevent over-fitting).

Robust learning algorithm

Input: Learning images containing outliers and occlusions.

1. Compute PV using SVD on the whole image set.
2. Detect outliers (pixels with large reconstruction error).
3. Compute PV from inliers using EM algorithm.
4. Repeat 1.-3. until change in outlier set is small.

Output: Principal subspace, learning images without outliers, detected outliers and occlusions.

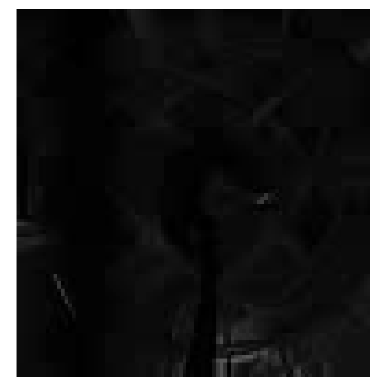
Experimental results – synthetic data

ground truth

added outliers

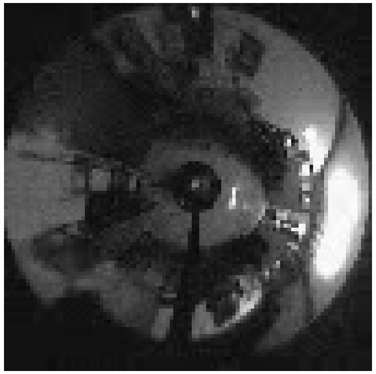
standard
PCA 2PC

standard
PCA 8PC

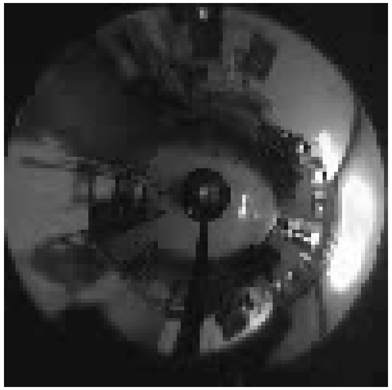


robust
PCA 8PC

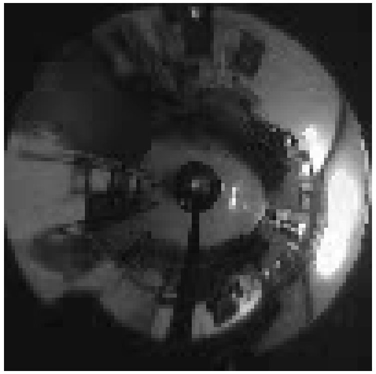
Experimental results – real data



input



standard PCA



robust PCA

outliers

Research issues

- Comparative studies (e.g., LDA versus PCA, PCA versus ICA)
- Robust learning of other representations (e.g. LDA, CCA)
- Integration of robust learning with modular eigenspaces
- Local versus Global subspace represenations
- Combination of subspace representations in a hierarchical framework

Further readings

- Recognizing objects by their appearance using eigenimages (SOFSEM 2000, LNCS 1963)
- Robust recognition using eigenimages (CVIU 2000, Special Issue on Robust Methods in CV)
- Hierarchical top down enhancement of robust PCA (SSPR 2002)
- Illumination insensitive eigenspaces (ICCV 2001)
- Mobile robot localization under varying illumination (ICPR 2002)
- Eigenspace of spinning images (OMNI 2000, ICPR 2000, ICAR 2001)
- Incremental building of eigenspaces (ICRA 2002, ICPR 2002)
- Multiple eigenspaces (Pattern Recognition, In press)
- Robust building of eigenspaces (ECCV 2002)
- Generalized canonical correlation analysis (ICANN 2001)