CS3451. Fall 08. Homework 1. Due Jan 23 in class.   NAME: ____________________ Grade:______ / 10.

Individual test. Do not look at other students' work. Please type and write legibly. Bring to class. All 2D.

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1) Explain precisely what the dot product measures. Then evaluate <1,2>•<3,4> :

/2 The dot product is a scalar. It measures the product of their lengths and of the cosine of the angle between them. Hence it is positive when that angle is less than 90°. It is zero when the two vectors are orthogonal to each other. <1,2>•<3,4>=3+8=11.

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2) Compute <1,2>.left. Explain precisely what it is :

/2 <1,2>.left = <–2,1>. It is obtained by rotating <1,2> ccw by 90°. Verify that the two vectors are orthogonal, since their dot-product is zero.

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3) Evaluate V^2, when V is the vector <3,4> . Explain what V^2 measures :

/2 V^2=25. It stands for V•V and measures the square of the norm of V.

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4) Compute the result R of rotating point P=(2,3) by 30° around point Q=(3,5). First provide an exact formulation (using fractions, roots, sin, cos… if necessary) and then a numerical approximation:

/3 QP=P–Q; c=cos(-30); s=sin(-30); I = <c,s>; J=I.left=<–s,c>; R=Q+(QP.x)I+(QP.y)J;

R=(3,5)–<c,s>–2<–s,c>; R=(3.1339746,2.767949);

pt P = new pt(2,3); pt Q = new pt(3,5); pt R = P.makeRotatedBy(-PI*30/180,Q); // see page 2 for details

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5) Let (x_1,y_1) be the coordinates of point P in $[I_1,J_1,O_1]$. How would you compute its coordinates (x_2,y_2) in $[I_2,J_2,O_2]$? Provide a series of assignments/steps that compute x_2 and y_2 using operators (+, –, scaling, •…) on points and/or vectors. Provide a brief comment on what each step computes.

/2 $P=O_1+ x_1I_1+ y_1J_1$; $x_2= O_2P•I_2$; $y_2=O_2P•J_2$;

/1 6) Provide a valid expression for a point P located 1/3 along the way from A to B : A+AB/3

/1 7) Vectors V and U are parallel when : V•U.left=0, *i.e. when V.xU.y=V.yU.x*

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8) A point R(t) starts at P and travels at constant velocity V. Compute the time t when it hits the line passing through point Q and tangent to T? Include the derivation and explain briefly each step.

/3 R(t)=P+tV; R(t) is on the line when **QR(t)•N=0**, where N=T.left is the normal to the line. Substituting R(t) : (R(t)–Q)•N=0 : (P+tV–Q)•N=0 : (P–Q+tV)•N=0 : (QP+tV)•N=0 : QP•N+tV•N=0 : t = – (QP•N) / (V•N)

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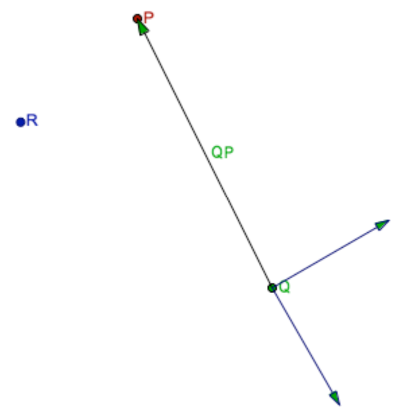
9) Assume that a disk(C_1,r) with velocity V_1 has just collided (i.e., is in tangential contact) with disk(C_2,r) that has velocity V_2. Explain how to compute their new velocities W_1 and W_2 after an elastic shock. (Assume both disks have the same mass.) Provide a formula or a series of assignments that evaluate W_1 and W_2 using operators (such as +, –, scaling, •) on points and/or vectors.

/4 $N=C_1C_1$.unit; $N_1=(V_1•N)N$; $N_2=(V_2•N)N$; $D= N_2–N_1$; $W_1=V_1+D$; $W_2=V_2–D$;

You can use processing to test your solutions to geometric constructions. Here's an example where I used Processing to test my solution to problem 4. The code in red is for visualization. Note that since the y-axis goes down in Processing, the coordinate system is inverted.

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pt P = new pt(200,300); fill(155,0,0); P.show(3); P.showLabel("P");
pt Q = new pt(300,500); fill(0,155,0); Q.show(3); Q.showLabel("Q");
vec QP = Q.makeVecTo(P); QP.showArrowAt(Q); mid(P,Q).showLabel("QP");
float c = cos(-PI*30/180), s =  sin(-PI*30/180);
vec I = new vec(c,s); stroke(0,0,100); I.makeScaledBy(100).showArrowAt(Q);
vec J = I.left(); J.makeScaledBy(100).showArrowAt(Q);
pt R = Q.makeClone(); R.translateBy(QP.x,I); R.translateBy(QP.y,J);
fill(0,0,155); R.show(3); R.showLabel("R"); R.scaleBy(0.01); R.write();
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