Smooth curves and surfaces are used for aesthetic, manufacturing, and analysis applications where discontinuities due to triangulated approximations would create misleading artifacts. I like to distinguish three classes of surfaces:

- implicit: f(x,y,z)=0, where f is often a polynomial of low degree (handy for computing intersections with rays)
- parametric surfaces: S(u,v)=(x(u,v),y(u,v),z(u,v)), where x, y, and z are often low degree polynomials in u and v
- generative surfaces, such as sweeps or subdivision surfaces, which are defined in terms of a construction procedure

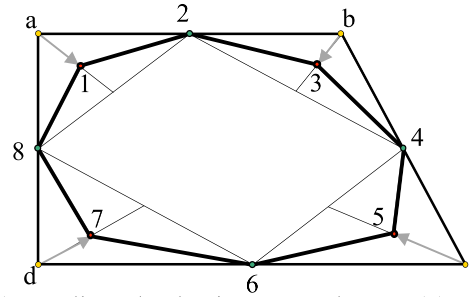
Piecewise cubic parametric curves and surfaces are popular in CAD, animation, and graphics. A point C(t) on curve C has coordinates (x(t),y(t),z(t)), where x, y, and z are cubic polynomials in t. The shape of C is defined by a **control polygon** with control points (i.e. vertices) $P_i$. We discuss below how to subdivide the control polygon and how to evaluate C(t). To define a bi-cubic surface, express each $P_i$ as a **curve** $P_i(s)$. As s is varied, C(t) sweeps out a surface S(t,s).

## 1. Split&tweak subdivision of control polygons a uniform cubic B-spline curves

Given a control polygon, for example (a,b,c,d), repeat the following sequence of two steps, until all consecutive 4-tuples of control points are nearly coplanar.

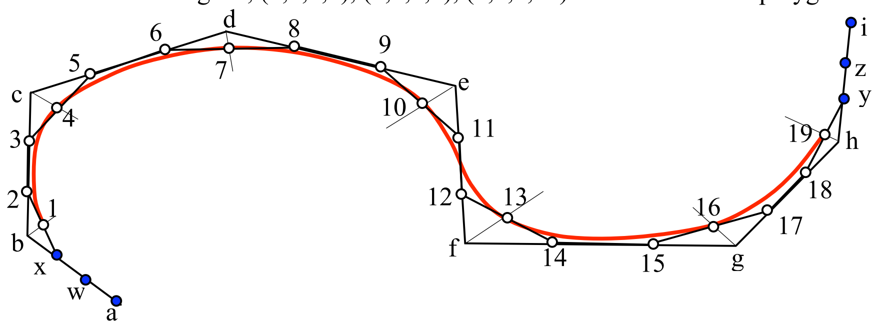
1. **Split**: **insert** a **new** control point in the **middle** of each **edge** (2,4,6,8)
2. **Tweak**: **move** the **old** control points **half-way** towards the **average** of their new **neighbors** (1,3,5,7)

The control polygon converges rapidly to the B-spline curve. This works whether the curve is closed or open.




## 2. Converting a uniform cubic Bspline into a series of cubic Bezier curves

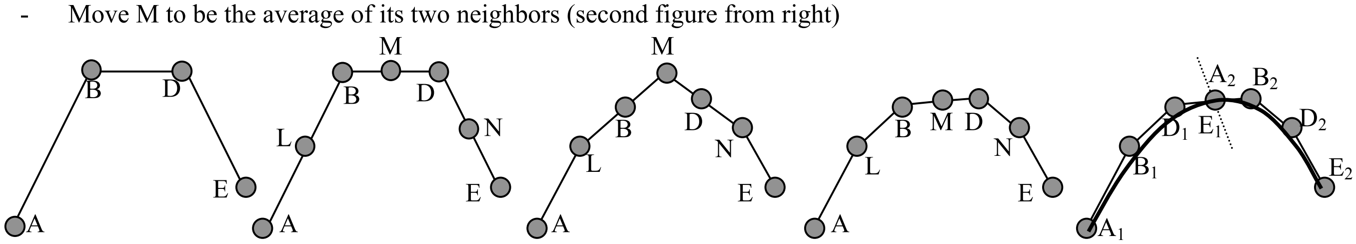
Given a control polygon with vertices a,b,c,… do: (1) insert new vertices (w,y,2,3,5…) to split each edge into 3 equal parts; (2) move the original vertices to the average of their immediate neighbors (b□ 1, c□ 4,…); and (3) delete the first and last 3 vertices (a,w,x,y,z,i). The consecutive trigons, (1,2,3,4), (4,5,6,7), (7,8,9,10)… are the control polygons of Bezier curves.




## 3. Subdividing a cubic Bezier control polygon

To replace the control trigon {A,B,D,E} with trigons {A,L,B,M} and {M,D,N,E}, each representing a portion of C:

- Insert points L, M, N at the centers of the three edges (second figure from left)
- Move B and D to be each the average of their two neighbors (center figure)
- Move M to be the average of its two neighbors (second figure from right)




This subdivision may be recursively applied to {A,L,B,M} and/or {M,D,N,E}, as desired.

## 4. Evaluating a point C(t) on a cubic Bezier curve

To compute C(t) perform the following sequence of operations: {slide(E), slide(D), slide(B), slide(E), slide(D), slide(E)}, where slide(K) replaces control point K by (1–t)J+tK, where J precedes K in the sequence {A,B,D,E}. Subscripts indicate order of slides in the figure. The result of the last slide, $E_6$, is C(t). Note that C starts at A, where it is tangent to AB and finishes at D, where it is tangent to CD. It is contained in the convex hull of {A,B,C,D}.