Lecture 14:
B+Tree

Logistics

- Programming assignment 3 (B+Tree) due on **Nov 2**
- Two-page project updates due on **Oct 29** (extra credit)
  - Publish code on public GitHub
  - Iterate over the code and add features incrementally
  - Focus on evaluating performance or other metrics

Recap

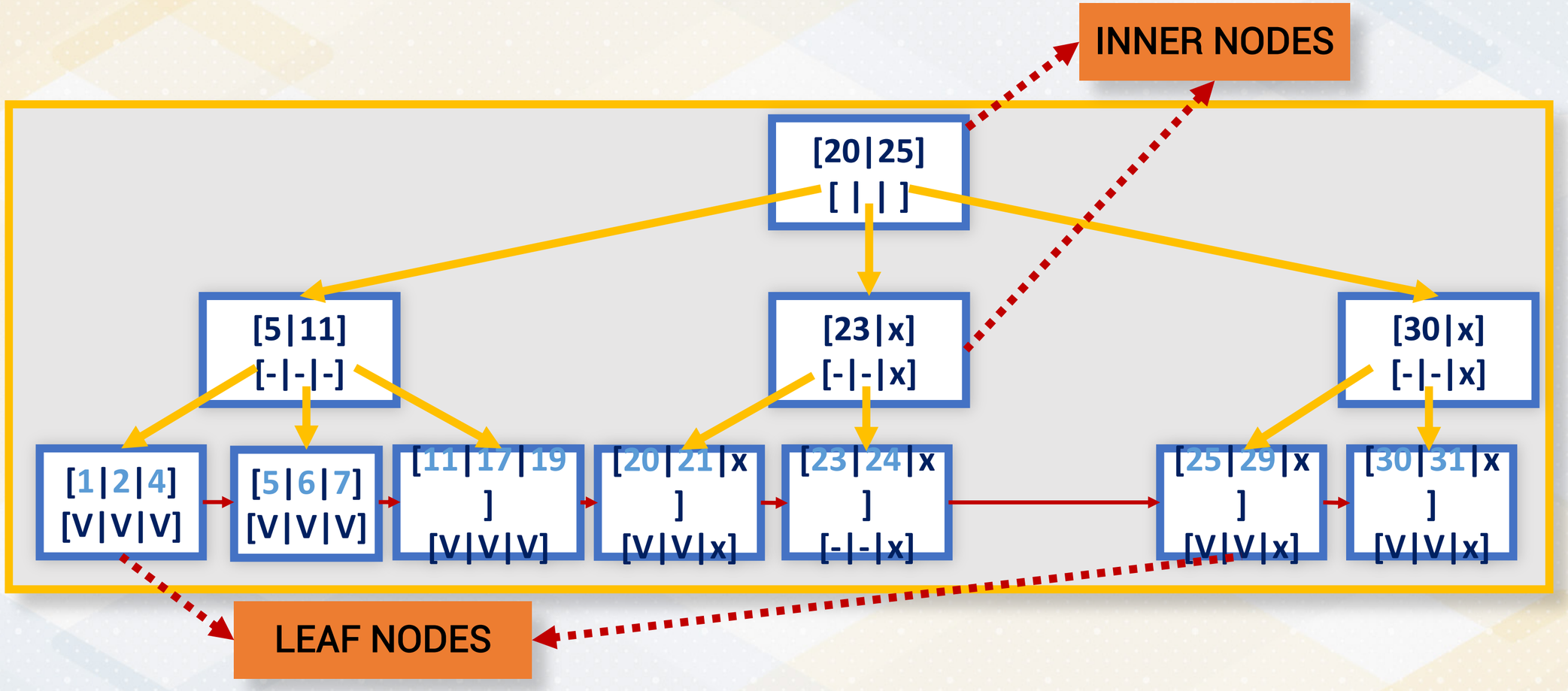
- Range Query
- Ordered Index
- B+Tree Template

Lecture Overview

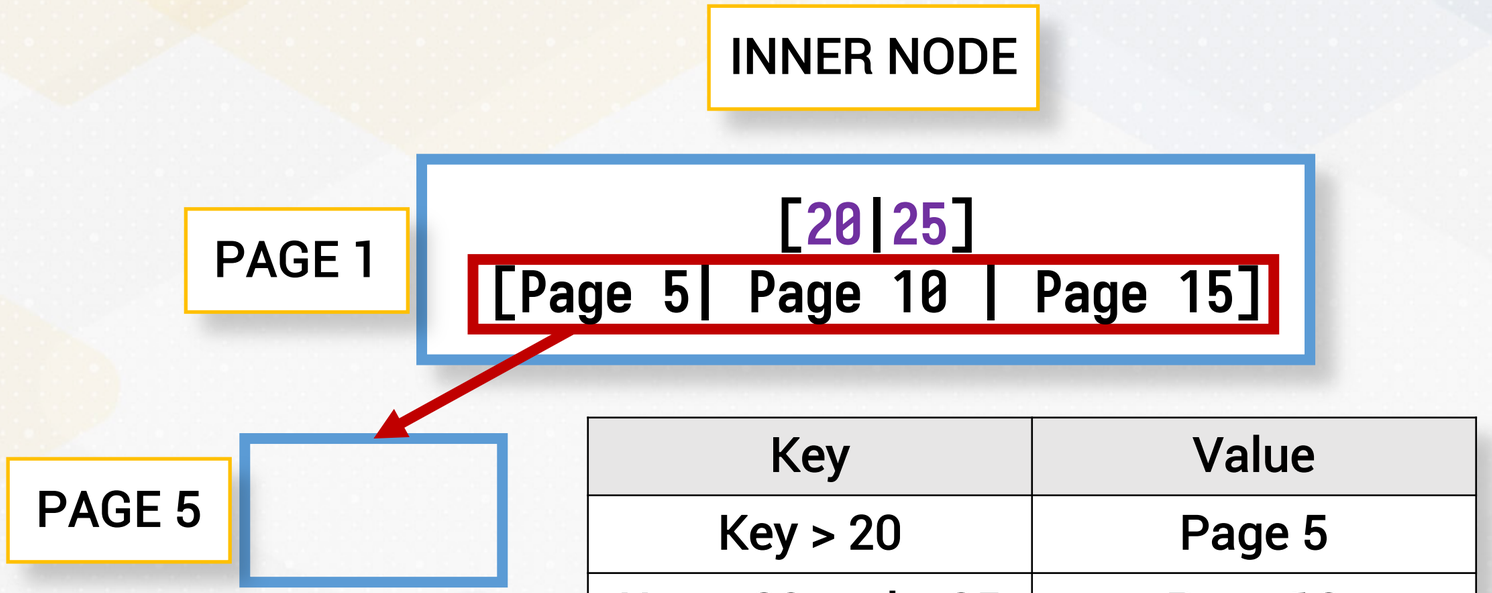
- B+Tree Structure
- Range Query Processing
- Node Split
- On-disk B+Tree

B+Tree Structure

B+Tree Structure

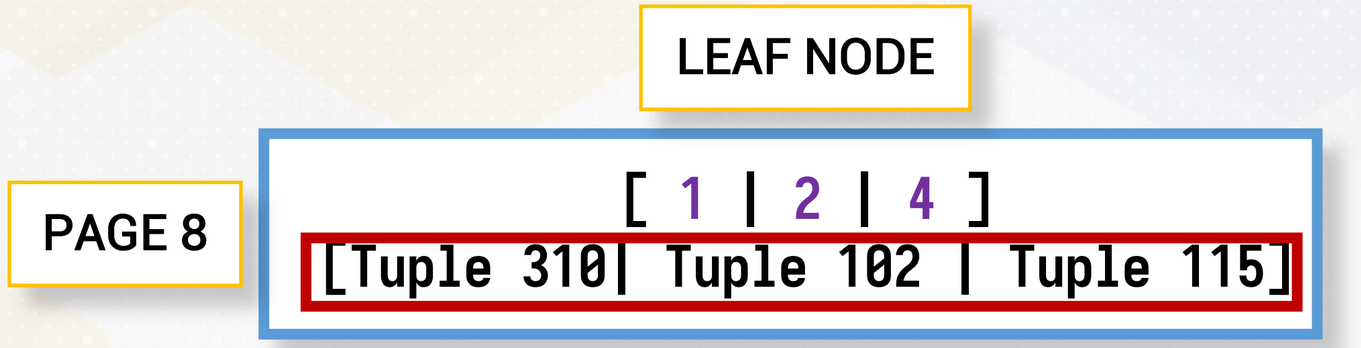


Inner Node



Key	Value
Key > 20	Page 5
Key = 20 and < 25	Page 10
Key > 25	Page 15

Leaf Node



Key	Value
Key = 1	Tuple 310
Key = 2	Tuple 102
Key = 4	Tuple 115

Key and Value Types

ENTRIES

Entry	Key	Value
Entry 1	“max_file_size"	“10 MB"
Entry 2	“default_language"	“English"
Entry 3	“session_timeout"	“30 minutes"

LEAF NODE

PAGE 8

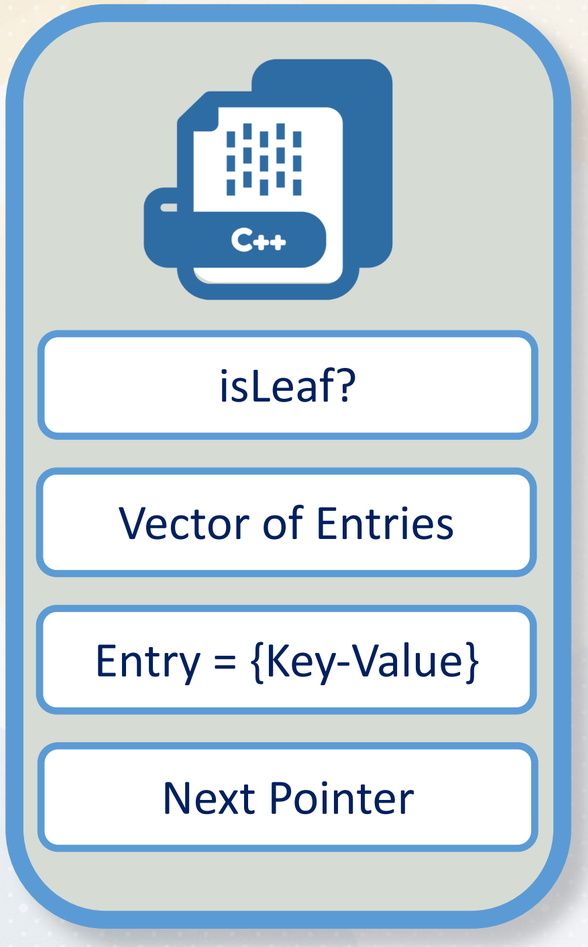
```
[ “max_file_size” | “default_language” | “session_timeout” ]
                [“10 MB” | “English” | “30 minutes”]
```

Entry in C++

```
struct Entry {
  Key key;
  Value value;  // Used only in leaf node entry
  NodePtr next; // Used only in inner node entry

  Entry(Key k, Value v) : key(k), value(v), next(nullptr) {}
};
```

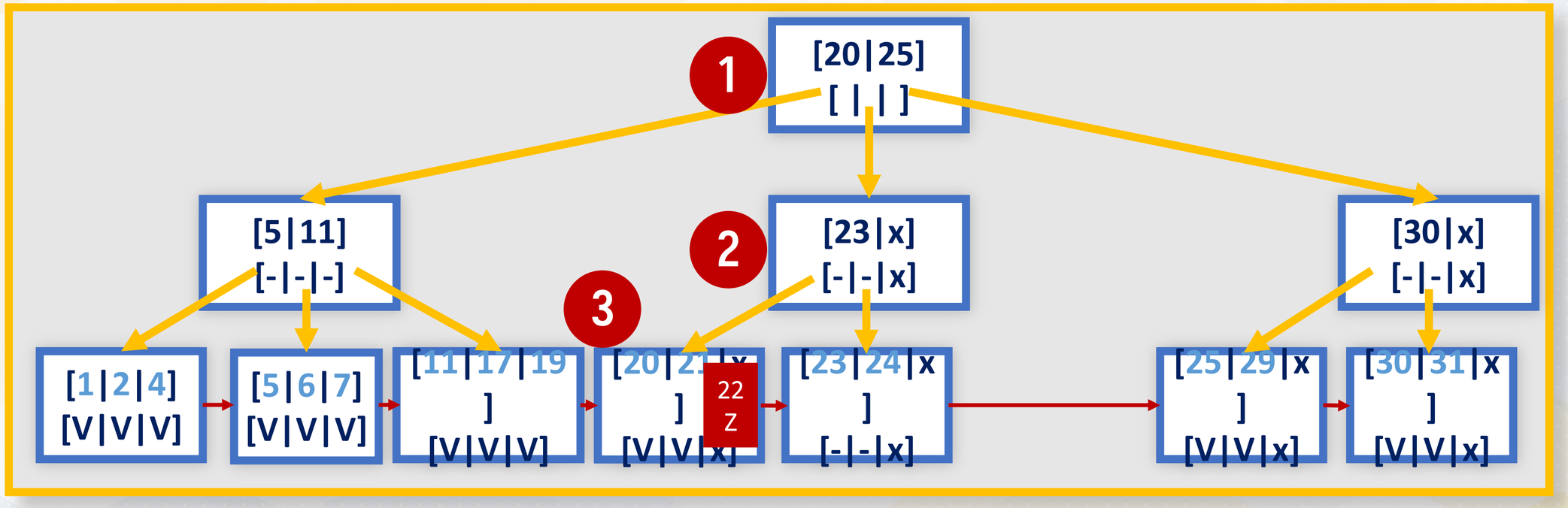
B+Tree Node in C++




```cpp
struct Node {
  bool isLeaf;
  std::vector<Entry> entries;
  std::unique_ptr<Node> next;

  Node(bool leaf) : isLeaf(leaf), next(nullptr) {}
};
```

Inserting a Key-Value Pair



Inserting a Key-Value Pair

Insertion process is recursive

```
void insertOrUpdate(const Key &key, const Value &value) {
    insertOrUpdateInternal(key, value, root);
}
```

Recursive Descent in an Inner Node

Find the appropriate child node by comparing keys.

```
void insertOrUpdateInternal(const Key &key, const Value &value, NodePtr &node) {
    ...
    else {
        bool found = false;
        for (auto it = node->entries.begin(); it != node->entries.end(); ++it) {
            if (key < it->key) {
                insertOrUpdateInternal(key, value, it->next);
                found = true;
                break;
            }
        }
    }
}
```

Insertion in a Leaf Node

If key does not exist in leaf node, insert it at the correct position to maintain sorted order.

```
void insertOrUpdateInternal(const Key &key, const Value &value, NodePtr &node) {
    if (node->isLeaf) {
      // Search for the key in the leaf node
      auto it = std::find_if(node->entries.begin(), node->entries.end(),
                   [&](const Entry &entry) { return entry.key == key; });
      // Key not found, proceed to insert in sorted order
      auto comp = [](const Entry &entry, const Key &k) { return entry.key < k;};
      it = std::lower_bound(node->entries.begin(), node->entries.end(), key,
comp);
      node->entries.insert(it, Entry(key, value));
    }
    ...
}
```

Binary Search using a Lambda Function

λ Function

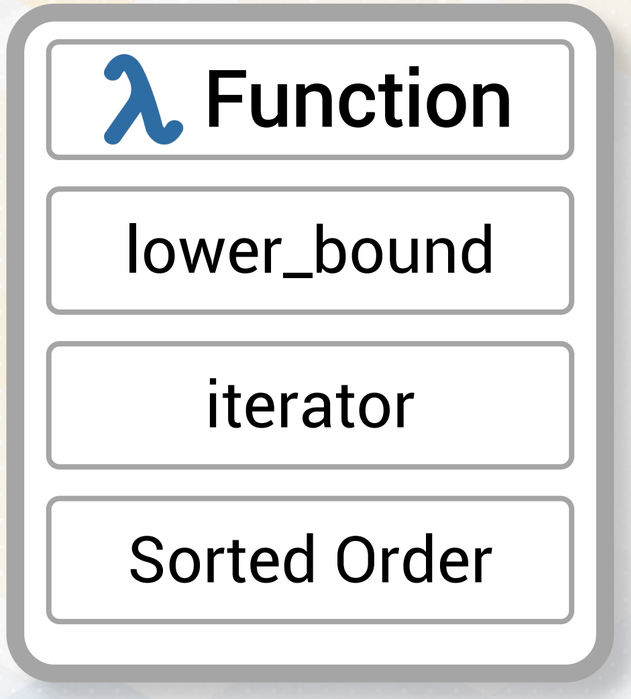
Anonymous

Short Functions

Comparator

```
// Key not found, proceed to insert in sorted order
auto comp = [](const Entry &entry, const Key &k)
                { return entry.key < k; };
it = std::lower_bound(node->entries.begin(),
                      node->entries.end(),
                      key, comp);
node->entries.insert(it, Entry(key, value));
```

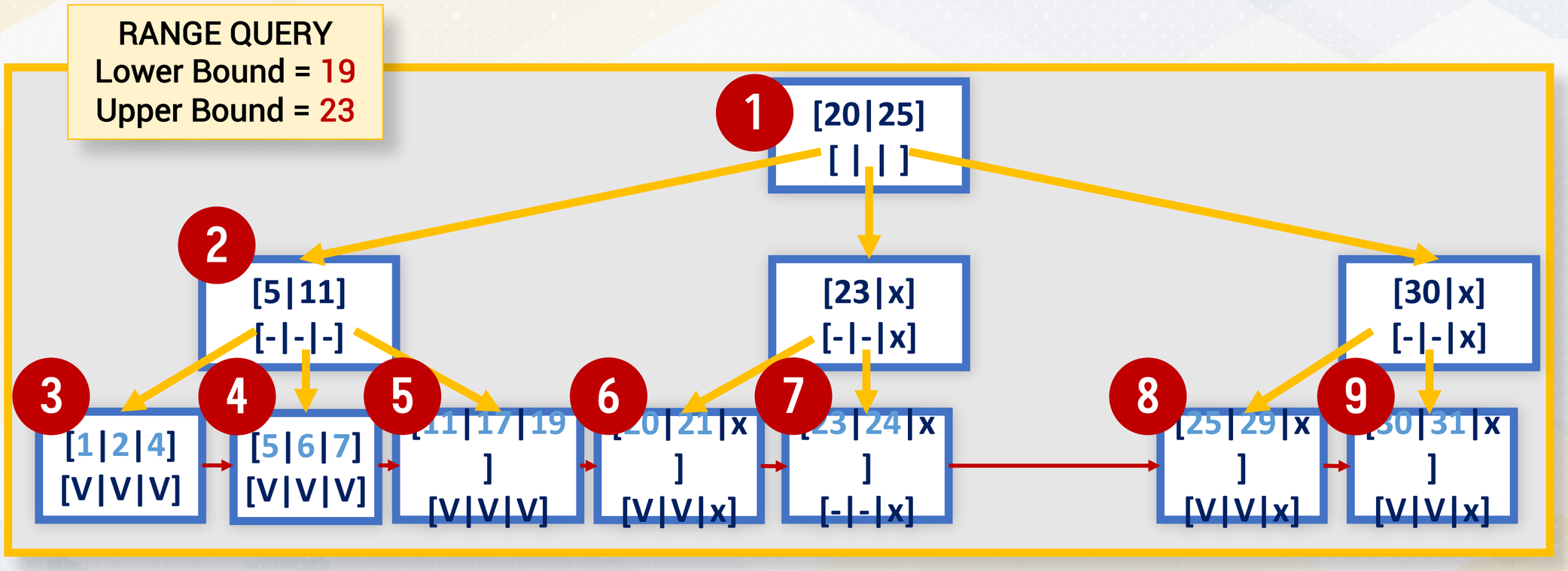
Binary Search using a Lambda Function




```
// Key not found, proceed to insert in sorted order
auto comp = [](const Entry &entry, const Key &k)
                { return entry.key < k; };
it = std::lower_bound(node->entries.begin(),
                      node->entries.end(),
                      key, comp);
node->entries.insert(it, Entry(key, value));
```

Range Query Processing

Inefficient Range Query Processing

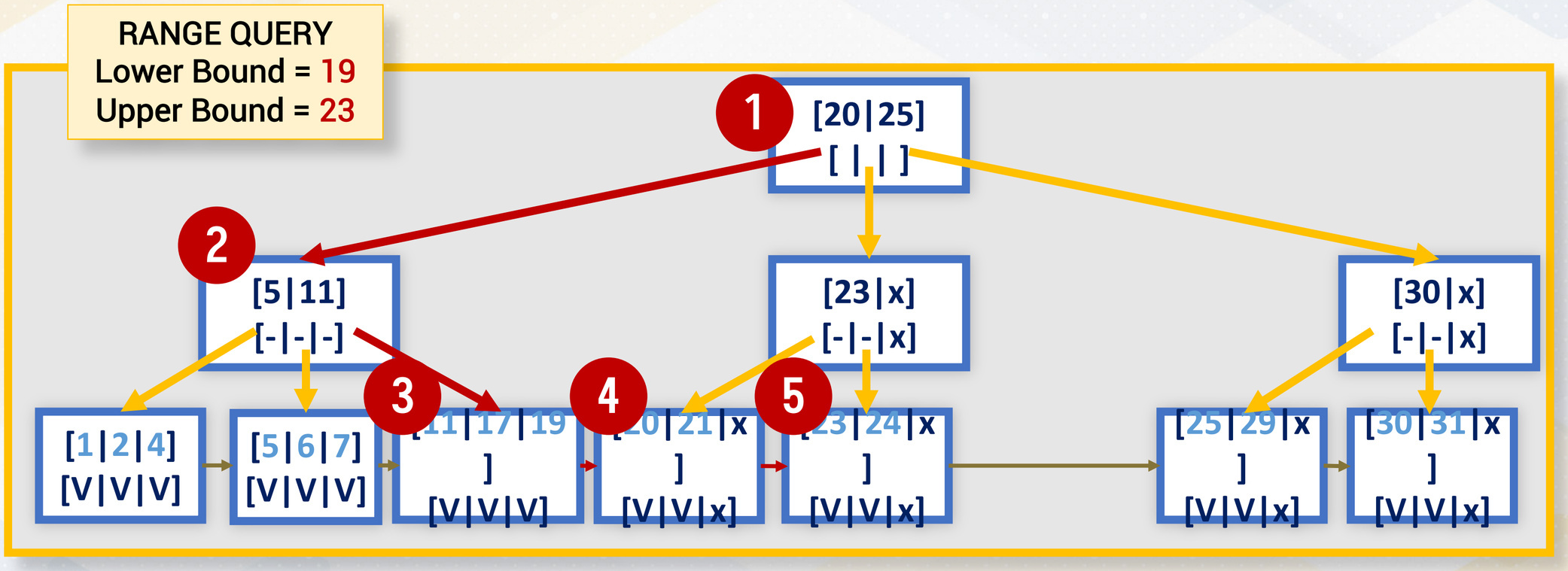


Inefficient Range Query Processing

The rangeQuery method begins at the root and traverses down to the left-most leaf, then continues horizontally through the leaves collecting all qualifying values.

```
std::vector<Value> rangeQuery(const Key &lowerBound,
                              const Key &upperBound) const {
  std::vector<Value> result;
  // Traverse to leftmost leaf
  while (!currentNode->isLeaf) {
    currentNode = currentNode->entries.front().next.get();
  }
  // Traverse all the leaf nodes
  while (currentNode != nullptr) {
    // Add relevant entries within lower and upper bounds
    currentNode = currentNode->next.get();
  }
}
```

Efficient Range Query Processing

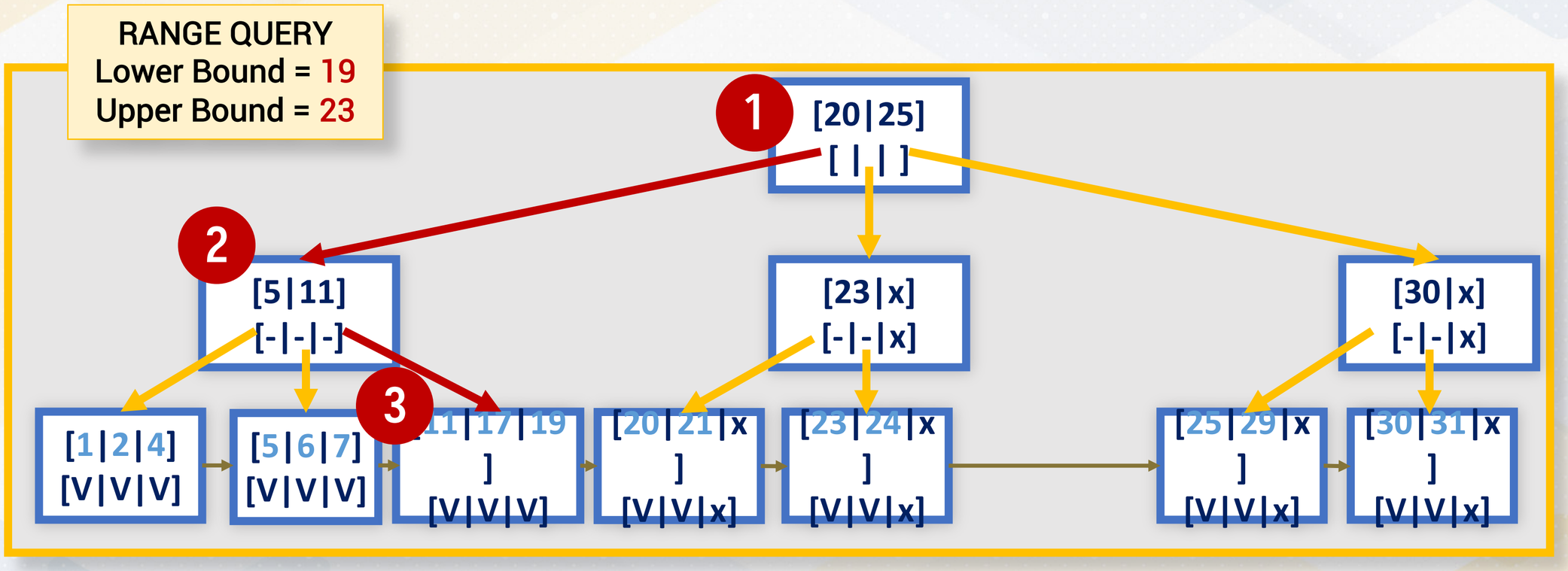


Vertical Traversal to Leaf Node

Move to appropriate child until a leaf node is reached by comparing lowerBound with node key, ensuring the starting point is as close as possible to lowerBound.

```
std::vector<Value> rangeQuery(const Key &lowerBound,
                              const Key &upperBound) const {
  auto node = root;
  while (node && !node->isLeaf) {
    size_t i = 0;
    while (i < node->keys.size() && lowerBound > node->keys[i]) {
      ++i;
    }
    node = node->children[i];
  }
  ...
}
```

Vertical Traversal to Leaf Node

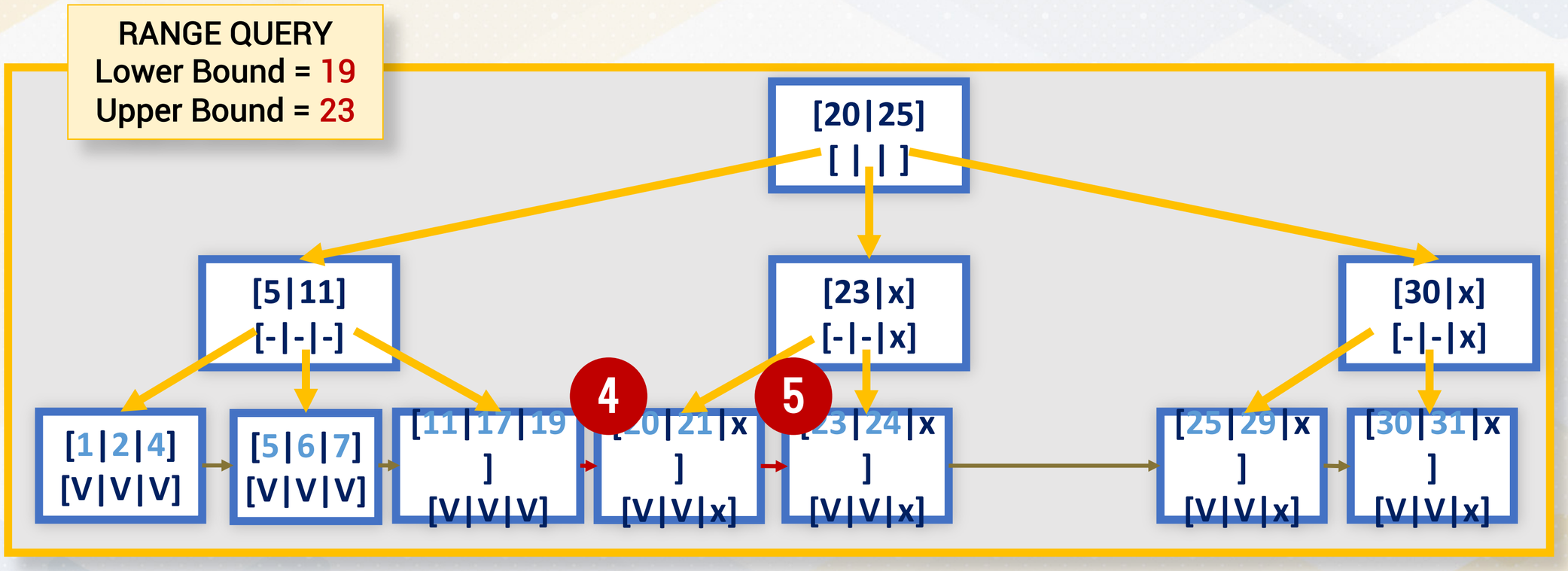


Horizontal Traversal Across Leaf Nodes

Once the appropriate leaf node is reached, the function iterates only through relevant nodes using the linked nature of leaf nodes in a B+Tree.

```
while (node) {
  for (size_t i = 0; i < node->keys.size(); ++i) {
    if (node->keys[i] > upperBound)
      return result;
    if (node->keys[i] >= lowerBound) {
      result.push_back(node->values[i]);
    }
  }
  node = node->next;
}
```

Horizontal Traversal Across Leaf Nodes



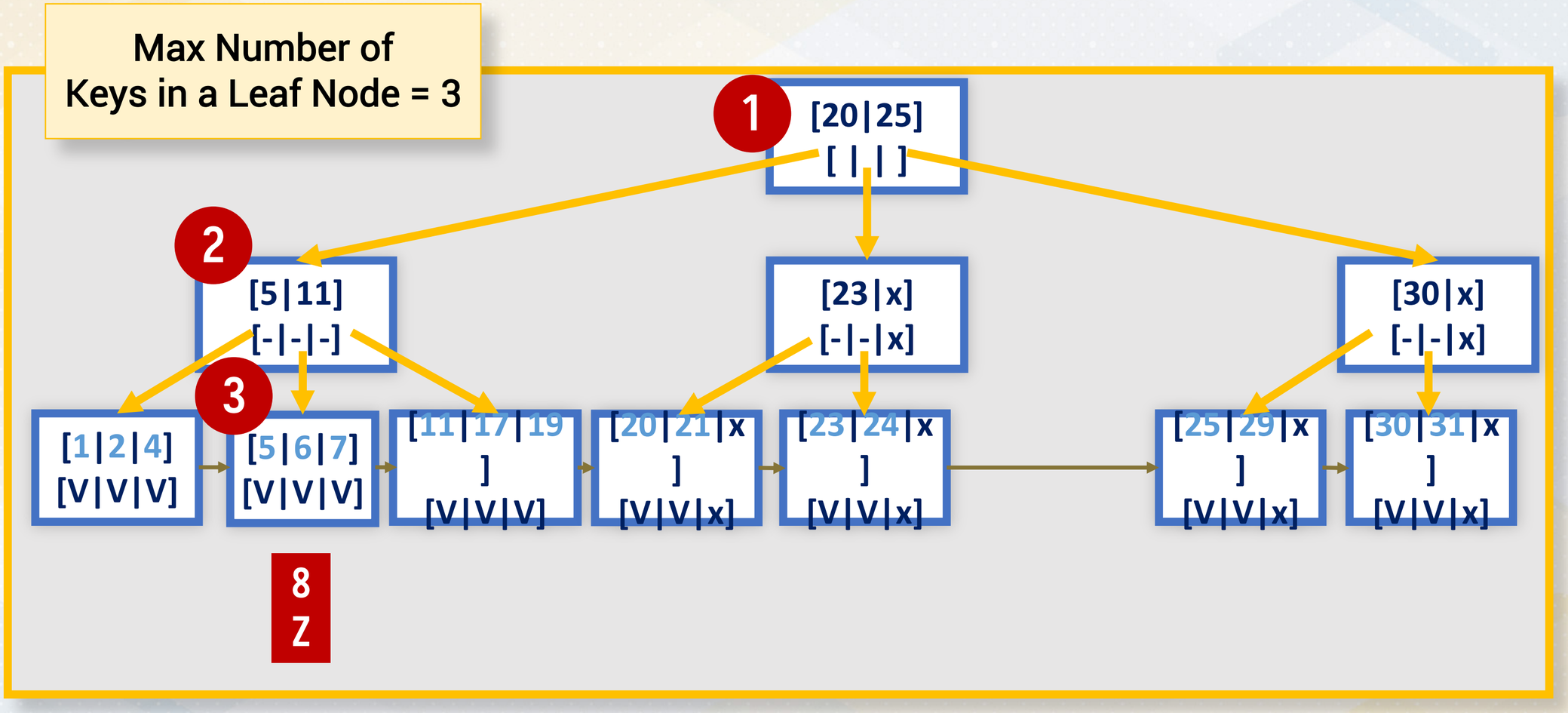
Node Split

Separate Vectors for Keys and Values

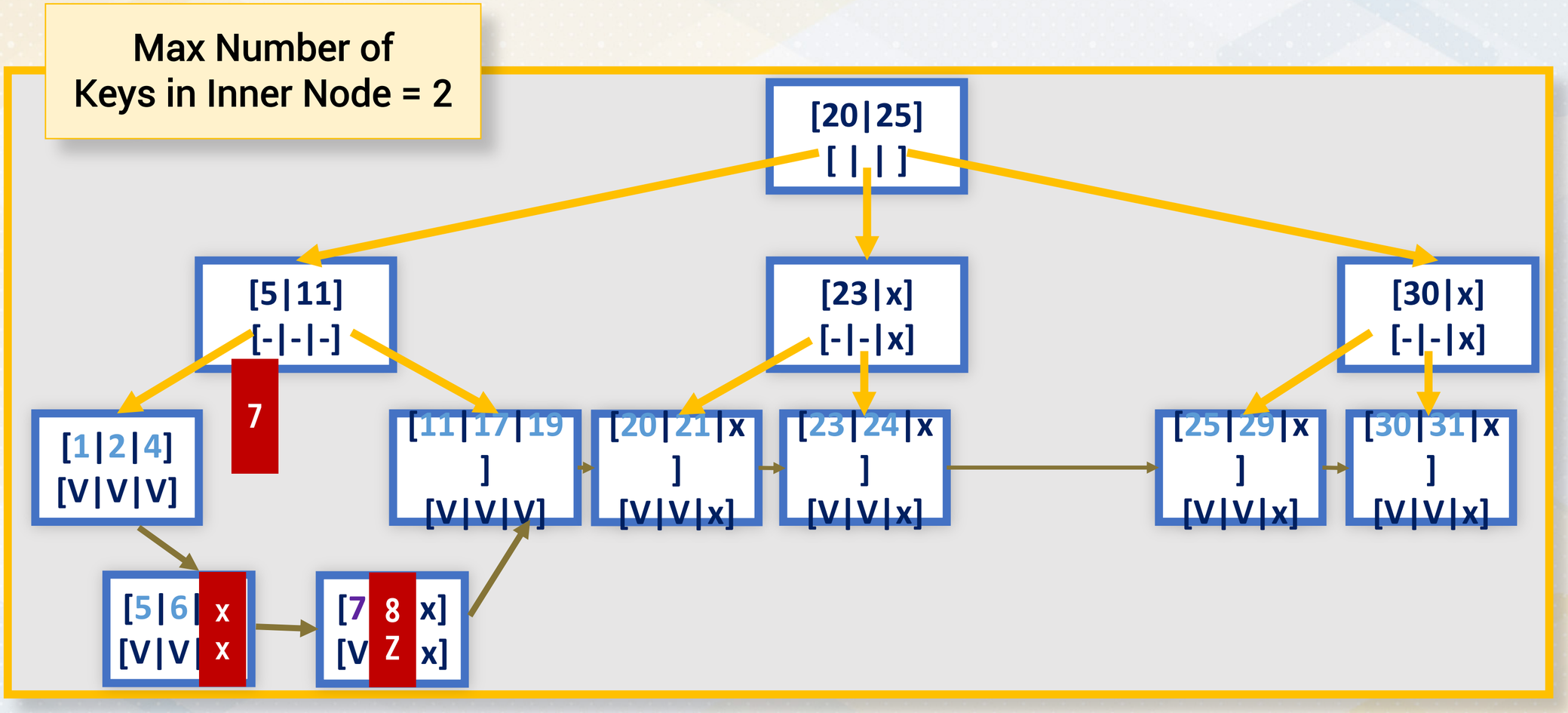
Node contains keys and values in separate vectors.

```
template <typename Key, typename Value> class BPlusTree {
  struct Node {
    std::vector<Key> keys;
    std::vector<Value> values;                    // Only used in leaf nodes
    std::vector<std::shared_ptr<Node>> children; // Only used in internal nodes
    std::shared_ptr<Node> next = nullptr;         // Next leaf node
    bool isLeaf = false;
    Node(bool leaf) : isLeaf(leaf) {}
  };
  size_t maxKeys; // Order of the tree
  std::shared_ptr<Node> root;
};
```

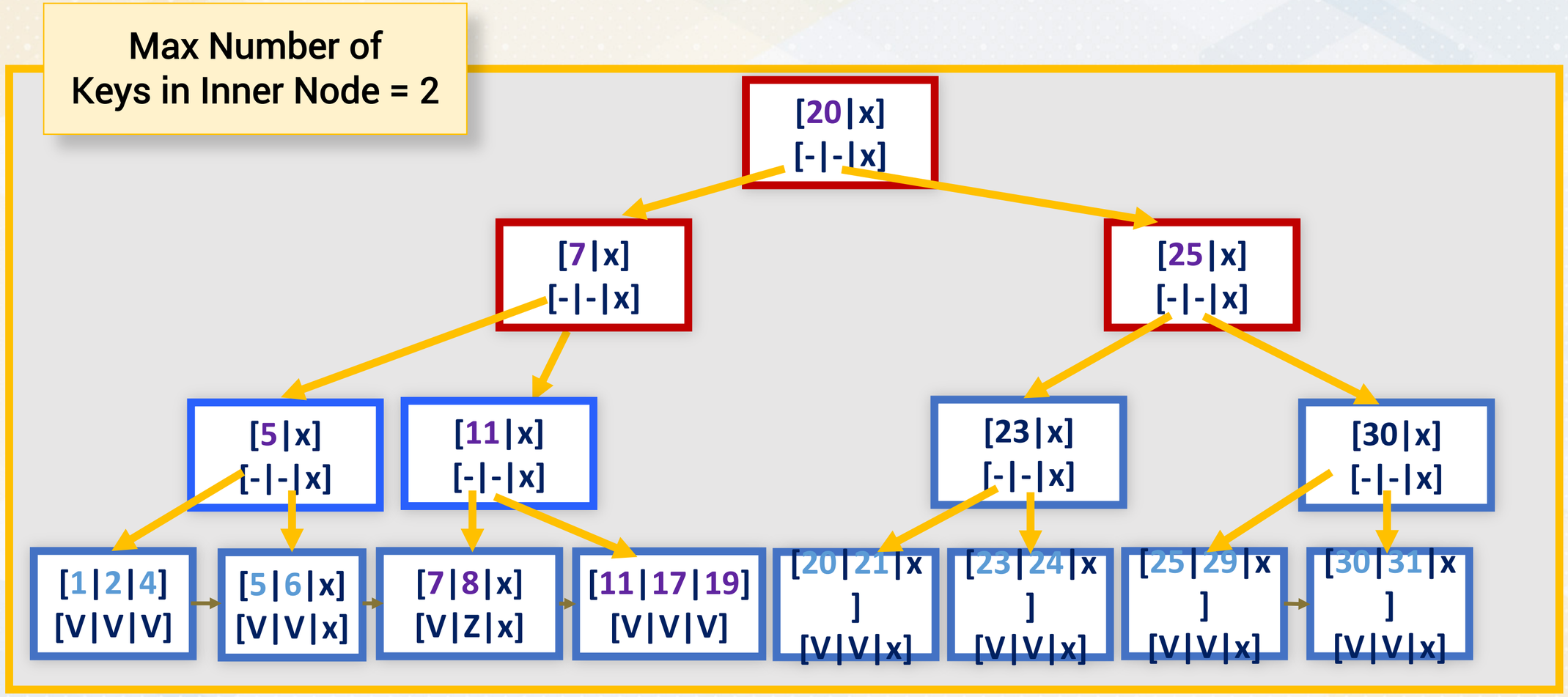
Node Split: Inserting [8, Z]



Node Split: Inserting [8, Z]



Node Split: Inserting [8, Z]



Node Split in C++

buzzDB

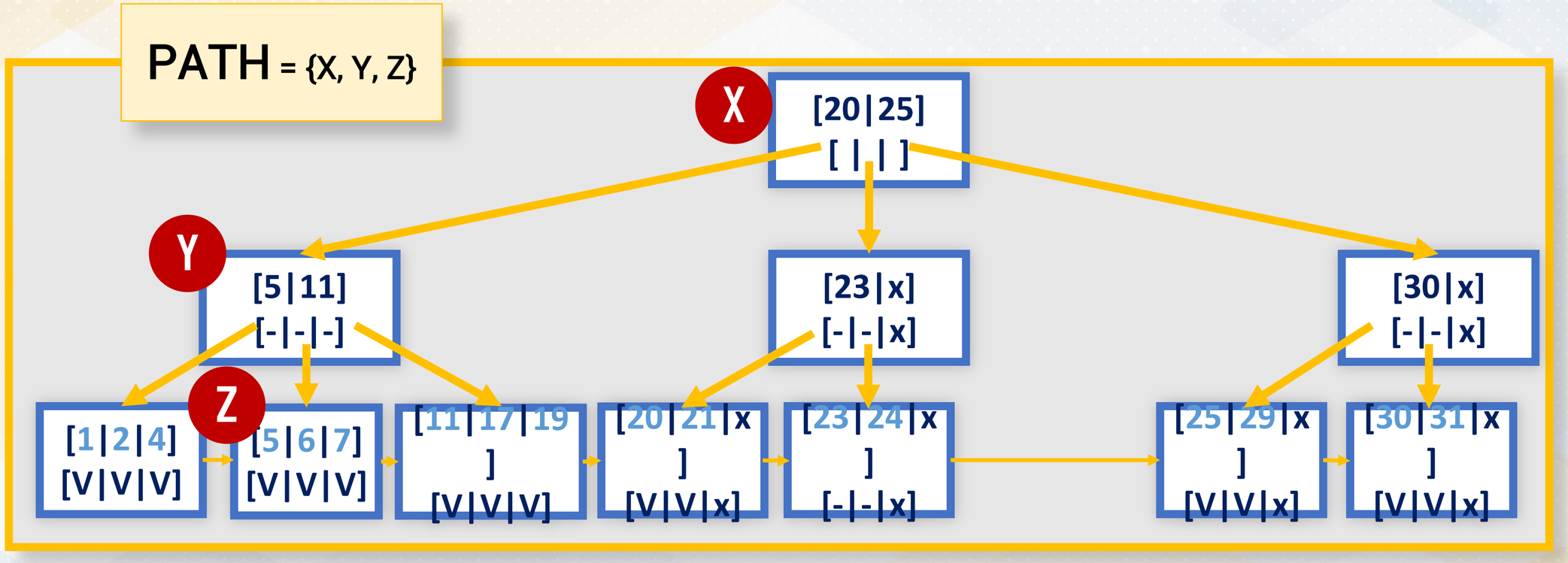
Node Exceeds Capacity → Split Node

```
if (node->keys.size() > maxKeys) {
  splitNode(path, node);
}
```

Insertion

```
auto node = root;
std::vector<std::shared_ptr<Node>> path; // Track the path for backtracking
while (!node->isLeaf) {
  path.push_back(node);
  auto it = std::upper_bound(node->keys.begin(), node->keys.end(), key);
  size_t index = it - node->keys.begin();
  node = node->children[index];
}
```

Path Tracking and Backtracking



Insertion in Leaf Node

```
auto it = std::lower_bound(node->keys.begin(), node->keys.end(), key);
if (it != node->keys.end() && *it == key) {
  size_t pos = std::distance(node->keys.begin(), it);
  node->values[pos] += value; // Update existing key's value
}
else {
  size_t pos = it - node->keys.begin();
  node->keys.insert(node->keys.begin() + pos, key);
  node->values.insert(node->values.begin() + pos, value);
}
```

Node Splitting in Leaf Node

```
if (node->isLeaf) {
  auto newNode = std::make_shared<Node>(true); // Create a new leaf node
  size_t mid = node->keys.size() / 2;          // Calculate the middle index
  // Move the second half of keys and values to the new node
  std::move(node->keys.begin() + mid, node->keys.end(),
            std::back_inserter(newNode->keys));
  std::move(node->values.begin() + mid, node->values.end(),
            std::back_inserter(newNode->values));
  newNode->next = node->next;
  node->next = newNode;
}
```

Node Splitting in Inner Node

```
else {                                                  // Handling internal nodes
  auto newNode = std::make_shared<Node>(false); // Create a new internal node
  size_t mid = node->keys.size() / 2;           // Middle index
  Key midKey = node->keys[mid]; // Key that will move to the parent
  // Move keys and children to the new node
  std::move(node->keys.begin() + mid + 1, node->keys.end(),
            std::back_inserter(newNode->keys));
  std::move(node->children.begin() + mid + 1, node->children.end(),
            std::back_inserter(newNode->children));
}
```

Backtracking to Parent Node

Path Vector

Determines Parent Node

Median Key Insertion

Parent Node May Exceed Capacity

Recursion

Can Travel Up to Root Node

```
else {
  auto parent = path.back(); // Get the parent node
  path.pop_back();           // Remove the last tracked node
  size_t pos = std::distance(
      parent->keys.begin(),
      std::lower_bound(parent->keys.begin(), parent->keys.end(), midKey));
  parent->keys.insert(parent->keys.begin() + pos, midKey); // Insert median key
  parent->children.insert(parent->children.begin() + pos + 1, newNode); // Insert
}
```

Backtracking to Root Node

Backtracking

- Empty Path Vector
- New Root Node
- Two Children
- Tree Height += 1

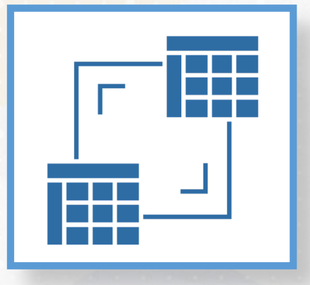
```
if (path.empty()) {
  // New root for the tree
  auto newRoot = std::make_shared<Node>(false);
  newRoot->keys.push_back(midKey);          // Add median key
  newRoot->children.push_back(node);        // Left child
  newRoot->children.push_back(newNode);     // Right child
  // Update root pointer
  root = newRoot;
}
```

On-Disk B+Tree

Need for On-Disk B+Tree

Scalability

ue to Disk
Capacity



Durability

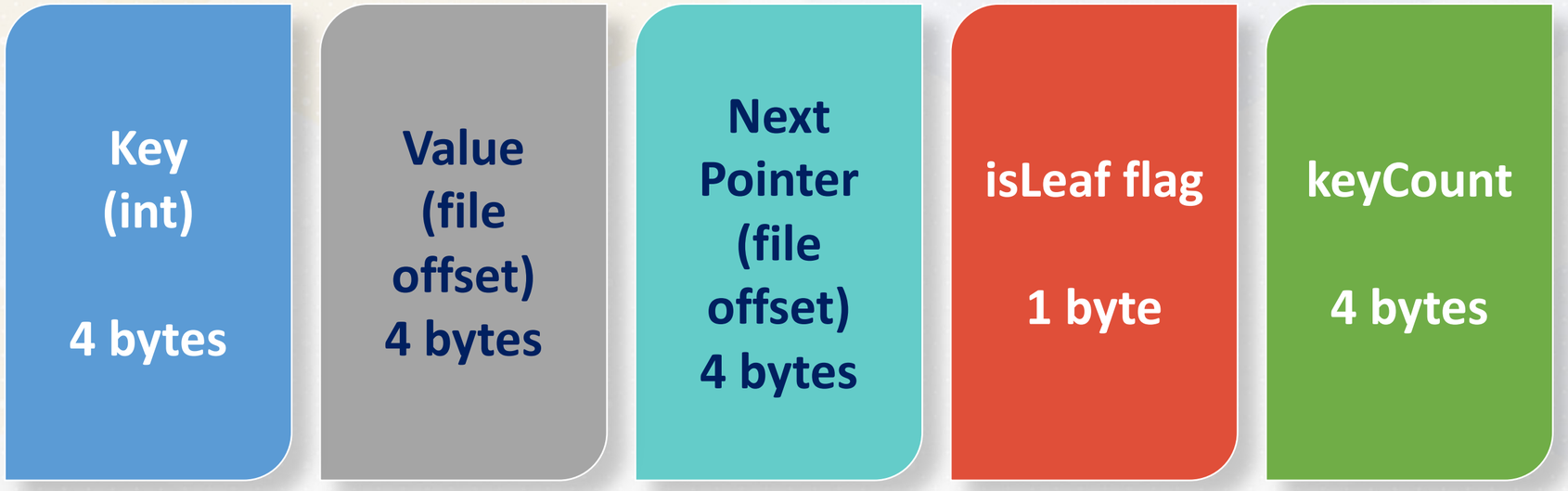
ue to Disk
Durability

Adaptation for Disk

```
struct Node {
  Key keys[MAX_KEYS];                 // Array to store keys
  uint32_t children[MAX_KEYS + 1]; // File offsets for child nodes
  uint32_t next;                      // File offset for the next leaf node
  bool isLeaf;
  int keyCount;
};

std::vector<uint8_t> serializeNodeToBytes(const DiskNode &node);
void deserializeNodeFromBytes(const std::vector<uint8_t> &buffer,
                              DiskNode &node);
```

Calculating Size of Node Struct

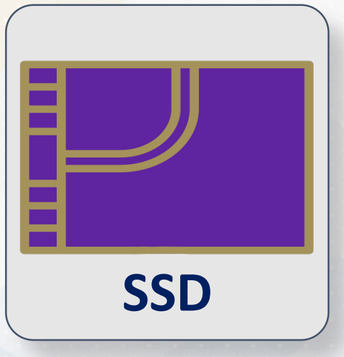


Size of Node struct with m keys =
m × (size of key + size of value) + size of an extra value +
size of next pointer + size of isLeaf + size of keyCount =
m × (4 + 4) + 4 + 4 + 1 + 4 = m × 8 + 13

Aligning Node Struct with Disk Page Size

4 kB Page

M = 510 keys



16 kB Page

M = 2046 keys

```
// 4 KB disk page size
m × 8 + 13 <= 4096 bytes
m ~= 510 keys

// 16 KB disk page size
m x 8 + 13 <= 16384 bytes
m ~= 2046 keys
```

Capacity of a B+Tree

Really Wide gh Fan Out

Page Size	Fan Out	Levels
4 kB	510	3

Leaf Pages	510 ^ (LEVELS -1) = 510 ^ (3 -1) = 510 ^ 2 PAGES
Max Keys	510 ^ (LEVELS) = 510 ^ 3 = 132 Million KEYS
B+Tree File Size	510 ^ 2 PAGES * 4 KB / PAGE = 1 GB

Capacity of a B+Tree (4 Levels)

Page Size	Fan Out	Levels
4 kB	510	4

Leaf Pages	510 ^ (LEVELS -1) = 510 ^ (4 -1) = 510 ^ 3 PAGES
Max Keys	510 ^ (LEVELS) = 510 ^ 4 = 67 Billion KEYS
B+Tree File Size	510 ^ 3 PAGES * 4 KB / PAGE = 0.5 TB

Serialization & Deserialization

Serialization

Convert an in-memory DiskNode structure to a byte stream suitable for disk

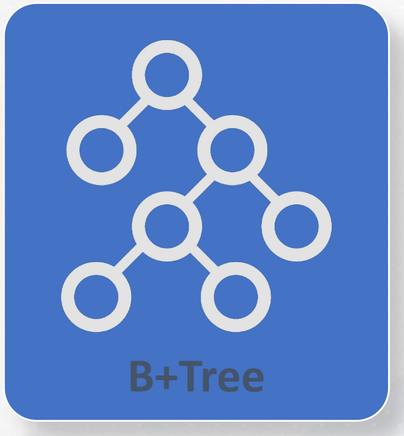
```
std::vector<uint8_t> serializeNodeToBytes(const DiskNode &node) {
  std::vector<uint8_t> buffer;
  // Assuming each key and offset is of fixed size, e.g., 4 bytes for int keys
  for (const auto &key : node.keys) {
    auto bytes = reinterpret_cast<const uint8_t *>(&key);
    buffer.insert(buffer.end(), bytes, bytes + sizeof(Key));
  }
  ...
  return buffer;
}
```

Deserialization

Convert a byte stream read from disk back to an in-memory Node

```
void deserializeNodeFromBytes(const std::vector<uint8_t> &buffer, DiskNode &node) {
  size_t offset = 0;
  // Assuming we know the number of keys and that Key is of a fixed size
  for (int i = 0; i < node.keyCount; ++i) {
    Key key;
    std::memcpy(&key, buffer.data() + offset, sizeof(Key));
    node.keys[i] = key;
    offset += sizeof(Key);
  }
  ...
}
```

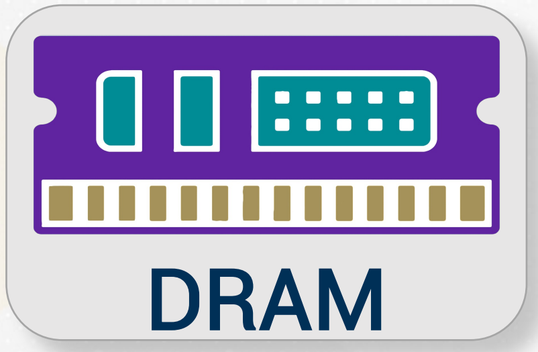
Node Storage in On-Disk File



FILE OFFSET	
0	NODE 1
4 KB	NODE 2
12 KB	NODE 3
16 KB	NODE 4
20 KB	NODE 5
24 KB	NODE 6
30 KB	NODE 7
...	...

Minimizing Disk Reads

Reduce Disk Operations | OOT Node Cached



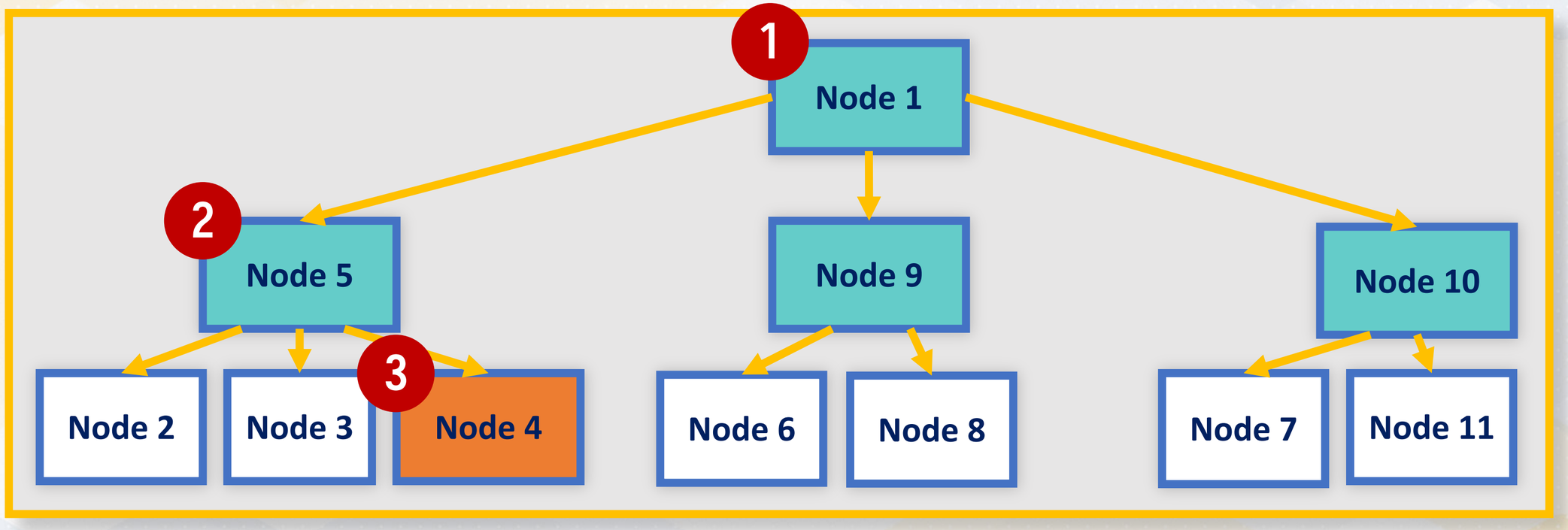
N 1	N 5	N 2
N 9	N 6	N 10

Cached Nodes

N 1	N 2	N 3	N 4
N 5	N 6	N 7	N 8
N 9	N 10	N 11	

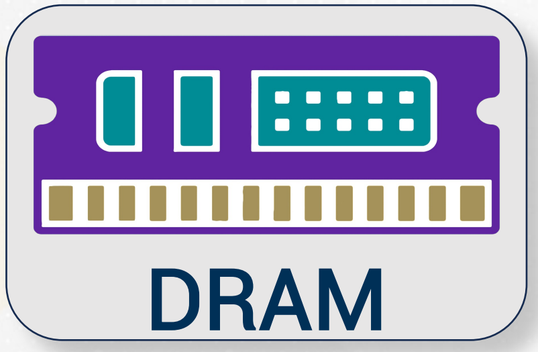
B+Tree

Minimizing Disk Reads: Finding a Key in Node 4



Minimizing Disk Writes

Node Eviction	Node evicted from buffer pool when it is full



N 1	N 5	N 2
N 9	N 6	N 10

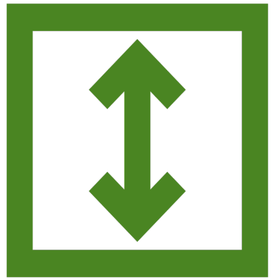
Cached Nodes

N 1	N 2	N 3	N 4
N 5	N 6	N 7	N 8
N 9	N 10	N 11	

B+Tree

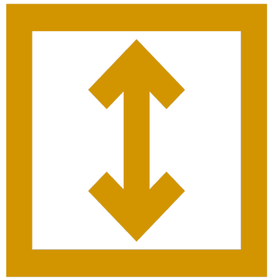
Advantages of B+Trees on Disk

B+Trees are designed for storage on disk



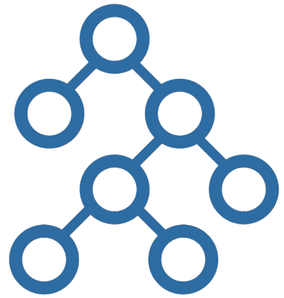
Uniform Height

All operations need same number of disk reads/writes



Short Height

Higher-level inner nodes are buffered leading to fewer page misses



Linked Leaves

Range queries can be processed efficiently

Conclusion

- Range Query
- Ordered Index
- B+Tree Template