Data Streaming Algorithms for Estimating Entropy of Network Traffic

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Outline

- Problem Definition and Applications
- Some Lower Bounds
- The Streaming Algorithm
- The Sieving Algorithm
- Results
- Open Questions, Conclusions, References

The Streaming Model - Notation

- A *stream* is an ordered tuple over the alphabet

$$[n] = \{1, 2, 3, \ldots, n\}.$$

- We denote the number of times that the item i is seen by m_i and the total number of items in the stream by m, i.e., $m = \sum_{i=1}^{n} m_i$.

- The number of distinct items in the stream (i.e., the number of items with non-zero count) is denoted by n_0.

- The tth item in the stream is represented by a_t. So, the stream is represented by $(a_1, a_2, a_2, \ldots, a_m)$, where each $a_t \in [n]$.

The Entropy of a Stream

The *sample entropy* (or just entropy) of a stream is defined to be

$$H \equiv -\sum_{i=1}^{n} (m_i/m) \log (m_i/m).$$

All logarithms are base 2 and, by convention, we define $0 \log 0 \equiv 0$.

The Entropy of a Stream

We will focus on computing the value $S \equiv \sum_{i=1}^{n} m_i \log m_i$ and note that

$$\begin{aligned} H &= -\sum_{i=1}^{n} \frac{m_i}{m} \log\left(\frac{m_i}{m}\right) \\ &= \frac{-1}{m}\left[\sum_i m_i \log m_i - \sum_i m_i \log m\right] \\ &= \log m - \frac{1}{m}\sum_i m_i \log m_i \\ &= \log m - \frac{1}{m} S, \end{aligned}$$

so that we can compute H from S if we know the value of m.

Motivation

- Entropy is a good measure of "diversity"
- Simple volume-based methods are insufficient to detect anomalies
- Several works have suggested entropy as a good metric to detect anomalies (Feinstein et al., Lakhina et al., Wagner et al., Xu et al.)
- Entropy can be used to detect a diverse spectrum of attacks, e.g., denial-of-service, distributed denial-of-service, port scans, worm attacks
- Having access to this information allows us to *profile* the anomalous behavior

Some Lower Bounds

Theorem: Any randomized streaming algorithm to compute the exact value of S when there are at most m items must use $\Omega(m)$ bits of space.

Proof Idea: Reduction from the communication complexity problem of set disjointness.

Theorem: Any deterministic streaming algorithm to approximate S with relative error less than $1/3$ must use $\Omega(m)$ bits of space.

Proof Idea: A counting argument to show that every sublinear algorithm makes an error of at least $1/3$ on some input.

(ϵ, δ)-Approximation

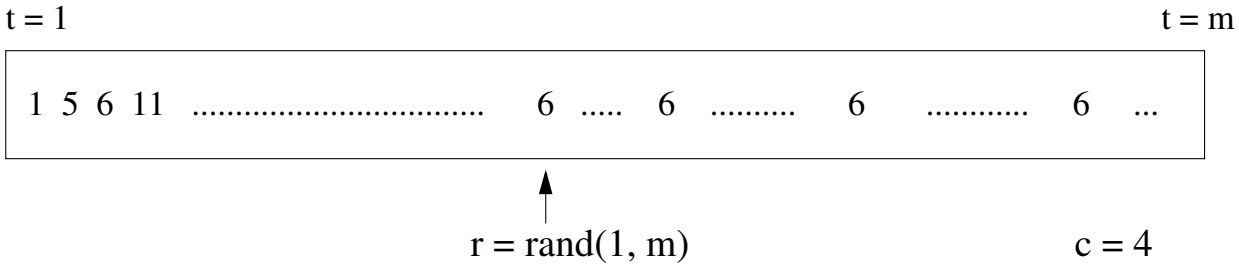
An (ϵ, δ)-approximation algorithm for X is one that returns an estimate X' with relative error more than ϵ with probability at most δ. That is

$$Pr(|X - X'| \geq X\epsilon) \leq \delta.$$

For example, the user may specify $\epsilon = 0.05, \delta = 0.01$ (i.e., at least 99% of the time the estimate is accurate to within 5% error). These parameters affect the space usage of the algorithm, so there is a tradeoff of accuracy versus space.

The Algorithm

The strategy will be to sample as follows:



and compute the following estimating variable:

$$X = m(c \log c - (c-1) \log (c-1)).$$

Algorithm Analysis

This estimator $X = m(c \log c - (c-1) \log (c-1))$ is an unbiased estimator of S since

$$\begin{aligned} E[X] &= \frac{m}{m} \sum_{i=1}^{n} \sum_{j=1}^{m_i} (j \log j - (j-1) \log (j-1)) \\ &= \sum_{i=1}^{n} m_i \log m_i \\ &= S. \end{aligned}$$

Algorithm Analysis, contd.

Next, we bound the variance of X:

$$
\begin{aligned}
Var(X) &= E(X^2) - E(X)^2 \leq E(X^2) \\
&= \frac{m^2}{m}[\sum_{i=1}^{n}\sum_{j=2}^{m_i}(j\log j - (j-1)\log(j-1))^2] \\
&\leq m\sum_{i=1}^{n}\sum_{j=2}^{m_i}(2\log j)^2 \leq 4m\sum_{i=1}^{n} m_i \log^2 m_i \\
&\leq 4m\log m(\sum_i m_i \log m_i) \leq 4(\sum_i m_i \log m_i)\log m(\sum_i m_i \log m_i) \\
&= 4S^2 \log m,
\end{aligned}
$$

assuming that, on average, each item appears in the stream at least twice.

Algorithm contd.

If we compute $s_1 = 32 \log m/\epsilon^2$ such estimators and compute their average Y, then by Chebyschev's inequality we have:

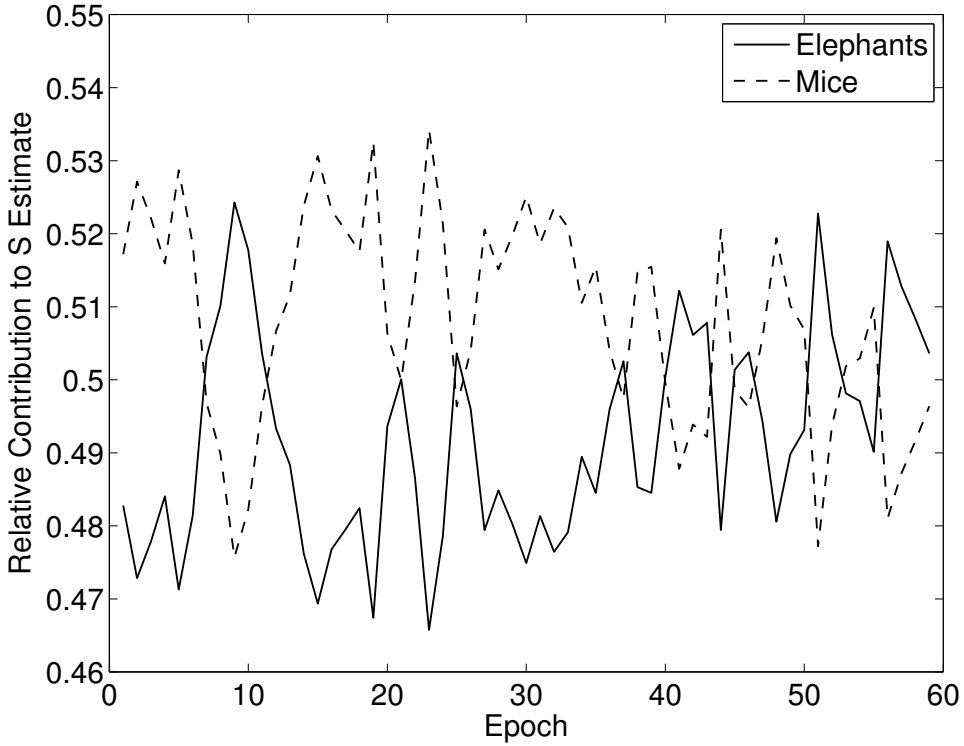
$$\begin{aligned} Pr(|Y - S| > \epsilon S) &\leq \frac{Var(Y)}{\epsilon^2 S^2} \\ &\leq \frac{4 \log m S^2}{s_1 \epsilon^2 S^2} = \frac{4 \log m}{s_1 \epsilon^2} \\ &\leq \frac{1}{8}. \end{aligned}$$

We do this averaging for $s_2 = 2 \log (1/\delta)$ groups. By a Chernoff bound we can get that at least $s_2/2$ averages have relative error at most ϵ with probability at least $1 - \delta$. Hence, the median of averages is an (ϵ, δ)-approximation.

The Sieving Algorithm

- KEY IDEA: Separating out the elephants decreases the variance, and hence the space usage, of the previous algorithm
- Each packet is now sampled with some fixed probability p
- If a particular item is sampled *two or more* times, it is considered an elephant and its exact count is estimated
- For all items that are not elephants we use the previous algorithm
- The entropy is estimated by adding the contribution from the elephants (from their estimated counts) and the ants (using the earlier algorithm)

The Sieving Algorithm, contd.



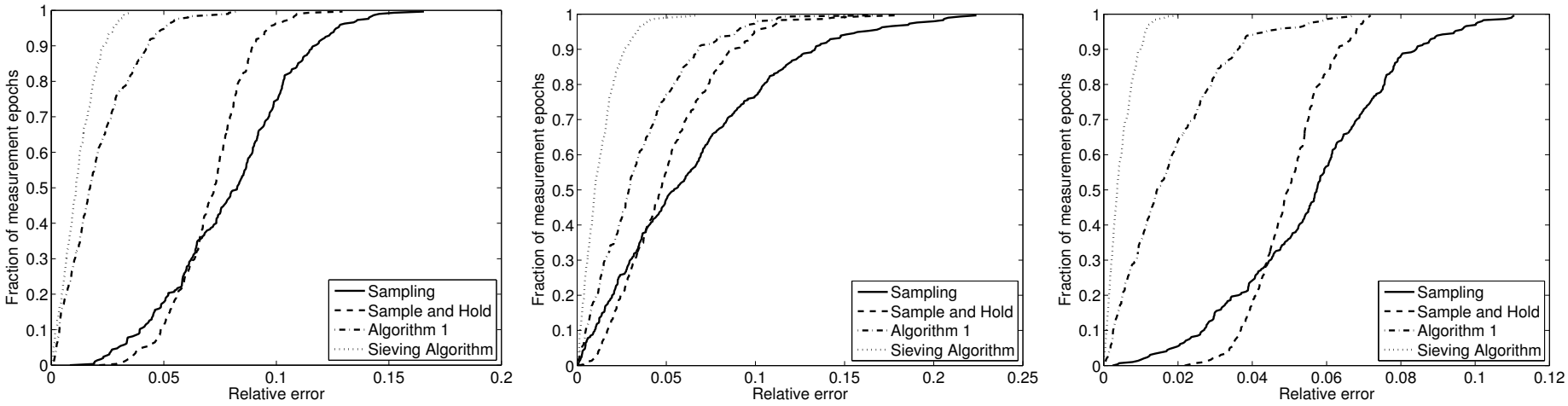
Evaluation Traces

-	Trace 1	Trace 2	Trace 3
Date	Feb. 2, 2004	Aug. 5, 2003	Apr. 24, 2003
Where	USC Los Netto	Mid-sized dept.	UNC Access Link
Epoch Length	1 min.	5 min.	1 min.
Trace Length	60 min.	60 min.	60 min.
# Packets per Epoch	1.7M	0.5M	2.5M
# Distinct Addresses	30,267	2,587	25,565
# Distinct Ports	15,165	4,672	8,080

Algorithms Compared

- Simple sampling
  - Sample packets at rate p
  - Counts are normalized by $1/p$ and used to compute entropy
- Sample and Hold (Estan and Varghese)
  - Sample positions and keep counts from that point on
  - Adjust counts by adding $1/p$ and sum to compute entropy
- Algorithm 1 - The Streaming Algorithm
  - Uses AMS-type estimator
- Algorithm 2 - The Sieving Algorithm
  - Estimates contribution of elephants and ants separately

Evaluation Results



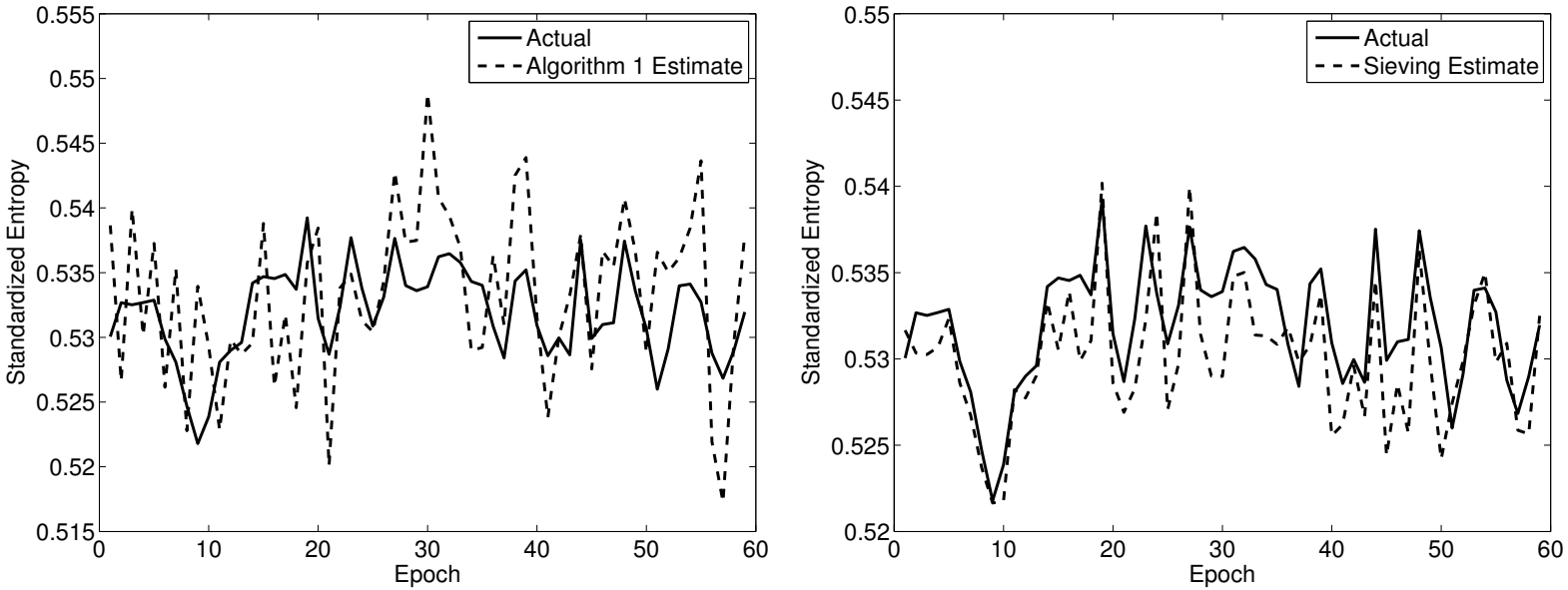
CDF of the three traces on destination address entropy

Evaluation Results

Distribution	Sample	Sample&Hold	Algo. 1	Sieving
DSTADDR	0.198	0.192	0.013	0.013
SRCADDR	0.054	0.049	0.017	0.004
DSTPORT	0.116	0.109	0.016	0.015
SRCPORT	0.069	0.062	0.016	0.005

Trace 3: Mean relative error in S estimate

Evaluation Results



Verifying the accuracy in estimating the standardized entropy

Open Problems and Conclusions

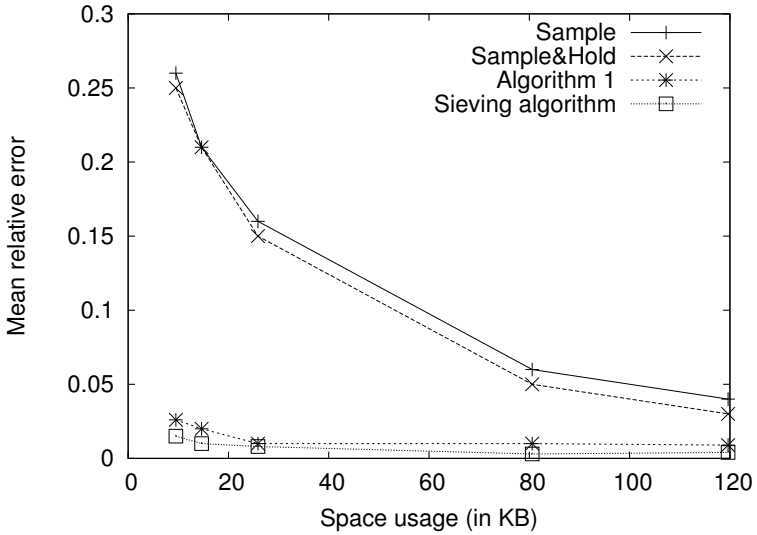
- There is much to be learnt about *streaming algorithms* and how they outperform traditional sampling methods
- The idea of separating out the elephants and performing computations for them and the rest of the stream separately may have a much wider use
- Are there methods out there to improve our results further? Can we do better if we make slightly stronger assumptions about the distribution of the stream?
- How hard would it be to maintain the entropy of arbitrary subpopulations? For example, to allow a network operator to study the behavior of a single source address *after* the data has been collected.

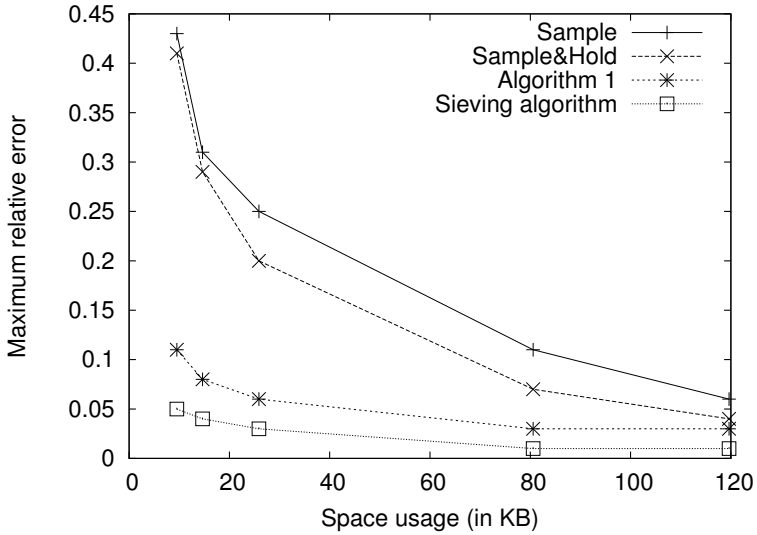
References

- N. Alon, Y. Matias, M. Szegedy. The space complexity of approximating the frequency moments. In *Proceedings of the 28th annual ACM Symposium on Theory of Computation (STOC 1996)*, New York, NY, USA, 1996. ACM Press.
- A. Chakrabarti, K. Do Ba, S. Muthukrishnan. Estimating entropy and entropy norm on data streams. In *Proceedings of the 23rd International Symposium on Theoretical Aspects of Computer Science (STACS 2006)*, Marseille, France, 2006. Springer LNCS.
- B. Kalyanasundaram and G. Schnitger. The probabilistic communication complexity of set intersection. In *SIAM Journal of Discrete Mathematics*, 5(4):545-557, 1992.

Thanks for your attention!

Extra slides - Varying Sampling Rate





Extra slides - Threshold for Sieving