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Efficiently Biasing PRMs with Passage Potentials

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***Abstract*— This paper presents a passage potential based biasing scheme for PRMs to specifically address the narrow passage problem. The biasing strategy fulfills minimum requirements for an efficient biasing, considering not only location issues, but also intensity, sparseness and applicability of the biasing criterion. Conforming to these features a particular family of passage potential functions has been defined and integrated within a basic PRM to achieve biasing. Simulations have demonstrated the reliable and successful implementation of the proposed architecture under several experimental settings and robot configurations.**

I. Introduction

Several difficult path planning problems have been solved using probabilistic roadmap mehods (PRMs) [1], [2]. The probabilistic completeness property of PRMs guarantees that nearly any sampling scheme will provide good results. In fact, any given problem can be solved considering a sufficiently large number of initial samples or enhancement steps [3]. However, for certain problem cases, the choice of sampling strategy can play a significant role in the performance of a sampling-based planner.

The key idea of the basic PRM is to create a roadmap in Q_{free} by uniformly distributing nodes in the robot's configuration space [3]. The roadmap's nodes are free configurations and its edges are paths between configurations. After the roadmap has been generated, planning queries can be answered by connecting user-defined initial and goal configurations to the roadmap. If the roadmap is successful in capturing the connectivity of Q_{free}, path planning may be reduced to a graph search.

However, the uniform sampling scheme used in the basic PRM does not work well in "difficult" planning settings in terms of efficiency. If a narrow passage exists in Q_{free} and it is absolutely necessary to go through that passage to solve a query, a uniform sampling planner must select a sample from a potentially very small set in order to answer the query. The sampling scheme tries to distribute nodes with constant density in Q_{free} and the volume spanned by the narrow regions of Q_{free} makes up only a small fraction of the total volume of Q_{free}. A number of different sampling methods have been designed to solve this problem [4], generally biasing the sampling in order to provide increased node density in narrow regions.

This paper proposes a PRM that uses biasing to improve the sampling distribution in narrow regions. The biasing is achieved by means of a particular family of passage potential functions specifically designed for this problem. Section 2 presents advantages of biasing schemes within PRMs and briefly introduces the work related to our chosen strategy for biasing. We then identify minimum requirements for an efficient biasing scheme, and consequently define particular passage potentials that fulfill these conditions. In Section 3 some implementation issues are commented, and experimental results confirming the validity of the proposed model and its performance are exposed. Some conclusions and possible improvements are finally discussed in Section 4.

II. Biasing PRMs

Due to the *probabilistic completeness* property of PRMs [3], any given problem can be solved considering a sufficiently large number of initial samples or enhancement steps. Therefore, it seems that a biased sampling scheme might not be necessary. However, we would prefer to use a biased sampling scheme for several practical reasons.

First, graph search time can be reduced [5]. If the planner is less likely to perform the enhancement step, or a smaller initial number of nodes is needed to capture the connectivity of the free configuration space Q_{free}, then roadmap connection and search time could be reduced. Another issue appears when there are various ways to reach the target and the shortest one contains a narrow passage. Given a large number of initial nodes, a classical PRM planner will probably find the long way. This behavior might be sometimes acceptable, however it will be usually preferable to find the shortest path.

Many different PRMs attempt to solve the narrow passage problem. Obstacle-based sampling methods consider that narrow passages are like thin corridors in Q_{free} surrounded by obstacles, and sample near the boundary of configuration-space obstacles [6], [7], [8]. Other biased planners sample directly inside narrow passages [9], [10], [11], [12].

The biased PRM (ABPRM) presented in [5] increases the node density along obstacle surfaces, and specially in narrow regions, using artificial potential fields [13]. The artificial potential field is computed from a partial solution of Laplace's equation in the workspace of the robot. Since in general Laplace's equation can be used to describe a potential of a particle in free space acted on only by gravitational forces, then a function $\phi(q)$ that satisfies Laplace's equation can be used to solve the path planning problem [14]. Specifically, the authors solved Laplace's equation numerically in the region of interest. They used the fact that, while solving for the potential ϕ, iterative methods cause in general the potential to rise more rapidly in grid points surrounded by boundary points, such as grid points in narrow regions. So they first compute a set of "unbiased" nodes uniformly distributed in Q_{free}, and then add some nodes biased by this potential function ϕ. This biasing scheme improves the sampling distribution. However, it seems that an ad-hoc, specially designed artificial potential field can be considered to better overcome the narrow passage problem in PRMs. In other words, we claim that within the same biasing framework, a specific family of potential functions with particular characteristics –aimed to fulfill requirements of the narrow passage problem– can be constructed.

A. Characteristics of an Efficient Biasing Scheme

We will enumerate in this section some characteristics that appear as ideal for any biasing scheme pointed to solve the narrow passage problem in an effective manner.

1) The first requirement indicates where to increase sampling by means of a stronger biasing. An efficient solution would generate samples inside narrow passages, but as far away as possible from the obstacle. This property is meant to avoid the robot to "crawl" near the edges of obstacles. While this is not an issue during planning for a point robots in a completely known environment, it is so when planning motions for real physical robots in an approximation of the real world [5]. In the real world, it is desirable for the robot to have some minimum clearance to the obstacles. The Generalized Voronoi Diagram (GVD) has this property [3]. The sampler can place points as close to the GVD as possible with the hope of aligning the whole robot with narrow passages. Although exact computation of the GVD is impractical for high-dimensional configuration spaces, it is possible to find samples on the GVD or close to it without computing it explicitly.
2) The second property qualitatively defines how the structure of the narrow passage should affect the distribution of the biasing. In fact, the expected biasing should have density and variance behaviors according to the closest surrounding obstacles. Due to the difficulty of placing samples in confined spaces, we pretend the biasing scheme to provide denser biasing in narrow and clutter regions. Specifically, if the passage is narrower, we want a stronger biasing –in fact, in the limit, if we let the passage to shrink to zero width the biasing must tend to "infinite." Moreover, the sparseness of the samples also needs to be controlled. A wider passage does not need the samples to be condensed necessarily near the GVD, some tolerance can exist and may be desirable for a better coverage of the passage. On the other hand, a narrower passage may oblige to compress the biasing near the GVD.
3) The third property simply restricts the applicability of the prior two concepts. We do not want to incorporate unnecessary computational overhead biasing points in regions where a classical uniform sampling can do it. It is clear that this issue is strongly related to the clearance given by the closest obstacles, i.e., it will be a measure determined by the width of the passages.

B. Definition of Passage Potentials for Efficient Biasing

Our criterion for biasing not only focuses on increasing the density in cluttered regions of the environment, but also on fulfulling certain requirements in the way this is done. We construct a passage potential function $\phi(\mathbf{q})$ associated with a given configuration according to the properties presented in Sec. II-A. As in [5], the idea to generate "biased" samples is to consider uniformly distributed samples first, and then keep only those $\mathbf{q} \in Q_{free}$ with probability:

$$P(\mathbf{q} \text{ is kept}) = \phi(\mathbf{q}), \text{ for } 0 \leq \phi(\mathbf{q}) \leq 1. \tag{1}$$

It is clear then that the probability of keeping a node $\mathbf{q}$ is given by the passage potential $\phi(\mathbf{q})$: if a denser sampling is sought in a certain region, the corresponding potential must be intense.

A natural initial approach would consider some information about the location of the GVD, in order to assign a high potential to samples that are close to it. The advantage of workspace GVD sampling is that the GVD captures well narrow passages in the workspace that typically lead to narrow passages in Q_{free} [3]. Fortunately, we do not need to calculate the entire GVD but only the "local GVD," the point in the GVD closest to a sample $\mathbf{q}$. Let's name this point of the GVD $\mu(\mathbf{q})$, and define it as:

$$\mu(\mathbf{q}) = (\mathbf{d}_1(\mathbf{q}) + \mathbf{d}_2(\mathbf{q}))/2,$$

where $\mathbf{d}_1(\mathbf{q})$ and $\mathbf{d}_2(\mathbf{q})$ correspond to the vectors to the two closest obstacles for the configuration $\mathbf{q}$. Now that we know where the local GVD is, we can quantify how dense and how sparse the biasing should be. We do this defining the passage potential $\phi(\mathbf{q})$ as a Gaussian distribution with mean $\mu(\mathbf{q})$:

$$\phi(\mathbf{q}) = \frac{1}{\sqrt{2\pi}\sigma(\mathbf{q})} e^{-\frac{(\mathbf{q}-\mu(\mathbf{q}))^T(\mathbf{q}-\mu(\mathbf{q}))}{2\sigma(\mathbf{q})^2}}, \tag{2}$$

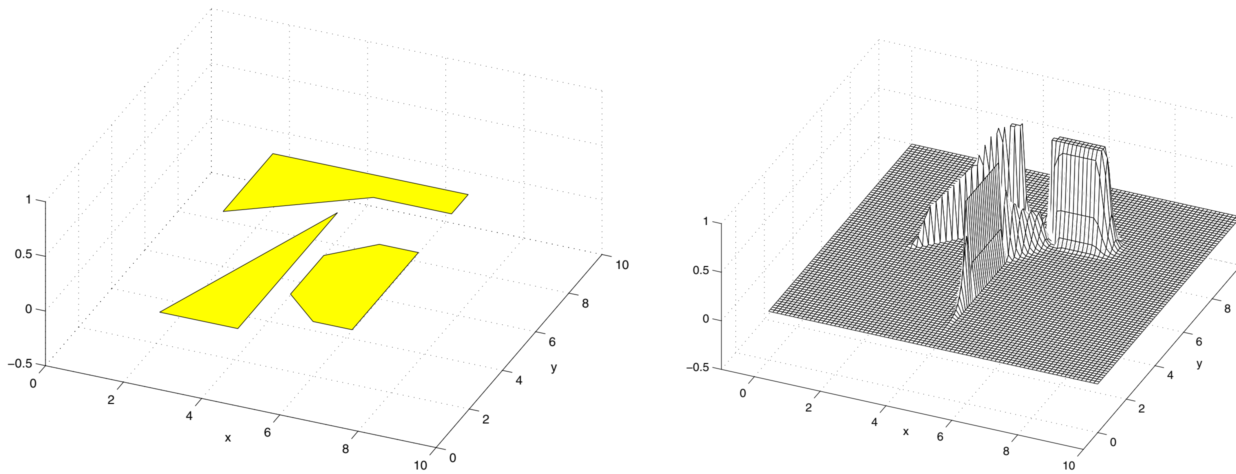
with standard deviation $\sigma(\mathbf{q})$:

$$\sigma(\mathbf{q}) = K||\mathbf{d}_1(\mathbf{q}) - \mathbf{d}_2(\mathbf{q})||,$$

and K an arbitrary gain constant. We finally scope the applicability of the passage potential of (2) to regions with sufficiently narrow passages. This is done redefining the passage potential as follows:

$$\phi(\mathbf{q}) = \begin{cases} \frac{1}{\sqrt{2\pi}\sigma(\mathbf{q})} e^{-\frac{(\mathbf{q}-\mu(\mathbf{q}))^T(\mathbf{q}-\mu(\mathbf{q}))}{2\sigma(\mathbf{q})^2}}, & ||\mathbf{d}_1(\mathbf{q}) - \mathbf{d}_2(\mathbf{q})|| \leq D \\ 0, & ||\mathbf{d}_1(\mathbf{q}) - \mathbf{d}_2(\mathbf{q})|| > D, \end{cases} \tag{3}$$

with D the scope threshold for applicability.



(a) (b)

Fig. 1. Validation of the biasing scheme: experimental obstacle layout in (a) and the corresponding passage potential function in (b).

It is clear then that a function $\phi(\mathbf{q})$ as defined in (3) will produce the desired biasing with all the sought properties:

- It will bias strongly near the GVD due to the fact that the passage potential $\phi(\mathbf{q})$ is defined as a Gaussian distribution with a mean that is indeed the closest point of the GVD.
- Since the standard deviation is proportional to the width of passages, the passage potential will distribute the biasing accordingly: it will intensify and concentrate the biasing when the width of the passage is small and reduce both intensity and concentration if the width is larger.
- The application of the scheme is constrained to those passages that are narrow enough, eliminating unnecessary computation.

III. Implementation

We have implemented a PRM that uses the biasing scheme presented in the previous section for a point robot and a multi-link planar robot moving among polygonal obstacles. The passage potential presented in Sec. II-B was for a robot represented as a single point, then we need to extend the approach for the case of an articulated multi-link manipulator. We do this as described in [3], selecting a subset of control points $\{\mathbf{r}_i\}$ on the i links of the robot, and then using forward kinematics to obtain the position of these points in the workspace. The potential for each of the points in the workspace is calculated evaluating (3), then the total potential for the manipulator at a given configuration $\mathbf{q}$ is obtained adding the individual potentials for each of the control points and normalizing the sum with respect to the total number of control points:

$$\phi(\mathbf{q}) = \frac{1}{n}\sum_{i=1}^{n} \phi_i(\mathbf{r}_i(\mathbf{q})).$$

Our planner first generates a set of nodes uniformly distributed throughout Q, keeping only those nodes $\mathbf{q} \in Q_{free}$ until M free-collision unbiased nodes are found. Then more nodes are uniformly distributed in the same way, keeping only those nodes $\mathbf{q} \in Q_{free}$ with probability given by (1). This is repeated until N free-collision biased nodes are obtained.

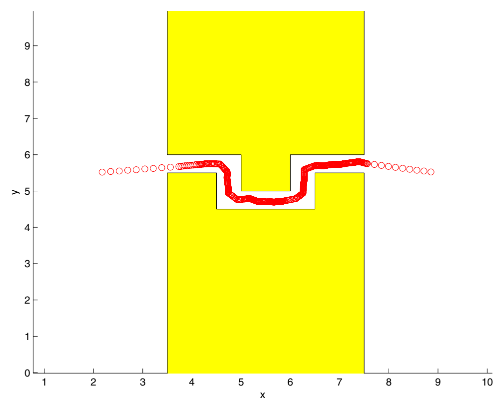
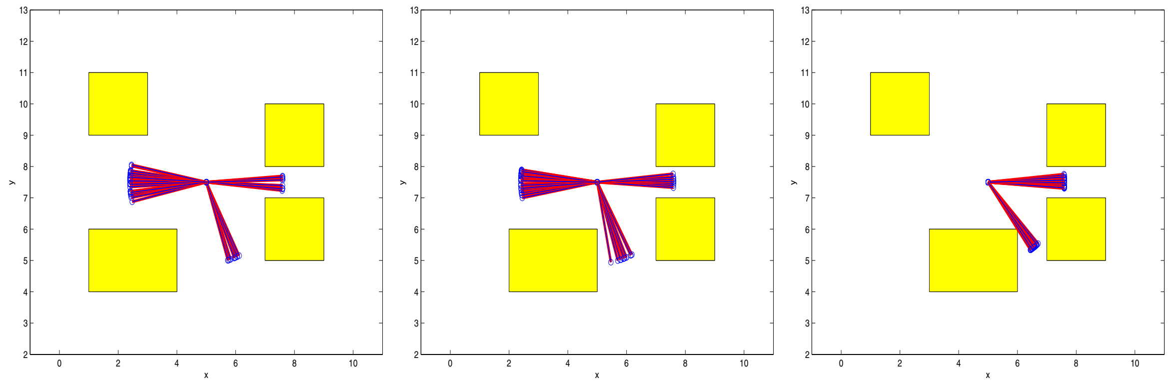


Fig. 3. Experimental setting: environment containing a narrow passage and a point robot moving through it.

A. Experimental Validation of the Biasing Scheme

The early experiments were meant to validate the correct behavior of the implemented biasing scheme. We first evaluated the passage potential for the case of a point robot moving in an experimental layout of polygonal obstacles. Fig. 1 shows this obstacle setting in (a) with the corresponding complete passage potential function in (b). Additionally, several other simulations were done for the multi-link robot. For instance, we considered a 1-link robot, at zero unbiased samples and an arbitrary number of biased samples. Some of these results are included in Fig. 2. For different obstacle settings the figures show the robot at all the biased sampled configurations.

From the experiments we can verify that the biasing scheme works as expected, distributing the sampling density accordingly to the biasing criteria defined in Sec. II. The sampling



(a) (b) (c)

Fig. 2. Experimental validation of the biasing scheme: images showing different obstacle settings together with the robot at all the biased sampled configurations (no unbiased samples in these simulations). The behavior of the biasing scheme is as expected.

TABLE I

PERFORMANCE COMPARISON FOR THE EXPERIMENTAL SETTING OF FIG. 3

# Samples	% Success
100 Unbiased	2
50/50 Unb/Biased	100
200 Unbiased	35
100/100 Unb/Biased	100
300 Unbiased	50
150/150 Unb/Biased	100
400 Unbiased	80
200/200 Unb/Biased	100
500 Unbiased	84
250/250 Unb/Biased	100
600 Unbiased	92
300/300 Unb/Biased	100
700 Unbiased	95
350/350 Unb/Biased	100

density is large in narrow passages and the concentration is tuned by the passages' width, with a distribution response centered at the corresponding GVD location. Moreover, if passages are too wide, the biasing scheme does not act at all in those areas.

B. More Experiments

Several other experiments were executed in order to compare the performance of the planner using the biasing scheme and the planner without biasing. Fig. 3 shows the image of an experimental setting containing a narrow passage and a point robot that moves through it. For this experimental setting, we have compared the basic unbiased PRM with the implemented biased PRM. We have computed the rate of success (the number of times the robot gets to the desired position from the initial one, over the total) for both the unbiased and the proposed biased PRM. We have run the experiment 100 times for an increasing number of total initial samples $(100, 200, 300, 400, 500, 600, 700)$ without considering enhancement in neither of the planners. Table I shows the results of this experiment.

The images in Fig. 4 show the performance of the biasing scheme for more complicated settings and multi-link robots. Each of the three rows corresponds to different experiments. The left column presents images of the robot at unbiased sampled configurations, the central column shows the robot at both unbiased and biased sampled configurations and the right column displays the robot path found using the combined sampling in the PRM . The upper row corresponds to an experiment that considers a 4-link robot, 150 unbiased sampled configurations and 100–50 for the combined unbiased-biased sampling. The central row considers a 5-link robot, 410 unbiased sampled configurations and 90–320 for the combined unbiased-biased sampling. Finally, the lower row considers a 7-link robot, 750 unbiased sampled configurations and 250–500 for the combined unbiased-biased sampling. In all of these cases, the unbiased sampling fails to capture the connectivity of Q_{free}, especially in those difficult regions corresponding to narrow passages. When the biasing scheme is included in the sampling procedure, the PRM notably improves its behavior, being capable of solving difficult path planning problems for robots with many degrees of freedom and among extremely confined passages.

IV. CONCLUSIONS

In this work we have developed a passage potential based biasing scheme for PRMs to specifically address the narrow passage problem. We have first identified certain desired characteristics for efficient biasing, considering not only location issues, but also intensity, sparseness and applicability of the biasing criterion. Conforming to these features a particular family of passage potential functions has been defined to achieve biasing. Experiments have demonstrated that the proposed approach works as expected, increasing the coverage of the Q_{free} specially in the presence of narrow passages.

The biasing scheme is of easy implementation, so it can be included as a particular component in other biasing scheme within a PRM. Since the formulated passage potential is

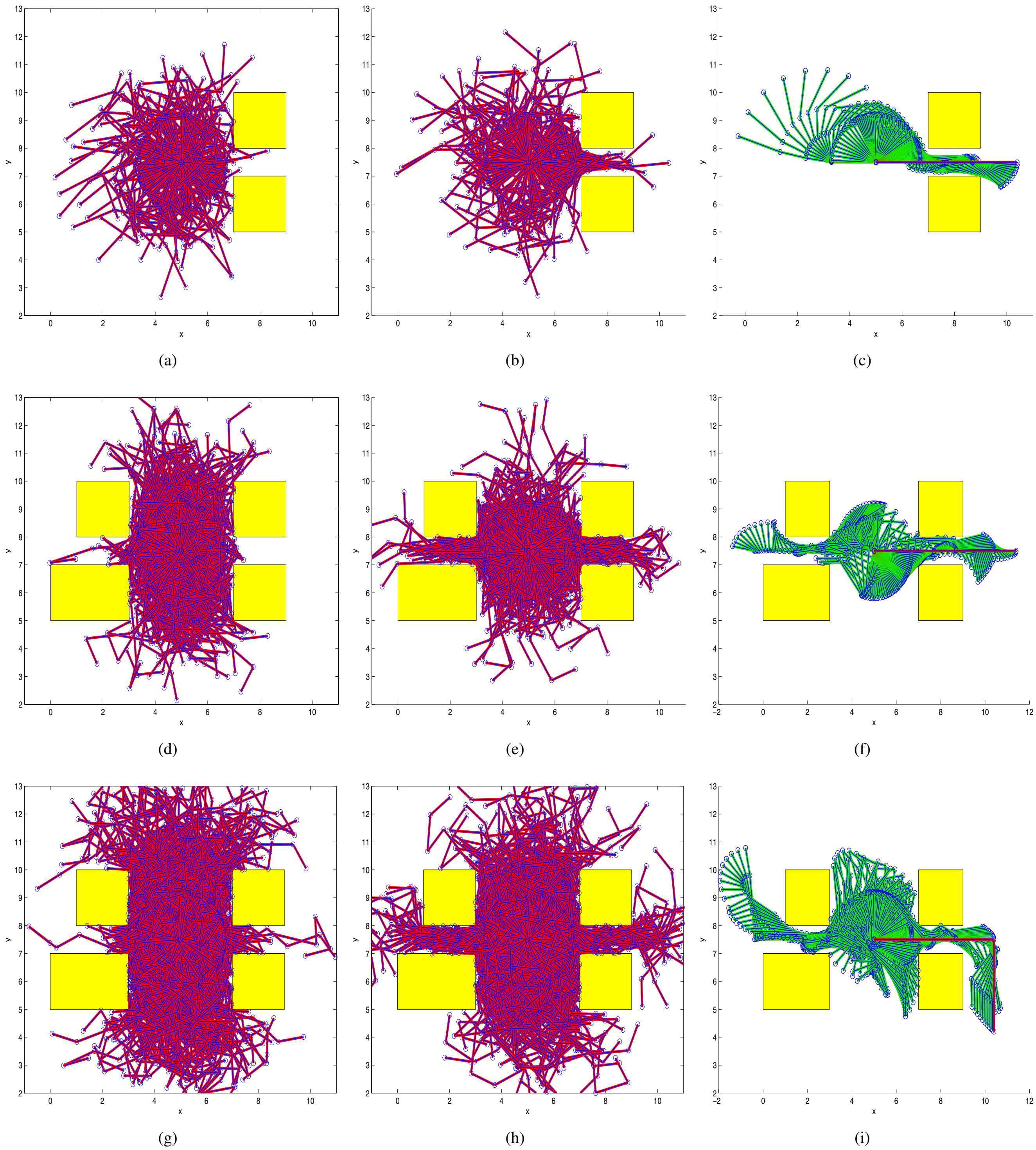


Fig. 4. Experiments considering robots with many links: 4-link robot at 150 unbiased sampled configurations (upper left), 100–50 for the combined unbiased-biased sampling (upper center) and path found using combined sampling (upper right); 90–320 for the combined unbiased-biased sampling (central) and path found using combined sampling (central right) and 7-link robot at 750 unbiased sampled configurations (lower left), 250–500 for the combined unbiased-biased sampling (lower center) and path found using combined sampling (lower right). (The number of neighbors used in the simulations is 75.)

defined in terms of distances to obstacles in the workspace, it can be used for 3D path planning problems. Moreover, the framework can be easily extended to grid-based implementations if the environment is populated with complex, irregular obstacles for which a polygonal representation might not be adequate.

Another major improvement of the system would be to extract all samples directly from a cumulative distribution generated from the passage potential function. This would greatly improve the planner performance, reducing the time consumed in the generation of samples.

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