

# Decision Making: Fuzzy Logic

2018-03-15

First, a bit of history, my 1965 paper on fuzzy sets was motivated by my feeling that the **then existing theories provided no means of dealing with a pervasive aspect of reality**—unsharpness (fuzziness) of class boundaries. Without such means, realistic models of human-centered and biological systems are hard to construct. My expectation was that fuzzy set theory would be welcomed by the scientific communities in these and related fields. **Contrary to my expectation, in these fields, fuzzy set theory was met with skepticism and, in some instances, with hostility.** What I did not anticipate was that, for many years after the debut of fuzzy set theory, its main applications would be in the realms of engineering systems and consumer products.

...

Underlying the concept of a linguistic variable is a fact which is widely unrecognized—a fact which relates to the concept of precision. **Precision has two distinct meanings—precision in value and precision in meaning.** The first meaning is traditional. The second meaning is not. The second meaning is rooted in fuzzy logic.

-- Lotfi Zadeh, 2013

# Questions (N-2)

1. What are the 2 most “complex” decision making techniques we’ve seen?
2. What are their strengths? Weaknesses?
3. What is the key (insight) to their success?
4. What does Planning have that (forward chaining) RBS do not?
5. When do we need a communication mechanism?

# Questions (N-1)

1. Cooperative problem solving / distributed expertise is using h\_\_\_ to d\_\_\_ problems into smaller parts.
2. R\_\_\_ experts rarely communicate/collaborate.
3. Three types of communication are...
4. The three main parts of a Blackboard are...
5. An Arbiter can be used to...

enables a computer to reason about linguistic terms and rules in a way similar to humans

## **FUZZY LOGIC**

**M&F 5.5; B CH 10**

1. Cut two slices of bread **medium thick**.
2. Turn the heat on the griddle on **high**.
3. Grill the slices on one side until **golden brown**.
4. Turn the slices over and add a **generous helping** of cheese.
5. Replace and grill until the top of the cheese is **slightly brown**.
6. Remove, sprinkle on a **small amount** of black pepper, and eat.

# Motivation

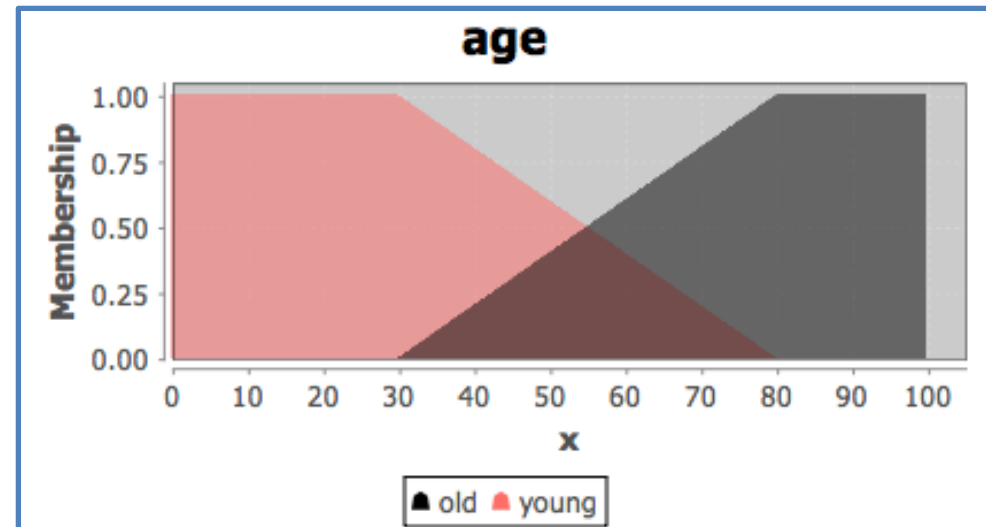
TRUE

FALSE

- Fuzzy logic: truth degrees, vagueness, subjectivity
  - Consider: “20% chance of rain” vs “partly cloudy”
  - Probability as likelihood, ignorance, uncertainty
- Linguistic: non-numeric expression of rules & facts (e.g. old)
  - Allows for use of vague human assessments in computing problems
- E.g.: Cautious vs Confident
  - FSM w/ 2 states – switching looks unnatural
  - Cautious (range), sneak slowly (range)
  - Confident, walk normally
- Caveat emptor:
  - relatively popular in games industry
  - debate in academic AI

# What is it?

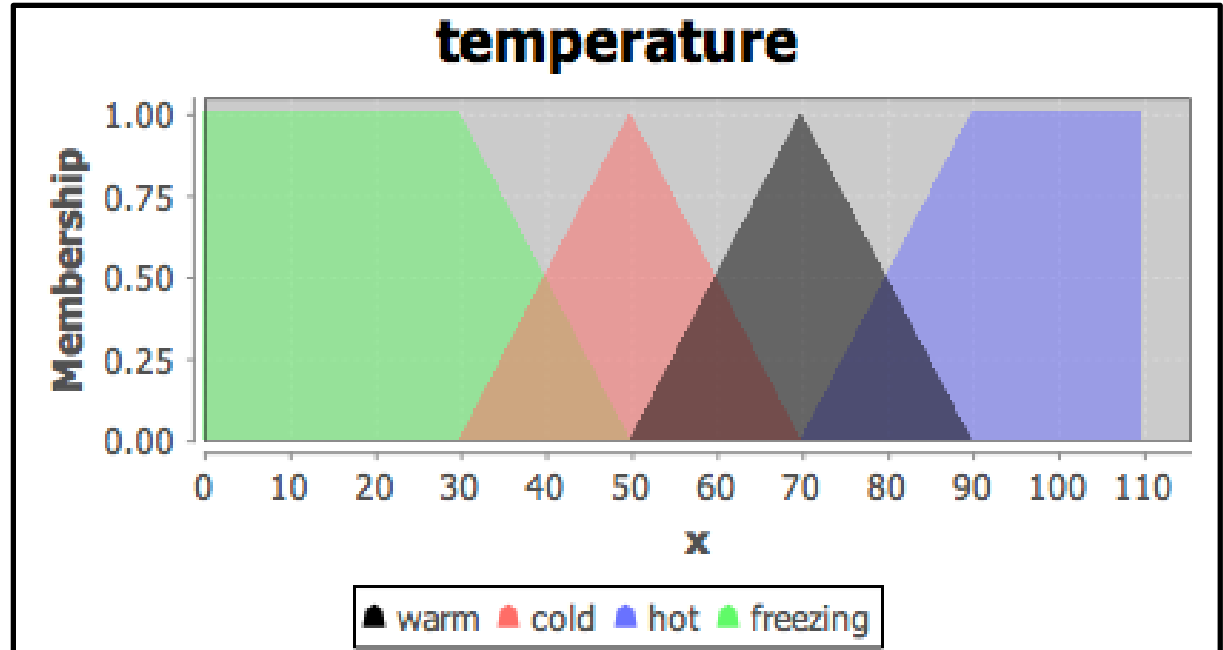
- Modeling of **imprecise concepts**
- Modeling of **imprecise dependencies**
- Superset of classical logic
- Introduces concept of “degree of membership”
- Uses “fuzzy” sets and rules
- Not probability





# Thermostat Example

- How to adjust the thermostat?
  - How do I feel now?
    - Freezing, Cool, Warm, Hot
  - How much do I adjust the temperature by?
    - Down a lot, down a little, keep same, up some, up a lot
- Inputs: Crisp values (e.g. 75F)
- Outputs:
  - Crisp values (turn temp down 5F)
  - Booleans, categories, nearly anything



**RULE 1: IF temperature IS hot THEN adjust\_temp IS down\_lots**

**RULE 1: IF temperature IS hot AND power\_cost IS expensive THEN adjust\_temp IS down\_little**



# When & Why it might be useful

- **Why use it?**
  - Easy to understand
  - Efficient way to represent linguistic and subjective attributes of the real world in computing
  - Generally easier and quicker to define than mapping out every possible situation
  - Generally easier to create versus a neural network
- **When to use it?**
  - Multi-parameter situations
  - Expert knowledge capture/representation
  - Behavioral systems
  - Approximate reasoning
  - Another option for non-linear control system

# A Brief History

- Proposal of Fuzzy Set Theory Introduced in 1965 by Lotfi A. Zadeh (UC Berkeley)
- Japanese were first to utilize for commercial applications in late 1970s-1980s (high-speed train, rice cookers)
- Use of Fuzzy Logic controllers really picked up late 1980s
- Research boom 1990s



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<https://en.wikipedia.org/w/index.php?curid=40290787>

# Useful?

- “as of March 4, 2013, there are
  - 26 research journals on theory or applications of fuzzy logic
  - 89,365 publications on theory or applications of fuzzy logic in the INSPEC database
  - 22,657 publications on theory or applications of fuzzy logic in the MathSciNet database
  - 16,898 patent applications and patents issued related to fuzzy logic in the USA
  - 7149 patent applications and patents issued related to fuzzy logic in Japan.”
- Successful use in/by
  - decision making, identification, pattern recognition, optimization, and control
  - engineers, mathematicians, computer software developers and researchers, natural scientists (biology, chemistry, earth science, and physics), medical researchers, social scientists, public policy analysts, business analysts, jurists
  - rice cookers and trains...

# Useful (cont')

- Appliances
  - Rice cooker, washing machine, climate control, vacuum, canon auto focus
- Aerospace
  - Altitude control of spacecraft & satellites, flow and mixture regulation deicing vehicles
- Automotive
  - Anti-lock braking system, traffic control, truck engine, transmissions (improving efficiency), governors, shift scheduling
- Industrial
  - Anti-sway control of cranes, climate control, positioning systems, coal power plant automated adaptations to coal quality, supervisory systems, humidity control, quality assurance, water purification, cement kiln controls
- Trains, including high-speed and monorail
- Image Processing
  - Monitoring glaucoma, edge detection, image stabilization
- Video Games & FX
  - Massive (Lord of the Rings), Video Game AI
- Medical
  - Diagnosis, control of arterial pressure during anesthesia,
- Chemical
  - pH Control, distillation processes, polymer extrusion productions, drying
- Signal Processing & Telecommunications
  - Adaptive filter for nonlinear channel equalization control of broadband noise
- Business
  - Decision support systems, personnel evaluation with large workforce
- Defense
  - Underwater target recognition, thermal imaging target recognition, naval decision support, control of hypervelocity interceptor, NATO decision support
  - Sonar systems: classification of DEMON spectra, automatic target tracking, airborne defense
- Marine
  - Autopilot, route selection, autonomous control underwater vehicles
- Financial
  - Banknote transfer, fund management, market predictions, securities trading
- Manufacturing
  - Optimization of cheese production
- Robotics

# Golfing Game Example

- Have to make Golfing game, ask expert...
  - When putting: if **ball far from hole**, and **green is slightly downward** from left to right THEN **hit ball firmly** and at **angle slightly** to left of flag
  - When driving: if **wind is strong**, and **blowing right to left**, and **hole is far away** THEN **hit ball hard** and at **angle far to right** of flag
- Close: [0m,2m), Medium: [2m-5m), Far: [5m
- Ball at 4.99m? We want a gradual shift.

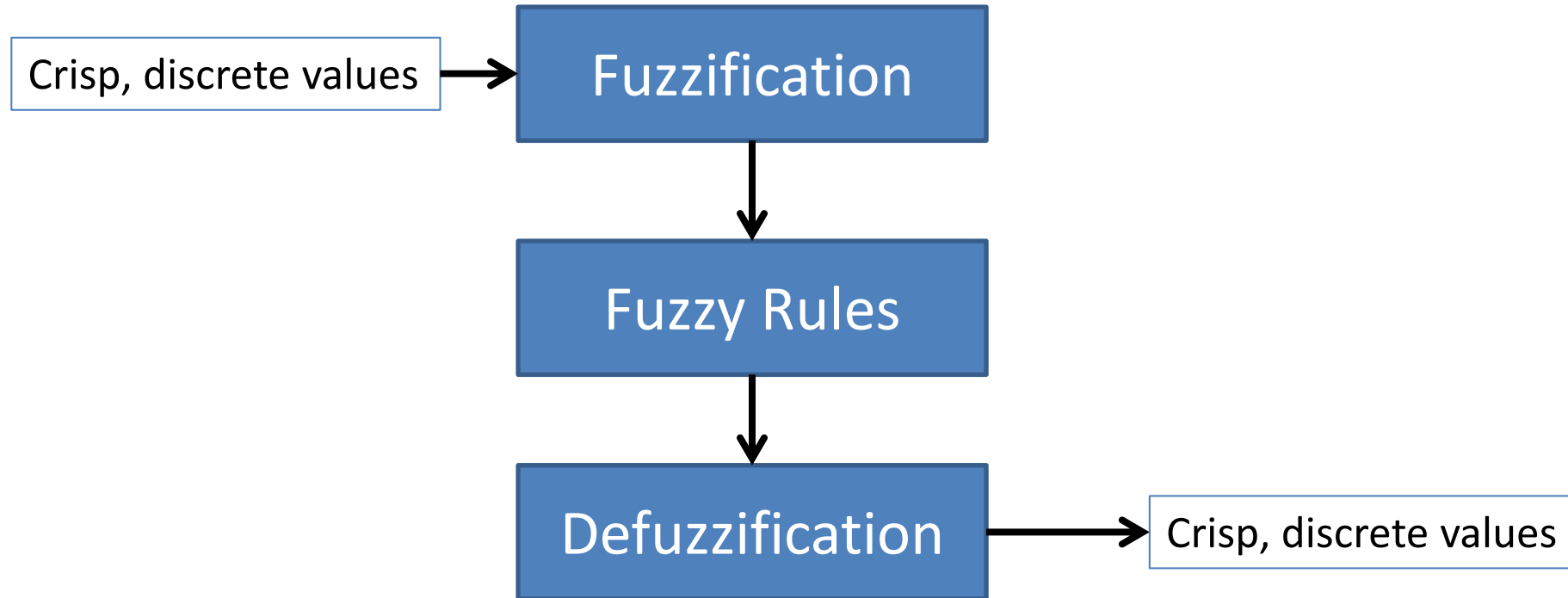
# The Principle

- Traditional (binary set) logic: predicates (F.O.L)
  - Predicates: { hurt( $t$ ), hungry( $t$ ) }
  - Constants: { steve, sadie, brian }
  - hungry(steve), !hurt(steve)
  - “Closed world assumption” (if not true, then false)
  - “classical sets” (either a member of set, or not)
- Fuzzy sets: “**degree of membership**” (DOM)
  - [0 to N] where N is completely in, 0 completely out
  - hungry(steve)[0.5], hurt(steve)[0.9]

# Fuzzy Sets

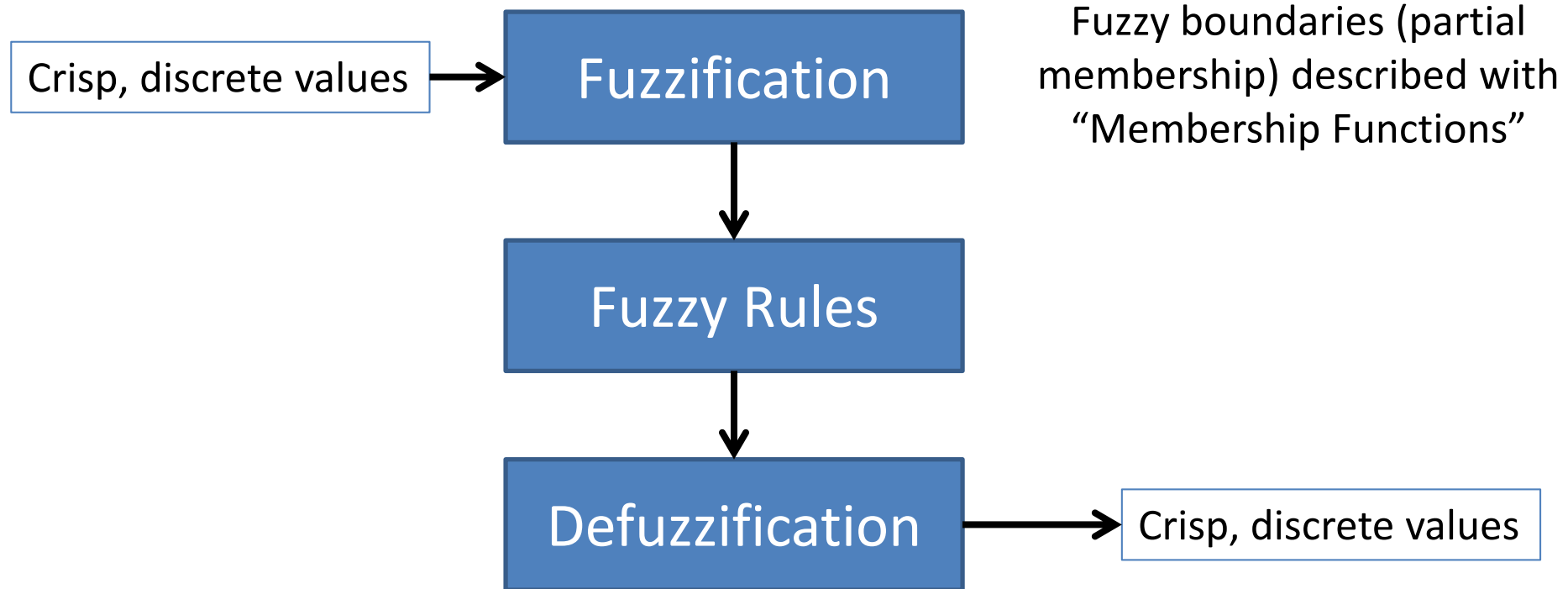
- DOM is usually  $[0 - 1]$  but...
  - Could use  $0 - 255$  (for int arithmetic speed)
  - Numbers are **NOT** probabilities **nor** percentages
- Fuzzy logic: truth degrees to model vagueness
- Probability theory: model non-determinism
- Mutual exclusion
  - Classical: some predicates are M.E. (hurt, healthy)
  - Fuzzy: can be a member of multiple sets
    - Can require DOM to sum to 1, but rare
    - Fuzzification to approximate; slightly off usually ok

# Fuzzy Rule-based Inference

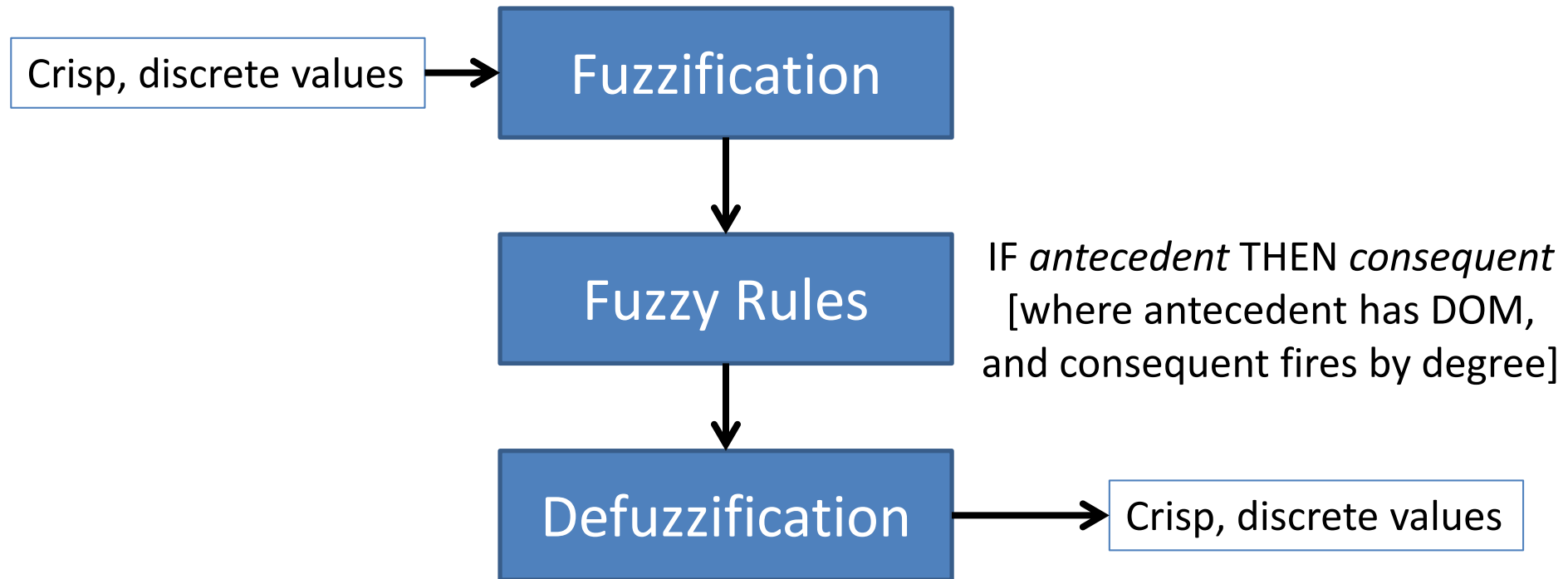




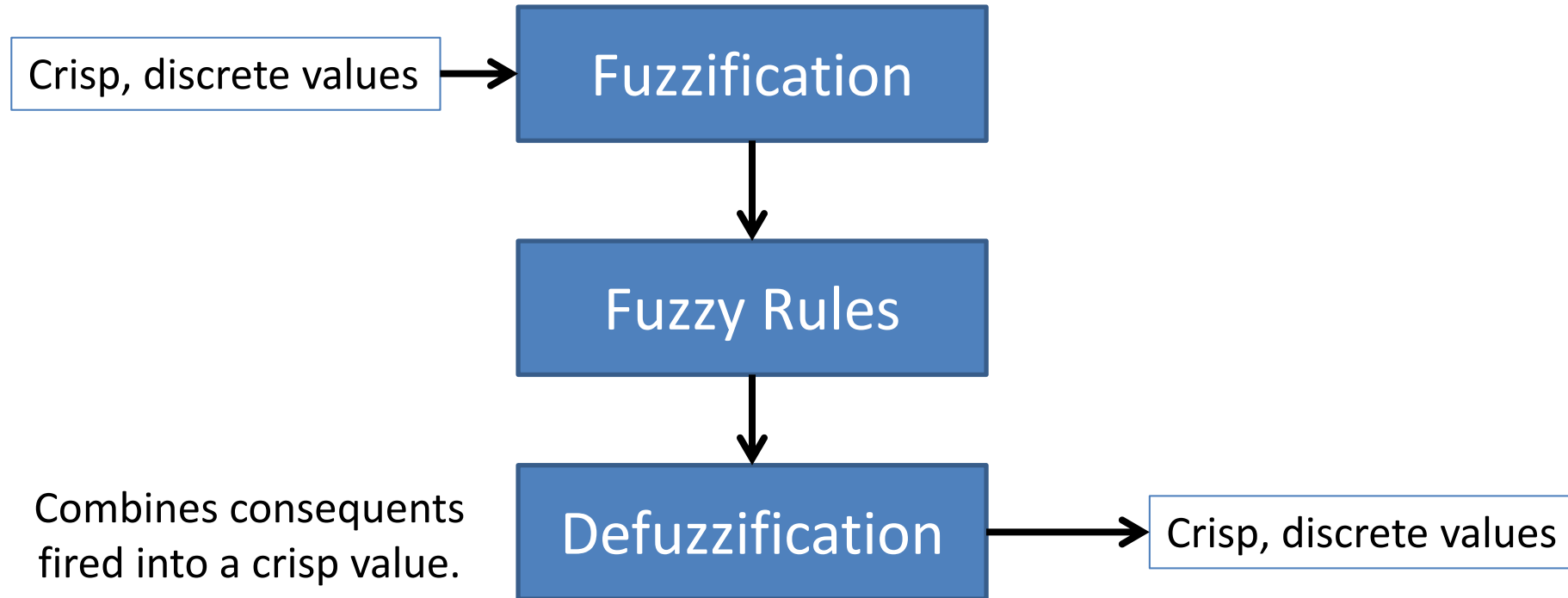
# Fuzzy Rule-based Inference



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# Fuzzy Rule-based Inference



# Fuzzy Inference

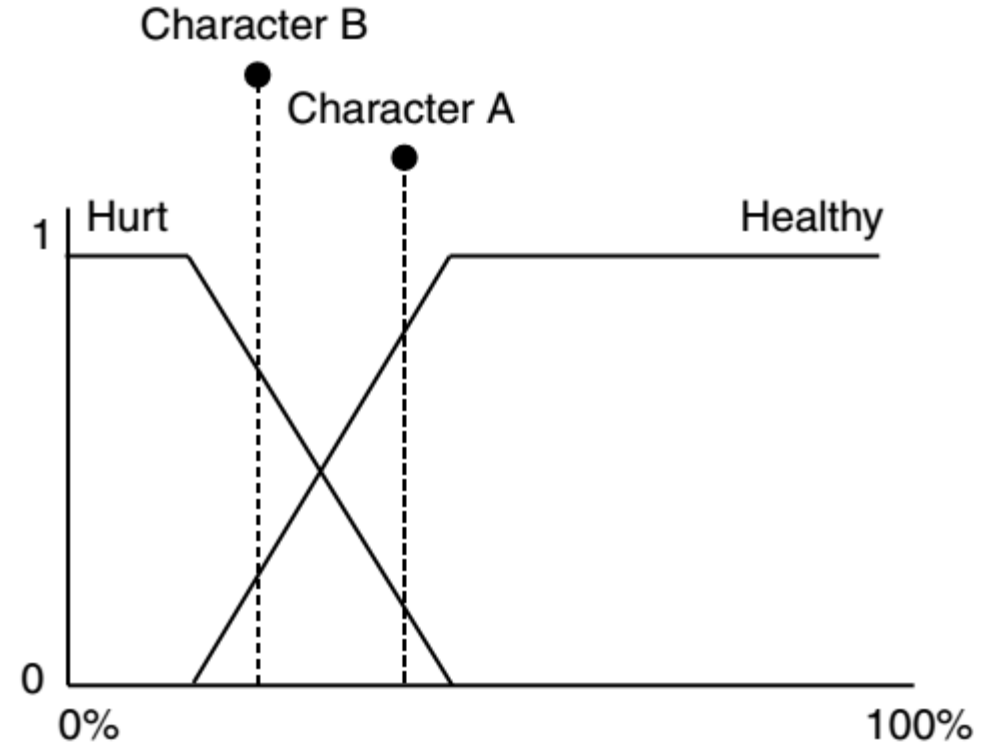
1. For each rule,
  - For each antecedent, calculate the **degree of membership** of the input data.
  - Calculate the rule's inferred conclusion based upon the values in previous step
2. Combine all the inferred conclusions into a single conclusion (a fuzzy set)
3. For crisp values, the conclusion from 2 must be defuzzified

# Fuzzy Rules

- Relate the known membership of certain fuzzy sets to generate new DOM values for other fuzzy sets
  - Vital: must create rule for each possible combination of antecedent sets
- e.g. “If I am close to the corner AND I am traveling fast, then I should brake”
- $m_{(\text{should brake})} = \min(m_{(\text{close to corner})}, m_{(\text{traveling quickly})})$
- Membership of should break with “close to corner” 0.6 and “traveling fast” 0.9?

# “Fuzzification”

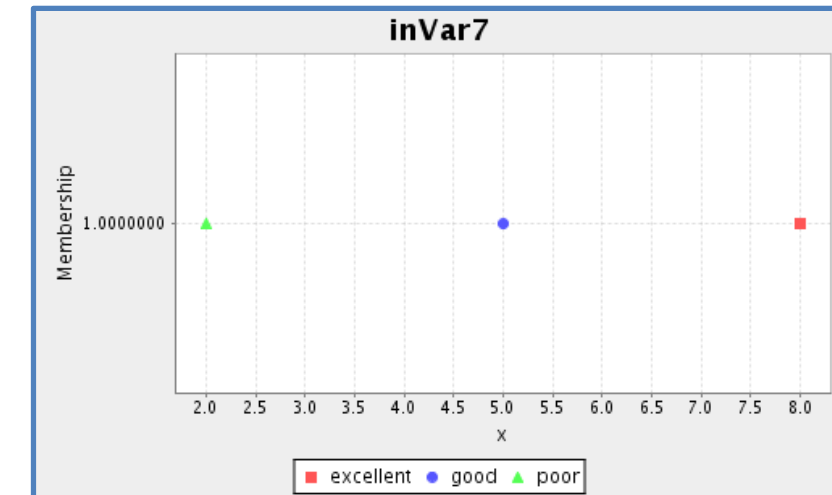
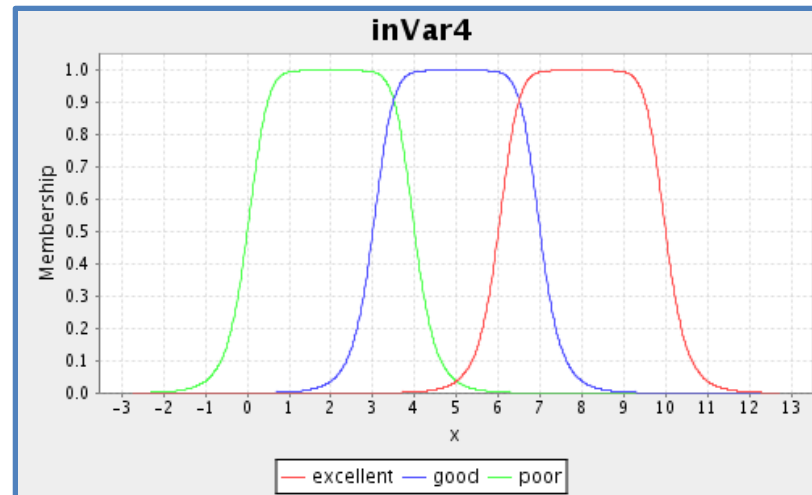
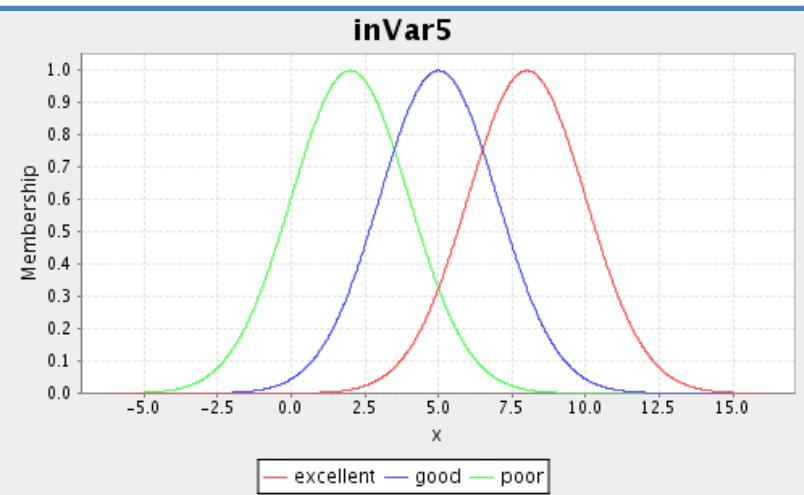
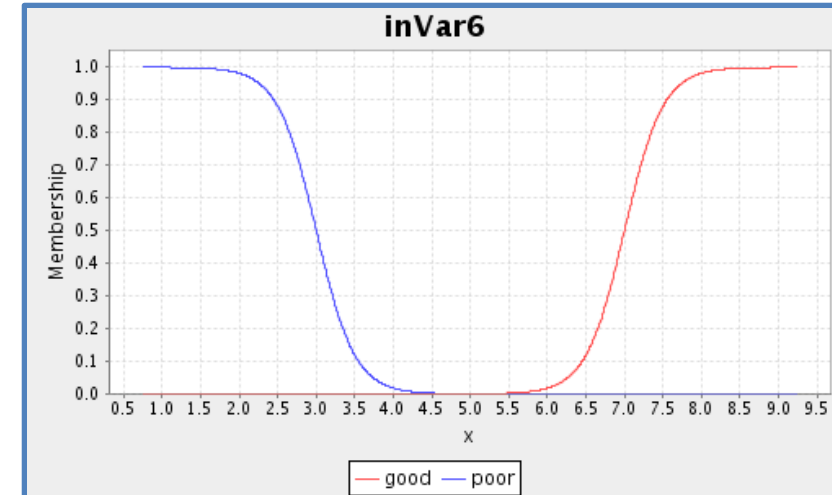
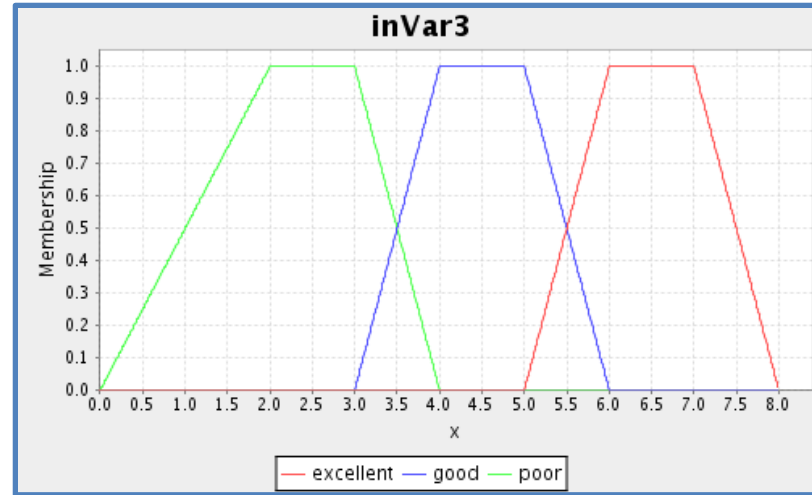
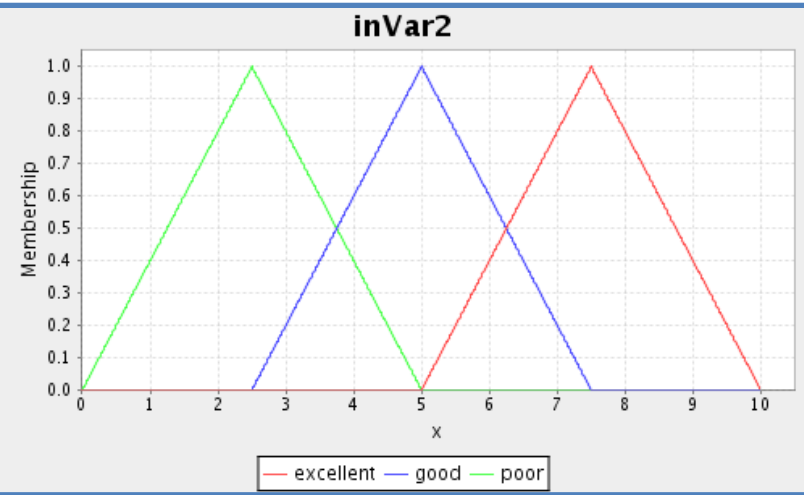
- Fuzzification:  
Game state  $\rightarrow$  DOM  
“Membership Function”
  - Triangular
  - Trapezoidal
  - S-Curve
  - Left/Right Shoulder
  - Singleton
  - **Note:** Vert. lines should sum to 1
- De-fuzz:  
DOM  $\rightarrow$  Game state
- Numeric Fuzzification:  
 $f(\text{numeric}) \rightarrow$  DOM



Health value  
Millington 5.38

$F(\text{health val}) \rightarrow \text{DOM}(\text{healthy})$   
 $F(\text{health val}) \rightarrow \text{DOM}(\text{hurt})$

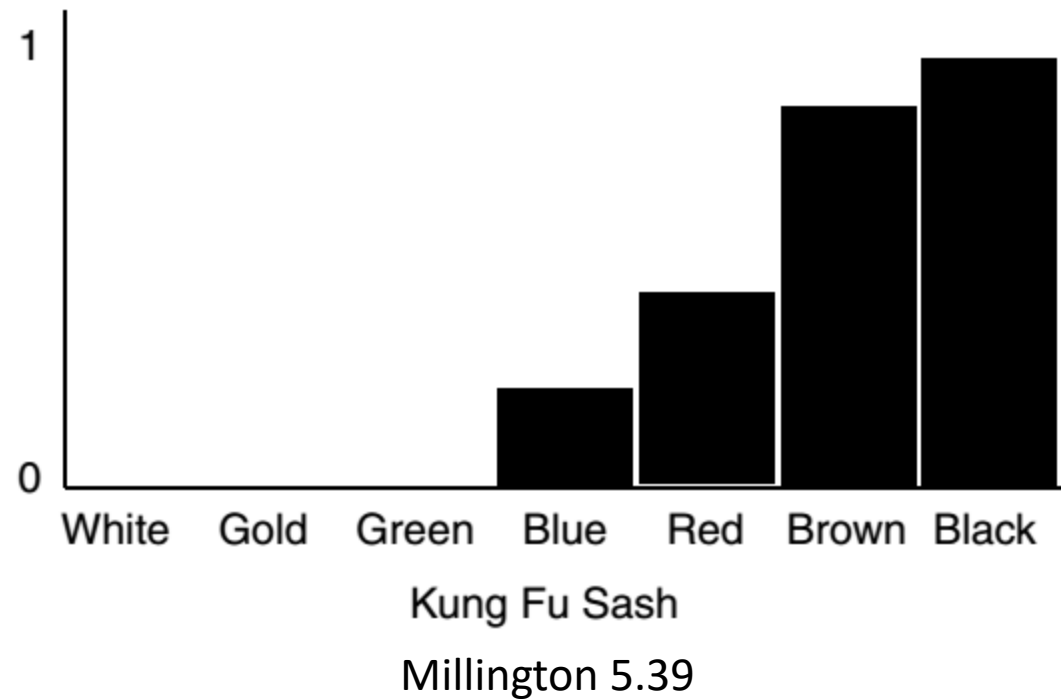
# Example membership functions



# Fuzzification of Small Sets/Enumerations

Store pre-determined membership values. E.g. kung fu game

- Boolean var
  - hasPwrflArtifact
- Enum var
  - DOM of fearsmFighter from one of set of sashes





# Set Operations (And, Or, Not)

- Boolean logic: True, False
- Fuzzy: DOM of a fuzzy set
  - Little rain (0.3) AND very cold (0.8)

A	B	A && B	A OR B
F	F	F	F
F	T	F	T
T	F	F	T
T	T	T	T

Fuzzy Logic

$$m_{(A \&\& B)} = \min(m_A, m_B)$$

$$m_{(A \text{ OR } B)} = \max(m_A, m_B)$$

$$m_{(\text{NOT } A)} = 1 - m_A$$

$$\text{Hedge: VERY} = (m_A)^2$$

$$\text{Hedge: FAIRLY} = (m_A)^{0.5}$$

# Fuzzy Logic

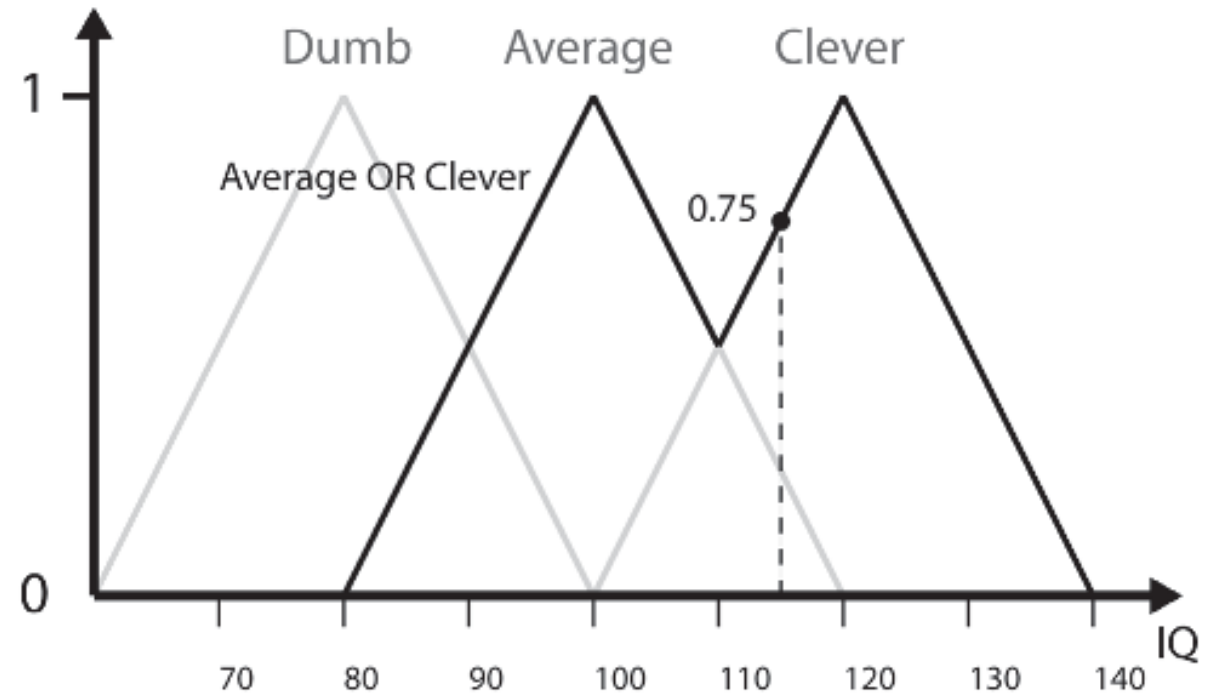
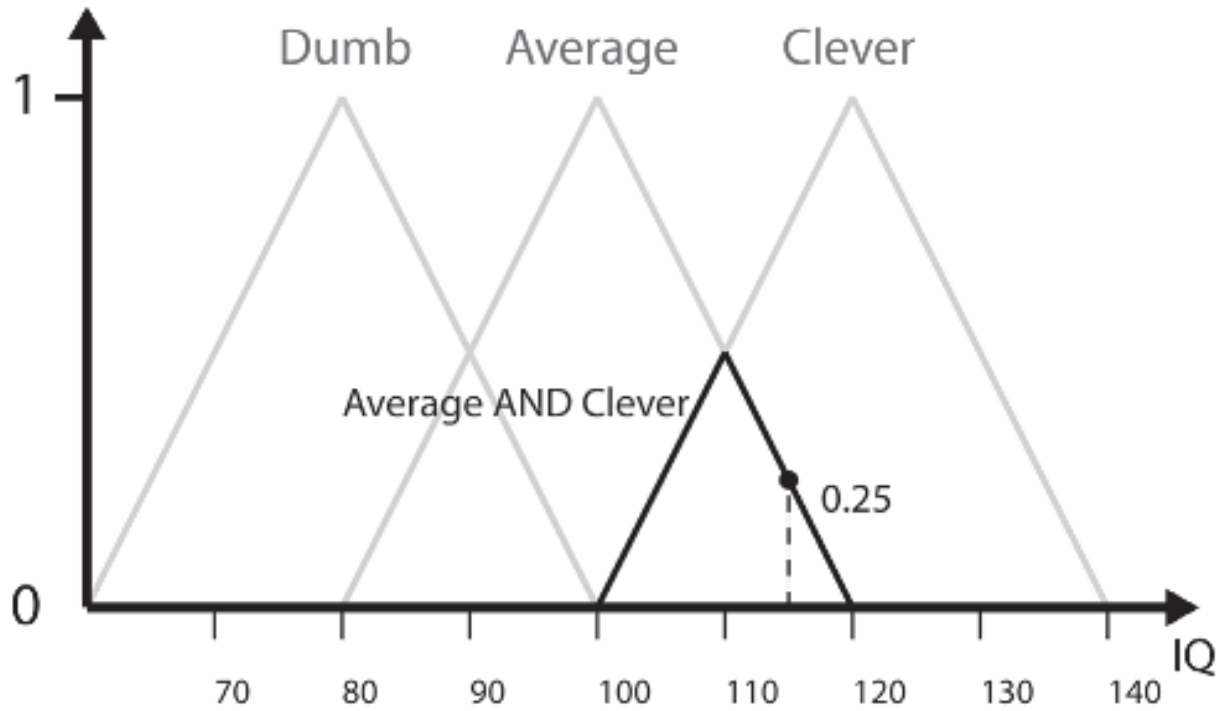
$$m_{(A \& B)} = \min(m_A, m_B)$$

$$m_{(A \text{ OR } B)} = \max(m_A, m_B)$$

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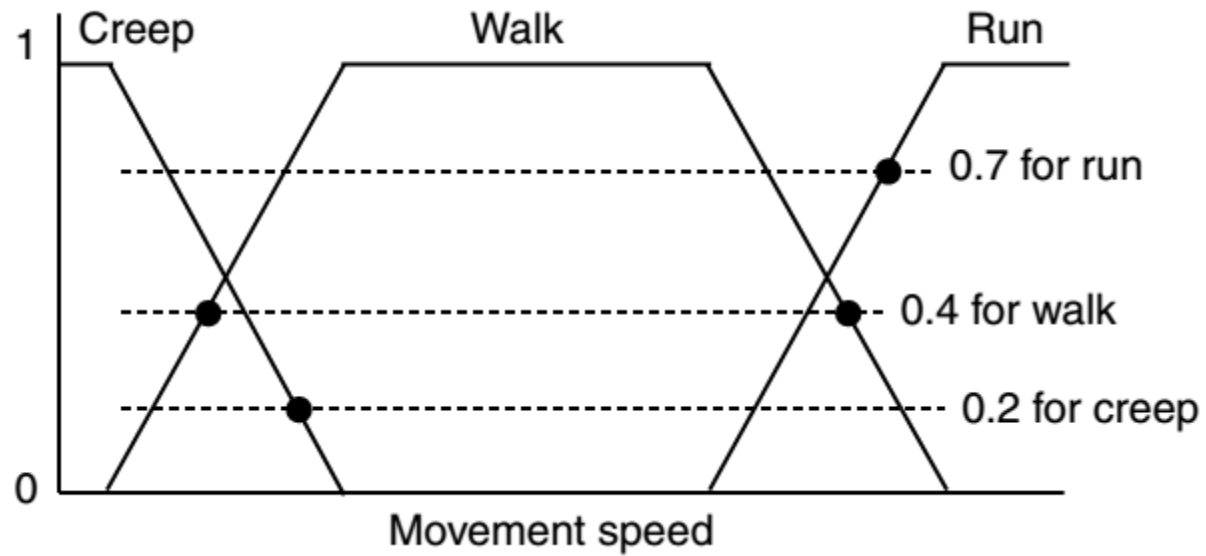


B Fig 10.6 & 10.7

# Defuzzification

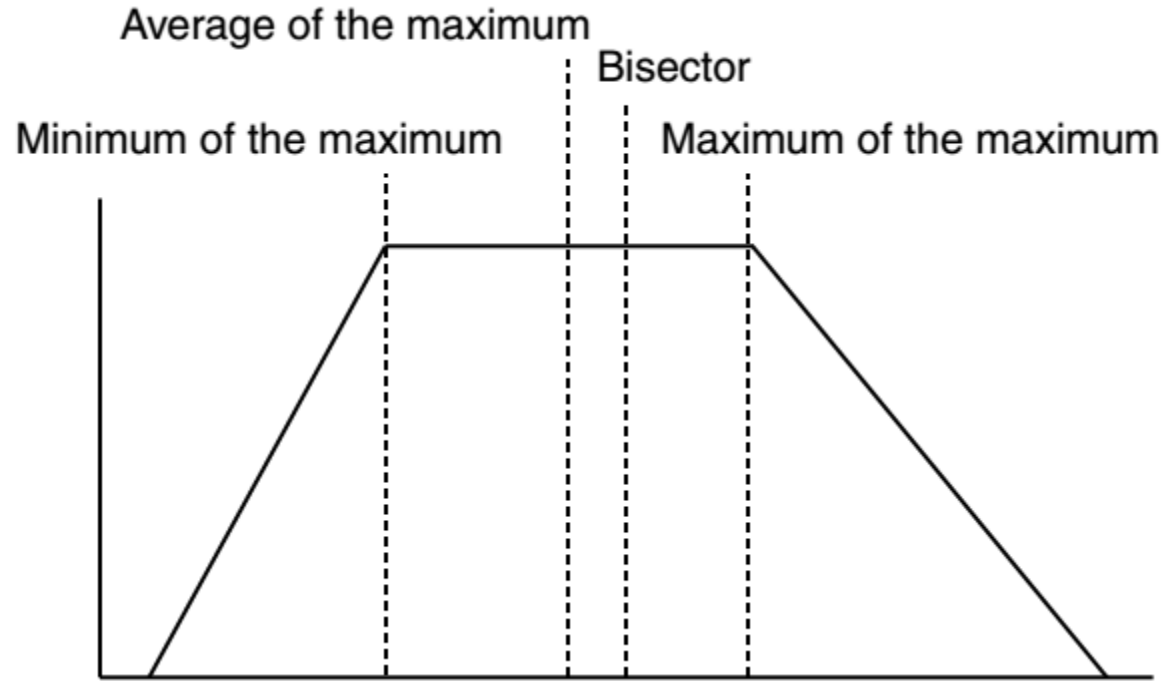
- Need to translate data back after applying whatever logic was needed
- Multiple approaches
  - Mean of maximum
  - Centroid
  - Average of Maxima
  - ...
- Problem: Turn a set of membership values into a (typically) single number

# Defuzzification

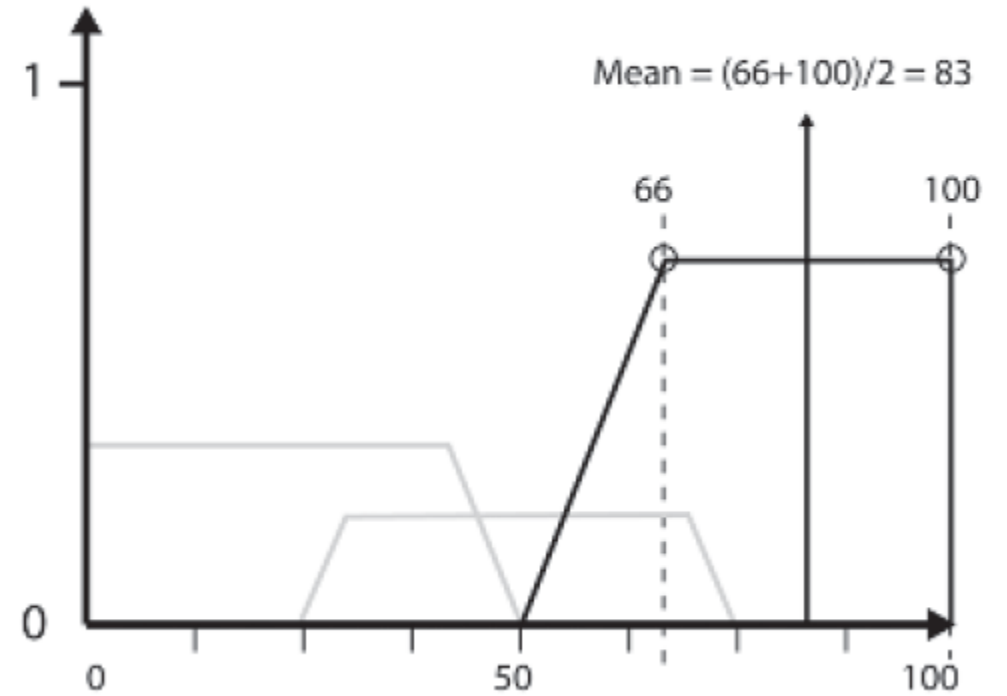


Millington 5.40

# Highest Membership



Millington 5.41

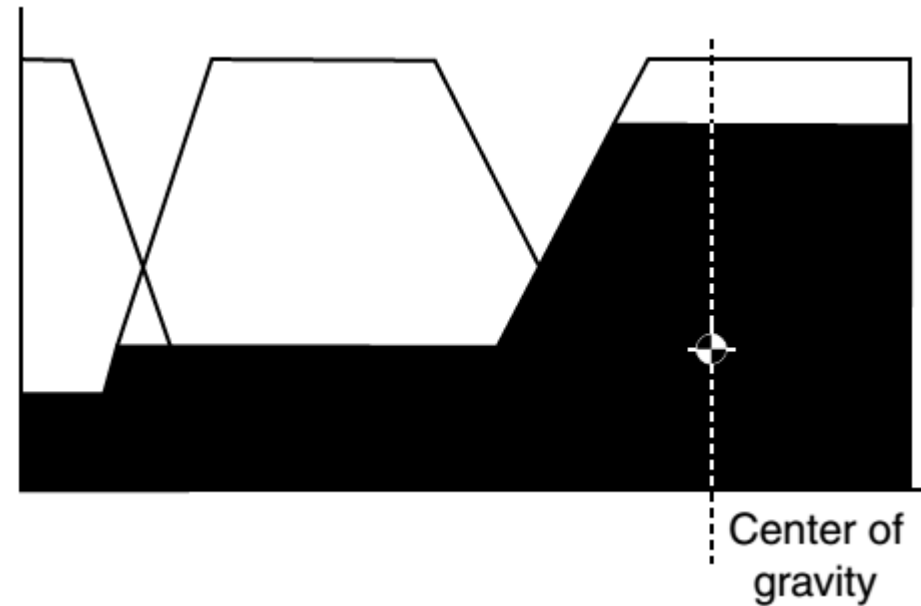


# Blending Based on Membership

- Use DOM as weights
  - 0.33 creep, 0.33 walk, 0.34 run
  - $0.33 * \text{characteristic creep speed} + 0.33 * \text{characteristic walk speed} + 0.34 * \text{characteristic run speed}$
  - Normalize values
- Can use minimum values (Smallest of Maximum method or Left of Maximum, LM)

# Center of Gravity

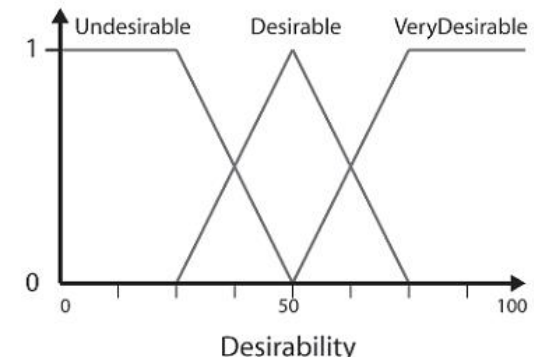
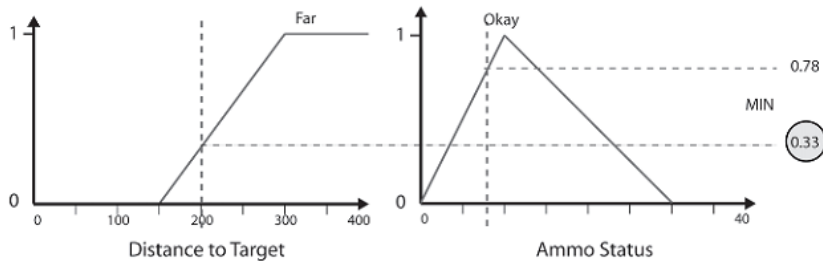
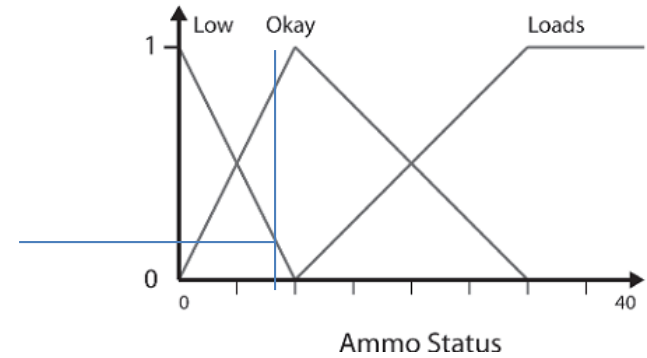
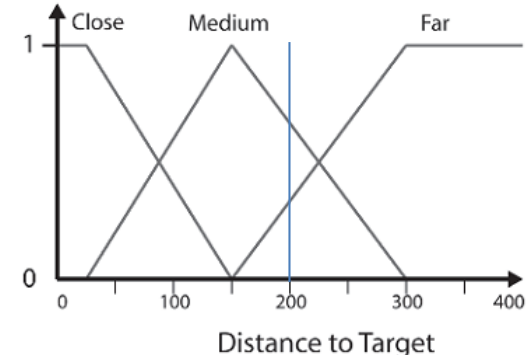
- Crop membership function at DOM value
- Integrate each in turn to find center of gravity
- Method often used, but is expensive
- Blending works about as well and is cheap



Millington 5.40

# Rocket Launcher Example: Target 200px, Ammo=8

- Rule 1. IF Target\_Far AND Ammo\_Loads THEN Desirable
- Rule 2. IF Target\_Far AND Ammo\_Okay THEN Undesirable
- Rule 3. IF Target\_Far AND Ammo\_Low THEN Undesirable
- Rule 4. IF Target\_Medium AND Ammo\_Loads THEN VeryDesirable
- Rule 5. IF Target\_Medium AND Ammo\_Okay THEN VeryDesirable
- Rule 6. IF Target\_Medium AND Ammo\_Low THEN Desirable
- Rule 7. IF Target\_Close AND Ammo\_Loads THEN Undesirable
- Rule 8. IF Target\_Close AND Ammo\_Okay THEN Undesirable
- Rule 9. IF Target\_Close AND Ammo\_Low THEN Undesirable



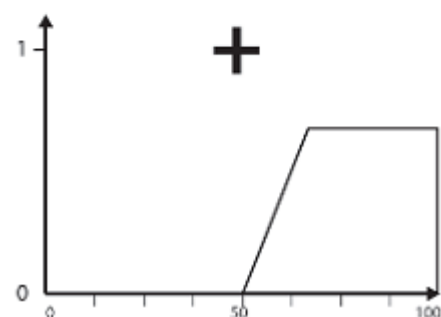
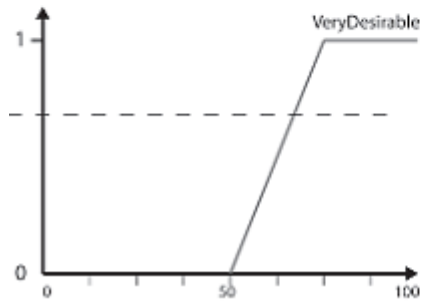
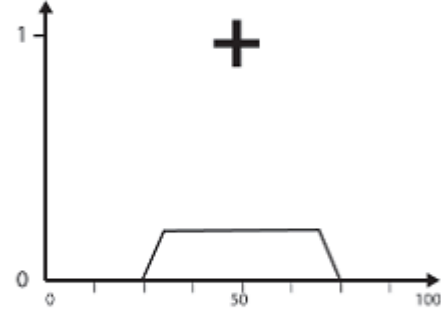
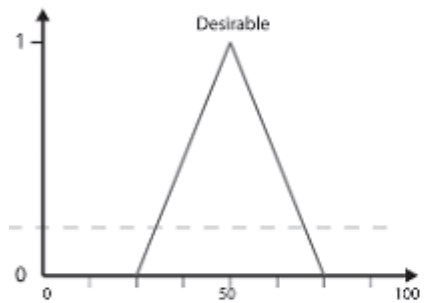
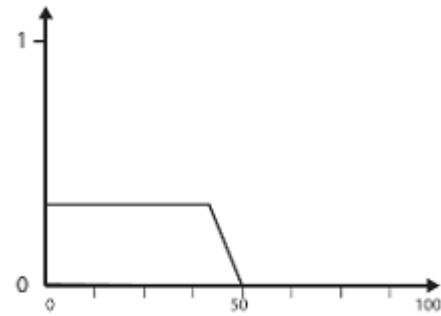
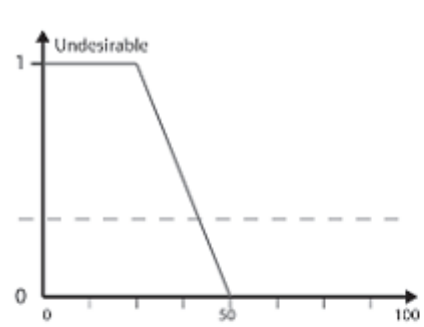
Fuzzy associative matrix

	Target_Close	Target_Medium	Target_Far
Ammo_Loads	Undesirable 0	Desirable 0.2	Undesirable 0.2
Ammo_Okay	Undesirable 0	VeryDesirable 0.67	Undesirable 0.33
Ammo_Low	Undesirable 0	VeryDesirable 0	Desirable 0

Consequent	Confidence
Undesirable	0.33
Desirable	0.2
Very Desirable	0.67



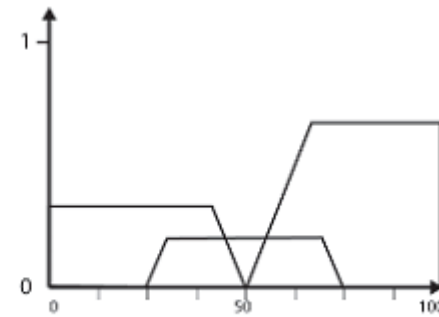
# See Buckland CH10



Desirability (consequent)

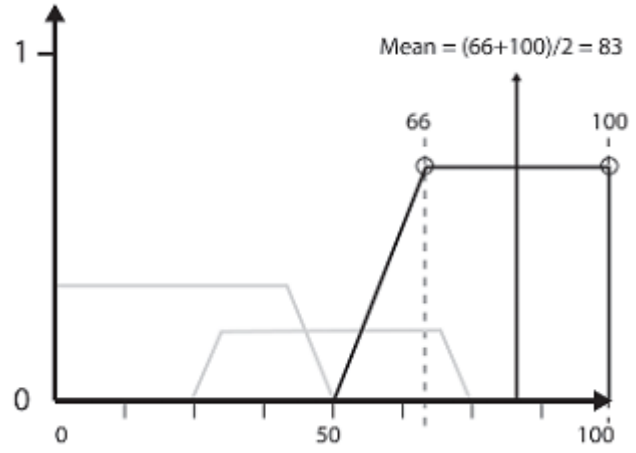
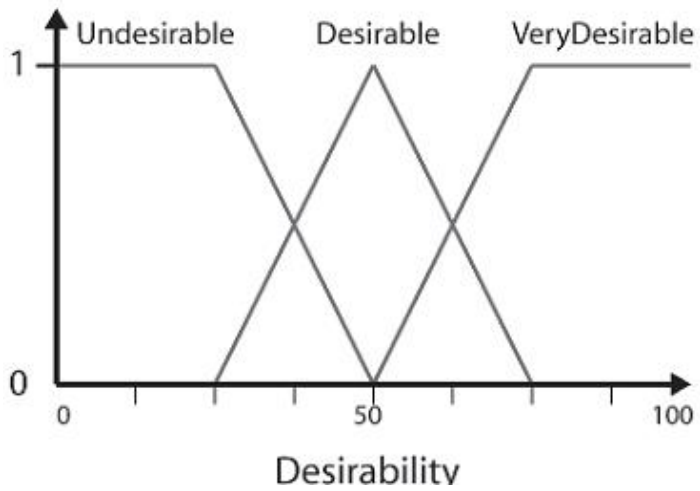
Desirability (inferred conclusion)

Consequent	Confidence
Undesirable	0.33
Desirable	0.2
Very Desirable	0.67

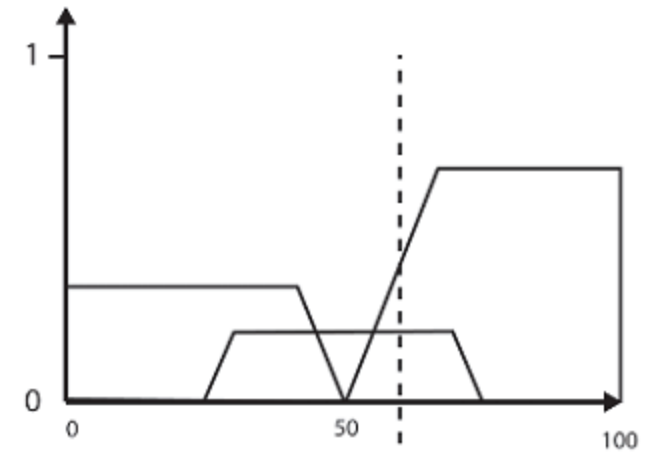


combined

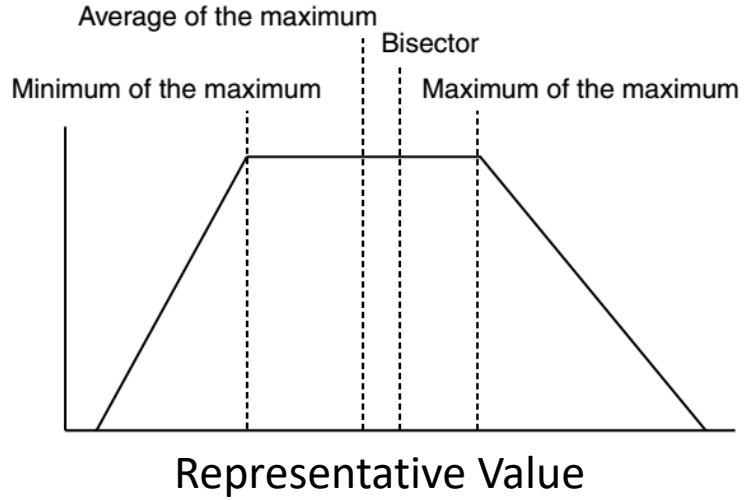
See Buckland CH10



Mean of Maximum



Centroid



Representative Value

Set	Representative Value	Confidence
Undesirable	12.5	0.33
Desirable	50	0.2
VeryDesirable	87.5	0.67

$$\begin{aligned}
 \text{Desirability} &= \frac{12.5 \times 0.33 + 50 \times 0.2 + 87.5 \times 0.67}{0.33 + 0.2 + 0.67} \\
 &= \frac{72.75}{1.2}
 \end{aligned}$$

$$\text{Desirability} = 60.625$$

representative value of a fuzzy set is the value where membership in that set is 1

# Fuzzy in Decision Making

- Can use in any system we'd use boolean logic
- Determine FSM transitions
- Define rules for RBS

# Fuzzy state machines

- Multiple interpretations
  - Any state machine with some element of fuzzy
- Example: crisp triggers, fuzzy states
  - Can be in any or all states with DOM
  - At each iteration, transitions belonging to all active states are given chance to trigger; fire transitions belonging to each state in decreasing DOM order
  - DOM of target is given by DOM of current state ANDed with degree of transition

# Scalability

- Weakness of this approach: combinatorial explosion
  - rule for each possible combination of antecedent sets
  - 10 input variables and 5 states ---> approx. 10 million rules
- (William) Combs Method; Boeing 1997
  - IF target\_far AND ammo\_loads THEN Desirable
  - IF target\_far THEN Desirable
  - OR
  - IF ammo\_loads THEN Desirable
- **See Buckland CH 10**

# Pros and Cons

- **Pro**

- Easy to understand; supports explanation
- Efficient way to represent **linguistic** and **subjective** attributes of the real world in computing.
- Supports smooth transitions between behaviors
- Generally easier to create versus a neural network

- **Cons**

- Defining set membership functions can be difficult
- Debugging knowledge can be difficult
- De-fuzzify step can have surprising subtleties

# See Also

- [http://videlectures.net/acai05\\_berthold\\_fl/](http://videlectures.net/acai05_berthold_fl/)
- Buckland 10, Millington 5.5
- Tools
  - Matlab
  - R (CRAN) packages (e.g. frbs)
  - jFuzzy logic
  - Fuzzy Control Language (FCL)
  - Octave & Fuzzy Logic Toolkit