Heat dissipation in wearable computers aided by thermal coupling with the user

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Wearable computers and PDA's are physically close to, or are in contact with, the user during most of the day. This proximity would seemingly limit the amount of heat such a device may generate, conflicting with user demands for increasing processor speeds and wireless capabilities. However, this paper explores significantly increasing the heat dissipation capability per unit surface area of a mobile computer by thermally coupling it to the user. In particular, a heat dissipation model of a forearm-mounted wearable computer is developed, and the model is verified experimentally. In the process, this paper also provides tools and novel suggestions for heat dissipation that may influence the design of a wearable computer.

1. Introduction: Heat dissipation for body-centered devices

Demand for higher computational power in notebook computers has forced hardware designers to plan processor heat dissipation carefully. However, as owners of high-end laptops will testify, the surface of the machine may still reach uncomfortable temperatures, especially upon momentary contact. As high-end computers are incorporated into smaller form factors, this problem will worsen. Wearable computers would seem to have particular difficulties since the computer housing may be in prolonged contact with skin. However, this paper suggests that wearable computers may provide a better form factor than today's notebooks in regard to heat dissipation.

An obvious approach to the problem of heat generation is to decrease the power required for high-end CPUs through higher integration, optimized instruction sets, and more exotic techniques such as "reversible computation" [42]. However, profit margins, user demand, and backwards compatibility concerns are pushing industry leaders to concentrate on processors requiring more than 5 W. In addition, the explosion of mobile peripherals, such as wireless Internet radios, video cameras, sound cards, body networks, scanners, and global positioning system (GPS) units creates an ever higher heat load as functionality increases. An example of this effect is the U.S. Army's modern (late 1990's) soldier, who is expected to dissipate 30 W on communications gear alone! Thus, even with improved technology, heat dissipation will continue to be an issue in the development of mobile devices.

Current systems try to insulate the user from heat sources, slowing or shutting down when internal temperatures get too high. However, the human body is one of the most effective and complex examples of thermoregulation in nature, capable of dissipating well over 2700 W of heat [10]. Thus, the human body itself might be used to help dissipate heat. To take advantage of this system, some background knowledge is necessary. The next section discusses the fundamentals of human heat regulation and thermal comfort, but for a more thorough discussion see [10,37]. Those readers who are familiar with the principles of thermoregulation should skip to the next section.

2. Thermoregulation in humans

In the extremes, the human body generates between 80 W to 10,000 W of power [27,36]. With proper preparation, it can survive in the hot Saharan desert or on the ice in Antarctica for extended periods. Yet, the body maintains its "core" temperature (the upper trunk and head regions) at $37 \,^{\circ}$ C, only varying $\pm 2 \,^{\circ}$ C while under stress (in medical extremes, $\pm 5 \,^{\circ}$ C may be observed) [10]. Obviously, the human body can be an excellent regulator of heat. However, the sedate body is comfortable in a relatively narrow range of environmental temperatures. Even so, the amount of heat that is exchanged in this comfort range can be significant when all the different modes are considered. Heat balance in the human body can be expressed by

$$M' - W' = Q'_{\text{evap}} + Q'_{\text{conv}} + Q'_{\text{rad}} + Q'_{\text{cond}} + Q'_{\text{stor}}, \quad (2.1)$$

where M' is the rate of heat production (due to metabolism), W' is the rate of useful mechanical work, Q'_{evap} is the rate of heat loss due to evaporation, Q'_{conv} is the rate of heat gained or lost (exchanged) due to convection, Q'_{rad} is the rate of heat exchanged by radiation, Q'_{cond} is the rate of heat exchanged by conduction, and Q'_{stor} is the rate of heat storage in the body. Thus, total body heat may increase or decrease resulting in changes in body temperature [10].

Body heat exchange is very dependent on the thermal environment. The thermal environment is characterized by ambient temperature (°C), dew point temperature (°C) and ambient vapor pressure (kg m⁻¹ s⁻²), air or fluid velocity (m s⁻¹), mean radiant temperature (°C) and effective radiant field (W m⁻²), clothing insulation (clo), barometric

pressure (kg m⁻¹ s⁻²), and exposure time. Ambient temperature is simply the temperature of the environment outside of the influence of the body. The dew point temperature is the temperature at which condensation first occurs when an air-water vapor mixture is cooled at a constant pressure. Ambient vapor pressure is also a measure of humidity and, for most cases, is the pressure exerted by the water vapor in the air. Air and fluid movement are the result of free buoyant motion caused by a warm body in cool air, forced ventilation of the environment, or body movement. Mean radiant temperature and the effective radiant field describe radiant heat exchange. Mean radiant temperature (MRT) is the temperature of an imaginary isothermal "black" enclosure in which humans would exchange the same amount of heat by radiation as in the actual nonuniform environment. Effective radiant field (ERF) relates the MRT or the surrounding surface temperatures of an enclosure to the air temperature. The "clo" is a unit of clothing insulation which represents the effective insulation provided by a normal business suit when worn by a resting person in a comfortable indoor environment. It is equivalent to a thermal resistance of 0.1547 $m^2 \circ C W^{-1}$ or a conductance of 6.46 W m^{$-2 \circ$}C⁻¹. Barometric pressure is caused by the atmosphere and usually expressed in kPa ($1000 \text{ kg m}^{-1} \text{ s}^{-2}$) or torr. While the following sections will address these variables where appropriate, the reader is encouraged to read [20] and [10] for a more extensive treatment.

For most discussions, the outer skin is considered to be the heat exchange boundary between the body and the thermal environment. Heat exchange terms reflect this, having units of $W m^{-2}$. A good approximation of an individual's skin surface area is given by the Dubois formula

$$A_{\rm D} = 0.202 m^{0.425} H^{0.725}, \qquad (2.2)$$

where A_D is the surface area in square meters, *m* is the body mass in kilograms, and *H* is the height in meters [20]. For convenience, we assume a user with a skin surface area of 1.8 m² and a mass of 70 kg.

2.1. Convection

In an environment where air temperature is cooler than that of the skin or clothing surface, the air immediately next to the body surface becomes heated by direct conduction. As the air heats, it becomes less dense and begins to rise. This occurs everywhere about the body and forms a microenvironment where heat is transferred by convection. This air flow is called the natural convection boundary layer [10].

Due to the complexity of the problem, a mathematical analysis of convection heat loss on the human body has not been developed. However, experimental approximations have been proposed. For natural convection in both seated and standing positions, Fanger [18] presents a convection coefficient h_c of

$$h_{\rm c} = 2.68(t_{\rm cl} - t_{\rm a})^{0.25} \tag{2.3}$$

in units of $Wm^{-2} \circ C^{-1}$, where t_{cl} is the clothing surface temperature and t_a is the ambient temperature.

Convection also occurs when a breeze is present. For uniform forced air flows under 2.6 m s⁻¹, Fanger [18] suggests an approximation of

$$h_{\rm c} = 12.1\sqrt{V},\tag{2.4}$$

where V is air velocity. When a slight breeze is present both the natural and forced air convection formulas should be calculated and the larger value used. In his book, Clark [10] presents a different experimental formula of

$$h_{\rm c} = 8.3\sqrt{V} \tag{2.5}$$

without providing a constraint on air flow speed. In addition, Clark states that h_c is doubled when the air flow is turbulent based on experimental evidence with appropriately human-sized and instrumented heated cylinders.

2.2. Radiation

Heat can be exchanged between two bodies by electromagnetic radiation, even through large distances. For the purposes of heat exchange to and from the human body, this paper is concerned with radiation from sources cooler than 100 °C. The Stefan–Boltzmann formula can be used to determine the total emissive power of a wavelength at absolute temperature T

$$W_{\rm b} = \epsilon \sigma T^4, \qquad (2.6)$$

where ϵ is the emittance of the body, and σ is the Stefan– Boltzmann constant (5.7 × 10⁻⁸ W m⁻² °K⁻⁴). The emittance of an object is the ratio of the actual emission of heat from a surface to that of a perfect black body, equally capable of emitting or absorbing radiation at any wavelength. The emittance for human skin and clothing are quite high in the longer wavelengths mainly involved at these temperatures, around 0.98 and 0.95, respectively. The units for W_b are W m⁻², so to calculate the heat energy emitted by the human body, again assuming 33 °C mean skin temperature and 1.8 m s⁻² surface area,

$$(\epsilon \sigma T^4)(A_{\rm D}) = 0.98 \times \left(5.7 \times 10^{-8} \frac{\rm W}{\rm m^2 \,^{\circ} K^4}\right) \times (306 \,^{\circ} \rm K)^4 1.8 \, \frac{\rm m}{\rm s^2} = 880 \, \rm W.$$
 (2.7)

In reality, the human body does not radiate this much heat. Instead it absorbs a portion of its own thermal radiation and is affected by surrounding surfaces. When calculating radiant heat transfer from the human body (or small object) to a surrounding room (or large container), the following approximation is useful:

$$R = \sigma \epsilon_1 \left(T_1^4 - T_2^4 \right) \frac{A_\mathrm{r}}{A_\mathrm{D}},\tag{2.8}$$

where T_1 is the absolute temperature of the body, T_2 is the temperature of the room, ϵ_1 is the emittance of the body (approx. 0.98), and the ratio A_r/A_D compares the area

exchanging radiative energy with the surroundings (A_r) to the total body surface area (A_D) . This ratio is 0.65 for a body sitting and 0.75 for a body standing. The max value is 0.95 for a body spread eagled. Thus, for a naked man sitting in a 25 °C room,

$$Q'_{\rm rad} = \sigma \epsilon_1 \left(T_1^4 - T_2^4 \right) \frac{A_{\rm r}}{A_{\rm D}} A_{\rm D}$$

= 5.7 × 10⁻⁸ $\frac{W}{m^2 \circ K^4} 0.98 \left((33 + 273 \circ K)^4 - (25 + 273 \circ K)^4 \right) 0.65 (1.8 \, {\rm m}^2) = 58 \, {\rm W}. (2.9)$

With maximum exposure of the body to the surroundings, the result becomes 86 W. Similarly, in a $15 \,^{\circ}$ C room, 122–180 W of dissipation may be expected.

Heat may also be re-gained by the body through radiation, in particular, solar radiation. Human skin and clothing have variable emissivity for many of the wavelengths generated by the sun (a 5760 °K source). In addition, the angle of the sun and orientation of the subject have significant effects on the heat transfer. However, empirical studies have shown that a semi-nude man walking in a desert has an effective 233 W solar load. When light colored clothing is worn, this can be lessened to 117 W [20].

2.3. Conduction

Conduction normally plays a small role in human heat regulation, except as the first stage of convection. Heat can be dissipated through contact with shoe soles, doorknobs, or through the surface underneath a reclining subject. Heat conduction through a plate of area A and thickness b is given by

$$Q'_{\rm cond} = \frac{kA(T_1 - T_2)}{b},$$
 (2.10)

where k is the thermal conductivity of the plate and T_1 and T_2 are the temperatures on either side of the plate. The sign of Q'_{cond} indicates the direction of heat flow.

2.4. Evaporation

When the body is sedentary, it loses heat during evaporation of water from the respiratory tract and from diffusion of water vapor through the skin (insensible or latent heat loss). When other modes of heat loss are insufficient, the body sheds excess heat through evaporation of sweat (sensible heat loss). The rate of heat lost through the evaporation process can be calculated by

$$Q'_{\rm evap} = \Delta m \lambda, \qquad (2.11)$$

where Δm is the rate of mass of water lost and λ is the latent heat of evaporation of sweat (2450 J g⁻¹). Thus, for the typical water loss of 0.008 g s⁻¹ through the respiratory tract, the heat loss is 20 W. In hot environments, sweat rates can be as high as 0.42 g s⁻¹ for unacclimatized persons and 1.11 g s⁻¹ for acclimatized persons, resulting in 1000 W to 2700 W of heat dissipation, respectively [10,39].

2.5. Heat storage

One term in equation (2.1) is still unexamined: Q'_{stor} . Heat storage in the body takes the form of a higher body temperature and can be calculated with the formula

$$S = mC\Delta T, \qquad (2.12)$$

where S is the energy stored, m is the body mass, C is the specific heat of the body (approx. $3.5 \times 10^3 \,\text{J kg}^{-1} \,^{\circ}\text{C}^{-1}$), and ΔT is the change in body temperature. Thus, for a 1 °C increase in a 70 kg man,

$$S = 70 \text{ kg} \left(3.5 \cdot 10^3 \frac{\text{J}}{\text{kg}^{\circ}\text{C}} \right) (1^{\circ}\text{C}) = 245,000 \text{ J} \quad (2.13)$$

of heat energy are stored. If this increase occurs over the course of an hour, the average power absorbed is $Q'_{\text{stor}} = 68 \text{ W}$. In this way the human body is its own buffer when adequate heat dissipation is not available or, conversely, when too much heat is being dissipated.

2.6. Skin temperature, thermal receptors, and damage

Skin temperature may vary wildly depending on the area measured. For example, while comfortable, a subject may have temperature readings of $25 \,^{\circ}$ C on the toes while the forehead is $34 \,^{\circ}$ C [23]. Even temperatures within a small region may show significant variation due to air flow [10]. How these temperatures are perceived by the body depends on the range and the context of the temperatures. Table 1 (adapted from [20]) summarizes typical responses to skin temperatures. The receptors in the skin are much more sensitive to changes in temperatures. Thus, momentary contact with a surface that is warmer than the skin will elicit a sensation that seems much hotter than would be felt with more constant contact. This, plus the fact that skin can be quite cool compared to normal body temperature, corresponds to the wide bounds on these ranges.

While contact with any surface above $43 \,^{\circ}$ C for an extended period of time risks burning, temporary contact can be made at higher temperatures. For 10 minutes, contact with a surface at a temperature of $48 \,^{\circ}$ C can be maintained. Metals and water at 50 $^{\circ}$ C can be in contact with the skin for 1 minute without a burn risk. In addition, concrete can be tolerated for 1 minute at 55 $^{\circ}$ C, and plastics and wood at 60 $^{\circ}$ C. At higher temperatures and shorter contact times, materials show a higher differentiation of burn risk [23].

Table 1Skin temperature sensations.		
Skin temp., °C	State	
45	tissue damage	
43-41	threshold of burning pain	
41-39	threshold of transient pain	
39–35	hot	
37–35	initial sense of warmth	
34–33	neutral	
33-15	increasing cold	
15–5	intolerably cold	

3. Thermal regulation in a forearm-mounted wearable computer

While the previous section discussed rules and principles in general, this section will concentrate on a specific example: a forearm-mounted wearable computer (inspired by BT's proposed "Office on the Arm" [40]). The goal is to model how much heat such a computer could generate if it is thermally coupled to the user. In order to perform this analysis, several conditions must be assumed.

First, the surface area of the forearm must be approximated. The forearm is about 3.5% of the body's surface area [20] or 0.063 m^2 for our assumed user. Note that this is approximately the surface area of the bottom of a smaller notebook computer. For convenience, it will be assumed that the computer fits snugly around the forearm as a sleeve for near perfect heat conduction and will have negligible thickness so that inner and outer surface areas will be approximately equal.

To provide an approximate bound on the amount of heat the forearm computer can generate, the free air dissipation of heat through convection and radiation must be calculated. For practical considerations, the assumed environment will be a relatively warm, humid day of $31 \,^{\circ}$ C (88 $^{\circ}$ F), relative humidity of 80%, and a maximum allowable surface temperature of the computer of $41.5 \,^{\circ}$ C. $41.5 \,^{\circ}$ C was chosen as a "safe" temperature based on a summary of the medical literature by Lele [7,24], a survey of heat shock protein (HSP) studies which use > $43 \,^{\circ}$ C water baths to encourage HSP production [16], and many reported physiological experiments where subjects were immersed in water baths for several hours at significantly higher temperatures [3,33,38]. Furthermore, similar temperatures can be measured from the bottom surfaces of modern notebook computers.

Using the guidelines from above

$$Q'_{\text{conv}} = h_{\text{c}}(A_{\text{fore}})(T_{\text{s}} - T_{\text{amb}})$$

= 2.68(41.5 °C - 31 °C)^{0.25} (0.063 m²)
×(41.5 °C - 31 °C) = 3.2 W. (3.1)

Assuming a surface emittance of 0.95 and 80% of the surface of the forearm computer "seeing" the environment for radiative exchange

$$Q'_{\rm rad} = R \cdot A_{\rm fore},\tag{3.2}$$

$$\begin{aligned} Q_{\rm rad}' &= \sigma \epsilon \big(T_1^4 - T_2^4 \big) \frac{A_{\rm r}}{A_{\rm D}} A_{\rm fore} \\ &= 5.7 \times 10^{-8} \frac{\rm W}{\rm m^{2\,{}^\circ}\rm K^4} 0.95 \big((41.5 + 273)^4 \\ &- (31 + 273)^4 \big) 0.80 (0.063 \,\rm m^2) = 3.4 \,\, \rm W. \, (3.3) \end{aligned}$$

Thus, in this environment, uncoupled from the body with no wind and no body motion, the forearm computer is limited to 6.6 W. From these calculations, a notebook computer could dissipate 13.2 W, having approximately twice the surface area. Note that this is in agreement with the 10–14 W heat production characteristic of passively cooled

Table 2 Skin structure.		
Depth, mm	Structure	
0-0.4	epidermis	
0.5	superficial venous plexi	
0.8	superficial arteriolar plexus	
1.4	superficial venous plexi	
2.2	subcutaneous fat begins	
2.5	subcutaneous arteries	
	accompanied by venae comitantes	

notebook computers common in 1994 and 1995. As an aside, Intel guidelines increase the heat limit to 23–25 W for notebook computers with aggressive active cooling [28].

However, once mounted on the arm, heat will be conducted from the computer to the arm. Most thermal coupling that must be considered is through the skin to the surface veins and arteries. Skin has a thermal conductivity of $0.37 \text{ Wm}^{-1} \circ \text{C}^{-1}$, and the body will maintain a temperature of $37 \circ \text{C}$ for blood coming from the body's core. However, the linear heat conduction equation above is inadequate for modeling the heat transport of the blood stream. In order to proceed in creating an appropriate model, the first step is to determine the rate of blood flow through the forearm.

The primary means of thermoregulation by the human body is the rerouting of blood flow from deeper blood vessels to more superficial skin vessels, or vice versa. Table 2 (from [8]) shows the approximate depths of skin blood vessels. Skin blood flow is increased to an area when the local temperature of that part is raised, when an irritant is applied, or when the body temperature as a whole is elevated [10]. In addition, if there is a sufficient rise in return blood temperature from a peripheral body part, the body as a whole will begin heat dissipation measures [4,13]. However, it is improbable that enough heat would be transfered via one forearm to incite such a response [21].

Skin blood flow is regulated by vasodilation and vasoconstriction nerves. Areas that act as heat sinks, like the hands [22], have almost exclusively vasoconstriction nerves. In these areas, the arterial flow into the area must be warm already to cause the relaxation of vasoconstriction. Larger areas, such as the forearm, have a mixture of vasodilators and vasconstrictors, making the prediction of skin blood flow difficult. However, empirical studies by Taylor et al. [38] suggest that maximum forearm blood flow occurs when the forearm skin is raised to 42 °C for 35–55 minutes. While there can be considerable variability among subjects depending on age, weight, blood pressure, and other factors, Taylor et al. found in their measurements that the average maximum skin blood flow in the forearm is

$\frac{21 \text{ ml}}{100 \text{ ml}_{\text{fore}} \cdot \text{min}}.$

This last set of units requires some explanation. In physiology literature, blood flow is normalized for the volume of tissue in which it is observed. In this case, the tissue is a



Figure 1. Blood flow geometries in the arm.

volume of the forearm. In many experiments, total forearm blood flow is measured, which includes blood flow through both skeletal muscle and skin. However, muscle blood flow does not change significantly with outside application of heat to the forearm [12,17,35]. Thus, as above, results are sometimes given in skin blood flow instead of total blood flow per volume of forearm [34]. Johnson and Proppe [22] provide a conversion factor: 100 ml of forearm roughly corresponds to 0.0050 m² of skin. Combining this figure with the specific heat of blood 0.064 W min g⁻¹ °C⁻¹ and its density 1.057 g ml⁻¹ [6] yields a striking maximum heat dissipation capability of the blood in the forearm of

$$\frac{22 \text{ ml}}{100 \text{ ml min}} \frac{100 \text{ ml}}{0.0050 \text{ m}^2} \frac{1.057 \text{ g}}{\text{ml}} \\ \times 0.064 \frac{\text{W min}}{\text{g}^{\circ}\text{C}} 0.063 \text{ m}^2 = 18 \frac{\text{W}}{^{\circ}\text{C}}.$$
 (3.4)

Equally amazing, estimates place total possible body transfer of heat through skin blood flow at a 1745 W [22]! Armed with these constants and empirical results, we can create a model for heat conduction in the forearm.

3.1. Derivation of heat flow in the forearm

We model the arm as a set of four concentric cylinders of increasing radius, based on the information in table 2 (see [9,31,32] for related bioheat models). Figure 1 demonstrates the variables used in this derivation. Blood originates from the body (at 37 $^{\circ}$ C), flows through the arterial layer and returns through the two venous layers. This model is similar to the double-pipe bayonet heat exchanger developed by Martin [26].

To begin, we define the fundamental heat flow rates in the arm.

$$-M_1'c_p \, \mathrm{d}T_1 = \mathrm{d}Q_{12}' - \mathrm{d}Q_{12}' \tag{3.5}$$

$$-M_2'c_{\rm p}\,\mathrm{d}T_2 = -\mathrm{d}Q_{12}' + \mathrm{d}Q_{23}',\tag{3.6}$$

$$-M'_{3}c_{\rm p}\,\mathrm{d}T_{3} = -\mathrm{d}Q'_{23} + \mathrm{d}Q'_{\rm out},\tag{3.7}$$

where $c_{\rm p}$ is the heat capacity of the blood. We define the heat flow rates as

$$dQ'_{12} = k(T_1 - T_2) dA_2, (3.8)$$

$$\mathrm{d}Q'_{23} = k(T_2 - T_3)\,\mathrm{d}A_3,\tag{3.9}$$

$$dQ'_{\rm in} = k(T_B - T_1) \, dA_1, \qquad (3.10)$$

$$dQ'_{\rm out} = k(T_3 - T_{\rm arm}) \, dA_4 = 0, \qquad (3.11)$$

where T_B is the temperature of the external heat bath, T_{arm} is the temperature of the inner arm, and T_1 , T_2 , and T_3 are the blood temperatures between the cylinders. A_1 , A_2 , A_3 , and A_4 are the areas of the outer to inner cylinders, respectively. dA_i is a cylindrical shell of blood layer *i* of length z/L over which one considers the differential heat flow rates across that surface.

Equation (3.11) was set to zero to simplify the calculation. Since blood mass is conserved, arterial blood flow must return along the two venous layers. We can thus define the blood flow rates as

$$M_1' = -M_2'/p, (3.12)$$

$$M_3' = -M_2'/q, (3.13)$$

where 1/p + 1/q = 1.

To ease the computation, the above equations can be reformulated in terms of dimensionless parameters. Define

$$N = \frac{kA_2}{M'_2 c_p}, \ dA_i = A_i \frac{dz}{L}, \ \xi = N \frac{z}{L},$$
$$\alpha = \frac{A_1}{A_2}, \ \beta = \frac{A_3}{A_2},$$

where L is the total length of the arm. Combining this with equations (3.5) through (3.13) yields

$$-\frac{\mathrm{d}T_1}{\mathrm{d}\xi} = -p(T_1 - T_2) + p\alpha(T_B - T_1), \qquad (3.14)$$

$$-\frac{\mathrm{d}T_2}{\mathrm{d}\xi} = -(T_1 - T_2) + \beta(T_2 - T_3), \qquad (3.15)$$

$$-\frac{dT_3}{d\xi} = q\beta(T_2 - T_3).$$
(3.16)

Let $\theta_i = (T_i - T_B)/(T_{in} - T_B)$. T_{in} is a constant to denote the temperature of the blood in the rest of the body as it flows into the artery of the arm. The derivative is $d\theta_i =$ $dT_i/(T_{in} - T_B)$. Finally this yields three dimensionless, coupled differential equations

$$\frac{\mathrm{d}\theta_1}{\mathrm{d}\xi} = p\theta_1(1+\alpha) - p\theta_2,\tag{3.17}$$

$$\frac{\mathrm{d}\theta_2}{\mathrm{d}\xi} = \theta_1 - \theta_2(1+\beta) + \beta\theta_3, \qquad (3.18)$$

$$\frac{\mathrm{d}\theta_3}{\mathrm{d}\xi} = -q\beta(\theta_2 - \theta_3). \tag{3.19}$$

The general solution is of the form

$$\theta_i = \sum_{j=1}^3 C_{i,j} \mathrm{e}^{\lambda_j \xi} \tag{3.20}$$

so

$$\frac{\mathrm{d}\theta_i}{\mathrm{d}\xi} = \sum_{j=1}^3 C_{i,j} \lambda_j \mathrm{e}^{\lambda_j \xi}.$$
(3.21)

Equations (3.17)–(3.19) can easily be decoupled in a matrix formalism. Rewriting the right side of equations (3.17) through (3.19) as a matrix, M, the eigenvalues of M are the λ_j 's. Thus, solving the cubic equation

$$\begin{vmatrix} p(1+\alpha) - \lambda & -p & 0\\ 1 & -(1+\beta) - \lambda & \beta\\ 0 & -q\beta & q\beta - \lambda \end{vmatrix}$$
(3.22)

will yield each λ_i .

The remaining part of the solution entails applying the boundary conditions. The boundary conditions are:

- The blood entering the artery at the elbow is in contact with a large heat bath (the body) that maintains the blood temperature entering down the arm at 37 °C.
- The blood mixes in the hand such that the blood temperature at the arm-hand junction is equal in the artery and two venous layers. In actuality, due to the hand, the exit temperature of the arterial flow will be slightly different (warmer) than the return venous flow. This boundary condition was imposed for simplicity and because it gives a lower bound on the heat exchange rate in the arm. (Most literature considers the hand to be a heat sink, which could significantly increase heat dissipation. However, Nagasaka et al. [29] provide evidence that vasoconstriction may occur in the fingers when exposed to local temperatures greater than body temperature, limiting the additional heat transfer.)

Mathematically, the first boundary condition can be written as

$$\theta_2(0) = C_{2,1} + C_{2,2} + C_{2,3} = 1. \tag{3.23}$$

Including this in equations (3.17) through (3.19) yields

$$\frac{\mathrm{d}\theta_1}{\mathrm{d}\xi}(0) = p\theta_1(0)(1+\alpha) - p, \qquad (3.24)$$

$$\frac{d\theta_2}{d\xi}(0) = \theta_1(0) - (1+\beta) + \beta\theta_3(0), \qquad (3.25)$$

$$\frac{d\theta_3}{d\xi}(0) = -q\beta(1-\theta_3(0)).$$
(3.26)

Since, at $\xi = 0$,

$$\theta_1(0) = \sum_{j=1}^3 C_{1,j},\tag{3.27}$$

$$\theta_3(0) = \sum_{j=1}^3 C_{3,j} \tag{3.28}$$

and equations (3.24)-(3.26) further simplify to

$$\sum_{j=1}^{3} C_{1,j}(p(1+\alpha) - \lambda_j) = p, \qquad (3.29)$$

$$\sum_{j=1}^{3} C_{1,j} - \sum_{j=1}^{3} C_{2,j} \lambda_j + \beta \sum_{j=1}^{3} C_{3,j} = 1 + \beta, \quad (3.30)$$

$$\sum_{j=1}^{3} C_{3,j}(q\beta - \lambda_j) = q\beta.$$
 (3.31)

The same can be done with the second boundary condition, occurring at z = L (and $\xi = N$),

$$\theta_1(N) = \theta_2(N) = \theta_3(N),$$
 (3.32)

which yields

$$\sum_{j=1}^{3} C_{1,j} e^{\lambda_j N} - \sum_{j=1}^{3} C_{2,j} e^{\lambda_j N} = 0, \qquad (3.33)$$

$$\sum_{j=1}^{3} C_{1,j} e^{\lambda_j N} - \sum_{j=1}^{3} C_{3,j} e^{\lambda_j N} = 0.$$
 (3.34)

Combining these results with equations (3.17) through (3.19) yields

$$\sum_{j=1}^{3} C_{1,j} e^{\lambda_j N} (\lambda_j - p\alpha) = 0, \qquad (3.35)$$

$$\sum_{j=1}^{3} C_{2,j} e^{\lambda_j N}(\lambda_j) = 0, \qquad (3.36)$$

$$\sum_{j=1}^{3} C_{3,j} e^{\lambda_j N}(\lambda_j) = 0.$$
 (3.37)

Equations (3.23), (3.29)–(3.31), and (3.33)–(3.37) can be combined into a matrix to evaluate the constants

$$M \cdot C = B, \tag{3.38}$$

$$M = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ \gamma_1 & \gamma_2 & \gamma_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & -\lambda_1 & -\lambda_2 & -\lambda_3 & \beta & \beta & \beta \\ 0 & 0 & 0 & 0 & 0 & 0 & \chi_1 & \chi_2 & \chi_3 \\ \upsilon_1 & \upsilon_2 & \upsilon_3 & -\upsilon_1 & -\upsilon_2 & -\upsilon_3 & 0 & 0 & 0 \\ \upsilon_1 & \upsilon_2 & \upsilon_3 & 0 & 0 & 0 & -\upsilon_1 & -\upsilon_2 & -\upsilon_3 \\ \upsilon_1 \mu_1 & \upsilon_2 \mu_2 & \upsilon_3 \mu_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \upsilon_1 \lambda_1 & \upsilon_2 \lambda_2 & \upsilon_3 \lambda_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \upsilon_1 \lambda_1 & \upsilon_2 \lambda_2 & \upsilon_3 \lambda_3 \end{bmatrix},$$

$$(3.39)$$

$$C = \begin{bmatrix} C_{1,1} \\ C_{1,2} \\ C_{1,3} \\ C_{2,1} \\ C_{2,2} \\ C_{2,3} \\ C_{3,1} \\ C_{3,2} \\ C_{3,3} \end{bmatrix}, \qquad B = \begin{bmatrix} 1 \\ p \\ 1 + \beta \\ q\beta \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \qquad (3.40)$$

where $v_j = e^{\lambda_j N}$, $\gamma_j = p(1 + \alpha) - \lambda_j$, $\chi_j = q\beta - \lambda_j$, and $\mu_j = \lambda_j - p\alpha$. The constants become $C = M^{-1} \cdot B$ and a final solution is obtained.

The actual physical data used to solve this problem are listed below:

- $T_{\rm in} = 37 \,{}^{\circ}{\rm C}, \, T_{\rm B} = 39 \,{}^{\circ}{\rm C}.$
- p = q = 2.
- $k = 0.37 \text{ W} \text{m}^{-1} \circ \text{C}^{-1}$ [1].
- $d_1 = 0.0005 \text{ m}, d_2 = 0.0008 \text{ m}, \text{ and } d_3 = 0.0014 \text{ m}.$
- Average forearm skin surface area is 0.063 m², and the average radius of the forearm is 0.035 m [10].
- Blood flow was calculated for a few values from the range of possible blood flows in the arm (5, 10, 15, and 22 ml/(100 ml_{fore} min) [10].

The average power transfer into the arm can be calculated by taking the mean integral of the temperature distribution in the outer vein and modifying equation (3.10):

$$Q_{\rm in}' = k \big(T_{\rm B} - \langle T_1 \rangle \big) A_1, \tag{3.41}$$

where

$$\langle T_1 \rangle = \frac{T_{\rm in} - T_{\rm B}}{N} \int_0^N \theta_1 \, \mathrm{d}\xi + T_{\rm B}$$

The power results for the different blood flows are shown in table 3.

While this derivation was performed for an applied temperature $T_{\rm B} = 39$ °C, the power rating increases linearly with the difference between the applied temperature and body temperature. Thus, at the hypothetical 41.5 °C, which should develop very close to maximum skin blood flow, we expect around 28 W of heat conduction through the forearm.

Table 3 Heat dissipation for various blood flow rates given a 39 °C external heat bath.

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Blood flow rate, $ml/(100 ml_{fore} min)$	Power, W	
5	2.96	
10	5.92	
15	8.87	
21	12.43	

3.2. Verification of the model

To verify the model in the last section, an experiment was devised to examine the conduction of heat away from the forearm's surface. Two open-topped 10 liter styrofoam containers were filled with water at 48.8 °C and placed side by side in a 25 °C temperature-controlled room. Magnetic stirrers were used to keep the water agitated. Two calibrated digital thermometers were placed diagonally across from each other in each bath. When the baths cooled to 43°C, a subject immersed his forearm into the "forearm bath," leaving his upper arm and hand out of the bath. The temperatures of both the forearm and control baths were recorded every 200 seconds until the baths cooled below body temperature. The subject was dressed in T-shirt, jeans, and boots. Before the experiment, the subject indicated he was overly warm even after remaining seated for an hour. Before and during the experiment, his body temperature remained constant and no visible perspiration was evident, though he claimed his forehead felt moist before immersion and during the early part of the experiment. This would seem to indicate the subject was near his physiological tolerance to heat before resorting to open sweating.

For each 200 second time period, the heat loss of each bath was calculated using the temperature corrected thermal capacity of water (approx. $4.179 \text{ J g}^{-1} \circ \text{C}^{-1}$) [41]. The result can be seen in figure 2. By interpolating the corresponding heat loss at a given temperature for the control water bath, the difference in heat loss between the two baths can be calculated. Figure 3 shows the increase in heat dissipation caused by conduction through the forearm.

Note that there is a "knee" in the graph at approximately 40.5 °C. Above this temperature, the forearm bath seems to dissipate, on average, 12.8 W more than the control bath. Under 40°C the forearm bath is actually dissipating less than the control path. Such a drastic change would be expected around 37 °C, when the body would be heating the water, but why would such a sudden change happen around 40°C? First, the amount of blood that is pumped to the surface veins and arteries of the forearm decreases as temperature decreases. A similar breakpoint is observed in the literature [3] for blood flow at this temperature. However, even given that the blood flow may be significantly reduced at these temperatures, why should the presence of the forearm inhibit heat dissipation before the water bath reaches body temperature? Obviously, since the subject's body temperature did not exceed 37 °C, the forearm could not be adding heat to the bath. Instead, the forearm likely blocked the natural radiative, convective, and evaporative heat dissipation of the bath. In future experiments, a thermally neutral dummy arm of the same volume should be inserted into the control bath, and both baths should be agitated more aggressively. To compensate for this effect, the results must be offset by at least the 12 W difference between the baths at body temperature. Thus, the total heat conducted away by the forearm is approximately 23 W at 41.5 °C. This experimental result is significantly smaller



Figure 2. Heat dissipation from forearm (o) and control (+) water baths.



Figure 3. Heat conduction through forearm versus water bath temperature.

than the predicted 28 W of the model. However, in the model we assumed that the interior of the arm would be held constant at 37 °C. In actuality, the interior muscle mass of the arm will reach 38 °C when the forearm is submerged in water at temperatures above 40 °C [3,12]. With this change, the model predicts approximately 22 W of heat conduction, which closely matches the experimental data. Other human limb heat transfer models that include muscle heating have been published since this paper was in review [32], but the above model provides a simple predictive tool and is specifically tailored for this task.

While this experiment involved one subject, the results coincide with temperature vs. blood flow experiments from the literature and also correlate nicely with the model proposed above. Note that since the body is very active in maintaining its core temperature, similar amounts of heat dissipation from the forearm may be available in all but the most adverse conditions. Even at a more conservative forearm temperature of 39 °C, a substantial amount of heat will be conducted away by the forearm as shown in the model above.

3.3. Discussion and practical issues

Given the above models and calculations, a forearm computer may generate up to

$$Q'_{\text{tot}} = Q'_{\text{conv}} + Q'_{\text{rad}} + Q'_{\text{cond}} = 30 \text{ W}$$
 (3.42)

in warm, still air and without body motion. This is summarized in figure 4 and is significantly higher heat dissipation per surface area than that of a normal passively



Figure 4. Forearm computer heat dissipation.

cooled notebook computer (approximately 450 W m⁻² versus 100 W m⁻²). Unfortunately, this calculation ignores several practical factors. The wearable would need to monitor its surface temperature to avoid exceeding the 41.5 °C limit, but would the user feel that 41.5 °C is too high a temperature for a sheath on the forearm? A simple way to address this problem is to provide the user with a physical knob to adjust the maximum operating temperature of the computer (up to safe limits) and a meter showing the fraction of full functionality available in current conditions. This interface makes the trade-off between heat generation and functionality explicit and accessible to the user.

Another concern is sweating under the sleeve due to exertion. Without a way for sweat to be released, the user may experience discomfort, similar to the sensation of sweating in rubber gloves. To alleviate this problem, a thin layer of heat conducting fabric can be used to "wick" the water trapped under the computer sleeve. Slits should be designed into the computer to allow evaporation of the water. The resulting evaporation will increase user comfort and increase cooling. The slits also provide the benefit of adding more surface area to the forearm computer, increasing cooling.

The above analysis assumed good thermal contact between the electronics and the forearm. In reality this may prove difficult given the obvious constraint of the user's comfort. A carefully chosen material for the wicking layer and a custom-fitted forearm sleeve may be sufficient for the needed heat conduction. In more exotic applications, phase-change materials might be used in the sleeve. However, the wearable computer will have "hot spots" which could cause discomfort [2,25]. A variant of a self-contained fluid heat-pipe may be needed to even out the temperature gradient. Forced air could also be used to transfer heat to the forearm. In practice, the actual computer in the forearm sheath may be the size of a credit card with the rest of the casing dedicated to the distribution of heat. By making these sections modular, upgrades are trivial, and the user could have fashionable casings designed to complement his or her wardrobe.

A side benefit to thermally coupling the computer to the user's forearm is that intermittent contacts of the surface with other body parts may be better tolerated. The user would have an innate sense that the computer cannot be burning him or else his forearm would be uncomfortable. This helps offset the effects of different relative temperatures of the skin surface. Careful selection of the computer casing material will also help this problem [23].

Previous sections assumed a static, reasonably constrained environment. In actuality, the user's thermal environment will change, often to the benefit of the computer. Small amounts of air flow can significantly increase heat dissipation. While walking, the air flow about the arm is significantly enhanced by the pendulum-like movement of the arm. In fact, the air flow along the forearm is turbulent for many situations, effectively doubling the heat dissipation of calmer air movement [10]. In addition, changes in ambient and skin temperature and the cooling effects of the user's sweating may be exploited in many cases (for example, when a sweating user enters an air conditioned building). With sensing of skin temperature and sweating, the forearm computer can regulate its own heat production according to the thermal environment. The temperature feedback mechanisms already common in microprocessor design suggest such a trend.

Mounting the computer on the forearm has many advantages including convenient access, higher availability of turbulent air flow due to the pendulum effect, and heat exchange through the hand. However, there are many potential mounting locations. The legs have a similar set of advantages as the arms with an even larger surface area but with less ease of access. Mounting on the head has the advantage of faster natural convective air flow while the user is sedentary and a constant flow of forced air while walking. Unfortunately, hair impedes heat conduction to the skin. Mounting the computer on "core" body areas is also a possibility, but it would result in lesser heat gradients in many instances. While the system analyzed above operates with skin regions at body temperature, the limbs are often colder due to lower ambient conditions, which increases their temporary heat storage capacity.

More aggressive systems might employ thermal regulation via active thermal reservoirs. For example, the heat capacity of the computer's batteries might be exploited. While charging, batteries could be chilled so that heat can be transferred into them during use [19]. The computer's heating of the batteries while running may also provide the benefit of increased battery life. In addition, by employing active cooling elements such as Peltier junctions, the computer might cool the batteries or components during times of low ambient temperature. Thus, the computer has access to a thermal reservoir during times of heat stress. Similarly, a water reservoir, perhaps stored in a sponge, could be used for evaporative cooling. Finally, software applications can be written with heat dissipation in mind. Disk maintenance, downloads, and batch jobs can be delayed until the computer senses a cooler environment. In this manner, performance is reserved for user interactions, and the effective average heat dissipation can be higher.

4. Conclusion

This paper provides first-order heat dissipation guidelines for developing wearable computer prototypes and introduces an unconventional way of thinking about cooling. By thermally coupling a forearm wearable computer with the user, heat dissipation can be significantly increased from today's industry limits, even in warm environments. The effective heat generation limit may be further increased by aggressive active management of thermal reservoirs such as the computer casing and batteries, monitoring of user and environmental conditions, software scheduling of background processes, and making the trade-off between heat generation and functionality explicit to the user.

Acknowledgements

We would like to thank the reviewers for their extremely helpful suggestions; Tavenner Hall for initial help in editing this paper and producing figures; Dr. Shuguang Zhang for his help in procuring the temperature controlled room and laboratory equipment; Keith Starner, Dr. H.F. Bowman, Dr. A. Pentland, Dr. R. Gonzalez, Dr. M. Kolka, and the members of the Natick Army Research Labs for help in finding background resources in these fields; BT for inspiring novel ways of approaching wearables; the wearables mailing list for commentary and encouragement; and Walter Bender for the prodding to substantiate an off-hand comment with research.

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