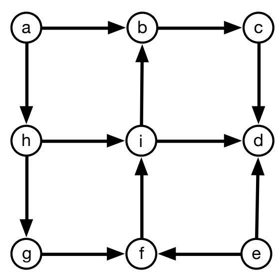
CS 4540, Fall 2014
Homework 9
due: Wednesday, November 19, 2014.

Problem 1: In Wednesday's class (November 12) we defined the following condition. For an undirected planar graph $G = (V, E)$, an orientation $\overrightarrow{G}$ satisfies condition $(\star)$ if

on every internal face f of G there is an odd number of edges that are clockwise in $\overrightarrow{G}$ with respect to f.

We proved that $(\star)$ implies that $\overrightarrow{G}$ is Pfaffian. Recall that proof. In the below figure, consider the even length cycle $C = a, b, c, d, e, f, g, h, a$ in G. And the edges show an orientation $\overrightarrow{G}$ that satisfies $(\star)$ but the cycle C has an even number of clockwise edges. What goes wrong in the proof or why doesn't the proof apply? Explain. Does there exist an orientation $\overrightarrow{G}$ of this graph which satisfies $(\star)$ and where C has an odd number of clockwise edges? Show such an orientation or give a nice proof why there does not exist one.



Problem 2:

At the end of Wednesday's class (November 12) we talked about the fact that: the number of permutations consisting of all even length cycles and corresponding to edges in G is equal to (the number of perfect matchings of G)^2. Prove this fact.

Do so by showing a bijection directly between the two, don't use the graph $\overleftrightarrow{G}$.