

Improved mixing condition for counting and sampling independent sets

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We study the hard-core model defined on independent sets of a given graph G . If I is an independent set of G , then I receives the weight $\lambda^{|I|}$ for a positive real parameter λ . For large λ , computing the normalization constant $Z(\lambda) = \sum_{I \text{ ind. set of } G} \lambda^{|I|}$ (a.k.a., partition function) is a well known computationally challenging problem. More concretely, in view of breakthrough results by Dror Weitz (2006) and Allan Sly (2010) for graphs G of maximum degree Δ there exists a critical value $\lambda(\Delta)$ such that the hardness of computing the partition function undergoes a computational transition. By improving upon the tree recursion used by Weitz to establish strong spatial mixing (and therefore polynomial approximability of the partition function), we develop an improved mixing condition that allows different ‘types’ among the recursion tree. Such approach applies to specific classes of graphs. For instance, we prove that strong spatial mixing holds for subgraphs of the square lattice for all $\lambda < 2.3882$, improving upon the previous bound of $\lambda < 1.6875$. We apply also our criteria to show that the presence of short cycles everywhere in the graph improves the domain in which strong spatial mixing holds up to $\lambda < \lambda(\Delta) + \epsilon$, where ϵ is a positive constant that depends on the size and density of such short cycles.

This talk is based on joint work with Jinwoo Shin, Prasad Tetali, Eric Vigoda and Linji Yang and recent joint work with Prasad Tetali.