CS 4644-DL / 7643-A
ZSOLT KIRA

Generative Models:

Denoising Diffusion Probabilistic Models (DDPMs)

Slides adapted from those by Danfei Xu

- **Assignment 3**
    - Due **March 9th 11:59pm EST**

- **Projects**
    - Project proposal due **March ~~15th~~ 17th**
    - Proposal description out on canvas @256

- Meta office hours today 3pm ET on embeddings

W8: Mar 1	Generative Models (Part I): Generative Adversarial Networks Slides (PDF)
W9: Mar 6	Project Planning Session
W9: Mar 8	Generative Models (Part II): Diffusion Models PS3/HW3 due Mar 9th 11:59pm (grace period Mar 11th), PS4/HW4 out (due Apr 2nd)
W10: Mar 13	Guest Lecture (Mido Assran, Meta) - JePA
W10: Mar 15	Guest Lecture (Michael Auli) - Self-supervised Learning for Audio Project Proposal Due Mar 17th 11:59pm

Taxonomy of Generative Models

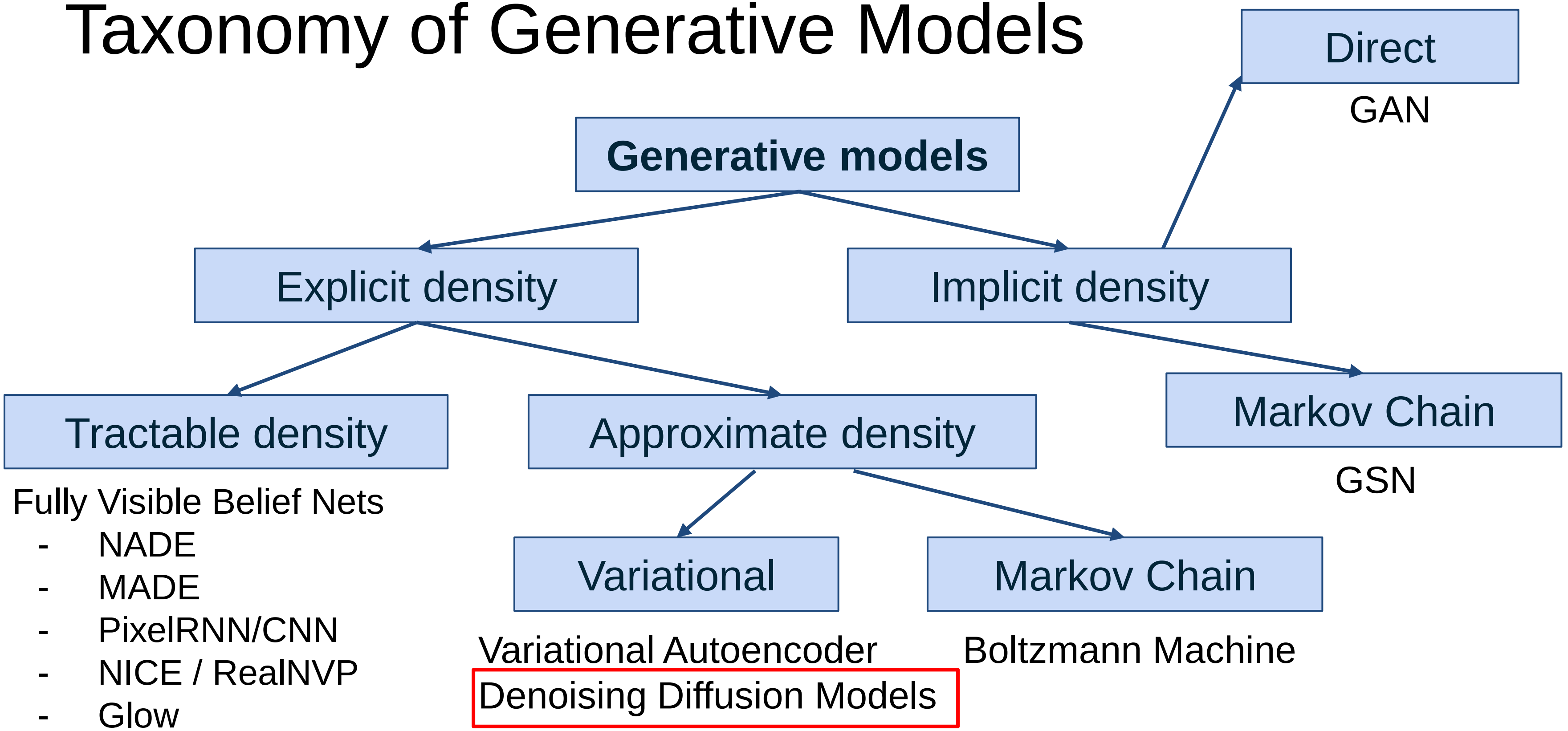

Figure copyright and adapted from Ian Goodfellow, Tutorial on Generative Adversarial Networks, 2017.

Denoising Diffusion Probabilistic Models (DDPMs)

And Conditional Diffusion Models

TEXT DESCRIPTION

An astronaut Teddy bears A bowl of soup

riding a horse lounging in a tropical resort in space playing basketball with cats in space

in a photorealistic style in the style of Andy Warhol as a pencil drawing

→

DALL·E 2

https://openai.com/dall-e-2/

Landscape Highlights of Diffusion Models (Nov 2022)

basic principles

- *Diffusion probabilistic models* (Sohl-Dickstein et al., 2015)
- *Noise-conditioned score network* (**NCSN**; Yang & Ermon, 2019)
- *Denoising diffusion probabilistic models* (**DDPM**; Ho et al. 2020)

conditional & high-res image generation

- *Classifier-guided conditional generation* (Dhariwal and Nichole, 2021)
- *Classifier-free Diffusion Guidance* (Ho and Salimans, 2022)
- *Latent-space Diffusion* (**StableDiffusion**; Rombach and Blattmann et al., 2022)

new applications

- *Planning with Diffusion for Flexible Behavior Synthesis* (**Diffuser**; Janner et al., 2022)
- *DreamFusion: Text-to-3D using 2D Diffusion* (Poole and Jain et al., 2022)
- *Make-A-Video: Text-to-Video Generation without Text-Video Data* (Singer et al., 2022)

How to make a new generative model

- **Setting:** Given unlabeled dataset of data, I want to learn to sample from P(x)
- Define the generative process
- Parameterize it
- Maximum likelihood (often + KL-divergence)
- Approximations
- Optimize parameters!
- Add conditioning, e.g. text

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The Denoising Diffusion Process

image from
dataset

x_0

The Denoising Diffusion Process

image from dataset

The “forward diffusion” process: add Gaussian noise each step

$x_0 \longrightarrow x_1 \longrightarrow$

...

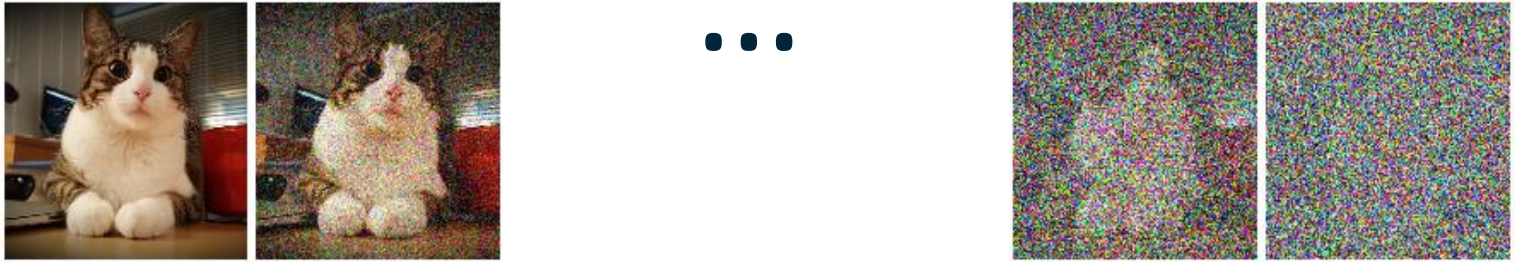
The Denoising Diffusion Process

image from dataset

The "forward diffusion" process: add Gaussian noise each step

noise $\mathcal{N}(0, I)$

$x_0 \longrightarrow x_1 \longrightarrow \cdots \longrightarrow x_{T-1} \longrightarrow x_T$



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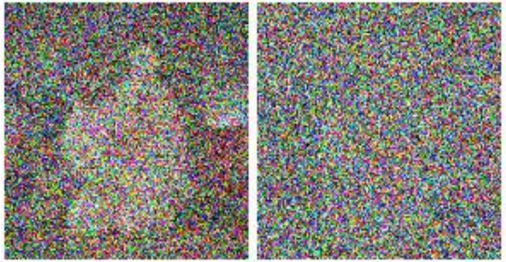


$x_0 \leftarrow x_1 \leftarrow \cdots \leftarrow x_{T-1} \leftarrow x_T$

The “denoising diffusion” process: generate an image from noise by *denoising* the gaussian noises

Ties/inspiration form Annealed Imporantce Sampling in physics

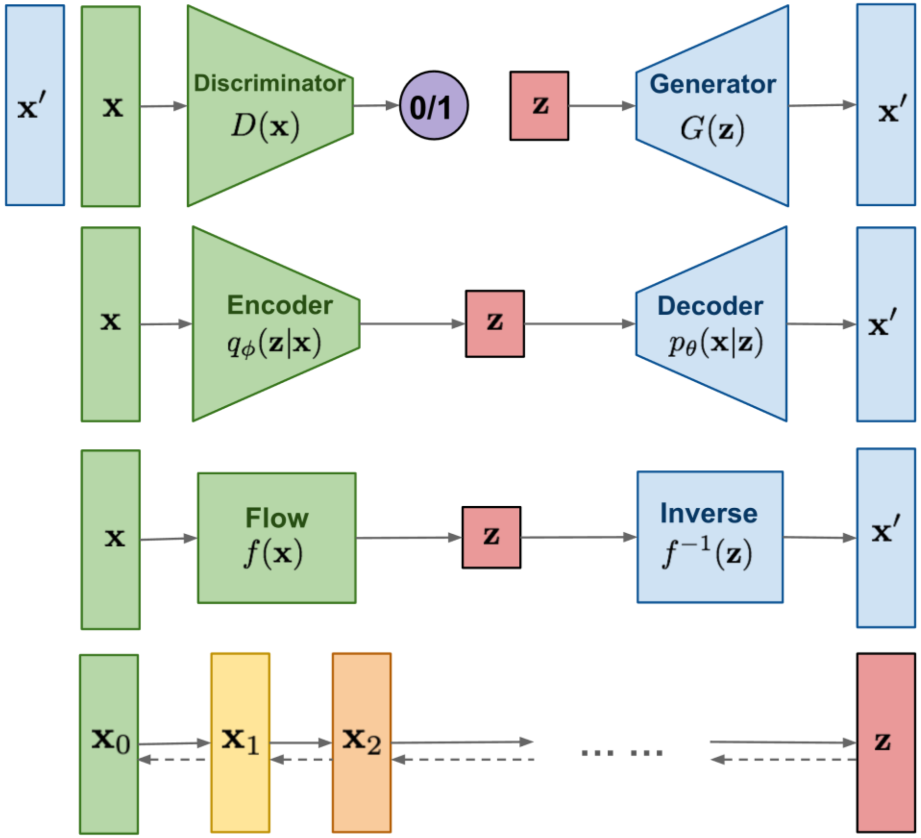
Comparison

GAN: Adversarial training

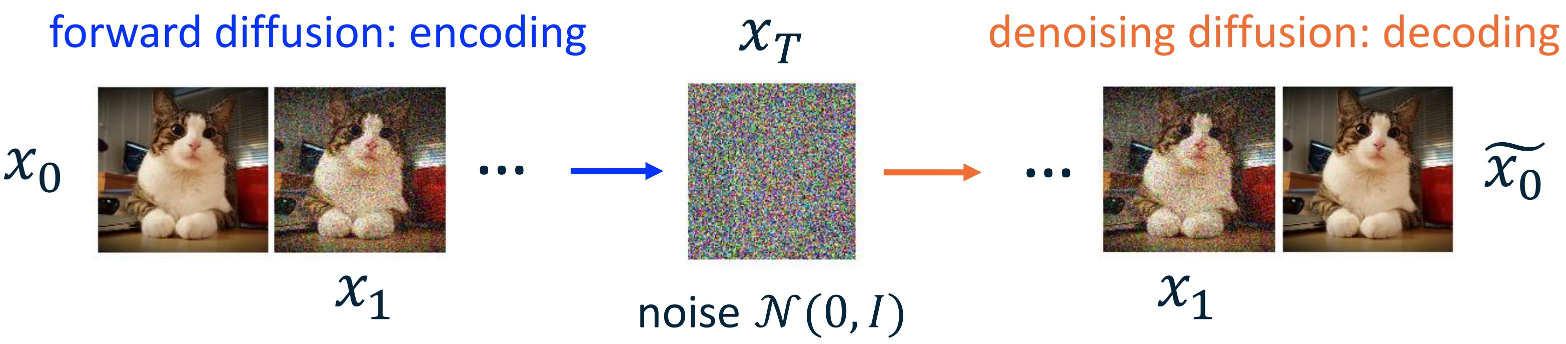
VAE: maximize variational lower bound

Flow-based models: Invertible transform of distributions

Diffusion models: Gradually add Gaussian noise and then reverse



Forward/Reverse Processes



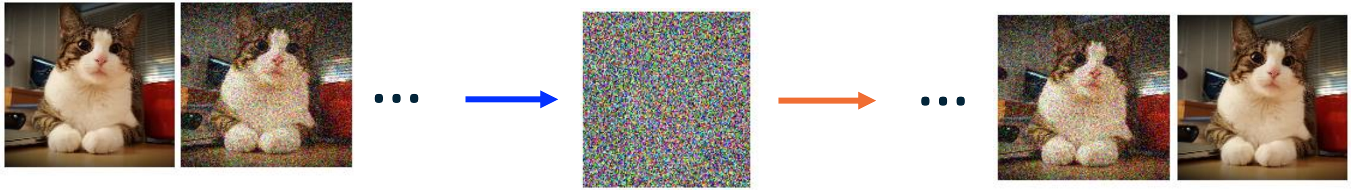
Forward/Reverse Processes

forward diffusion: encoding

x_T

denoising diffusion: decoding

x_0



$\widetilde{x_0}$

x_1

noise $\mathcal{N}(0, I)$

x_1

Known / predefined:

$q(x_{1:T}|x_0)$

Unknown / learned:

$$p_\theta(x_{0:T}) = p(x_T)\prod_{t=1}^{T} p_\theta(x_{t-1}|x_t)$$

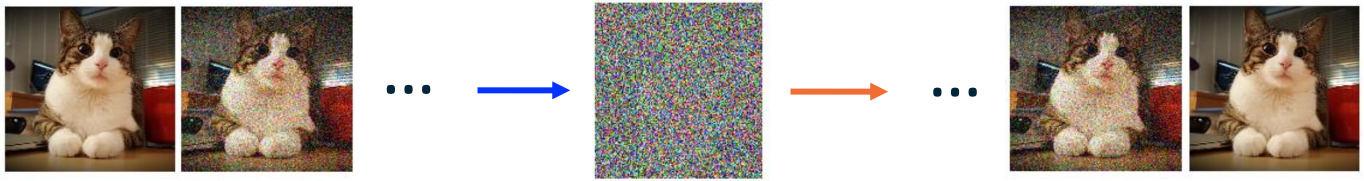
Input: x_t

Output: Noise mean to remove, sample & use w/ x_t to get x_{t-1}

Forward/Reverse Processes

forward diffusion: encoding x_T denoising diffusion: decoding

x_0



x_1 noise $\mathcal{N}(0, I)$ x_1 $\widetilde{x_0}$

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Unknown / learned:

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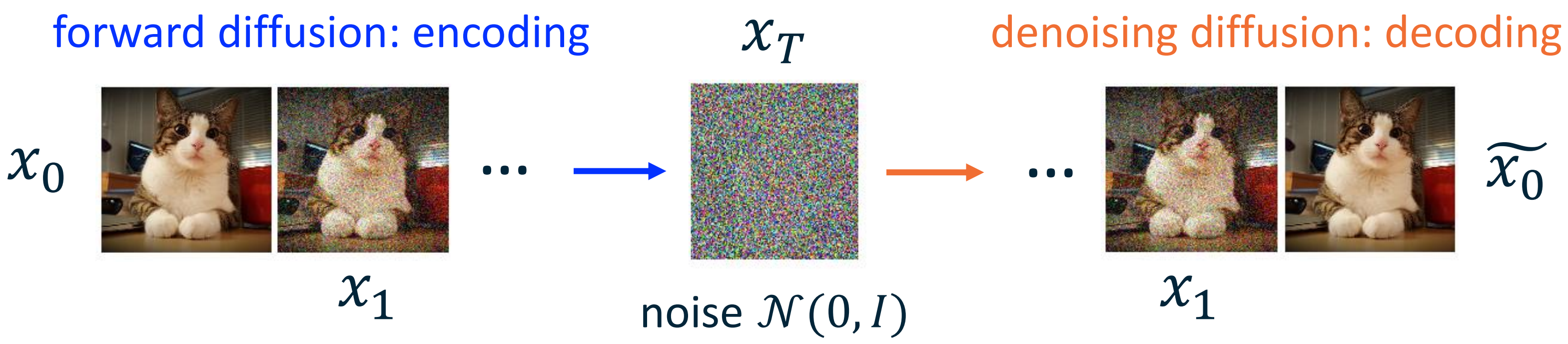
Use the denoising decoding process to generate new images.

Forward/Reverse Processes



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The Diffusion (Encoding) Process

The **known** forward process $x_0 \longrightarrow x_1 \longrightarrow \cdots \longrightarrow x_T$

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Probability Chain Rule (Markov Chain)

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$$q(x_{1:T}|x_0) = \prod_{t=1}^{T} q(x_t|x_{t-1})$$ Probability Chain Rule (Markov Chain)

$$q(x_t|x_{t-1}) = \mathcal{N}(x_t; (\sqrt{1-\beta_t})x_{t-1}, \beta_t I)$$ Conditional Gaussian

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Notation: A Gaussian distribution "for" x_t

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β_t is the *variance schedule* at the diffusion step t

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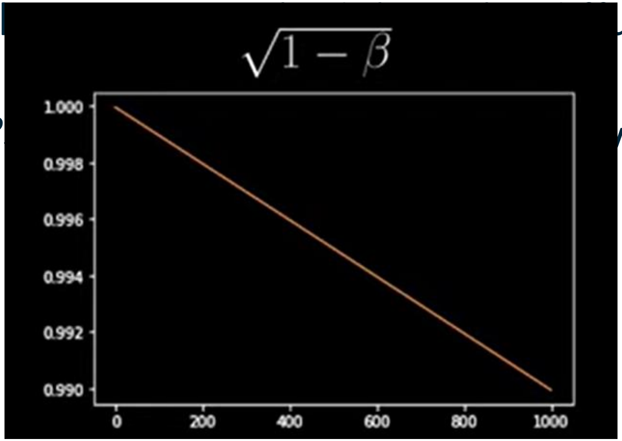
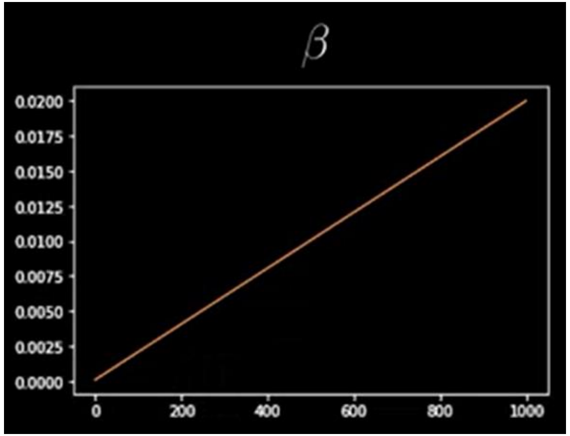
β_t is t … usion

$0 < \beta$ … value … $= 1000$





https://www.youtube.com/watch?v=HoKDTa5jHvg&t=517s

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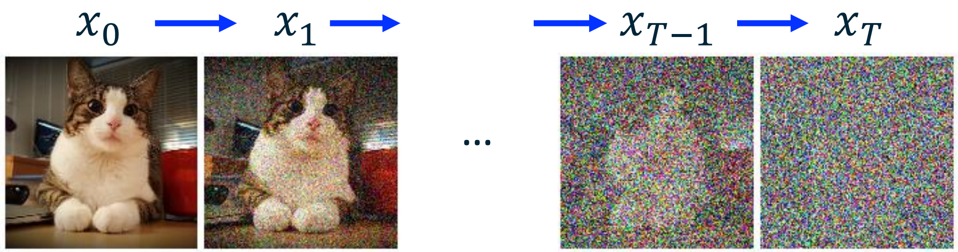
β_t is the *variance schedule* at the diffusion step t

$0 < \beta_1 < \beta_2 < \cdots < \beta_T < 1$, typical value range $[0.0001, 0.02]$, with $T = 1000$



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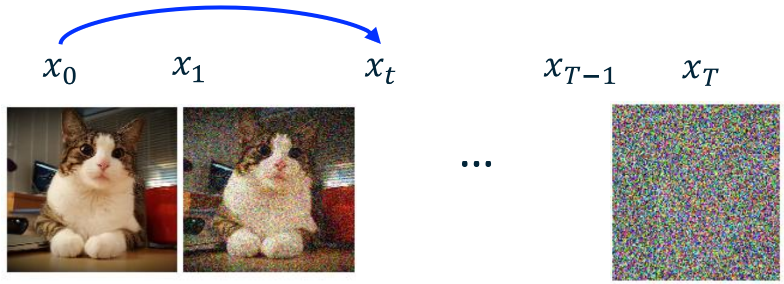
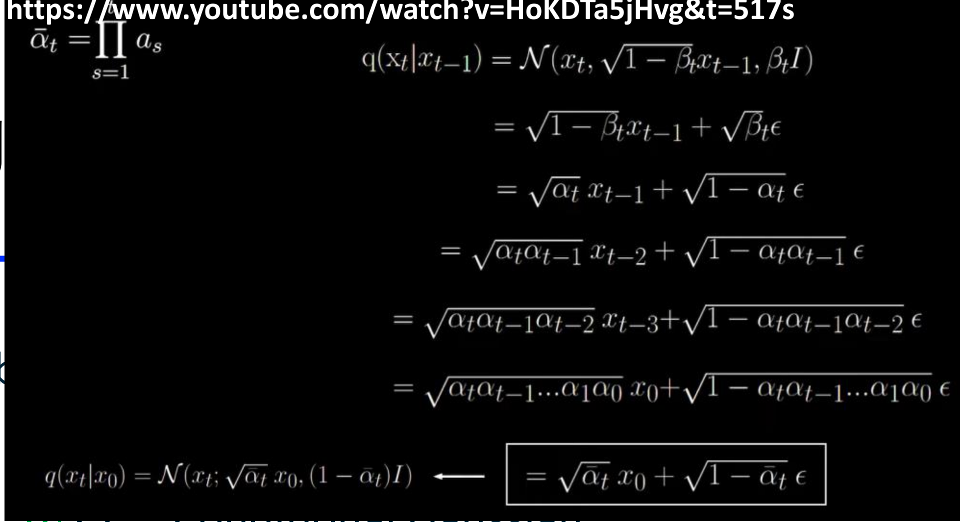
Nice property: samples from an *arbitrary forward step* are also Gaussian-distributed!

$$q(x_t|x_0) = \mathcal{N}(x_t; \sqrt{\bar{\alpha}_t}x_0, (1-\bar{\alpha}_t)I)$$

, where $\alpha_t = (1-\beta_t)$, $\bar{\alpha}_t = \prod_{s=1}^{t} \alpha_s$



The Diffusion (Encoding



The **known** forward process x_0

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$$q(x_t|x_0) = \mathcal{N}(x_t; \sqrt{\bar{\alpha}_t} x_0, (1-\bar{\alpha}_t)I)$$

Gaussian reparameterization trick:

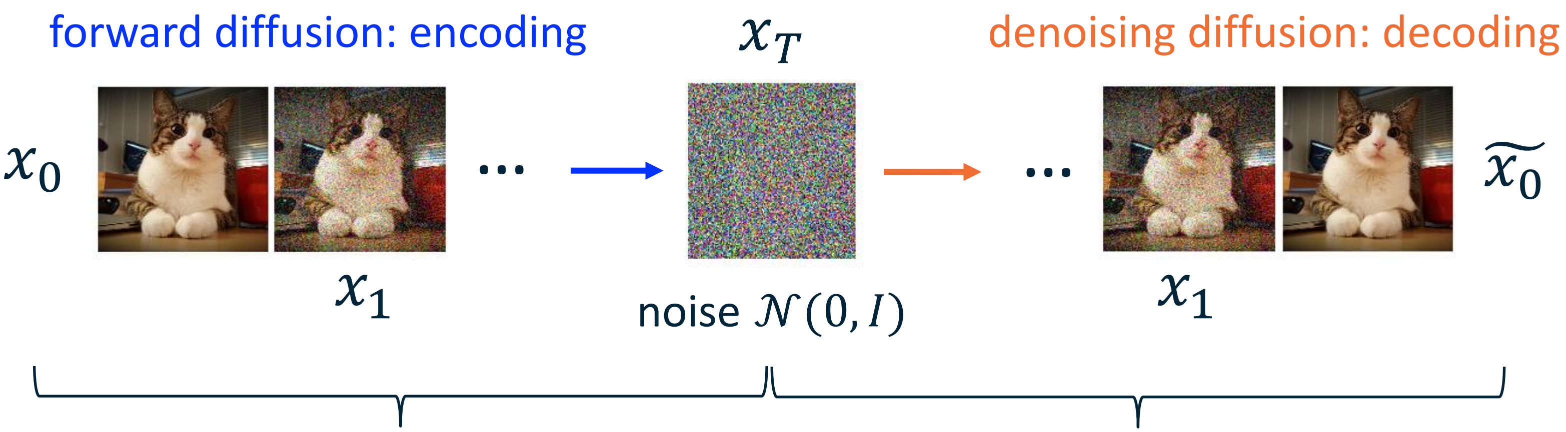
$$z = \mu + \epsilon * \sigma, \epsilon \sim N(0,1)$$

$$x_t = \sqrt{\bar{\alpha}_t} x_0 + \sqrt{1-\bar{\alpha}_t}\epsilon, \qquad \epsilon \sim \mathcal{N}(0, I)$$

Intuition: We know all distributions in forward process, and can in fact directly compute for any t based on X_0

(square root appears because reparameterization trick has just σ)

The Diffusion and Denoising Process



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Unknown / learned:

$$p_\theta(x_{0:T}) = p(x_T) \prod_{t=1}^{T} p_\theta(x_{t-1} | x_t)$$

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The **learned** denoising process $x_0 \leftarrow x_1 \leftarrow \cdots \leftarrow x_T$

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Conditional Gaussian

Want to learn time-dependent mean

Assume fixed / known variance (simplification)

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How do we form a learning objective?

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High-level intuition: derive a *ground truth denoising distribution* $q(x_{t-1}|x_t, x_0)$ and train a neural net $p_\theta(x_{t-1}|x_t)$ to match the distribution.

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What does it look like? $q(x_{t-1}|x_t, x_0) = \mathcal{N}\left(x_{t-1}; \mu_q(t), \Sigma_q(t)\right)$

$$\mu_q(t) = \frac{1}{\sqrt{\alpha_t}}\left(x_t - \frac{\beta_t}{\sqrt{(1-\bar{\alpha}_t)}}\epsilon\right), \qquad \epsilon \sim \mathcal{N}(0, I)$$

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$\leftarrow$ Recall: Gaussian reparameterization trick

The "ground truth" noise that brought x_{t-1} to x_t

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What does it look like? $q(x_{t-1}|x_t, x_0) = \mathcal{N}\left(x_{t-1}; \mu_q(t), \Sigma_q(t)\right)$

Assuming identical variance $\Sigma_q(t)$, we have:

$$\text{argmin}_\theta D_{KL}(q(x_{t-1}|x_t, x_0)\big|\big|p_\theta(x_{t-1}|x_t)\big) = \text{argmin}_\theta w||\mu_q(t) - \mu_\theta(x_t, t)||^2$$

Should be variance-dependent, but constant works better in practice

The Denoising (Decoding) Process

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Simplified learning objective: $\text{argmin}_\theta||\epsilon - \epsilon_\theta(x_t, t)||^2$

Predict the one-step noise that was added (and remove it)!

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$$p_\theta(x_{0:T}) = p(x_T) \prod_{t=1}^{T} p_\theta(x_{t-1}|x_t)$$

Probability Chain Rule (Markov Chain)

$$p_\theta(x_{t-1}|x_t) = \mathcal{N}(x_{t-1}; \mu_\theta(x_t, t), \Sigma_q(t))$$

Conditional Gaussian

Assume fixed / known variance

How did we arrive at the learning objective?
Let's go back to the basics of variational models …

(Quick) Derivation!

Variational Inference → Simplify to KL → Reverse Process => Normal → Bayes + Reparameterization → Remove (variance-dependent) constant → Predict the noise!!!

$p(x) = \int p(x|z)p(z)dz$ Intractable to estimate!

Deep Unsupervised Learning using Nonequilibrium Thermodynamics, Sohl-Dickstein *et al.*, 2015

Variational Inference

$p(x) = \int p(x|z)p(z)dz$ Intractable to estimate!

$$\log p(x) = \mathrm{E}_q\left[\log \frac{p(x|z)p(z)}{q(z|x)}\right] + D_{KL}(q(z|x)||p(z|x))$$

$$\geq \mathrm{E}_q\left[\log \frac{p(x|z)p(z)}{q(z|x)}\right]$$

Evidence Lower Bound (ELBO)

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Evidence Lower Bound (ELBO)

$$\log p(x_0) \geq \mathrm{E}_q\left[\log \frac{p(x_0|x_{1:T})p(x_{1:T})}{q(x_{1:T}|x_0)}\right]$$

$x = x_0, \ z = x_{1:T}$

Deep Unsupervised Learning using Nonequilibrium Thermodynamics, Sohl-Dickstein *et al.*, 2015

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$$\log p(x_0) \geq \mathrm{E}_q\left[\log \frac{p(x_0|x_{1:T})p(x_{1:T})}{q(x_{1:T}|x_0)}\right]$$

$x = x_0,\ z = x_{1:T}$

… (derivation omitted, see Sohl-Dickstein *et al.*, 2015 Appendix B)

$$= \mathrm{E}_q\left[\log \frac{p(x_T) \prod_{t=1}^{T} p_\theta(x_{t-1}|x_t)}{\prod_{t=1}^{T} q(x_t|x_{t-1})}\right]$$

← reverse denoising

← forward diffusion

Deep Unsupervised Learning using Nonequilibrium Thermodynamics, Sohl-Dickstein *et al.*, 2015

Variational Inference → Simplify to KL

$p(x) = \int p(x|z)p(z)dz$ Intractable to estimate!

$$\log p(x) = \mathrm{E}_q\left[\log \frac{p(x|z)p(z)}{q(z|x)}\right] + D_{KL}(q(z|x)||p(z|x))$$

$$\geq \mathrm{E}_q\left[\log \frac{p(x|z)p(z)}{q(z|x)}\right]$$ Evidence Lower Bound (ELBO)

$$\log p(x_0) \geq \mathrm{E}_q\left[\log \frac{p(x_0|x_{1:T})p(x_{1:T})}{q(x_{1:T}|x_0)}\right]$$ $x = x_0,\ z = x_{1:T}$

… (derivation omitted, see Sohl-Dickstein *et al.,* 2015 Appendix B)

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$$= -\mathrm{E}_q[D_{KL}(q(x_T|x_0)||p(x_T))] - \sum_{t=2}^{T} D_{KL}(q(x_{t-1}|x_t, x_0)||p_\theta(x_{t-1}|x_t)) + \log p_\theta(x_0|x_1)$$

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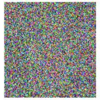
fixed



Easy to optimize / sometimes omitted

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Maximize the agreement between the predicted reverse diffusion distribution p_θ and the "ground truth" reverse diffusion distribution q

Deep Unsupervised Learning using Nonequilibrium Thermodynamics, Sohl-Dickstein *et al.*, 2015

Variational Inference → Simplify to KL → Reverse Process => Normal

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$$q(x_{t-1}|x_t) = \boldsymbol{q(x_{t-1}|x_t, x_0)} \quad \textbf{(markov assumption)}$$

$$= \frac{q(x_t|x_{t-1}, x_0)q(x_{t-1}|x_0)}{q(x_t|x_0)} \quad \text{(Bayes rule)}$$

$$= \frac{\mathcal{N}(x_t; \sqrt{\alpha_t}x_{t-1}, \beta_t I)\mathcal{N}(x_{t-1}; \sqrt{\bar{\alpha}_{t-1}}x_{t-1}, (1-\bar{\alpha}_{t-1})I)}{\mathcal{N}(x_t; \sqrt{\bar{\alpha}_t}x_0, (1-\bar{\alpha}_{t-1})I)}$$

$$\propto \mathcal{N}\left(x_{t-1}; \frac{\sqrt{\alpha_t}(1-\bar{\alpha}_{t-1})x_t + \sqrt{\bar{\alpha}_{t-1}}(1-a_t)x_0}{1-\sqrt{\bar{\alpha}_t}}, \Sigma_q(t)\right) \quad \text{(Property of Gaussian)}$$

Deep Unsupervised Learning using Nonequilibrium Thermodynamics, Sohl-Dickstein *et al.*, 2015

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$$\mu_q(t) = \frac{1}{\sqrt{\alpha_t}}\left(x_t - \frac{\beta_t}{\sqrt{(1-\bar{\alpha}_t)}}\epsilon\right), \qquad \epsilon \sim \mathcal{N}(0, I)$$

Proof using bayes rule and gaussian reparameterization trick

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Proof using bayes rule and gaussian reparameterization trick

The "ground truth" noise that brought x_{t-1} to x_t

Deep Unsupervised Learning using Nonequilibrium Thermodynamics, Sohl-Dickstein *et al.*, 2015

Variational Inference → Simplify to KL → Reverse Process => Normal → Bayes + Reparameterization → Remove (variance-dependent) constant

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Minimize the difference of distribution means (assuming identical variance)

$$\mathrm{argmin}_\theta w||\mu_q(t) - \mu_\theta(x_t, t)||^2$$

Deep Unsupervised Learning using Nonequilibrium Thermodynamics, Sohl-Dickstein *et al.*, 2015

Learning the Denoising Process

The **learned** denoising process $x_0 \leftarrow x_1 \leftarrow \cdots \leftarrow x_T$

$$p_\theta(x_{0:T}) = p(x_T)\prod_{t=1}^{T} p_\theta(x_{t-1}|x_t)$$

$$p_\theta(x_{t-1}|x_t) = \mathcal{N}(x_{t-1}; \mu_\theta(x_t, t), \Sigma(t))$$

Conditional Gaussian

Learning objective: $\text{argmin}_\theta ||\mu_q(t) - \mu_\theta(x_t, t)||^2$

$$\mu_q(t) = \frac{1}{\sqrt{\alpha_t}}\left(x_t - \frac{\beta_t}{\sqrt{(1-\bar{\alpha}_t)}}\epsilon\right), \qquad \epsilon \sim \mathcal{N}(0, I)$$

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Do we actually need to learn the entire $\mu_\theta(x_t, t)$?

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known during inference

Unknown during inference

Recall: this is the “ground truth” noise that brought x_{t-1} to x_t

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known during inference

Unknown during inference

Recall: this is the “ground truth” noise that brought x_{t-1} to x_t

Idea: just learn ϵ with $\epsilon_\theta(x_t, t)$!

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Simplified learning objective: $\text{argmin}_\theta ||\epsilon - \epsilon_\theta(x_t, t)||^2$

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Recall: the simplified t-step forward sample:

$$x_t = \sqrt{\bar{\alpha}_t}x_0 + \sqrt{1 - \bar{\alpha}_t}\epsilon$$

Variational Inference → Simplify to KL → Reverse Process => Normal → Bayes + Reparameterization → Remove (variance-dependent) constant → Predict the noise!!!

Learning the Denoising Process

The **learned** denoising process $x_0 \leftarrow x_1 \leftarrow \cdots \leftarrow x_T$

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Conditional Gaussian

Simplified learning objective: $\text{argmin}_\theta ||\epsilon - \epsilon_\theta(\sqrt{\bar{\alpha}_t}x_0 + \sqrt{1-\bar{\alpha}_t}\epsilon, t)||^2$

The Denoising (Decoding) Process

The **learned** denoising process $x_0 \leftarrow x_1 \leftarrow \cdots \leftarrow x_T$

$$p_\theta(x_{t-1}|x_t) = \mathcal{N}(x_{t-1}; \mu_\theta(x_t, t), \Sigma_q(t))$$

Simplified learning objective: $\mathbf{argmin}_\theta ||\epsilon - \epsilon_\theta(\boldsymbol{x}_t, \boldsymbol{t})||^2$

Predict the one-step noise that was added (and remove it)!

The Denoising (Decoding) Process

The **learned** denoising process $x_0 \leftarrow x_1 \leftarrow \cdots \leftarrow x_T$

$$p_\theta(x_{0:T}) = p(x_T)\prod_{t=1}^{T} p_\theta(x_{t-1}|x_t)$$

Probability Chain Rule (Markov Chain)

$$p_\theta(x_{t-1}|x_t) = \mathcal{N}(x_{t-1}; \mu_\theta(x_t, t), \Sigma_q(t))$$

Conditional Gaussian

We know how to learn ($\mu_\theta(x_t, t)$)

Assume fixed / known variance ($\Sigma_q(t)$)

Inference time: $\mu_\theta(x_t, t) = \frac{1}{\sqrt{\alpha_t}}\left(x_t - \frac{\beta_t}{\sqrt{(1-\bar{\alpha}_t)}}\epsilon_\theta(x_t, t)\right)$

The Denoising (Decoding) Process

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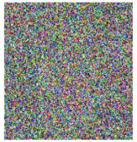
Conditional Gaussian

We know how to learn ($\mu_\theta(x_t, t)$)

Assume fixed / known variance ($\Sigma_q(t)$)

$x_T \sim \mathcal{N}(0, I)$



$p_\theta(x_T|x_{T-1})$ →

x_{T-1}

$p_\theta(x_{T-1}|x_{T-2})$ → … $p_\theta(x_1|x_0)$ →

x_0

Generate new images!

The Denoising Diffusion Algorithm

Algorithm 1 Training

1: **repeat**
2: $\mathbf{x}_0 \sim q(\mathbf{x}_0)$
3: $t \sim \text{Uniform}(\{1, \ldots, T\})$
4: $\boldsymbol{\epsilon} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$
5: Take gradient descent step on

$$\nabla_\theta \left\| \boldsymbol{\epsilon} - \boldsymbol{\epsilon}_\theta(\sqrt{\bar{\alpha}_t}\mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_t}\boldsymbol{\epsilon}, t) \right\|^2$$

6: **until** converged

The Denoising Diffusion Probabilistic Models, Ho *et al.*, 2020

The Denoising Diffusion Algorithm

Algorithm 1 Training

repeat
  $\mathbf{x}_0 \sim q(\mathbf{x}_0)$
  $t \sim \mathrm{Uniform}(\{1, \ldots, T\})$
  $\boldsymbol{\epsilon} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$
  Take gradient descent step on
  $$\nabla_\theta \left\| \boldsymbol{\epsilon} - \boldsymbol{\epsilon}_\theta(\sqrt{\bar{\alpha}_t}\mathbf{x}_0 + \sqrt{1-\bar{\alpha}_t}\boldsymbol{\epsilon}, t) \right\|^2$$
until converged

Algorithm 2 Sampling

$\mathbf{x}_T \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$
for $t = T, \ldots, 1$ **do**
  $\mathbf{z} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ if $t > 1$, else $\mathbf{z} = \mathbf{0}$
  $\mathbf{x}_{t-1} = \frac{1}{\sqrt{\alpha_t}} \left(\mathbf{x}_t - \frac{1-\alpha_t}{\sqrt{1-\bar{\alpha}_t}} \boldsymbol{\epsilon}_\theta(\mathbf{x}_t, t) \right) + \sigma_t \mathbf{z}$
end for
return $\mathbf{x}_0$

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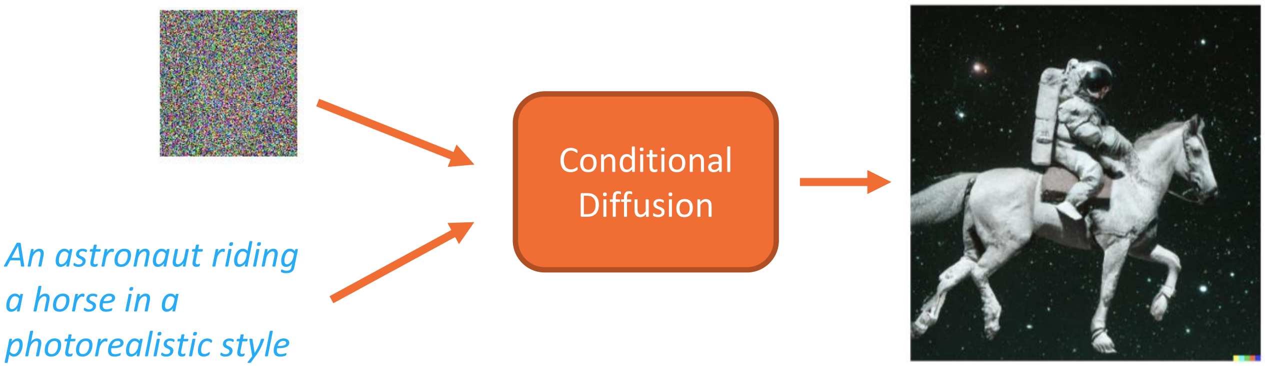
4: $\mathbf{x}_{t-1} = \frac{1}{\sqrt{\alpha_t}}\left(\mathbf{x}_t - \frac{1-\alpha_t}{\sqrt{1-\bar{\alpha}_t}}\boldsymbol{\epsilon}_\theta(\mathbf{x}_t, t)\right) + \sigma_t \mathbf{z}$
5: **end for**
6: **return** $\mathbf{x}_0$

$$p_\theta(x_{t-1}|x_t) = \mathcal{N}(x_{t-1}; \mu_\theta(x_t, t), \Sigma(t))$$

$$x_t = \sqrt{\bar{\alpha}_t}x_0 + \sqrt{1-\bar{\alpha}_t}\epsilon, \qquad \epsilon \sim \mathcal{N}(0, I)$$

The Denoising Diffusion Probabilistic Models, Ho *et al.*, 2020

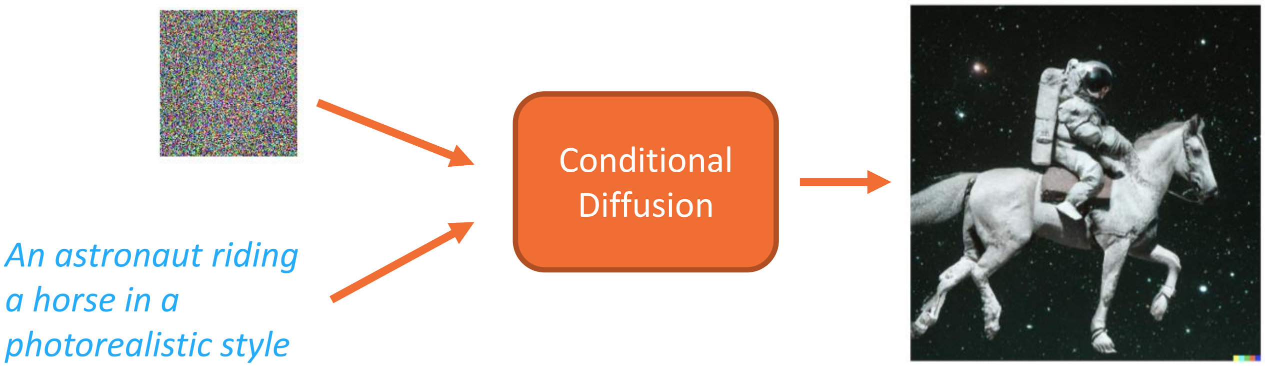
Conditional Diffusion Models



Simple idea: just condition the model on some text labels y!

$$\epsilon_\theta(x_t, y, t)$$

Conditional Diffusion Models

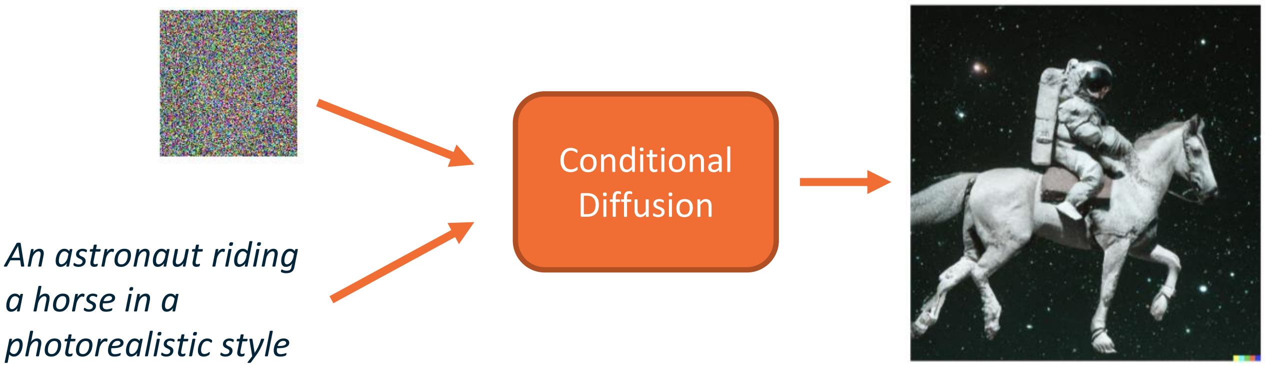


Simple idea: just condition the model on some text labels y!

$$\epsilon_\theta(x_t, y, t)$$

Problem: Very blurry generation

Classifier-guided Diffusion



Better idea: use the *gradients* from a image captioning model $f_\varphi(y|x_t)$ to guide the diffusion process!

$$\bar{\epsilon}_\theta(x_t, t) = \epsilon_\theta(x_t, t) - \sqrt{1 - \bar{\alpha}_t}\nabla_{x_t}\log f_\varphi(y|x_t)$$

Dhariwal & Nichol, 2021

Classifier guidance

Using the gradient of a trained classifier as guidance

Algorithm 1 Classifier guided diffusion sampling, given a diffusion model $(\mu_\theta(x_t), \Sigma_\theta(x_t))$, classifier $p_\phi(y|x_t)$, and gradient scale s.

Input: class label y, gradient scale s — Score model — Classifier gradient
$x_T \leftarrow$ sample from $\mathcal{N}(0, \mathbf{I})$
for all t from T to 1 **do**
  $\mu, \Sigma \leftarrow \mu_\theta(x_t), \Sigma_\theta(x_t)$
  $x_{t-1} \leftarrow$ sample from $\mathcal{N}(\mu + s\Sigma \nabla_{x_t} \log p_\phi(y|x_t), \Sigma)$
end for
return x_0

- Train unconditional Diffusion model
- Take your favorite classifier, depending on the conditioning type
- During inference / sampling mix the gradients of the classifier with the predicted score function of the unconditional diffusion model.

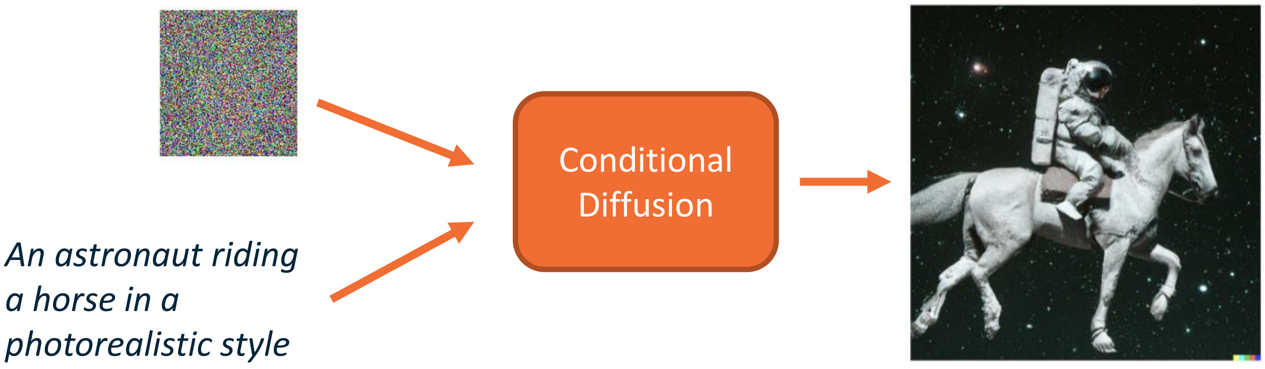
Classifier guidance

Using the gradient of a trained classifier as guidance

$$\nabla_x \log p_\gamma(x \mid y) = \nabla_x \log p(x) + \gamma \nabla_x \log p(y \mid x).$$

Samples from an unconditional diffusion model with classifier guidance, for guidance scales 1.0 (left) and 10.0 (right), taken from Dhariwal & Nichol (2021).

Classifier-free Guided Diffusion



Classifier-free Guided Diffusion: estimate the gradient of the classifier model with conditional diffusion models!

$$\nabla_{x_t} \log f_\varphi(y|x_t) = -\frac{1}{\sqrt{1-\bar{\alpha}_t}}\left(\epsilon_\theta(x_t, t, y) - \epsilon_\theta(x_t, t)\right)$$

Ho and Salimans, 2022

Classifier-free guidance

Trade-off for sample quality and sample diversity

Non-guidance

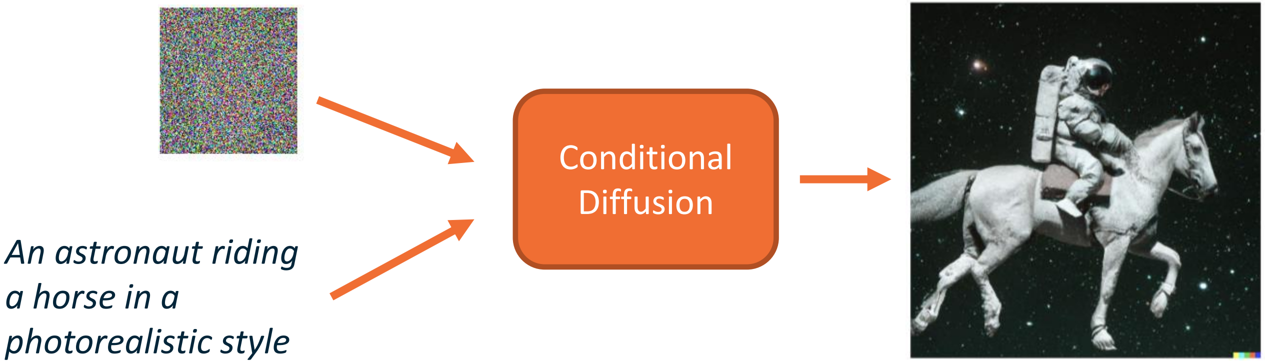
Guidance scale = 1

Guidance scale = 3

Large guidance weight (ω) usually leads to better individual sample quality but less sample diversity.

Ho & Salimans, "Classifier-Free Diffusion Guidance", 2021.

Classifier-free Guided Diffusion



Classifier-free Guided Diffusion: estimate the gradient of the classifier model with conditional diffusion models!

$$\nabla_{x_t} \log f_{\varphi}(y|x_t) = -\frac{1}{\sqrt{1-\bar{\alpha}_t}}\left(\epsilon_\theta(x_t, t, y) - \epsilon_\theta(x_t, t)\right)$$

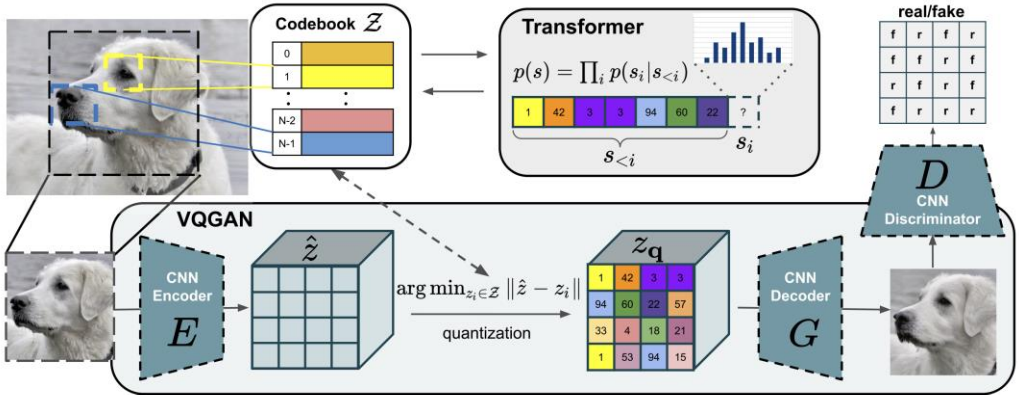
$$\bar{\epsilon}_\theta(x_t, t, y) = (w+1)\epsilon_\theta(x_t, t, y) - w\epsilon_\theta(x_t, t)$$

Ho and Salimans, 2022

Latent-space Diffusion

Problem: Hard to learn diffusion process on high-resolution images

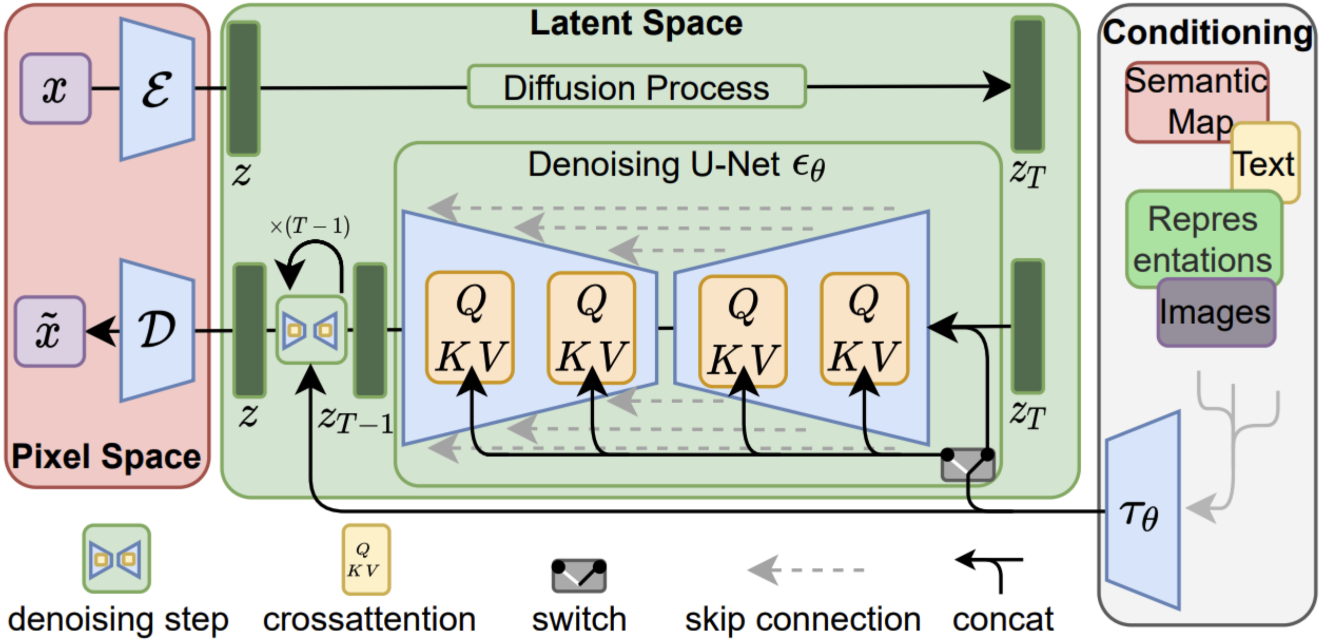
Solution: learn a low-dimensional latent space using a transformer-based autoencoder and *do diffusion on the latent space*!



The latent space autoencoder

Esser and Rombach *et al.*, 2021

“StableDiffusion”



Rombach and Blattmann *et al.*, 2022

“StableDiffusion”

Layout-Conditional Generation

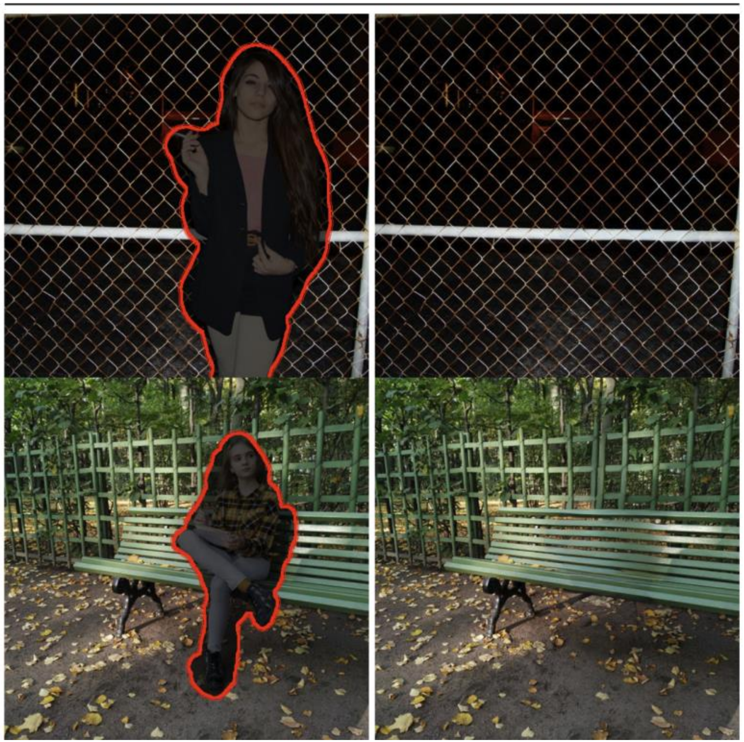
Rombach and Blattmann *et al.*, 2022

“StableDiffusion”

Segmentation-Conditional Generation

Rombach and Blattmann *et al.*, 2022

“StableDiffusion”



Inpainting

Rombach and Blattmann *et al.*, 2022

https://openai.com/dall-e-2/

Additional resources / tutorials

- Overview of the research landscape: What are Diffusion Models?
- More math! Understanding Diffusion Models: A Unified Perspective
- Tutorial with hands-on example: The Annotated Diffusion Model
- Nice introduction videos:
  - What are Diffusion Models?
  - Diffusion Models | Math Explained
  - Three hours of the math! https://www.youtube.com/watch?v=rLepfNziDPM
- CVPR Tutorial: Denoising Diffusion-based Generative Modeling: Foundations and Applications
- Score functions:
  - In general
  - For Diffusion models

Summary

- Denoising Diffusion model is a type of generative model that learns the process of “denoising” a known noise source (Gaussian).
- We can construct a learning problem by deriving the evidence lower bound (ELBO) of the denoising process.
- The learning objective is to minimize the KL divergence between the “ground truth” and the learned denoising distribution.
- A simplified learning objective is to estimate the noise of the forward diffusion process.
- The diffusion process can be guided to generate targeted samples.
- Can be applied to many different domains. Same underlying principle.
- Very hot topic!