Topics:

Convolutional Neural Networks

CS 4644-DL / 7643-A ZSOLT KIRA

Assignment 2

- Implement convolutional neural networks
- Resources (in addition to lectures):
 - DL book: Convolutional Networks
 - CNN notes https://www.cc.gatech.edu/classes/AY2022/cs7643 spring/assets/L10 cnns notes.pdf
 - Backprop notes
 https://www.cc.gatech.edu/classes/AY2023/cs7643 spring/assets/L10 cnns backprop notes.pdf
 - HW2 Tutorial, Conv backward @176
 - Slower OMSCS lectures on dropbox: Module 2 Lessons 5-6 (M2L5/M2L6) (https://www.dropbox.com/sh/iviro188gq0b4vs/AADdHxX_Uy1TkpF_yvlzX0nPa?dl=0)

Project

Info on @178

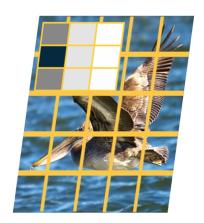
$$X(0:2,0:2) = \begin{bmatrix} 200 & 150 & 150 \\ 100 & 50 & 100 \\ 25 & 25 & 10 \end{bmatrix} \qquad K' = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

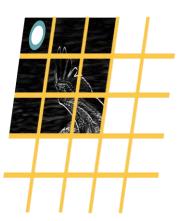
$$\mathsf{K}' = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$



 $X(0:2,0:2) \cdot K' = 65 + bias$

Dot product (element-wise multiply and sum)



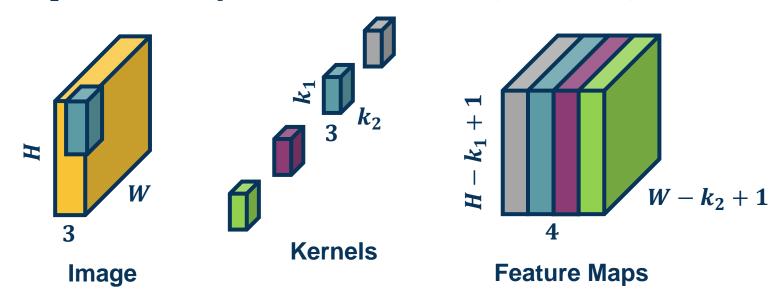




Number of parameters with N filters is: $N * (k_1 * k_2 * 3 + 1)$

Example:

$$k_1 = 3, k_2 = 3, N = 4 input channels = 3, then $(3 * 3 * 3 + 1) * 4 = 112$$$





Need to incorporate all upstream gradients:

$$\left(\frac{\partial L}{\partial y(0,0)}, \frac{\partial L}{\partial y(0,1)}, \dots, \frac{\partial L}{\partial y(H,W)}\right)$$

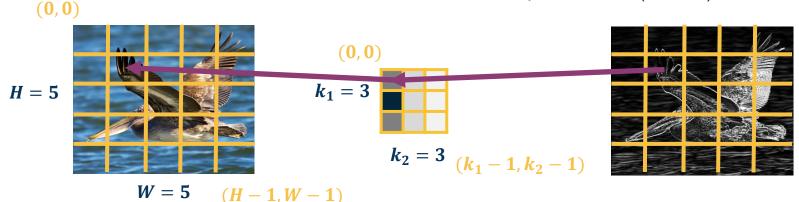
Chain Rule:

$$\frac{\partial L}{\partial k(a,b)} = \sum_{r=0}^{H-1} \sum_{c=0}^{W-1} \frac{\partial L}{\partial y(r,c)} \frac{\partial y(r,c)}{\partial k(a,b)}$$

Sum over all output pixels

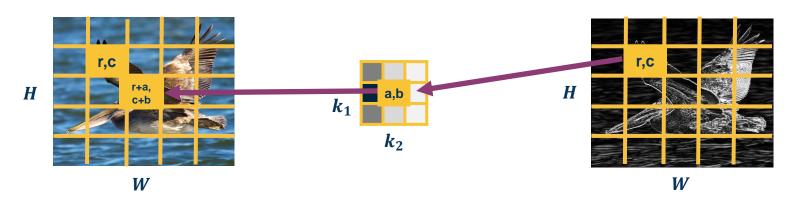
Upstream gradient (known)

We will compute



$$\frac{\partial y(r,c)}{\partial k(a,b)} = x(r+a,c+b)$$

$$\frac{\partial L}{\partial k(a,b)} = \sum_{r=0}^{H-1} \sum_{c=0}^{W-1} \frac{\partial L}{\partial y(r,c)} x(r+a,c+b)$$

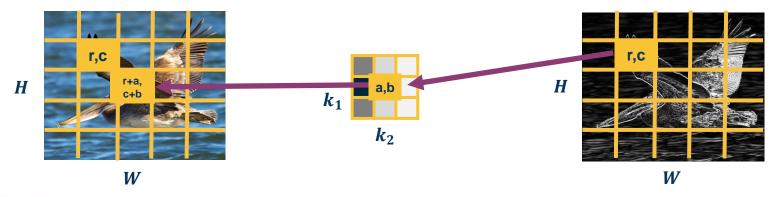


$$\frac{\partial y(r,c)}{\partial k(a,b)} = x(r+a,c+b)$$

$$\frac{\partial L}{\partial k(a,b)} = \sum_{r=0}^{H-1} \sum_{c=0}^{W-1} \frac{\partial L}{\partial y(r,c)} x(r+a,c+b)$$

Does this look familiar?

Cross-correlation between upstream gradient and input! (until $k_1 \times k_2$ output)





$$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial y} \quad \frac{\partial y}{\partial x}$$

Gradient for input (to pass to prior layer)

Calculate one pixel at a time $\frac{\partial L}{\partial x(r',c')}$

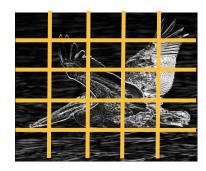
What does this input pixel affect at the output?

Neighborhood around it (where part of the kernel touches it)

$$H = 5$$

$$r',c'$$

$$k_1 = 3$$
 $k_2 = 3$
 $(k_1 - 1, k_2 - 1)$



$$W = 5 \qquad (H - 1, W - 1)$$

Chain rule for affected pixels (sum gradients):

$$\frac{\partial L}{\partial x(r',c')} = \sum_{Pixels \, p} \frac{\partial L}{\partial y(p)} \frac{\partial y(p)}{\partial x(r',c')}$$

$$\frac{\partial L}{\partial x(r',c')} = \sum_{a=0}^{k_1-1} \sum_{b=0}^{k_2-1} \frac{\partial L}{\partial y(?,?)} \frac{\partial y(?,?)}{\partial x(r',c')}$$

$$H = 5$$

$$W = 5$$

$$x(r',c') * k(0,0) \Rightarrow y(r',c')$$

$$x(r',c') * k(1,1) \Rightarrow y(r'-1,c'-1)$$
...
$$x(r',c') * k(a,b) \Rightarrow y(r'-a,c'-b)$$

$$(r'-k_1+1,c'-k_2+1)$$

1

2

 r',c'

3

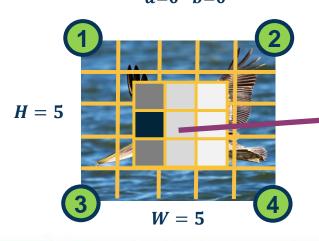


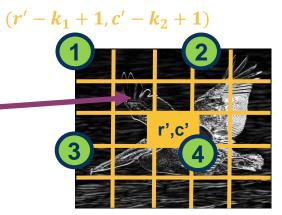
Chain rule for affected pixels (sum gradients):

$$\frac{\partial L}{\partial x(r',c')} = \sum_{Pixels\ p} \frac{\partial L}{\partial y(p)} \frac{\partial y(p)}{\partial x(r',c')}$$

Let's derive it analytically this time (as opposed to visually)

$$\frac{\partial L}{\partial x(r',c')} = \sum_{a=0}^{k_1-1} \sum_{b=0}^{k_2-1} \frac{\partial L}{\partial y(r'-a,c'-b)} \frac{\partial y(r'-a,c'-b)}{\partial x(r',c')}$$







Plugging in to earlier equation:

$$\frac{\partial L}{\partial x(r',c')} = \sum_{a=0}^{k_1-1} \sum_{b=0}^{k_2-1} \frac{\partial L}{\partial y(r'-a,c'-b)} \frac{\partial y(r'-a,c'-b)}{\partial x(r',c')}$$

$$=\sum_{a=0}^{k_1-1}\sum_{b=0}^{k_2-1}\frac{\partial L}{\partial y(r'-a,c'-b)}k(a,b)$$

Again, all operations can be implemented via matrix multiplications (same as FC layer)!

Does this look familiar?

Convolution between upstream gradient and kernel!

(can implement by flipping kernel and cross- correlation)



- Convolutions are mathematical descriptions of striding linear operation
- In practice, we implement cross-correlation neural networks! (still called convolutional neural networks due to history)
 - Can connect to convolutions via duality (flipping kernel)
 - Convolution formulation has mathematical properties explored in ECE
- Duality for forwards and backwards:
 - Forward: Cross-correlation
 - Backwards w.r.t. K: Cross-correlation b/w upstream gradient and input
 - Backwards w.r.t. X: Convolution b/w upstream gradient and kernel
 - In practice implement via cross-correlation and flipped kernel
- All operations still implemented via efficient linear algebra (e.g. matrixmatrix multiplication)

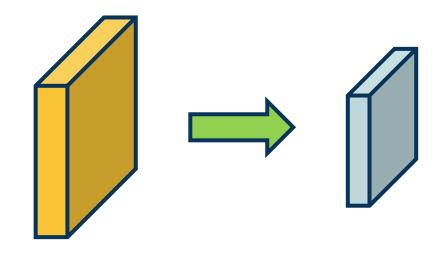


Pooling Layers



- Dimensionality reduction is an important aspect of machine learning
- Can we make a layer to explicitly down-sample image or feature maps?

Yes! We call one class of these operations pooling operations



Parameters

- kernel_size the size of the window to take a max over
- stride the stride of the window. Default value is kernel size
- **padding** implicit zero padding to be added on both sides

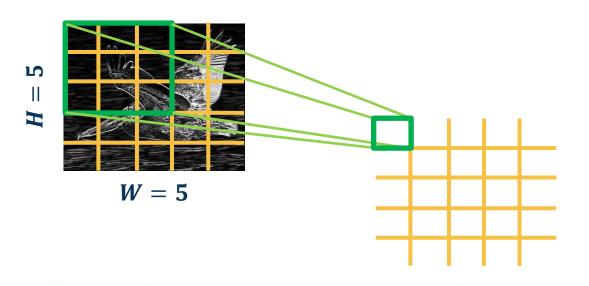
From: https://pytorch.org/docs/stable/generated/torch.nn.MaxPool2d.html#torch.html#to



Example: Max pooling

Stride window across image but perform per-patch max operation

$$X(0:2,0:2) = \begin{bmatrix} 200 & 150 & 150 \\ 100 & 50 & 100 \\ 25 & 25 & 10 \end{bmatrix} \longrightarrow \max(0:2,0:2) = 200$$

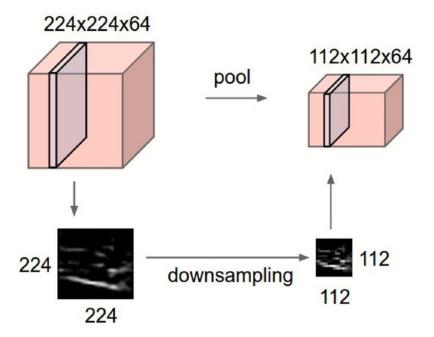


How many learned parameters does this layer have?

None!



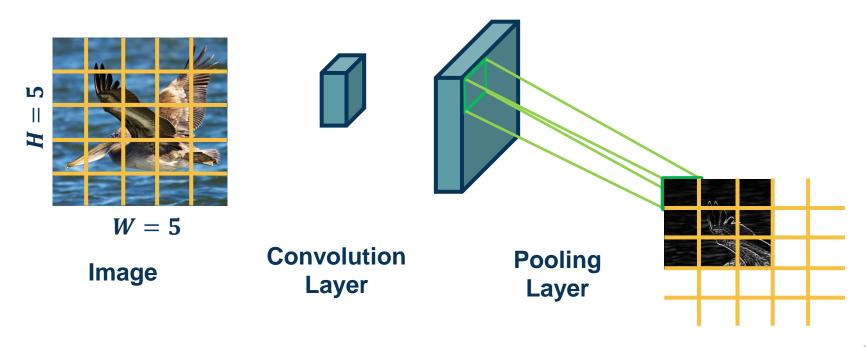
- makes the representations spatially smaller
- saves computation (GPU mem & speed), allows go deeper
- operates over each activation map independently:



From: Slides by CS 231n, Dantei Xu



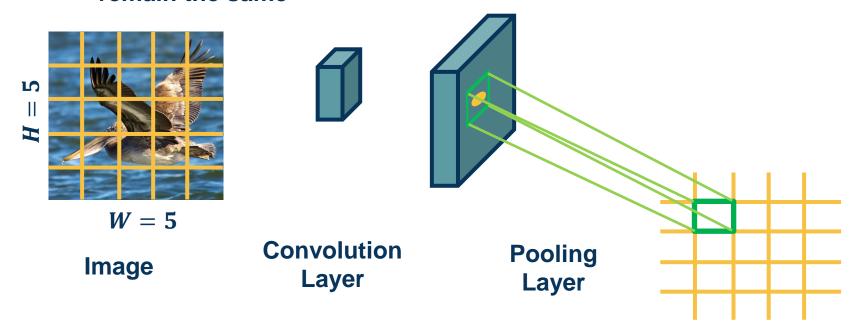
Since the **output** of convolution and pooling layers are **(multi-channel) images**, we can sequence them just as any other layer





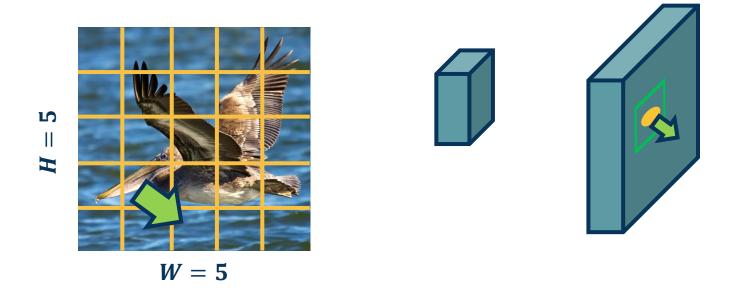
This combination adds some **invariance** to translation of the features

If feature (such as beak) translated a little bit, output values still
 remain the same



Convolution by itself has the property of equivariance

 If feature (such as beak) translated a little bit, output values move by the same translation

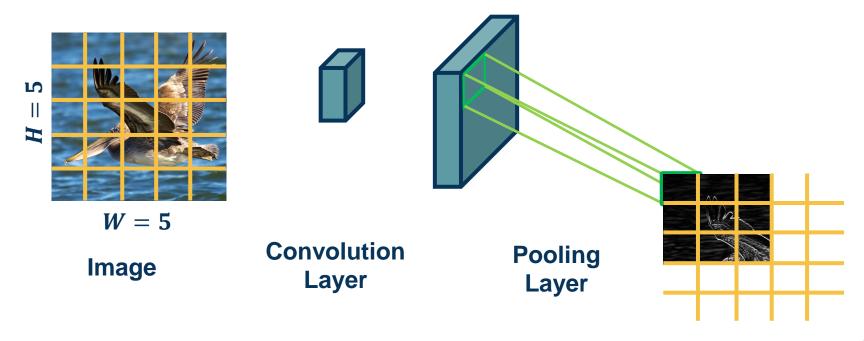




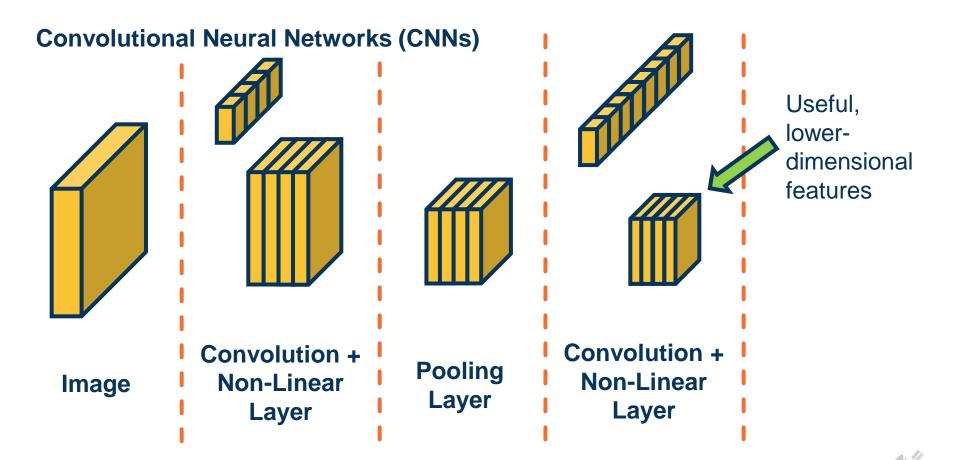
Simple
Convolutional
Neural
Networks



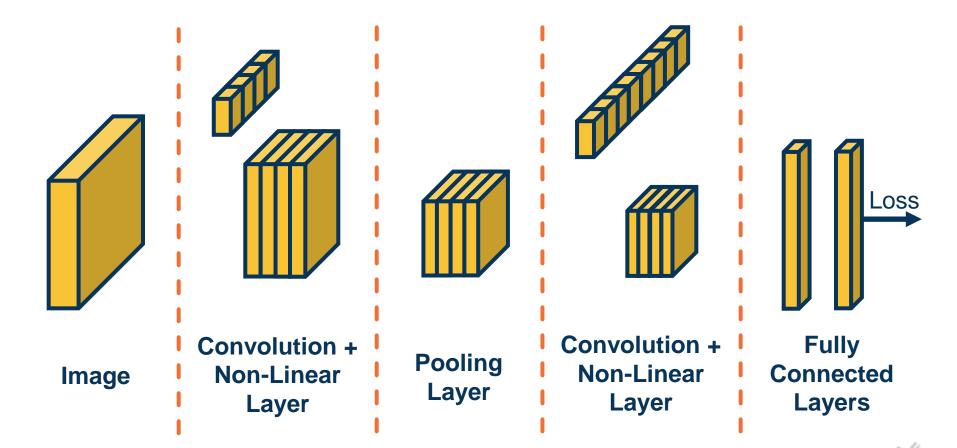
Since the **output** of convolution and pooling layers are **(multi-channel) images**, we can sequence them just as any other layer

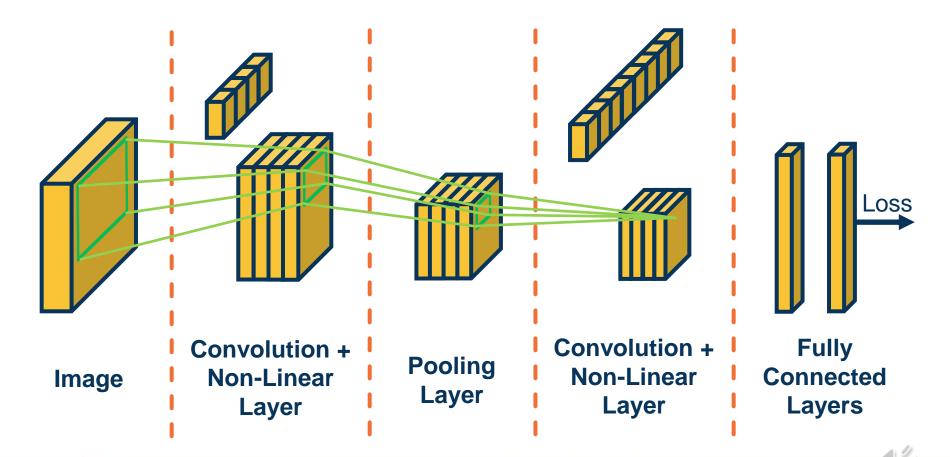


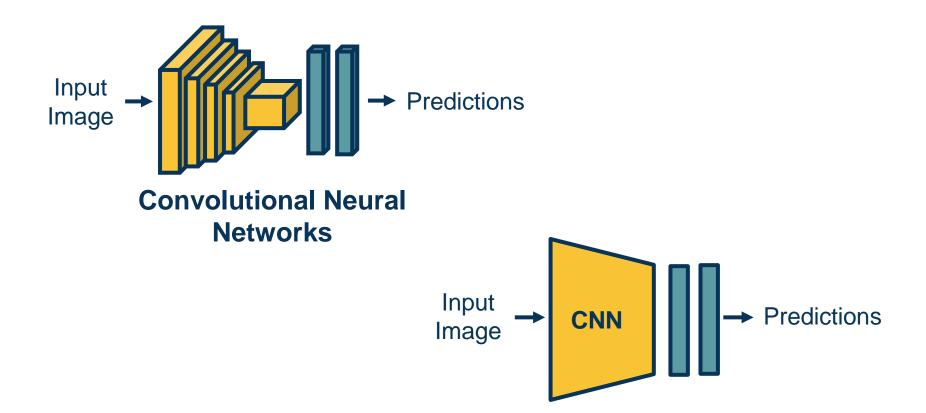












These architectures have existed **since 1980s**

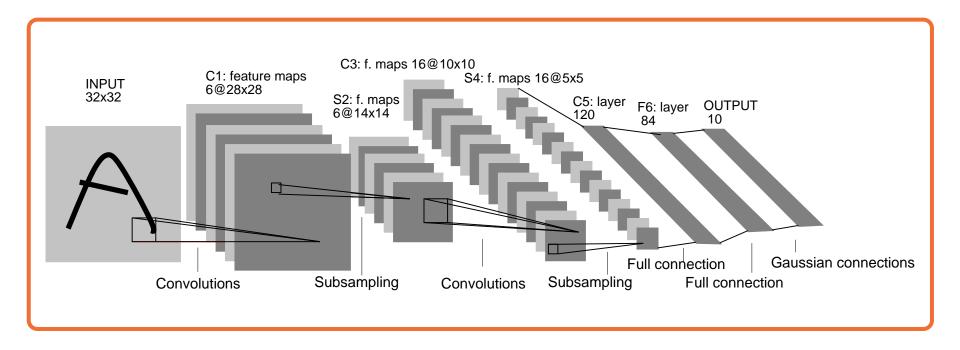


Image Credit: Yann LeCun, Kevin Murchy



Handwriting Recognition



Image Credit:
Yann LeCun
Georgan

Translation Equivariance (Conv Layers) & Invariance (Output)



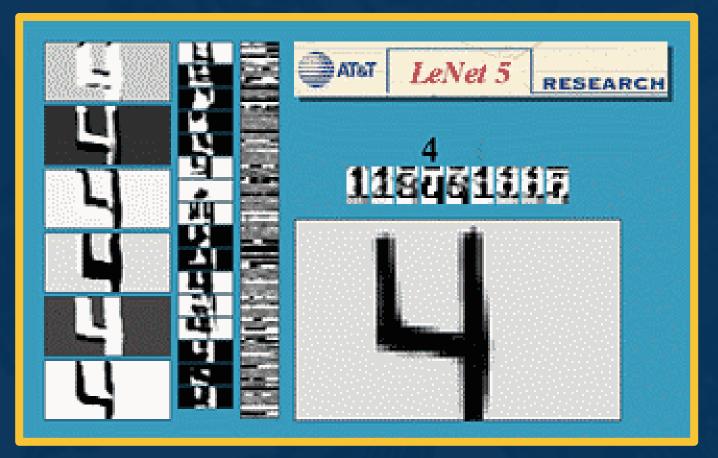


(Some) Rotation Invariance

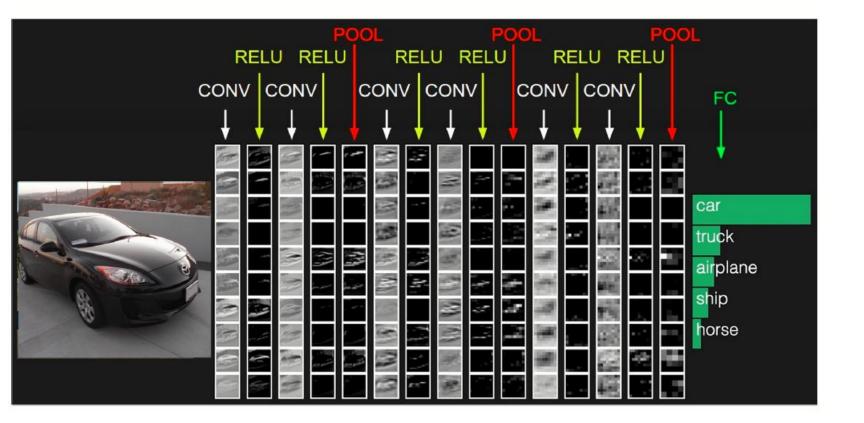




(Some) Scale Invariance









Advanced Convolutional Networks





The **ImageNet** dataset contains 14,197,122 annotated images according to the WordNet hierarchy. ImageNet Large Scale Visual Recognition Challenge (ILSVRC) is a benchmark for image classification and object detection based on the dataset.



The Importance of Benchmarks

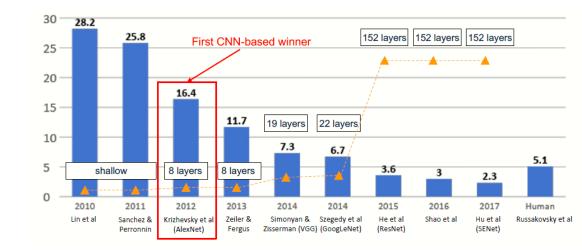




From: https://paperswithcode.com

Case Studies

- AlexNet
- VGG
- GoogLeNet
- ResNet



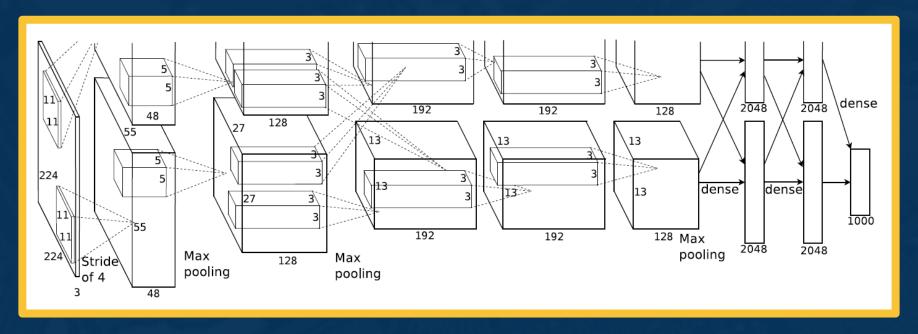
Also....

- SENet
- Wide ResNet
- ResNeXT

- DenseNet
- MobileNets
- NASNet
- EfficientNet
- ConvNeXt v1/v2



AlexNet - Architecture



From: Krizhevsky et al., ImageNet Classification with Deep ConvolutionalNeural Networks, 2012.



Case Study: AlexNet

[Krizhevsky et al. 2012]

Architecture:

CONV1

MAX POOL1

NORM1

CONV2

MAX POOL2

NORM2

CONV3

CONV4

CONV5

Max POOL3

FC6

FC7

FC8

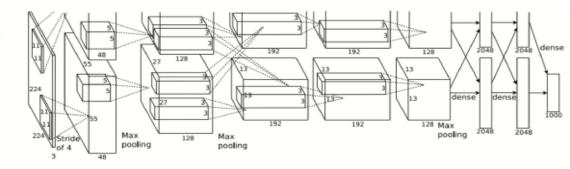
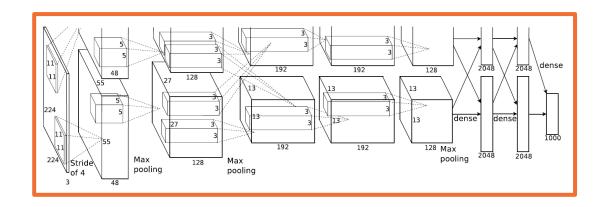


Figure copyright Alex Krizhevsky, Ilya Sutskever, and Geoffrey Hinton, 2012. Reproduced with permission.



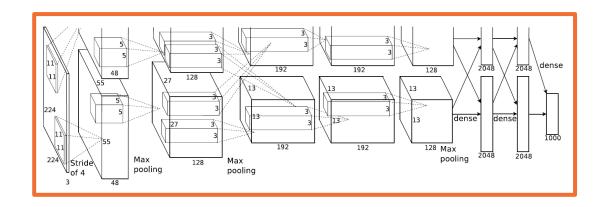


First layer (CONV1): 96 11x11 filters applied at stride 4

W' = (W - F + 2P) / S + 1

=>

Q: what is the output volume size? Hint: (227-11)/4+1 = 55

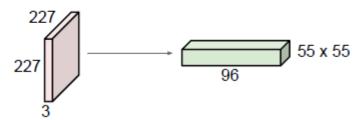


First layer (CONV1): 96 11x11 filters applied at stride 4

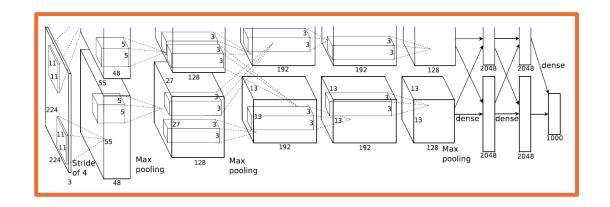
W' = (W - F + 2P) / S + 1

=>

Output volume [55x55x96]





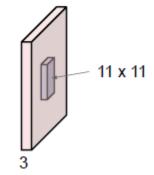


First layer (CONV1): 96 11x11 filters applied at stride 4

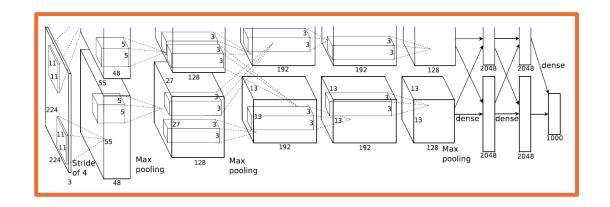
=>

Output volume [55x55x96]

Q: What is the total number of parameters in this layer?





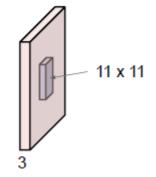


First layer (CONV1): 96 11x11 filters applied at stride 4

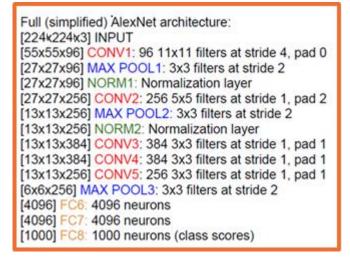
=>

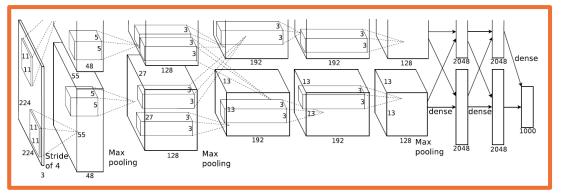
Output volume [55x55x96]

Parameters: (11*11*3 + 1)*96 = **35K**









Key aspects:

- ReLU instead of sigmoid or tanh
- Specialized normalization layers
- PCA-based data augmentation
- Dropout
- Ensembling



Small filters, Deeper networks

8 layers (AlexNet)
-> 16 - 19 layers (VGG16Net)

Only 3x3 CONV stride 1, pad 1 and 2x2 MAX POOL stride 2

11.7% top 5 error in ILSVRC'13 (ZFNet)

-> 7.3% top 5 error in ILSVRC'14

Softmax	
FC 1000	
FC 4098	
FC 4098	
Pool	
3x3 conv, 256	
3x3 conv, 384	
Pool	
3x3 conv, 384	
Pool	
5x5 conv, 256	
11x11 conv, 96	
Input	
AlexNet	

	Sommax
	FC 1000
Softmax	FC 4096
FC 1000	FC 4096
FC 4096	Pool
FC 4096	3x3 conv, 512
Pool	3x3 conv, 512
3x3 conv, 512	3x3 conv, 512
3x3 conv, 512	3x3 conv, 512
3x3 conv, 512	Pool
Pool	3x3 conv, 512
3x3 conv, 512	3x3 conv, 512
3x3 conv, 512	3x3 conv, 512
3x3 conv, 512	3x3 conv, 512
Pool	Pool
3x3 conv, 258	3x3 conv, 256
3x3 conv, 256	3x3 conv, 256
Pool	Pool
3x3 conv, 128	3x3 conv, 128
3x3 conv, 128	3x3 conv, 128
Pool	Pool
3x3 conv, 64	3x3 conv, 64
3x3 conv, 64	3x3 conv, 64
Input	Input
VGG16	VGG19



Q: Why use smaller filters? (3x3 conv)

Stack of three 3x3 conv (stride 1) layers has same **effective receptive field** as one 7x7 conv layer

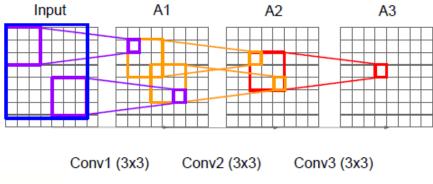
Q: What is the effective receptive field of three 3x3 conv (stride 1) layers?

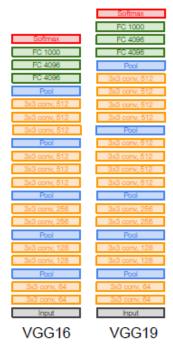


FC 1000 FC 4096



Q: What is the effective receptive field of three 3x3 conv (stride 1) layers?





But deeper, more non-linearities

And fewer parameters: $3 * (3^2C^2)$ vs. 7^2C^2 for C channels per layer



```
(not counting biases)
INPUT: [224x224x3]
                     memory: 224*224*3=150K params: 0
CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*3)*64 = 1.728
CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*64)*64 = 36,864
POOL2: [112x112x64] memory: 112*112*64=800K params: 0
CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*64)*128 = 73,728
CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*128)*128 = 147,456
POOL2: [56x56x128] memory: 56*56*128=400K params: 0
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*128)*256 = 294,912
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589.824
POOL2: [28x28x256] memory: 28*28*256=200K params: 0
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*256)*512 = 1,179,648
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296
POOL2: [14x14x512] memory: 14*14*512=100K params: 0
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
POOL2: [7x7x512] memory: 7*7*512=25K params: 0
FC: [1x1x4096] memory: 4096 params: 7*7*512*4096 = 102,760,448
FC: [1x1x4096] memory: 4096 params: 4096*4096 = 16,777,216
FC: [1x1x1000] memory: 1000 params: 4096*1000 = 4.096.000
```

ConvNet Configuration					
A	A-LRN	В	C	D	E
11 weight	11 weight	13 weight	16 weight	16 weight	19 weight
layers	layers	layers	layers	layers	layers
	i	nput (224×2)	24 RGB image	e)	
conv3-64	conv3-64	conv3-64	conv3-64	conv3-64	conv3-64
	LRN	conv3-64	conv3-64	conv3-64	conv3-64
			pool		
conv3-128	conv3-128	conv3-128	conv3-128	conv3-128	conv3-128
		conv3-128	conv3-128	conv3-128	conv3-128
			pool		
conv3-256	conv3-256	conv3-256	conv3-256	conv3-256	conv3-256
conv3-256	conv3-256	conv3-256	conv3-256	conv3-256	conv3-256
			conv1-256	conv3-256	conv3-256
					conv3-256
			pool		
conv3-512	conv3-512	conv3-512	conv3-512	conv3-512	conv3-512
conv3-512	conv3-512	conv3-512	conv3-512	conv3-512	conv3-512
			conv1-512	conv3-512	conv3-512
					conv3-512
			pool		
conv3-512	conv3-512	conv3-512	conv3-512	conv3-512	conv3-512
conv3-512	conv3-512	conv3-512	conv3-512	conv3-512	conv3-512
			conv1-512	conv3-512	conv3-512
					conv3-512
	maxpool				
	FC-4096				
	FC-4096				
FC-1000					
soft-max					

Table 2: Number of parameters (in millions).

Network	A,A-LRN	В	C	D	Е
Number of parameters	133	133	134	138	144

From: Simonyan & Zimmerman, Very Deep Convolutional Networks for Large-Scale Image Recognition From: Slides by Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231r/



```
(not counting biases)
INPUT: [224x224x3]
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POOL2: [112x112x64] memory: 112*112*64=800K params: 0
CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*64)*128 = 73,728
CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*128)*128 = 147,456
POOL2: [56x56x128] memory: 56*56*128=400K params: 0
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POOL2: [28x28x256] memory: 28*28*256=200K params: 0
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*256)*512 = 1,179,648
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296
POOL2: [14x14x512] memory: 14*14*512=100K params: 0
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
POOL2: [7x7x512] memory: 7*7*512=25K params: 0
FC: [1x1x4096] memory: 4096 params: 7*7*512*4096 = 102,760,448
FC: [1x1x4096] memory: 4096 params: 4096*4096 = 16,777,216
FC: [1x1x1000] memory: 1000 params: 4096*1000 = 4.096.000
```

Most memory usage in convolution layers

Most parameters in FC layers

From: Simonyan & Zimmerman, Very Deep Convolutional Networks for Large-Scale Image Recognition From: Slides by Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231r/



Case Study: VGGNet

[Simonyan and Zisserman, 2014]

Details:

- ILSVRC'14 2nd in classification, 1st in localization
- Similar training procedure as Krizhevsky 2012
- No Local Response Normalisation (LRN)
- Use VGG16 or VGG19 (VGG19 only slightly better, more memory)
- Use ensembles for best results
- FC7 features generalize well to other tasks

Still very expensive!

TOTAL memory: 24M * 4 bytes ~= 96MB / image

(only forward! ~*2 for bwd)

TOTAL params: 138M parameters

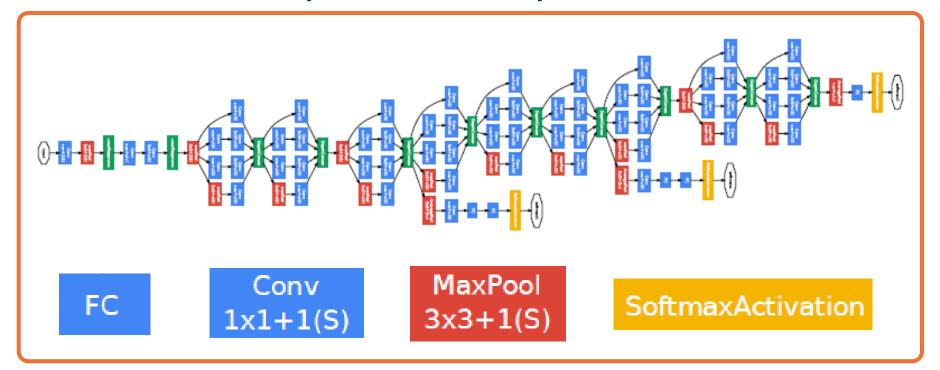


		FC 1000
	Softmax	FC 4098
fc8	FC 1000	FC 4098
fc7	FC 4098	Pool
fc6	FC 4098	3x3 conv, 512
	Pool	3x3 conv. 512
conv5-3	3x3 conv, 512	3x3 conv. 512
conv5-2	3x3 conv, 512	3x3 conv. 512
conv5-1	3x3 conv. 512	Pool
	Pool	3x3 conv. 512
conv4-3	3x3 conv. 512	3x3 conv, 512
conv4-2	3x3 conv, 512	3x3 conv, 512
conv4-1	3x3 conv, 512	3x3 conv. 512
	Pool	Pool
conv3-2	3x3 conv, 256	3x3 conv, 256
conv3-1	3x3 conv. 256	3x3 conv. 258
	Pool	Pool
conv2-2	3x3 conv. 128	3x3 conv. 128
conv2-1	3x3 conv. 128	3x3 conv. 128
	Pool	Pool
conv1-2	3x3 conv, 64	3x3 conv, 64
conv1-1	3x3 conv, 64	3x3 conv, 64
I	Input	Input
	1/00/40	1/00/10

VGG16 VGG19



But have become deeper and more complex

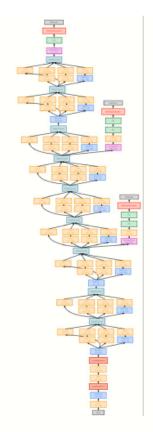




[Szegedy et al., 2014]

Deeper networks, focus on computational efficiency

- ILSVRC'14 classification winner (6.7% top 5 error)
- 22 layers
- Only 5 million parameters!
 12x less than AlexNet
 27x less than VGG-16
- Efficient "Inception" module
- No FC layers





[Szegedy et al., 2014]

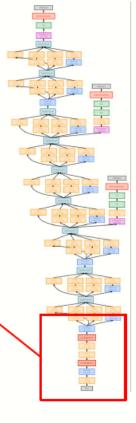
Deeper networks, focus on computational efficiency

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Stem Network: aggressively reduce the input feature volume

- Conv 7 x 7 x 64 with stride 2
- MaxPool
- Conv 1 x 1 x 64
- Conv 3 x 3 x 192
- MaxPool

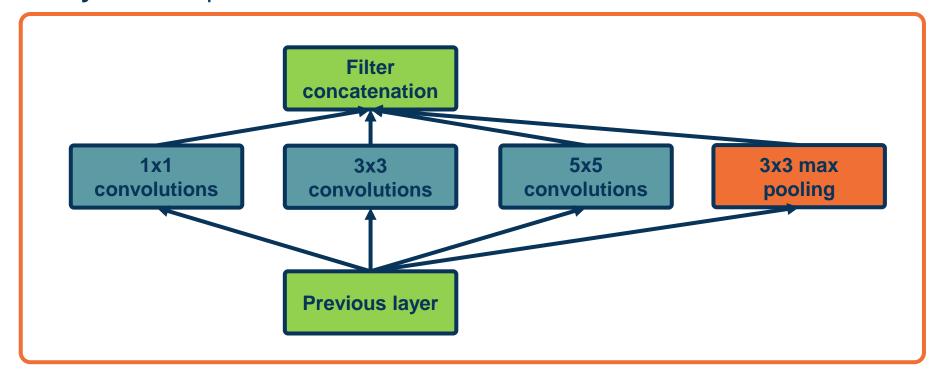
Reduce 224 x 224 spatial solution to 28 x 28 with just 418 MFLOP! (Comparing to 7485 MFLOP of VGG)



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Key idea: Repeated blocks and multi-scale features

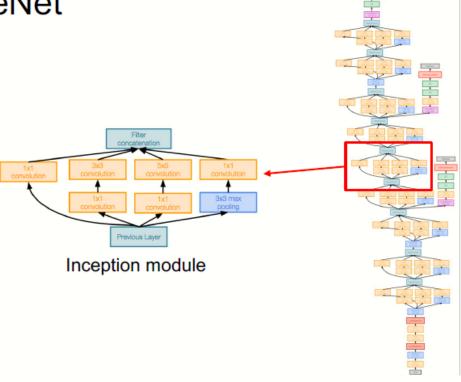




[Szegedy et al., 2014]

"Inception module": design a good local network topology (network within a network) and then stack these modules on top of each other

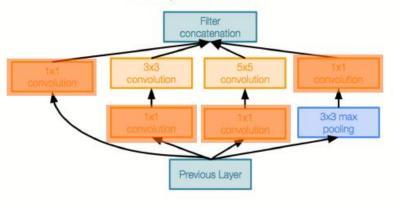
Multiple conv filter size diversifies learned features





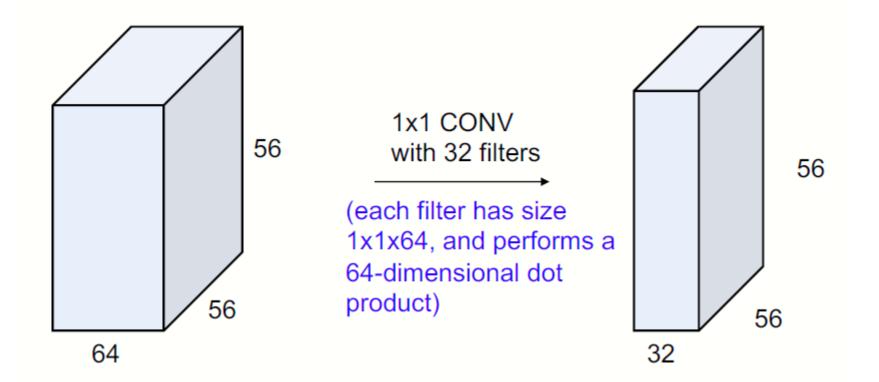
[Szegedy et al., 2014]

Inception module



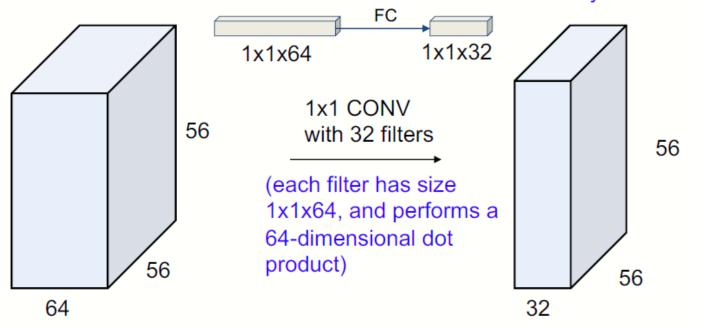
Uses 1x1 "Bottleneck" layers to reduce channel dimension before expensive conv (we will revisit this with ResNet!)





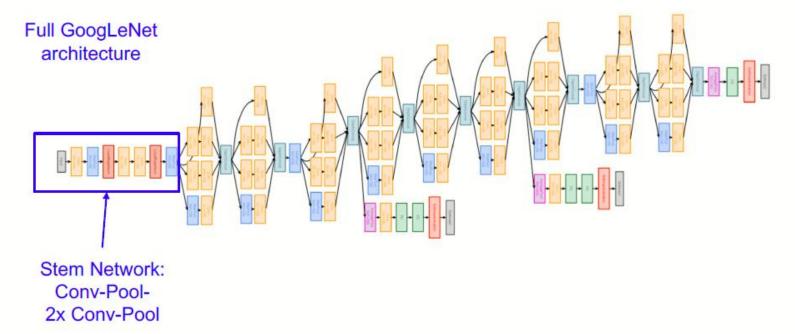


Alternatively, interpret it as applying the same FC layer on each input pixel



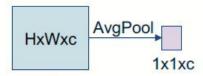


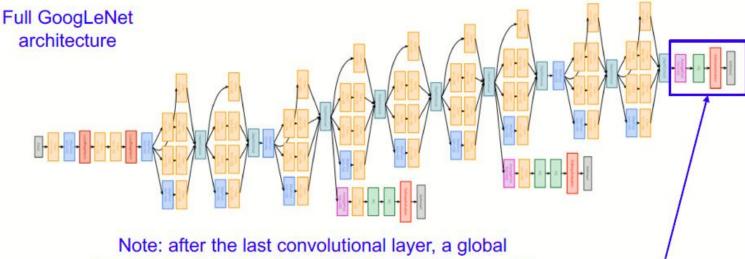
[Szegedy et al., 2014]





[Szegedy et al., 2014]





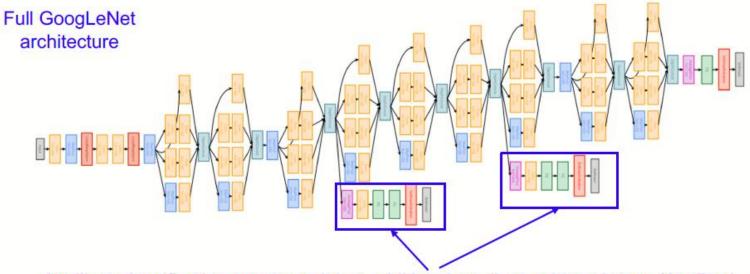
Note: after the last convolutional layer, a global average pooling layer is used that **spatially** averages across each feature map, before final FC layer. No longer multiple expensive FC layers!

(Also used in ResNet)

Classifier output



[Szegedy et al., 2014]



Auxiliary classification outputs to inject additional gradient at lower layers (AvgPool-1x1Conv-FC-FC-Softmax)

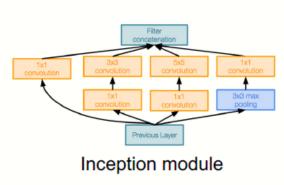
Why?

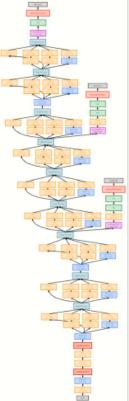


[Szegedy et al., 2014]

Deeper networks, with computational efficiency

- 22 layers
- Efficient "Inception" module
- Avoids expensive FC layers
- 12x less params than AlexNet
- 27x less params than VGG-16
- ILSVRC'14 classification winner (6.7% top 5 error)







 Convolutional neural networks (CNNs) stack pooling, convolution, nonlinearities, and fully connected (FC) layers

- Feature engineering => architecture engineering!
 - Tons of small details and tips/tricks
 - Considerations: Memory, compute/FLO, dimensionality reduction, diversity of features, number of parameters/capacity, etc.

