Topics:

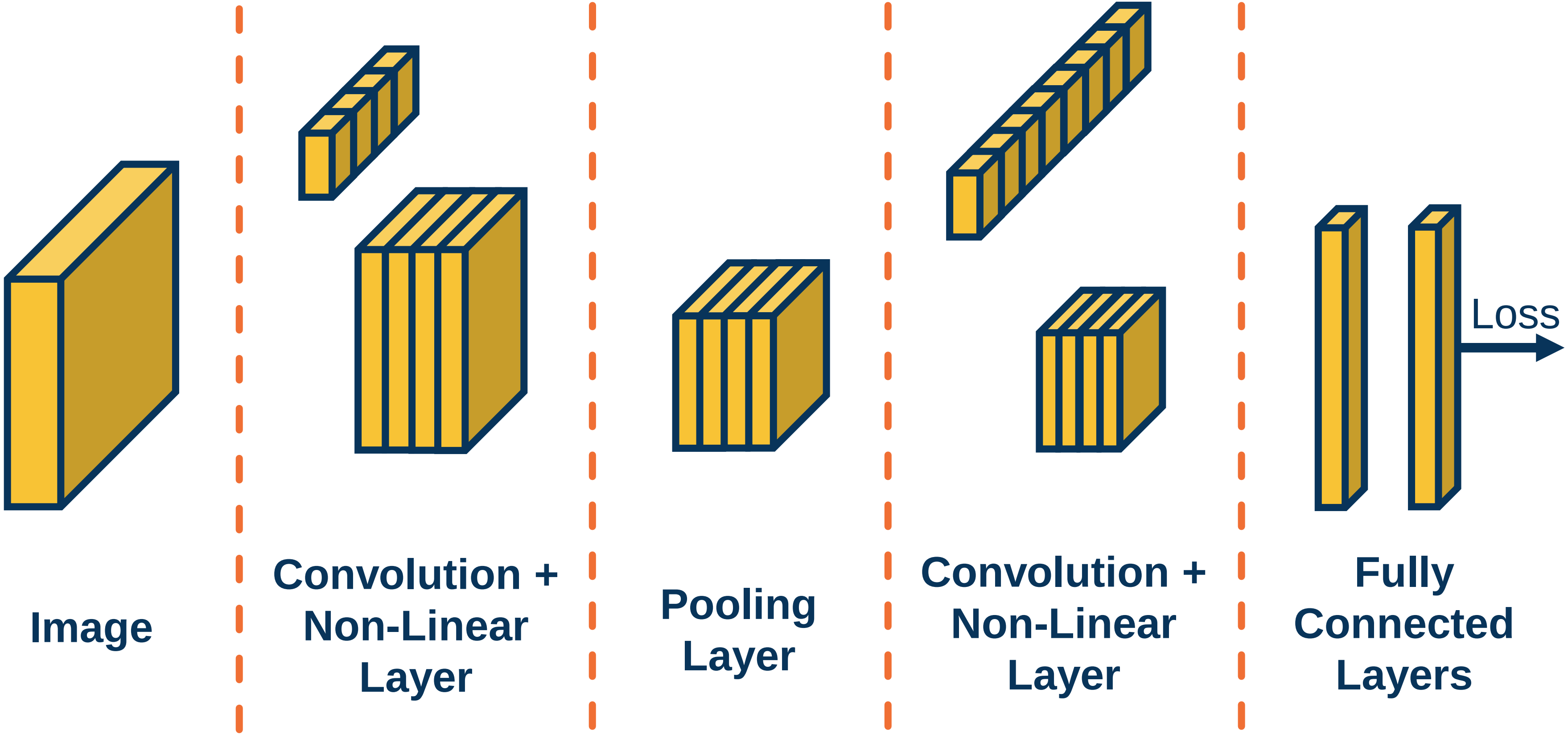
- Recurrent Neural Networks (RNNs)
- Long Short-Term Memory

CS 4644-DL / 7643-A
ZSOLT KIRA

- **Assignment 2 – Due TODAY! (grace period 19th)**
  - Implement convolutional neural networks
  - Resources (in addition to lectures):
    - DL book: Convolutional Networks
    - CNN notes https://www.cc.gatech.edu/classes/AY2022/cs7643_spring/assets/L10_cnns_notes.pdf
    - Backprop notes https://www.cc.gatech.edu/classes/AY2023/cs7643_spring/assets/L10_cnns_backprop_notes.pdf
    - **HW2 Tutorial (@176), Conv backward (@181)**
    - Slower OMSCS lectures on dropbox: Module 2 Lessons 5-6 (M2L5/M2L6) (https://www.dropbox.com/sh/iviro188gq0b4vs/AADdHxX_Uy1TkpF_yvIzX0nPa?dl=0)

- **FB/Meta Office hours Friday 02/21 3pm EST!**
  - Attention/language Models

- **GPU resources**: PACE-ICE announced



Adding a Fully Connected Layer

layer name	output size	18-layer	34-layer			
conv1	112×112					
conv2_x	56×56					
		$\begin{bmatrix} 3\times3, 64 \\ 3\times3, 64 \end{bmatrix}\times2$	$\begin{bmatrix} 3\times3, 64 \\ 3\times3, 64 \end{bmatrix}\times3$			
conv3_x	28×28	$\begin{bmatrix} 3\times3, 128 \\ 3\times3, 128 \end{bmatrix}\times2$	$\begin{bmatrix} 3\times3, 128 \\ 3\times3, 128 \end{bmatrix}\times4$			
conv4_x	14×14	$\begin{bmatrix} 3\times3, 256 \\ 3\times3, 256 \end{bmatrix}\times2$	$\begin{bmatrix} 3\times3, 256 \\ 3\times3, 256 \end{bmatrix}\times6$			
conv5_x	7×7	$\begin{bmatrix} 3\times3, 512 \\ 3\times3, 512 \end{bmatrix}\times2$	$\begin{bmatrix} 3\times3, 512 \\ 3\times3, 512 \end{bmatrix}\times3$			
	1×1	ave				
FLOPs		1.8×10^9	3.6×10^9	3.8×10^9	7.6×10^9	11.3×10^9

```
>>> import torch
>>> from torchvision.models import resnet18
>>> model = resnet18()
>>> summary(model2, (3, 224, 224), device='cpu')
----------------------------------------------------------------
        Layer (type)               Output Shape         Param #
================================================================
            Conv2d-1         [-1, 64, 112, 112]           9,408
       BatchNorm2d-2         [-1, 64, 112, 112]             128
              ReLU-3         [-1, 64, 112, 112]               0
         MaxPool2d-4           [-1, 64, 56, 56]               0
            Conv2d-5           [-1, 64, 56, 56]          36,864
       BatchNorm2d-6           [-1, 64, 56, 56]             128
              ReLU-7           [-1, 64, 56, 56]               0
            Conv2d-8           [-1, 64, 56, 56]          36,864
       BatchNorm2d-9           [-1, 64, 56, 56]             128
             ReLU-10           [-1, 64, 56, 56]               0
       BasicBlock-11           [-1, 64, 56, 56]               0
           Conv2d-12           [-1, 64, 56, 56]          36,864
      BatchNorm2d-13           [-1, 64, 56, 56]             128
             ReLU-14           [-1, 64, 56, 56]               0
           Conv2d-15           [-1, 64, 56, 56]          36,864
      BatchNorm2d-16           [-1, 64, 56, 56]             128
             ReLU-17           [-1, 64, 56, 56]               0
       BasicBlock-18           [-1, 64, 56, 56]               0
           Conv2d-19          [-1, 128, 28, 28]          73,728
      BatchNorm2d-20          [-1, 128, 28, 28]             256
             ReLU-21          [-1, 128, 28, 28]               0
           Conv2d-22          [-1, 128, 28, 28]         147,456
```

ResNet Details

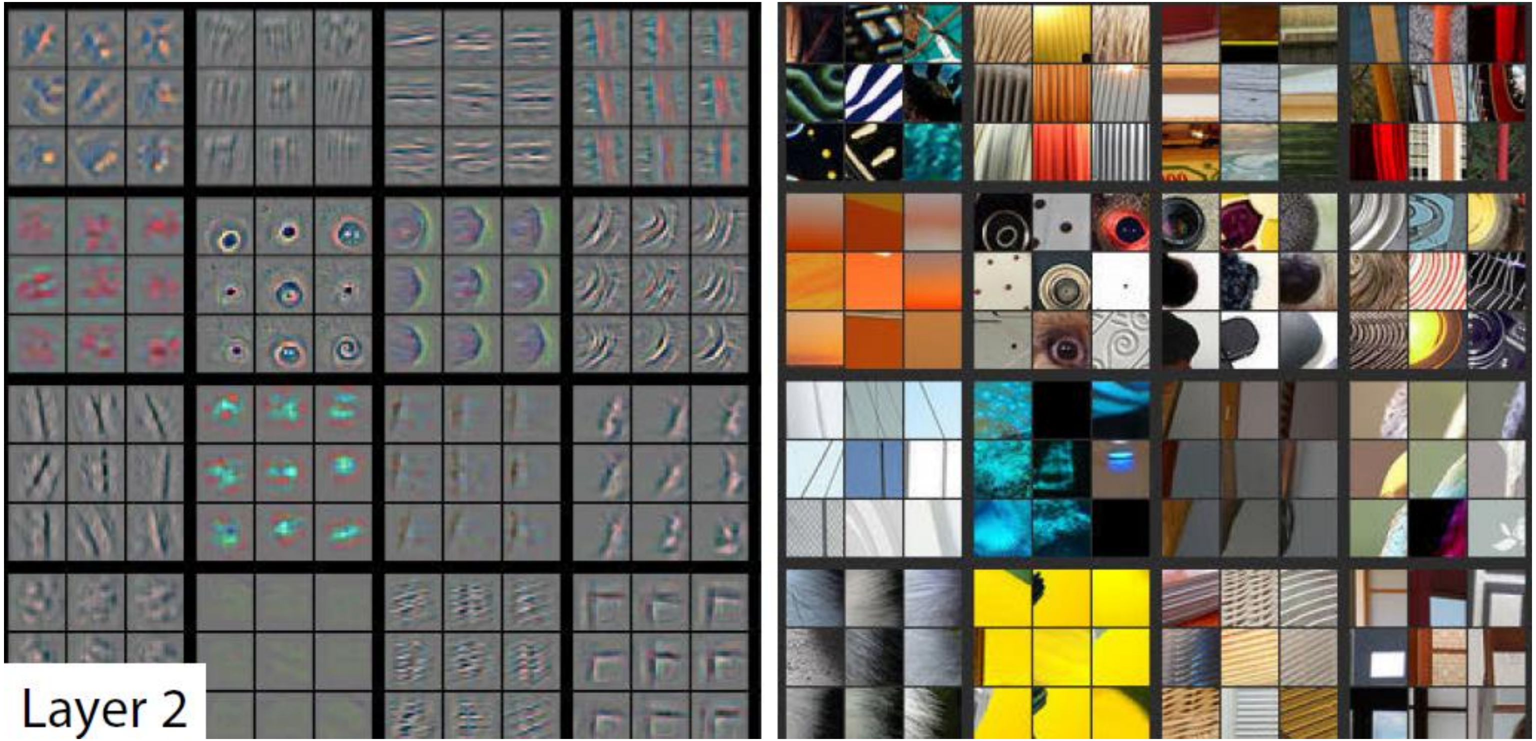
Step 3: (Continue to) train on new dataset

- **Finetune:** Update all parameters
- **Freeze** feature layer: Update only last layer weights (used when not enough data)

Replace last layer with new fully-connected for output nodes per new category

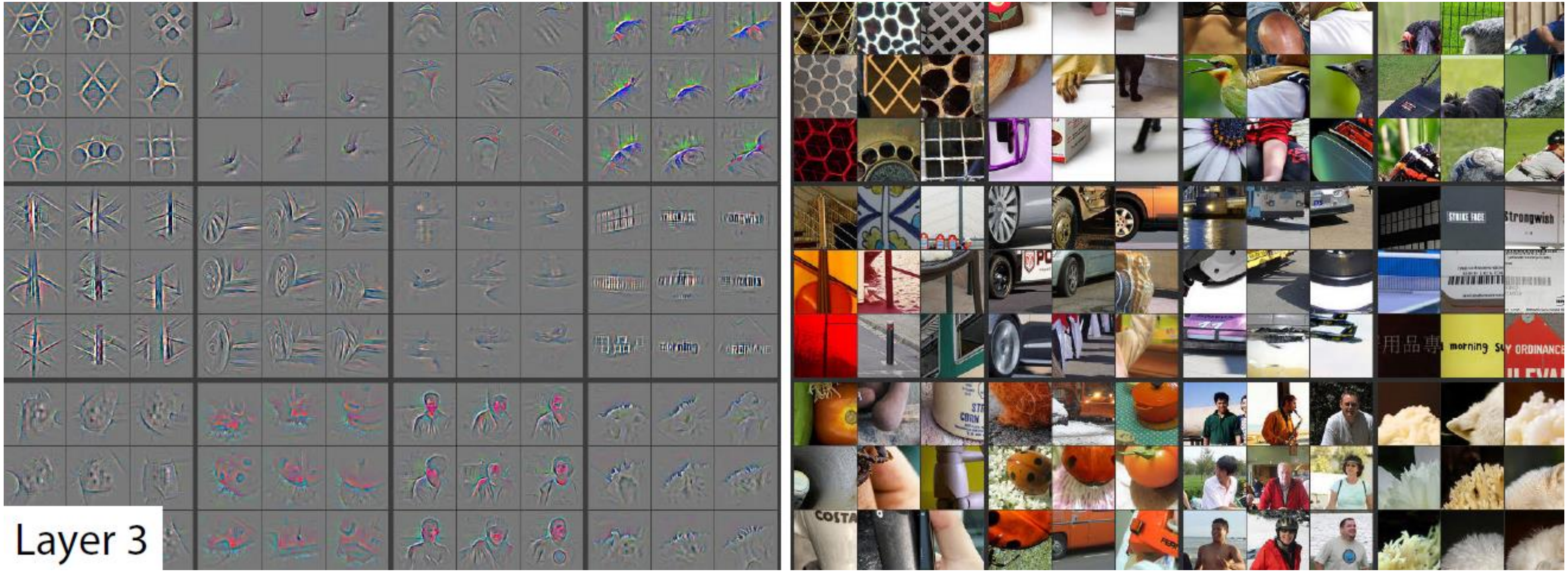
What do CNNs Learn?

VGG Layer-by-Layer Visualization



From: “Visualizing and Understanding Convolutional Networks, Zeiler & Fergus, 2014.

VGG Layer-by-Layer Visualization



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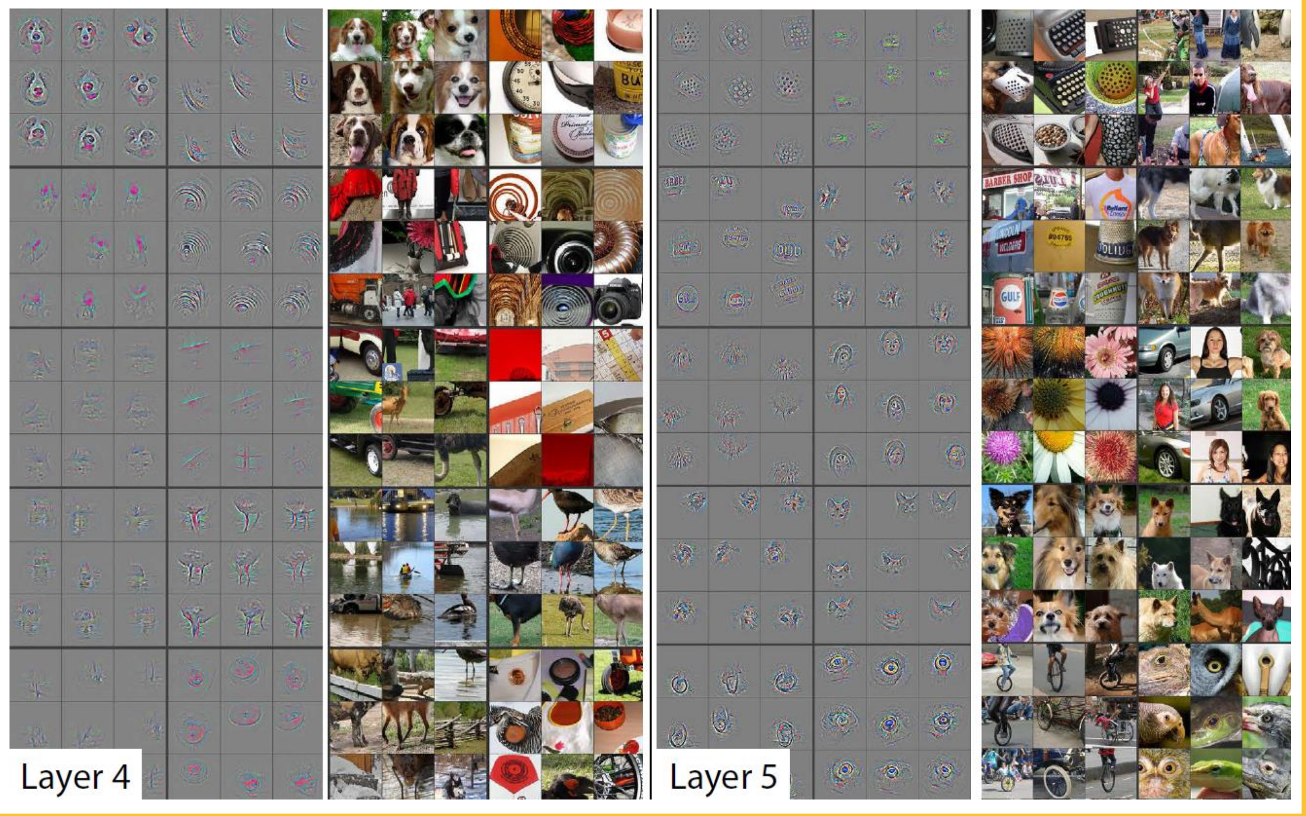
VGG Layer-by-Layer Visualization



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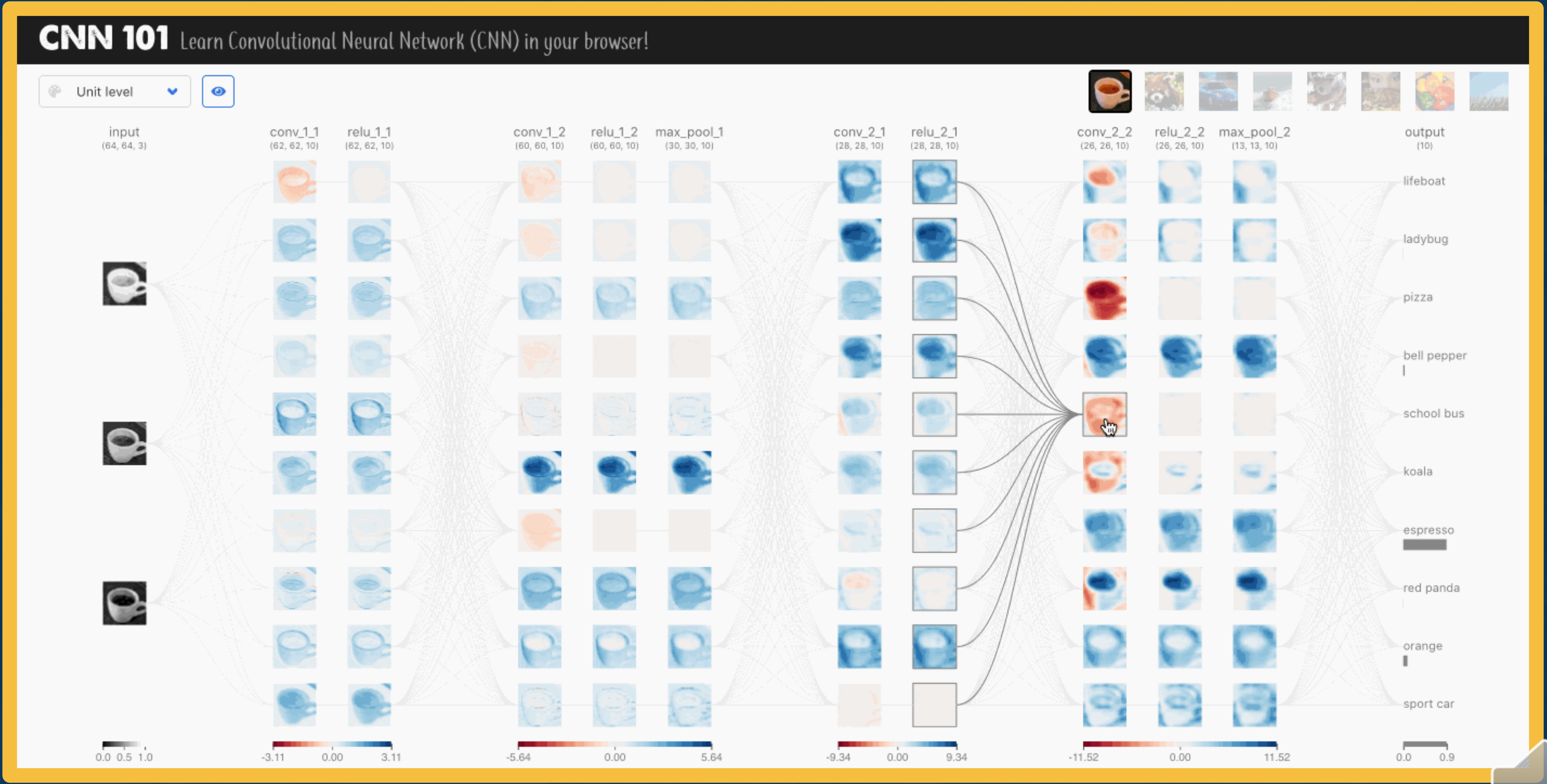
CNN101 and CNN Explainer

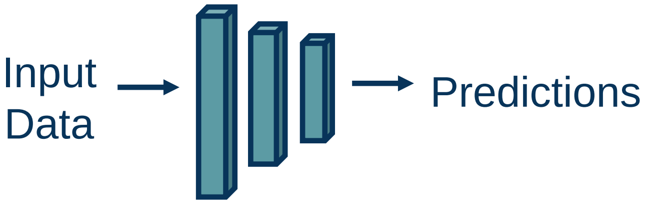


https://poloclub.github.io/cnn-explainer/

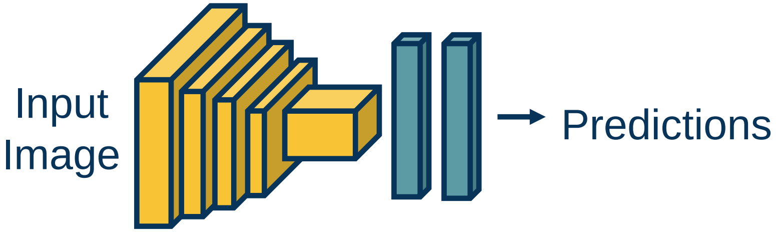
https://fredhohman.com/papers/cnn101

Module 3
Introduction

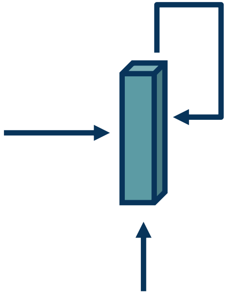
The Space of Architectures



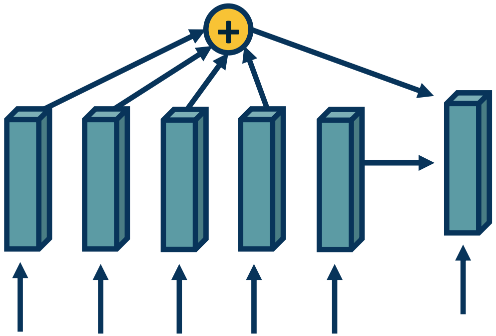
Fully Connected Neural Networks



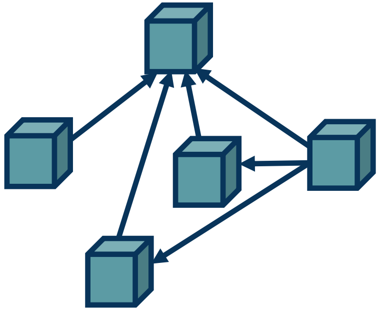
Convolutional Neural Networks



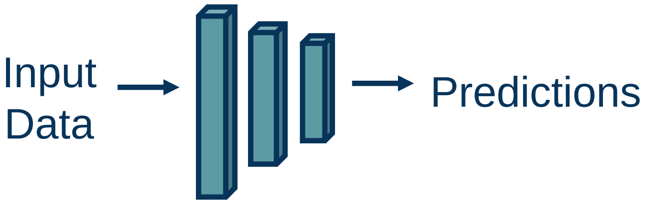
Recurrent Neural Networks



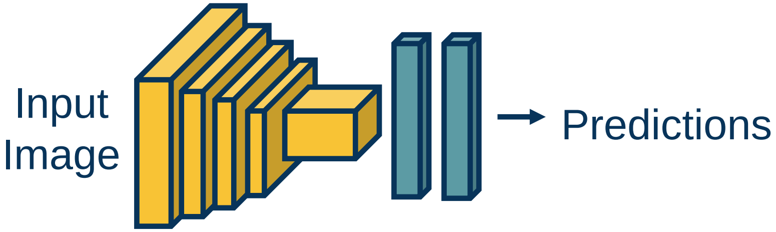
Attention-Based Networks



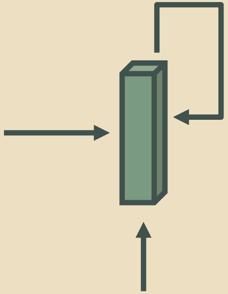
Graph-Based Networks



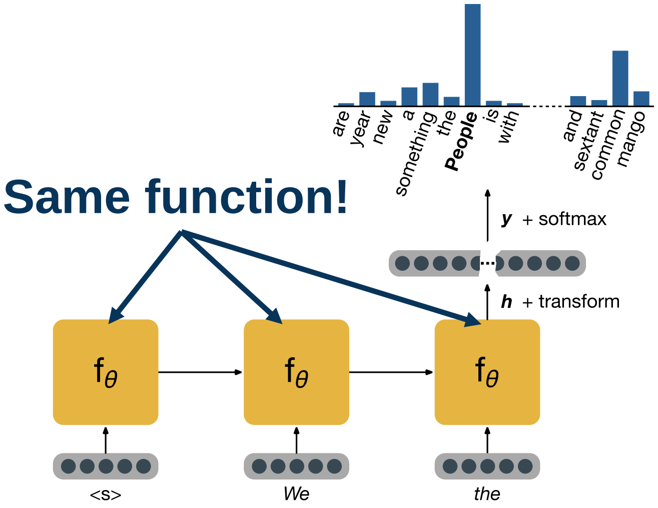
Fully Connected Neural Networks



Convolutional Neural Networks



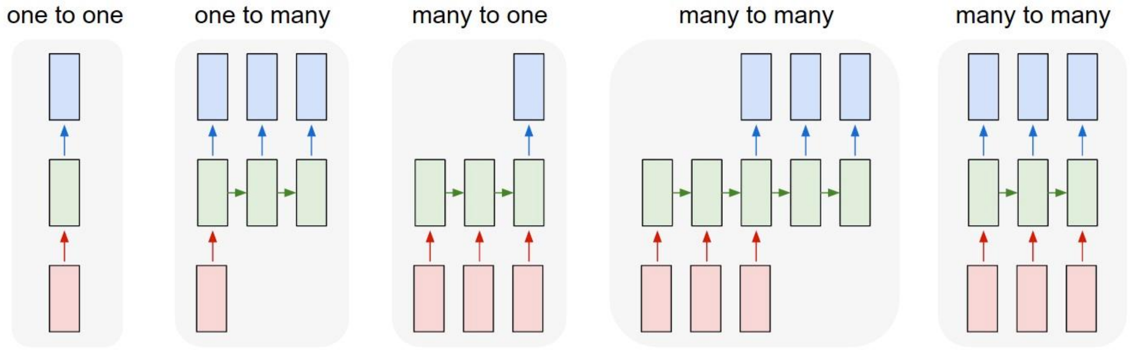
Recurrent Neural Networks



Recurrent Neural Networks & Transformers

Georgia Tech

New Topic: RNNs



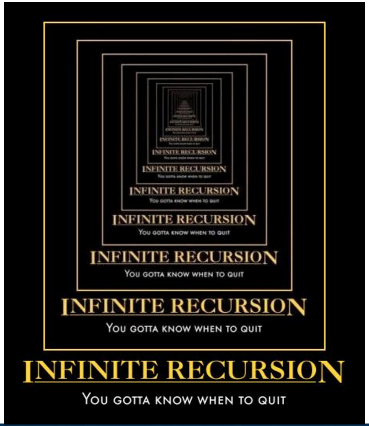


Image Credit: Andrei Karpathy

Why model sequences?

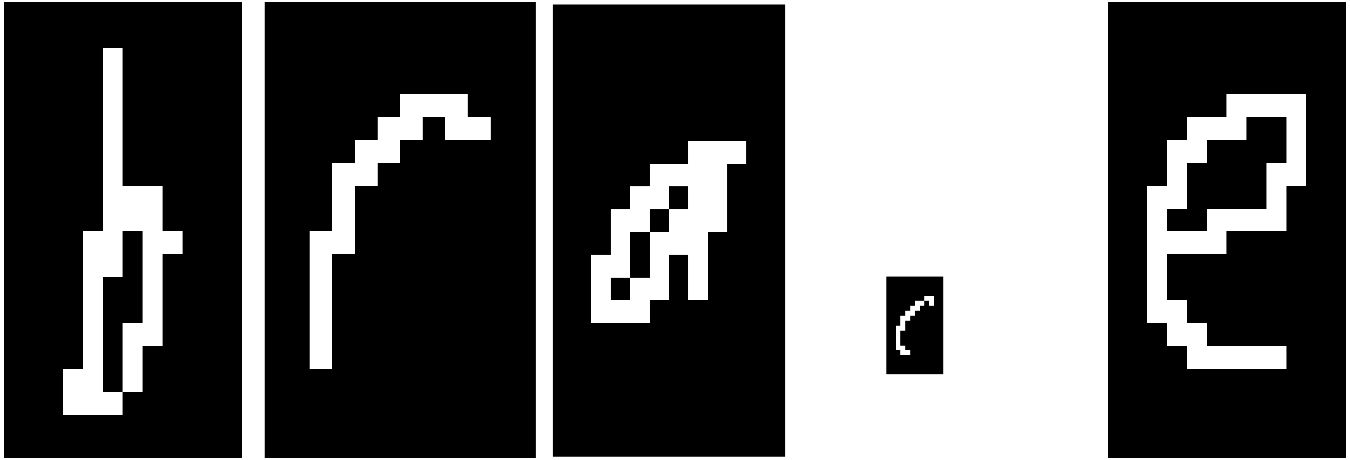


Figure Credit: Carlos Guestrin

Sequences are everywhere…

FOREIGN MINISTER.

THE SOUND OF

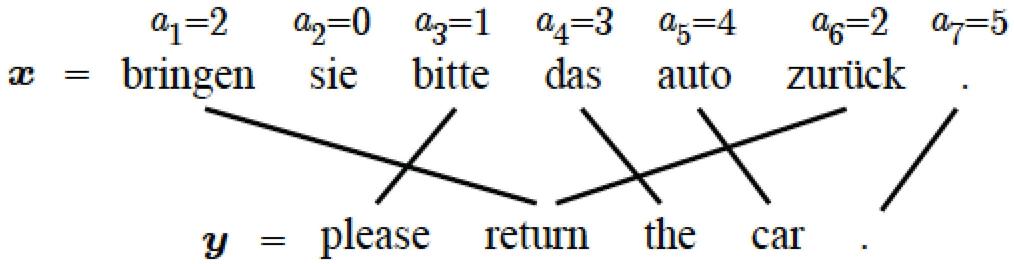
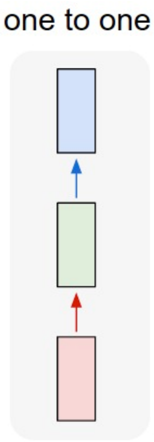


Image Credit: Alex Graves and Kevin Gimpel

Sequences in Input or Output?

- It’s a spectrum…



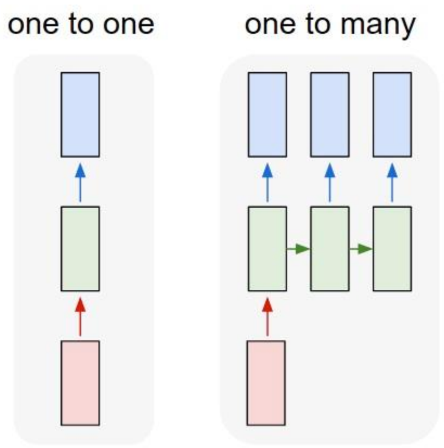
Input: No sequence

Output: No sequence

Example: “standard” classification / regression problems

Sequences in Input or Output?

- It's a spectrum…



Input: No sequence

Output: No sequence

Example: "standard" classification / regression problems

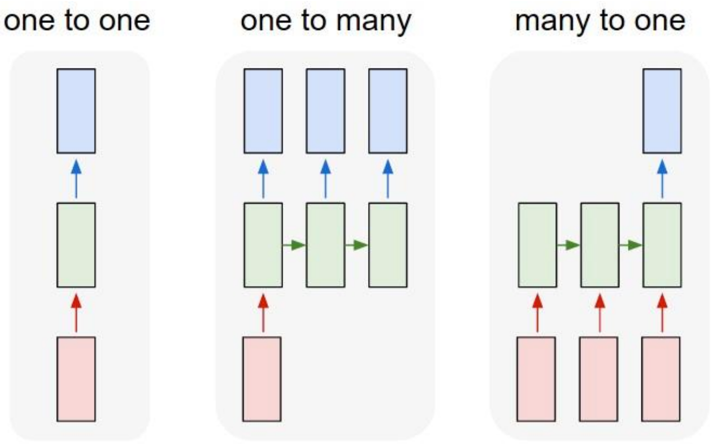
Input: No sequence

Output: Sequence

Example: Im2Caption

Sequences in Input or Output?

- It's a spectrum…



Input: No sequence

Output: No sequence

Example: "standard" classification / regression problems

Input: No sequence

Output: Sequence

Example: Im2Caption

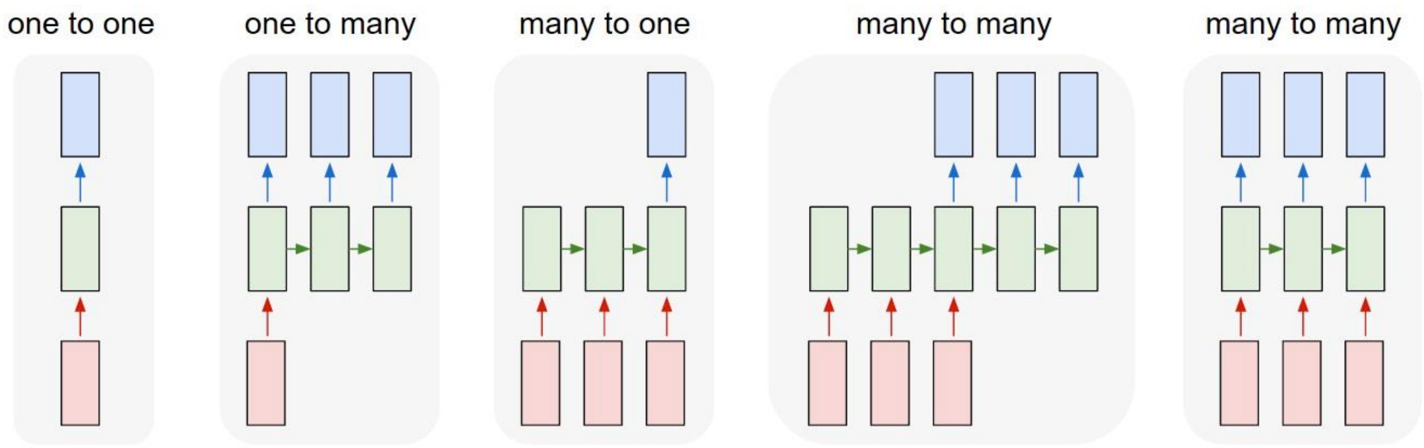
Input: Sequence

Output: No sequence

Example: sentence classification, multiple-choice question answering

Sequences in Input or Output?

- It's a spectrum…



Input: No sequence

Output: No sequence

Example: "standard" classification / regression problems

Input: No sequence

Output: Sequence

Example: Im2Caption

Input: Sequence

Output: No sequence

Example: sentence classification, multiple-choice question answering

Input: Sequence

Output: Sequence

Example: machine translation, video classification, video captioning, open-ended question answering

What's wrong with MLPs?

- Problem 1: Can't model sequences
  - Fixed-sized Inputs & Outputs
  - No temporal structure

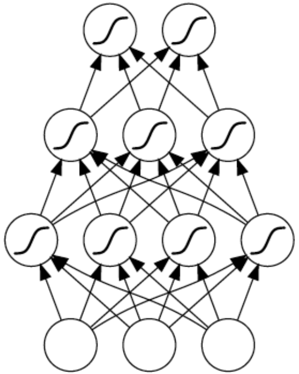


Image Credit: Alex Graves, book

What’s wrong with MLPs?

- Problem 1: Can’t model sequences
  - Fixed-sized Inputs & Outputs
  - No temporal structure

- Problem 2: Pure feed-forward processing
  - No “memory”, no feedback

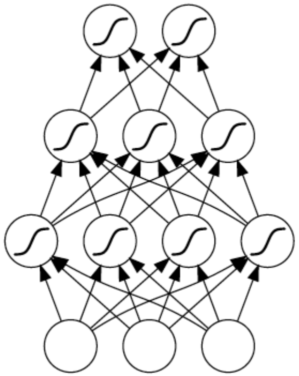


Image Credit: Alex Graves, book

3 Key Ideas

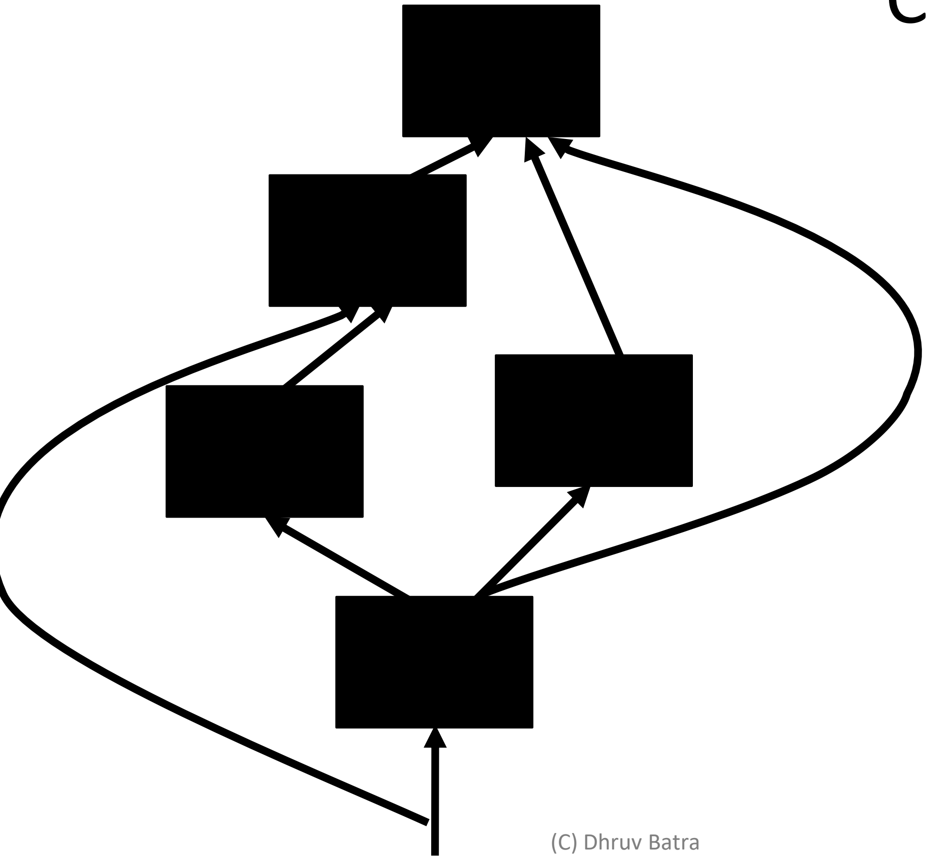
- The notion of memory (state)
  - We want to propagate information across the sequence
  - We will do this with *state*, represented by a vector (embedding/representation)
    - Key idea will be mixing new inputs with this state, to yield a new state
    - All represented as vector operations
  - Just as a CNN represents an image with the final hidden vector/embedding before the final classifier

3 Key Ideas

- The notion of memory (state)

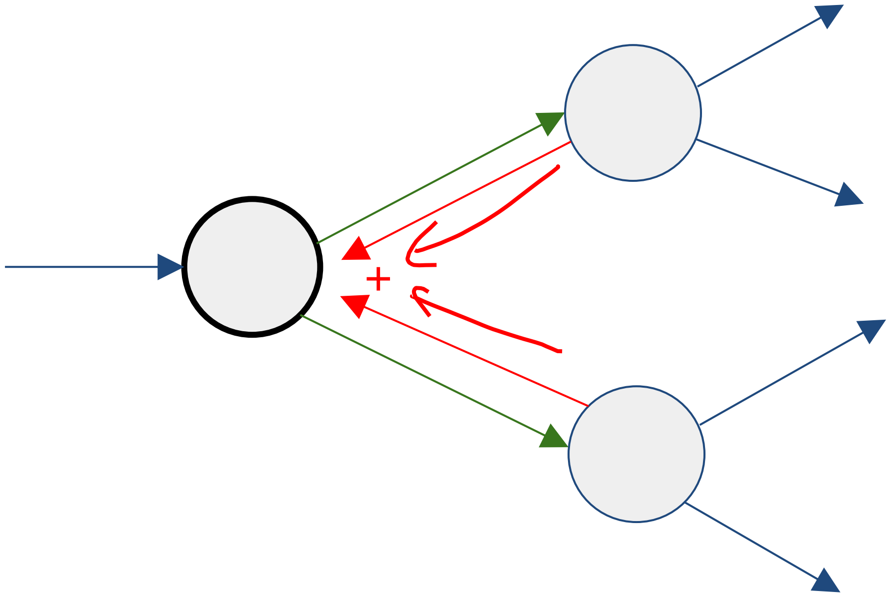
- Parameter Sharing
  - in computation graphs = adding gradients

Computational Graph



Slide Credit: Marc'Aurelio Ranzato

Gradients add at branches



3 Key Ideas

- The notion of memory (state)

- Parameter Sharing
  – in computation graphs = adding gradients

- “Unrolling”
  – in computation graphs with parameter sharing

New Words

- Recurrent Neural Networks (RNNs)

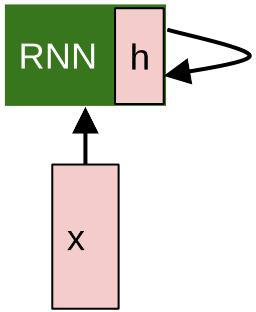
- Recursive Neural Networks
  - General family; think graphs instead of chains

- Types:
  - "Vanilla" RNNs (Elman Networks)
  - Long Short Term Memory (LSTMs)
  - Gated Recurrent Units (GRUs)
  - …

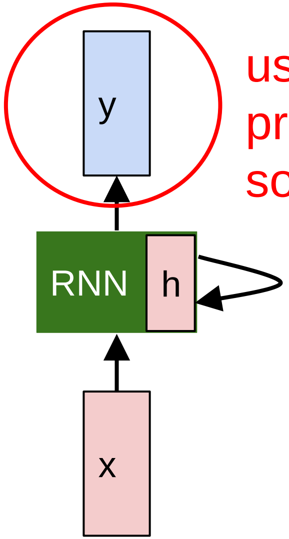
- Algorithms
  - BackProp Through Time (BPTT)
  - BackProp Through Structure (BPTS)

Recurrent Neural Network

- Idea: Input is a **sequence** and we will process it sequentially though a neural network module with *state*
- For each timestep (element of sequence):



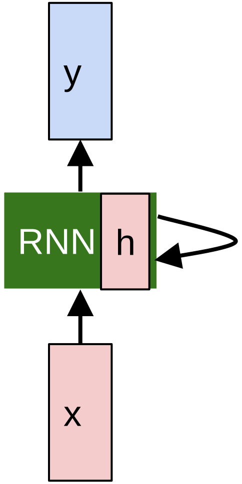
Recurrent Neural Network



usually want to
predict a vector at
some time steps

(Vanilla) Recurrent Neural Network

The state consists of a single *“hidden”* vector **h**:



$$y_t = W_{hy}h_t + b_y$$

$$h_t = f_W(h_{t-1}, x_t)$$

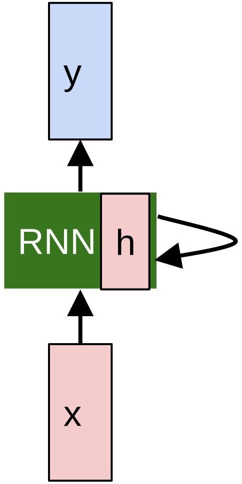
$$\downarrow$$

$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t + b_h)$$

Sometimes called a “Vanilla RNN” or an “Elman RNN” after Prof. Jeffrey Elman

(Vanilla) Recurrent Neural Network

The state consists of a single *"hidden"* vector **h**:



$$y_t = W_{hy} h_t + b_y$$

$$\begin{aligned} h_t &= \tanh(W_{hh} h_{t-1} + W_{xh} x_t) \\ &= \tanh\left(\begin{pmatrix} W_{hh} & W_{hx} \end{pmatrix} \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} \right) \\ &= \tanh\left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} \right) \end{aligned}$$

Recurrent Neural Network

We can process a sequence of vectors **x** by applying a **recurrence formula** at every time step:

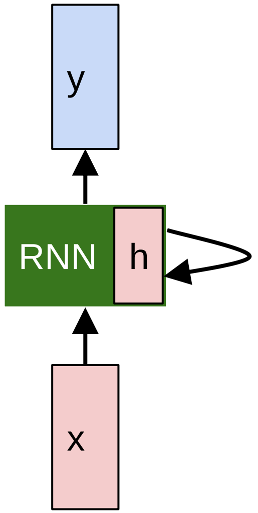
$$\boxed{h_t} = \boxed{f_W}(\boxed{h_{t-1}}, \boxed{x_t})$$

new state

old state

input vector at some time step

some function with parameters W



Recurrent Neural Network

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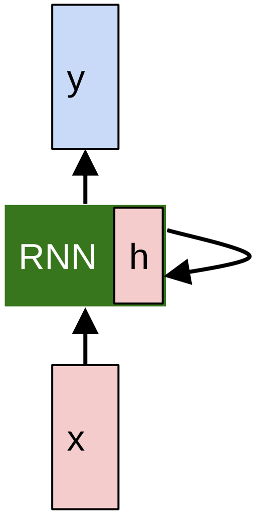
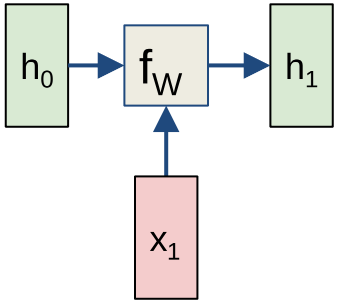
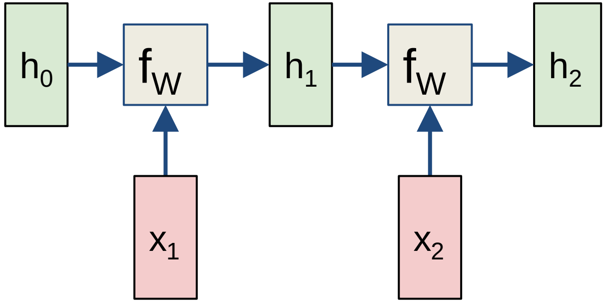


Notice: the same function and the same set of parameters are used at every time step.

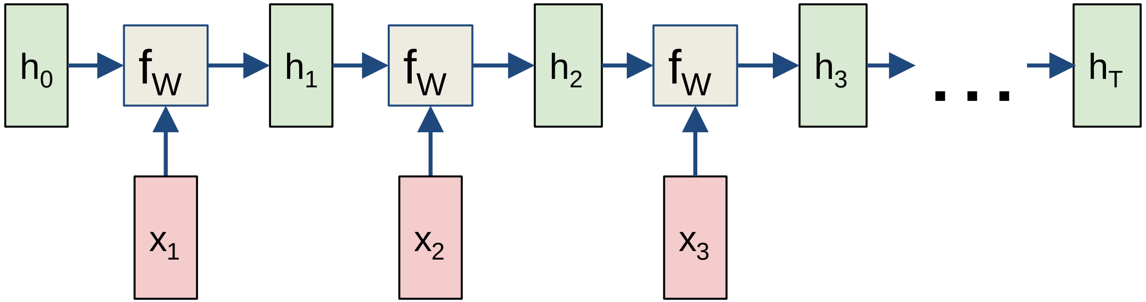
RNN: Computational Graph



RNN: Computational Graph

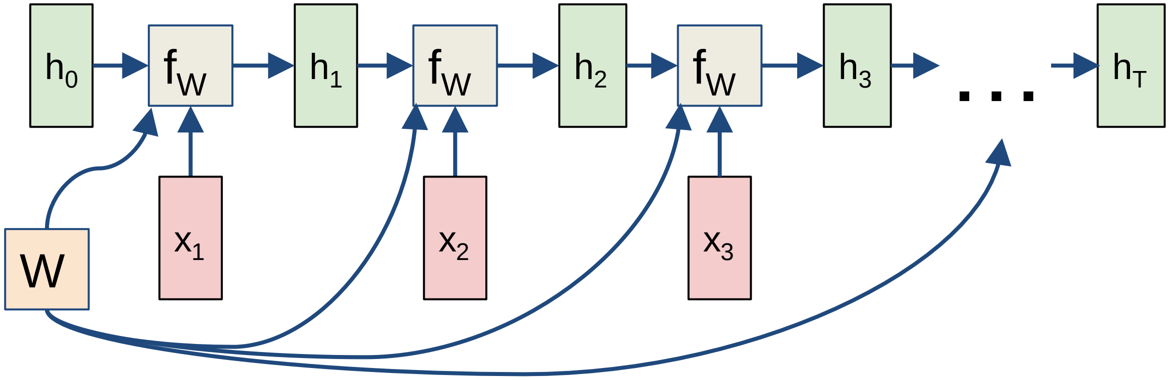


RNN: Computational Graph

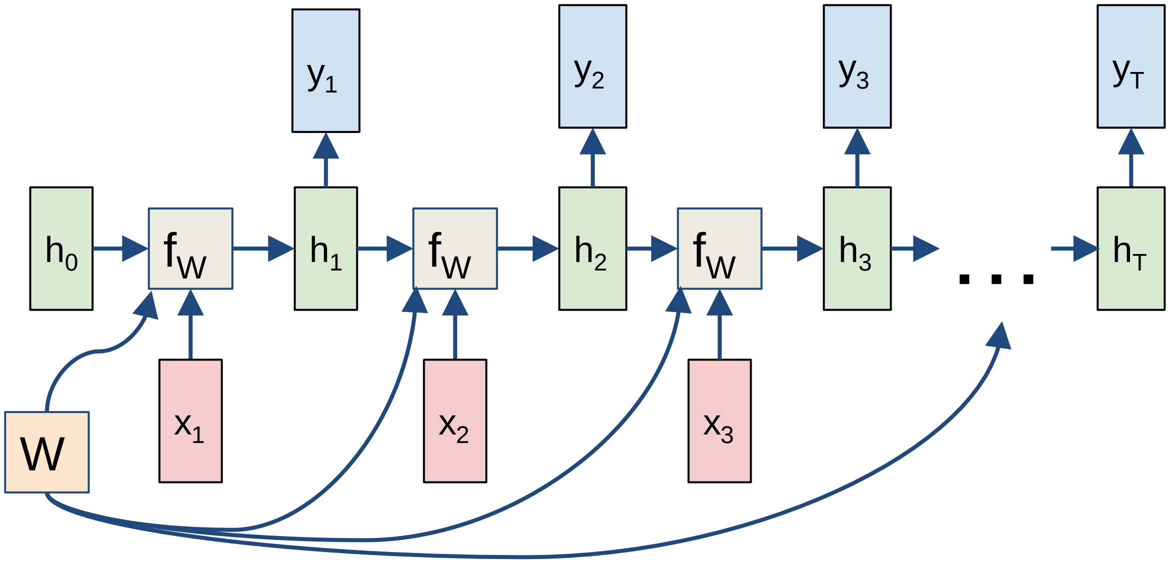


RNN: Computational Graph

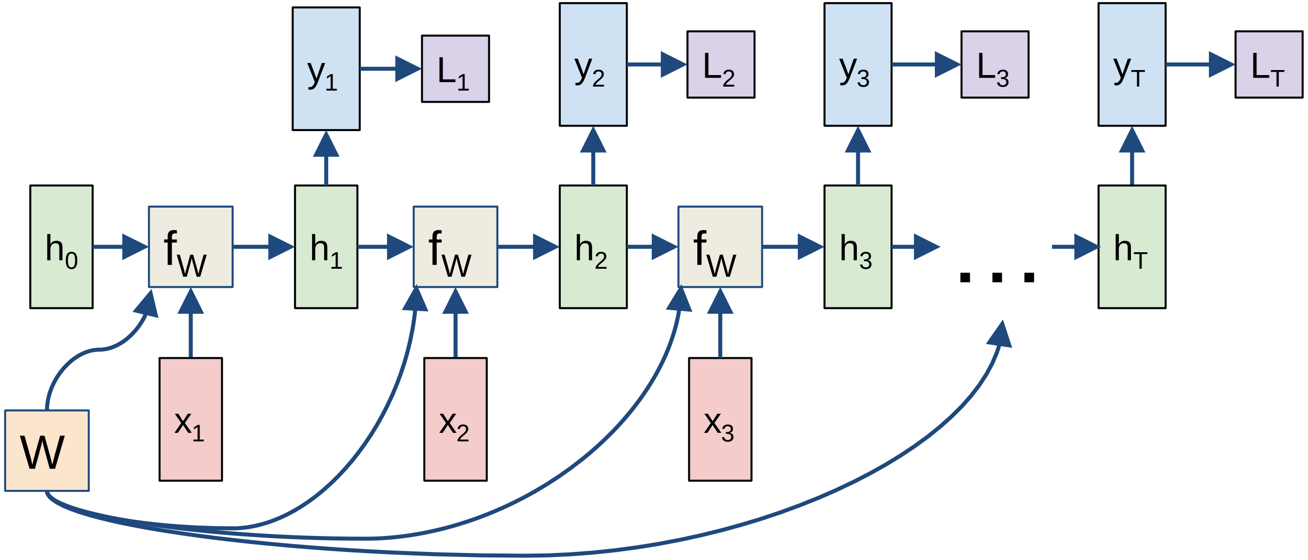
Re-use the same weight matrix at every time-step



RNN: Computational Graph: Many to Many



RNN: Computational Graph: Many to Many



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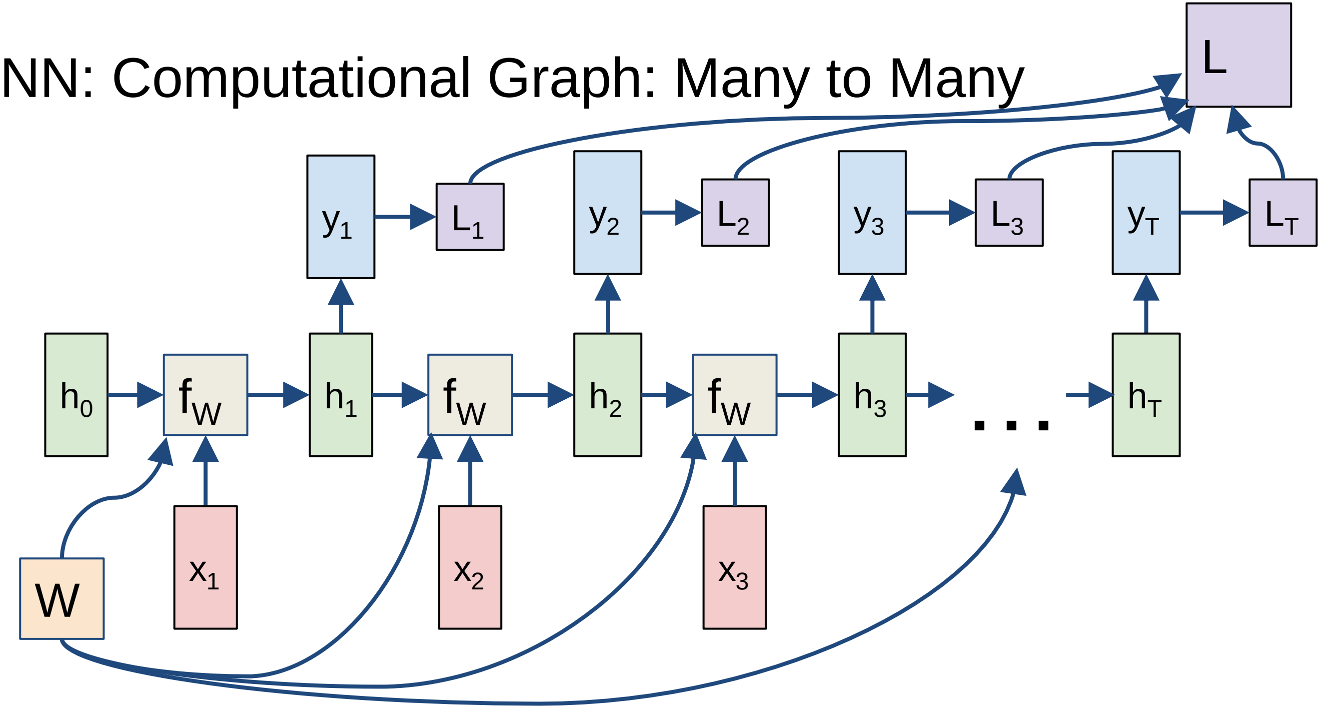
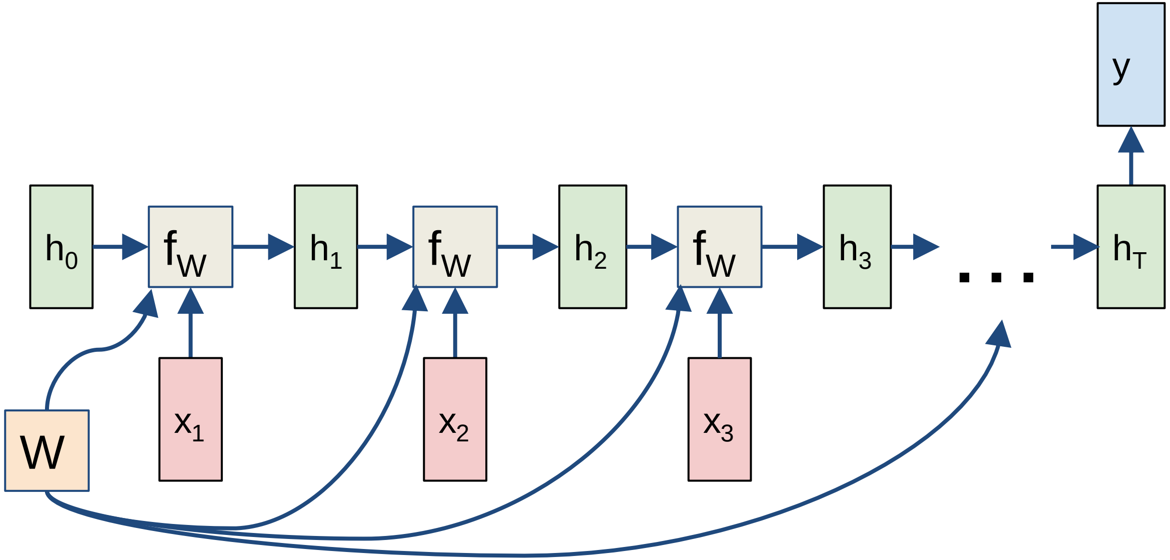
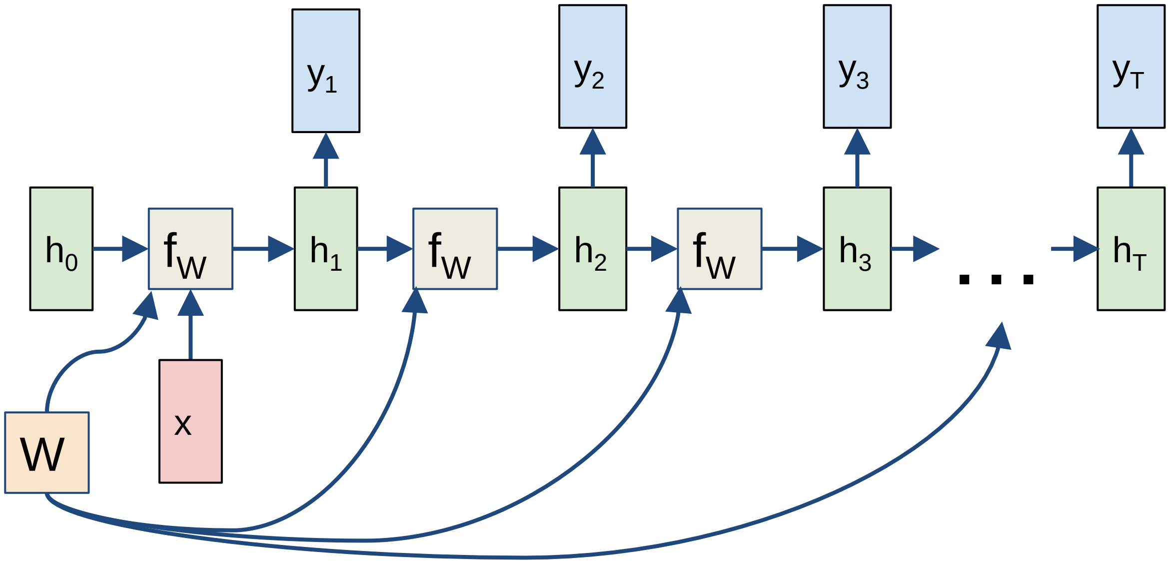


RNN: Computational Graph: Many to One

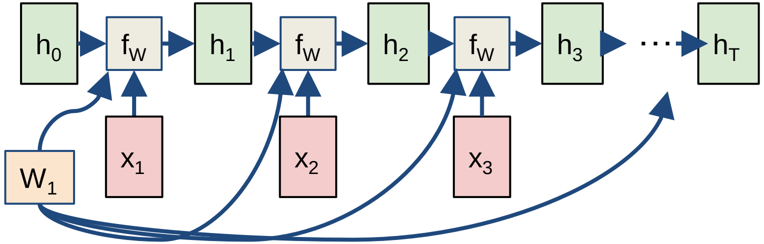


RNN: Computational Graph: One to Many



Sequence to Sequence: Many-to-one + one-to-many

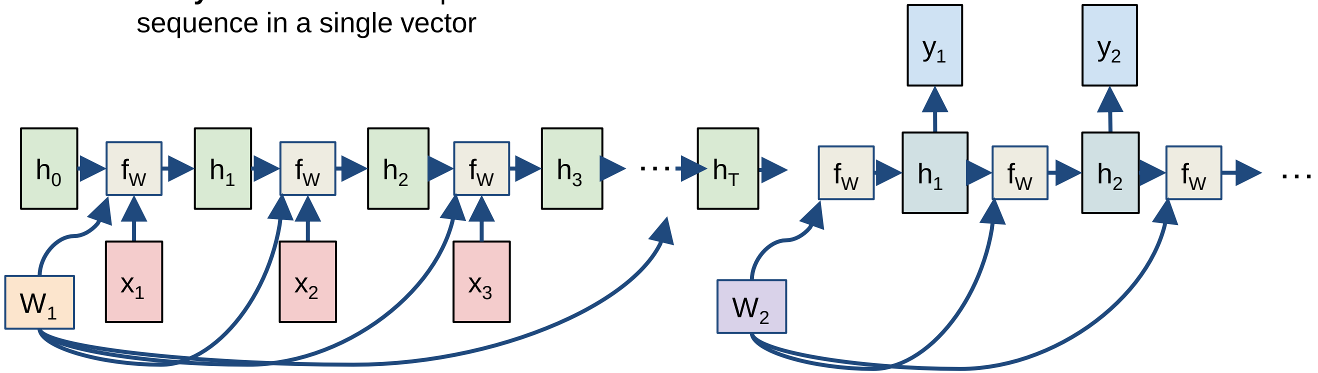
Many to one: Encode input sequence in a single vector



Sequence to Sequence: Many-to-one + one-to-many

One to many: Produce output sequence from single input vector

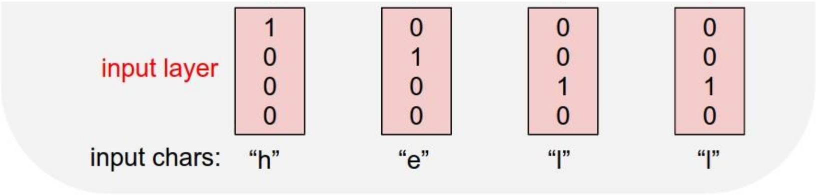
Many to one: Encode input sequence in a single vector



Example: Character-level Language Model

Vocabulary:
[h,e,l,o]

Example training sequence:
“hello”



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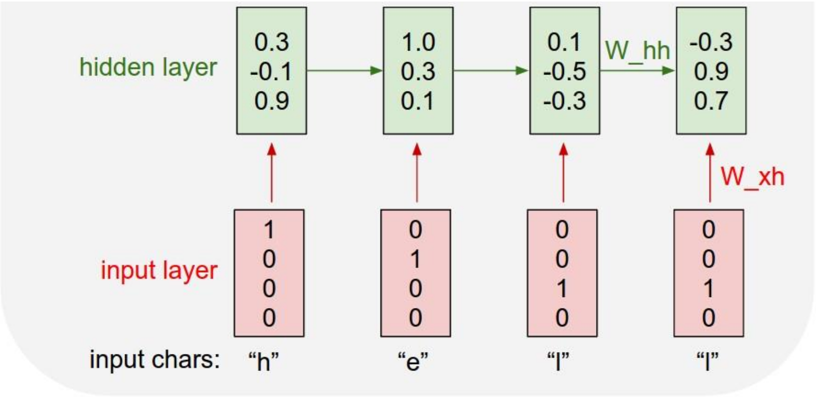
$$\boxed{h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t + b_h)}$$



Example: Character-level Language Model

Vocabulary: [h,e,l,o]

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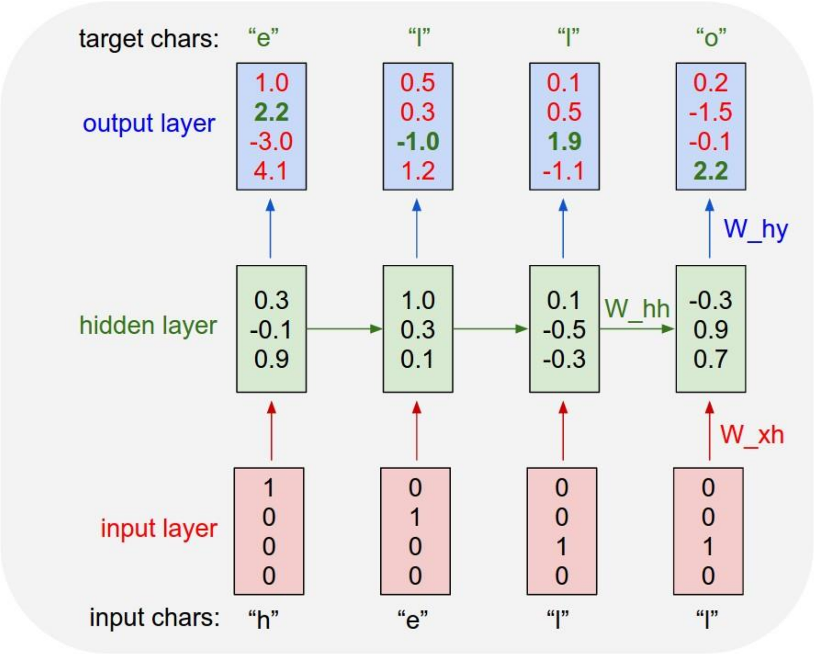
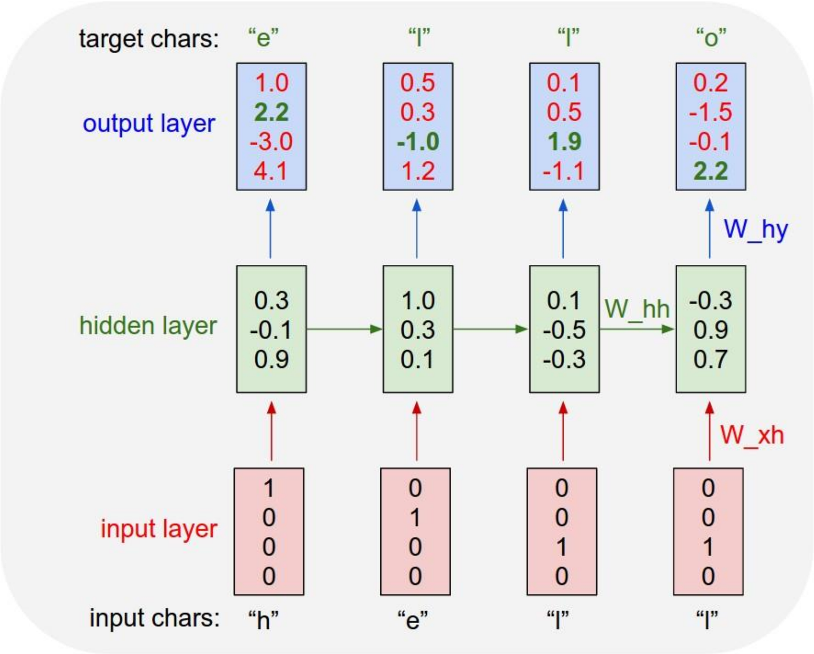


Training Time: MLE / “Teacher Forcing”

Example:
Character-level
Language Model

Vocabulary:
[h,e,l,o]

Example training
sequence:
“hello”

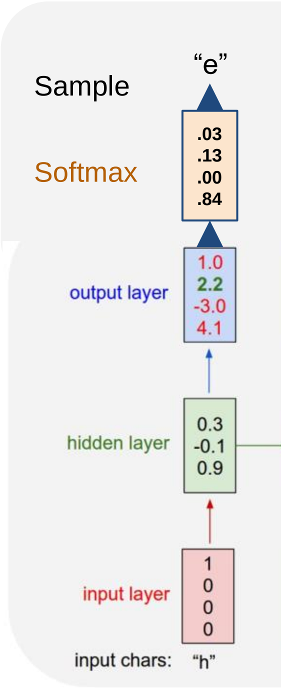


Test Time: Sample / Argmax / Beam Search

Example: Character-level Language Model Sampling

Vocabulary: [h,e,l,o]

At test-time sample characters one at a time, feed back to model

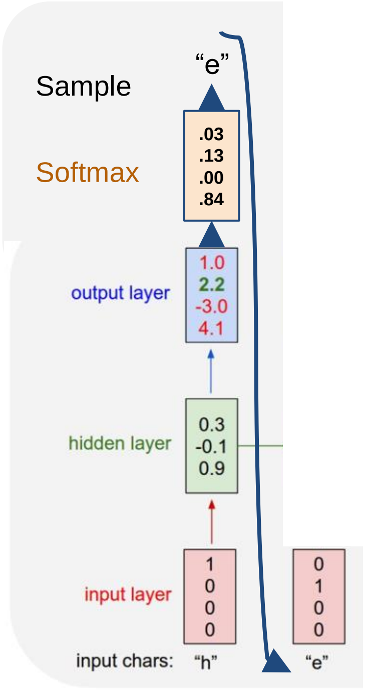


Test Time: Sample / Argmax / Beam Search

Example: Character-level Language Model Sampling

Vocabulary: [h,e,l,o]

At test-time sample characters one at a time, feed back to model



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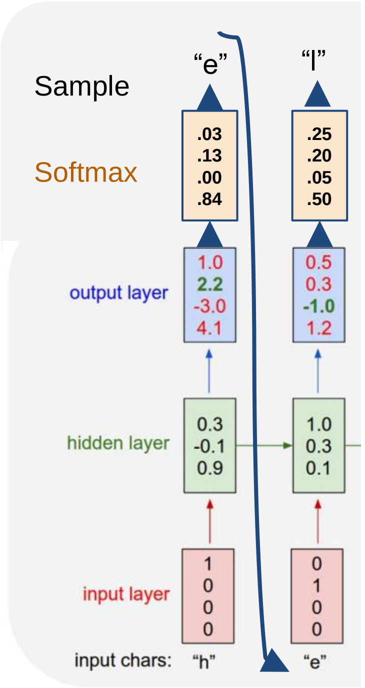

Test Time: Sample / Argmax / Beam Search

Example: Character-level Language Model Sampling

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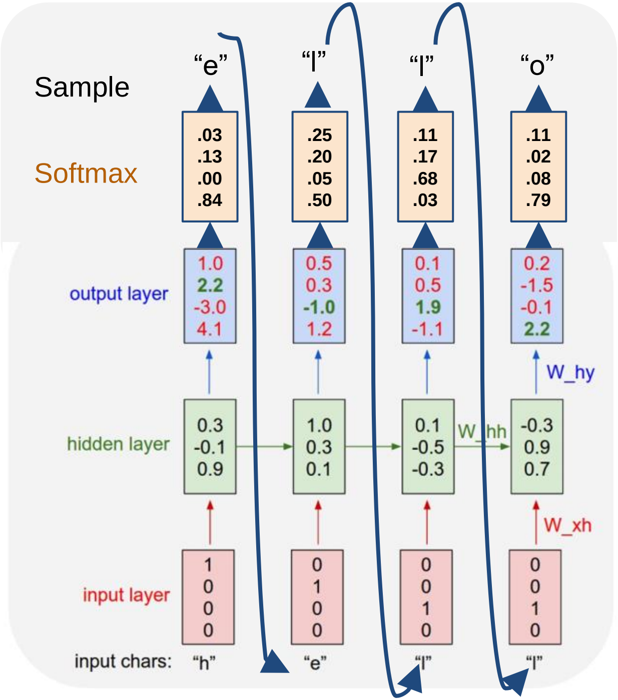
At test-time sample characters one at a time, feed back to model

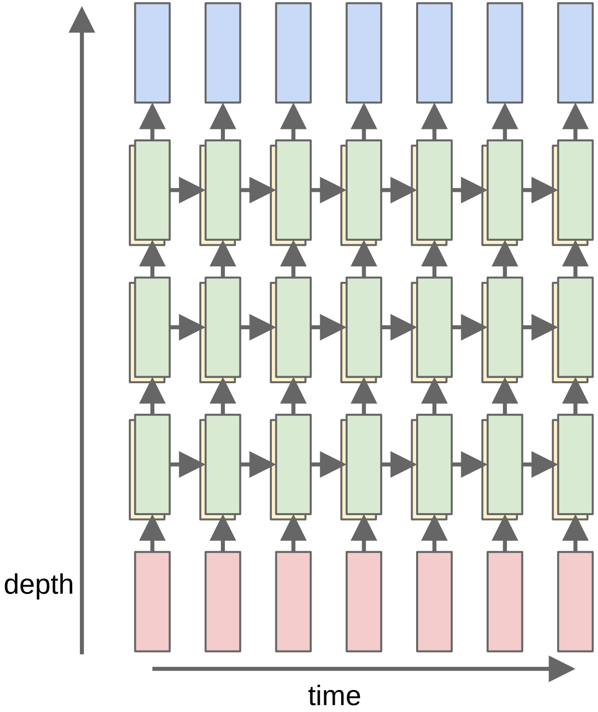


Can also feed in predictions during training (student forcing)

Multilayer RNNs

$$\boxed{h_t^l = \tanh W^l \begin{pmatrix} h_t^{l-1} \\ h_{t-1}^l \end{pmatrix}}$$

$h \in \mathbb{R}^n$ $\quad W^l \; [n \times 2n]$



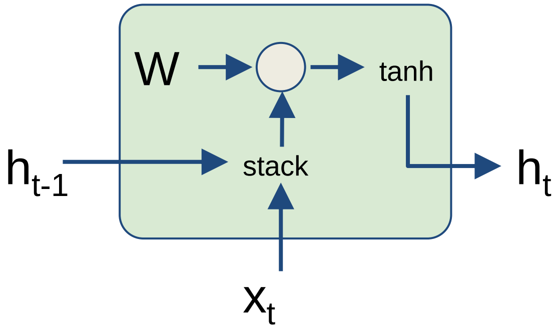
- **Training:** A large corpus of text from the web
    - Note: No annotation required! It's just "the text"

- **Inference:** Just generate me new text
    - Can condition on some initial input (**prompt**)

```
#include <asm/io.h>
#include <asm/prom.h>
#include <asm/e820.h>
#include <asm/system_info.h>
#include <asm/setew.h>
#include <asm/pgproto.h>

#define REG_PG    vesa_slot_addr_pack
#define PFM_NOCOMP  AFSR(0, load)
#define STACK_DDR(type)     (func)

#define SWAP_ALLOCATE(nr)     (e)
#define emulate_sigs()  arch_get_unaligned_child()
#define access_rw(TST)  asm volatile("movd %%esp, %0, %3" : : "r" (0));   \
  if (__type & DO_READ)

static void stat_PC_SEC __read_mostly offsetof(struct seq_argsqueue, \
          pC>[1]);

static void
os_prefix(unsigned long sys)
{
#ifdef CONFIG_PREEMPT
  PUT_PARAM_RAID(2, sel) = get_state_state();
  set_pid_sum((unsigned long)state, current_state_str(),
           (unsigned long)-1->lr_full; low;
}
```

Vanilla RNN Gradient Flow

Bengio et al, “Learning long-term dependencies with gradient descent is difficult”, IEEE Transactions on Neural Networks, 1994
Pascanu et al, “On the difficulty of training recurrent neural networks”, ICML 2013

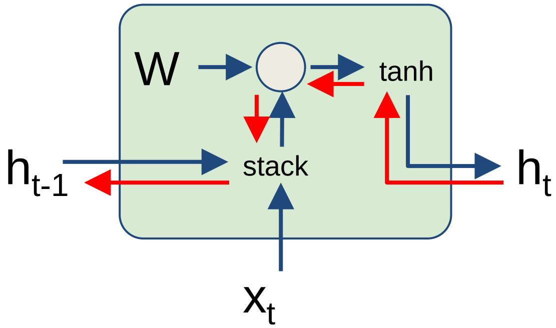


$$
\begin{aligned}
h_t &= \tanh(W_{hh}h_{t-1} + W_{xh}x_t) \\
&= \tanh\left(\begin{pmatrix} W_{hh} & W_{hx} \end{pmatrix} \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right) \\
&= \tanh\left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right)
\end{aligned}
$$

Vanilla RNN Gradient Flow

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013

Backpropagation from h_t to h_{t-1} multiplies by W (actually W_{hh})



$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t)$$
$$= \tanh\left(\begin{pmatrix} W_{hh} & W_{hx} \end{pmatrix} \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right)$$
$$= \tanh\left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right)$$

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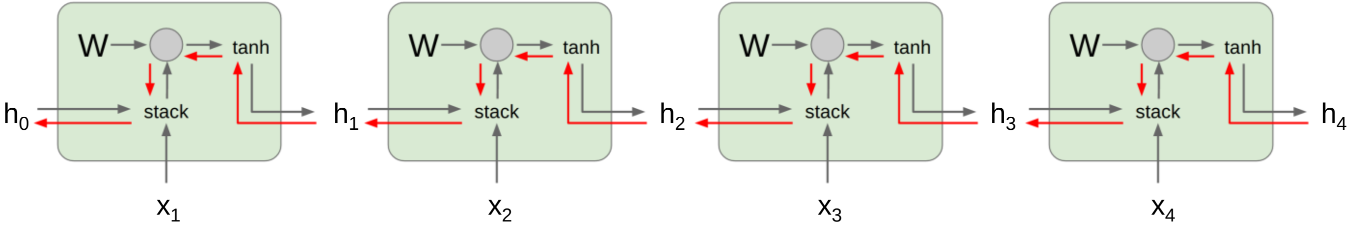
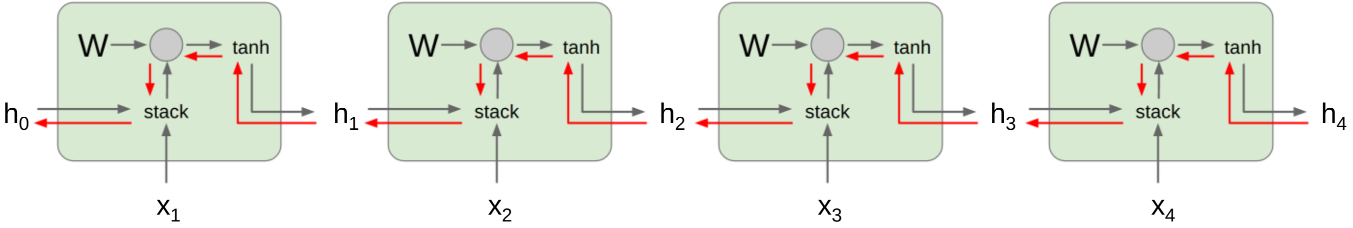


Computing gradient of h_0 involves many factors of W (and repeated tanh)

Vanilla RNN Gradient Flow

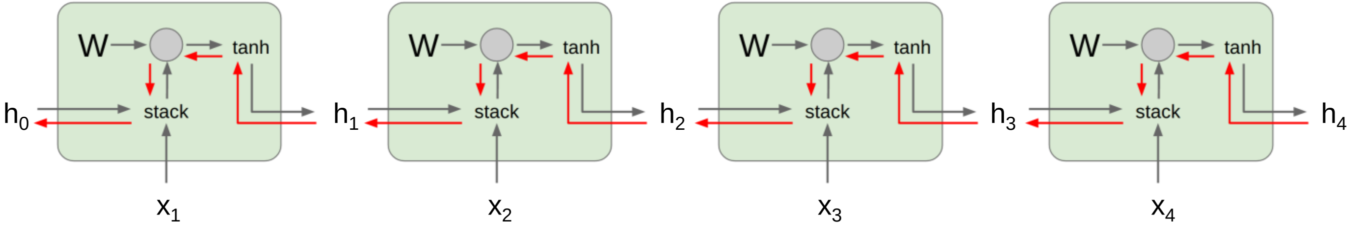
Bengio et al, “Learning long-term dependencies with gradient descent is difficult”, IEEE Transactions on Neural Networks, 1994
Pascanu et al, “On the difficulty of training recurrent neural networks”, ICML 2013



$$\frac{\partial h_t}{\partial h_{t-1}} = tanh'(W_{hh}h_{t-1} + W_{xh}x_t)W_{hh}$$

Vanilla RNN Gradient Flow

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013



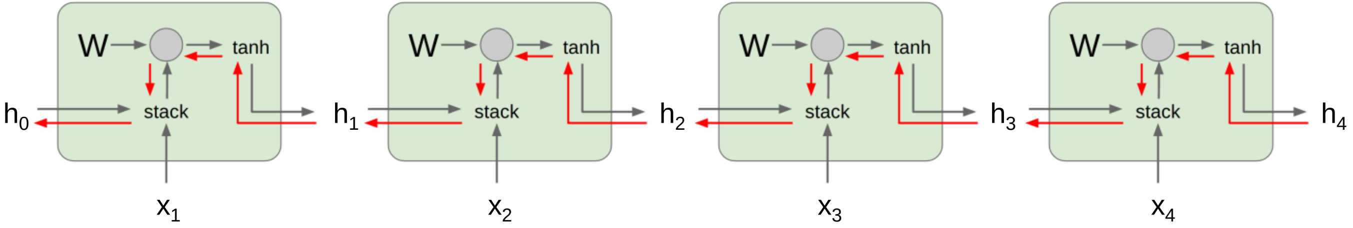
$$\frac{\partial L}{\partial W} = \sum_{t=1}^{T} \frac{\partial L_t}{\partial W}$$

$$\boxed{\frac{\partial h_t}{\partial h_{t-1}} = tanh'(W_{hh}h_{t-1} + W_{xh}x_t)W_{hh}}$$

$$\frac{\partial L_T}{\partial W} = \frac{\partial L_T}{\partial h_T}\frac{\partial h_t}{\partial h_{t-1}} \cdots \frac{\partial h_1}{\partial W} = \frac{\partial L_T}{\partial h_T}\left(\prod_{t=2}^{T} \frac{\partial h_t}{\partial h_{t-1}}\right)\frac{\partial h_1}{\partial W}$$

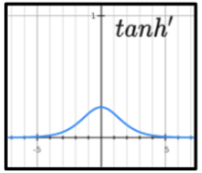
Vanilla RNN Gradient Flow

Bengio et al, “Learning long-term dependencies with gradient descent is difficult”, IEEE Transactions on Neural Networks, 1994
Pascanu et al, “On the difficulty of training recurrent neural networks”, ICML 2013



$$\frac{\partial L}{\partial W} = \sum_{t=1}^{T} \frac{\partial L_t}{\partial W}$$

Always < 1
Vanishing gradients



$$\frac{\partial L_T}{\partial W} = \frac{\partial L_T}{\partial h_T} \frac{\partial h_t}{\partial h_{t-1}} \cdots \frac{\partial h_1}{\partial W} = \frac{\partial L_T}{\partial h_T} \left(\prod_{t=2}^{T} \frac{\partial h_t}{\partial h_{t-1}}\right) \frac{\partial h_1}{\partial W}$$

Vanilla RNN Gradient Flow

Bengio et al, “Learning long-term dependencies with gradient descent is difficult”, IEEE Transactions on Neural Networks, 1994
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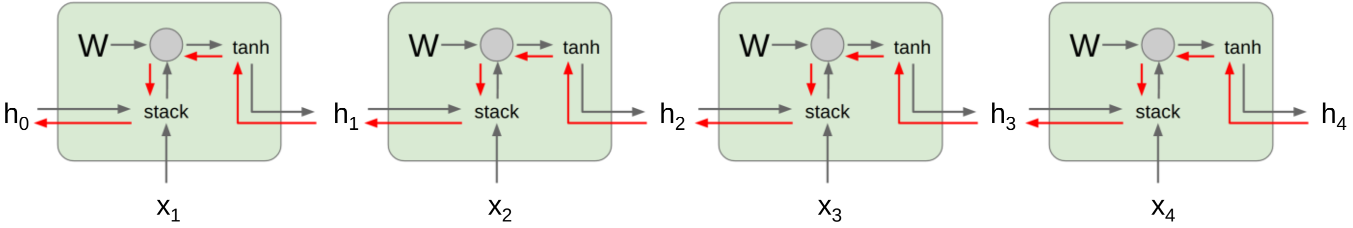


Computing gradient of h_0 involves many factors of W
(and repeated tanh)

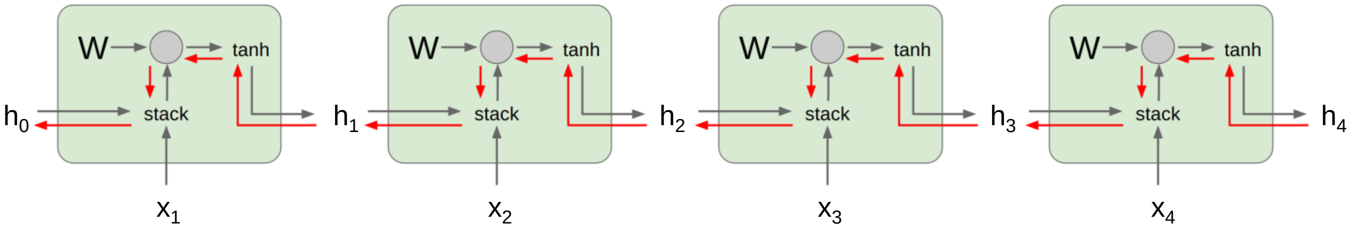
With no non-linearity:

Largest singular value > 1:
Exploding gradients

Largest singular value < 1:
Vanishing gradients

Vanilla RNN Gradient Flow

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013



Computing gradient of h_0 involves many factors of W (and repeated tanh)

Largest singular value > 1:
Exploding gradients

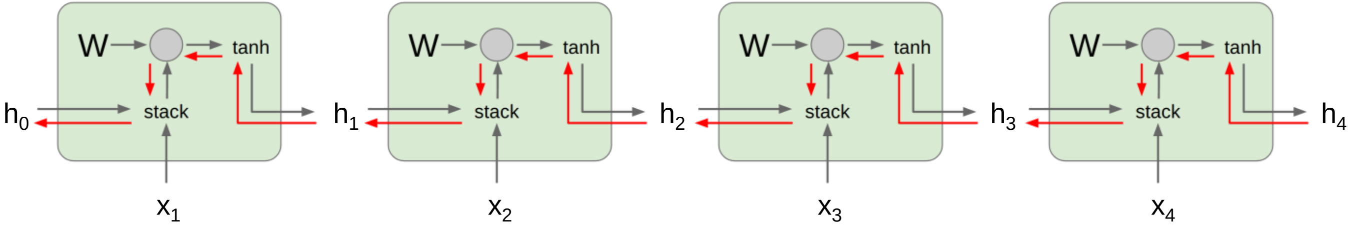
Largest singular value < 1:
Vanishing gradients

Gradient clipping: Scale gradient if its norm is too big

```
grad_norm = np.sum(grad * grad)
if grad_norm > threshold:
  grad *= (threshold / grad_norm)
```

Vanilla RNN Gradient Flow

Bengio et al, “Learning long-term dependencies with gradient descent is difficult”, IEEE Transactions on Neural Networks, 1994
Pascanu et al, “On the difficulty of training recurrent neural networks”, ICML 2013



Computing gradient of h_0 involves many factors of W (and repeated tanh)

Largest singular value > 1:
Exploding gradients

Largest singular value < 1:
Vanishing gradients

→ Change RNN architecture

Long Short Term Memory (LSTM)

Vanilla RNN

$$\boxed{h_t = \tanh\left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right)}$$

LSTM

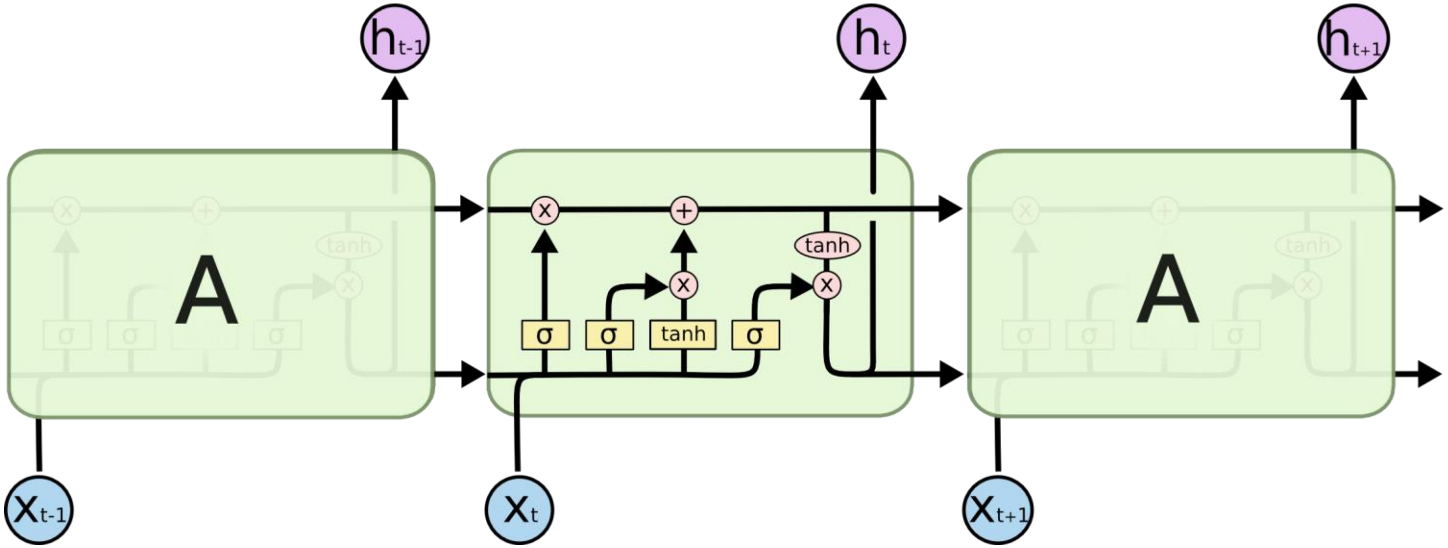
$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

$$c_t = f \odot c_{t-1} + i \odot g$$

$$h_t = o \odot \tanh(c_t)$$

Hochreiter and Schmidhuber, “Long Short Term Memory”, Neural Computation 1997

Meet LSTMs



Intuition: "Gating" mechanism similar to residual, but more complex memory operations (forget/read, write)

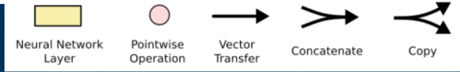


Image Credit: Christopher Olah (http://colah.github.io/posts/2015-08-Understanding-LSTMs/)

LSTMs Intuition: Memory

- Cell State / Memory

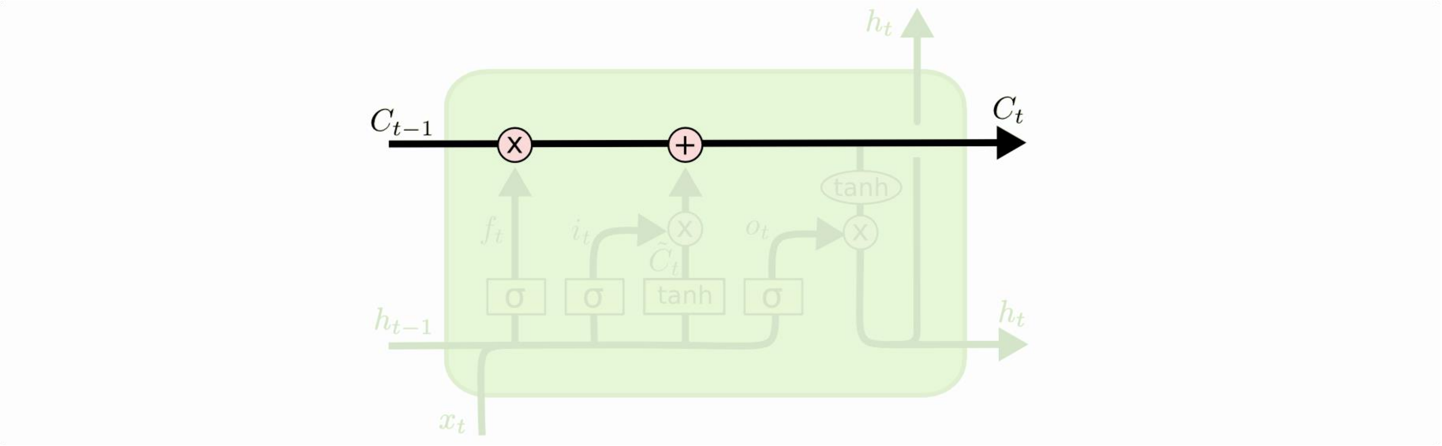
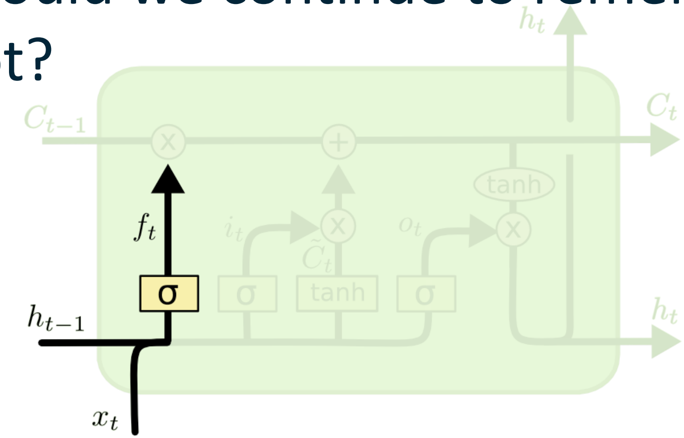


Image Credit: Christopher Olah (http://colah.github.io/posts/2015-08-Understanding-LSTMs/)

LSTMs Intuition: Forget Gate

- Should we continue to remember this "bit" of information or not?

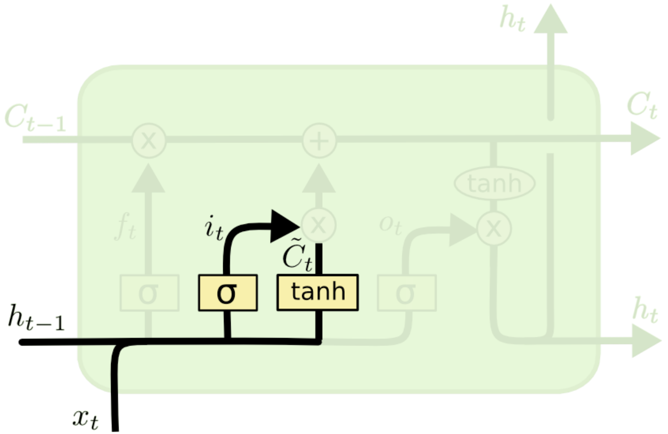


$$f_t = \sigma\left(W_f \cdot [h_{t-1}, x_t] \ + \ b_f\right)$$

Image Credit: Christopher Olah (http://colah.github.io/posts/2015-08-Understanding-LSTMs/)

LSTMs Intuition: Input Gate

- Should we update this “bit” of information or not?
  - If so, with what?



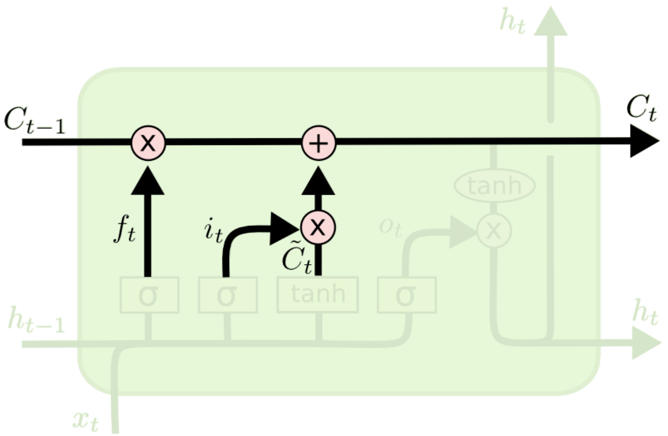
$$i_t = \sigma\left(W_i \cdot [h_{t-1}, x_t] \ + \ b_i\right)$$

$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] \ + \ b_C)$$

Image Credit: Christopher Olah (http://colah.github.io/posts/2015-08-Understanding-LSTMs/)

LSTMs Intuition: Memory Update

- Forget that + memorize this

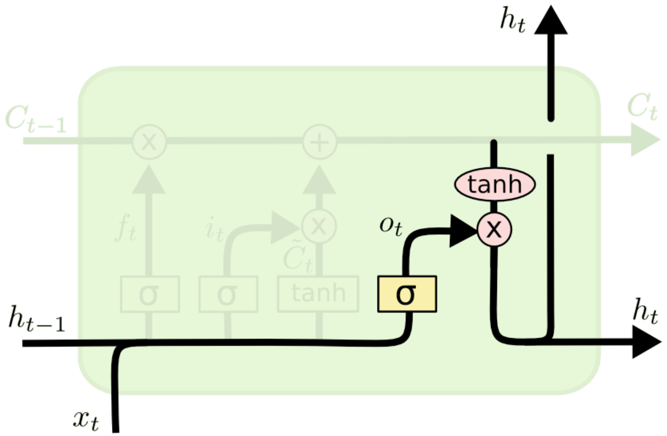


$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

Image Credit: Christopher Olah (http://colah.github.io/posts/2015-08-Understanding-LSTMs/)

LSTMs Intuition: Output Gate

- Should we output this “bit” of information to “deeper” layers?

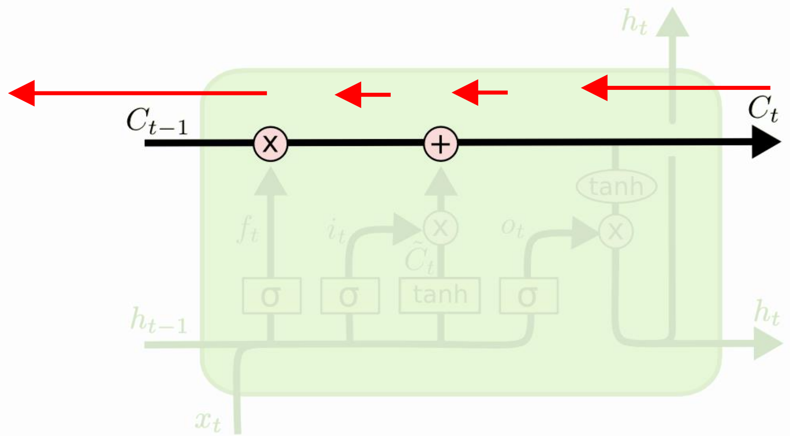


$$o_t = \sigma\left(W_o\ [h_{t-1}, x_t]\ +\ b_o\right)$$

$$h_t = o_t * \tanh\left(C_t\right)$$

Image Credit: Christopher Olah (http://colah.github.io/posts/2015-08-Understanding-LSTMs/)

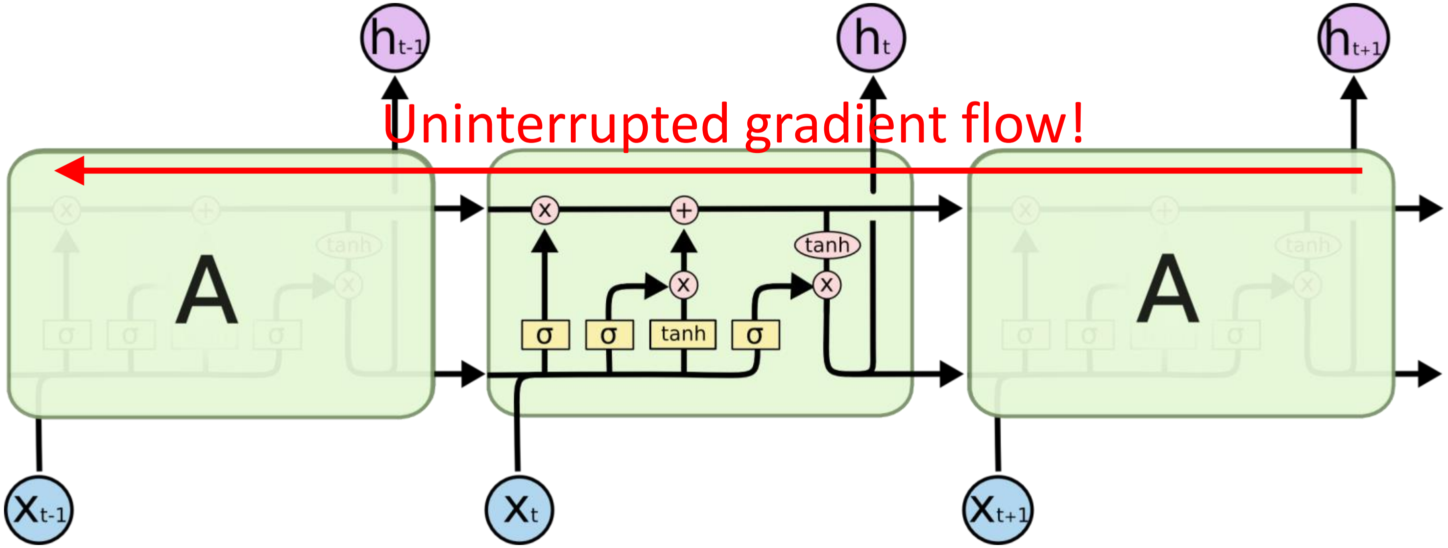
LSTMs Intuition: Additive Updates



Backpropagation from c_t to c_{t-1} only elementwise multiplication by f, no matrix multiply by W

Image Credit: Christopher Olah (http://colah.github.io/posts/2015-08-Understanding-LSTMs/)

LSTMs Intuition: Additive Updates



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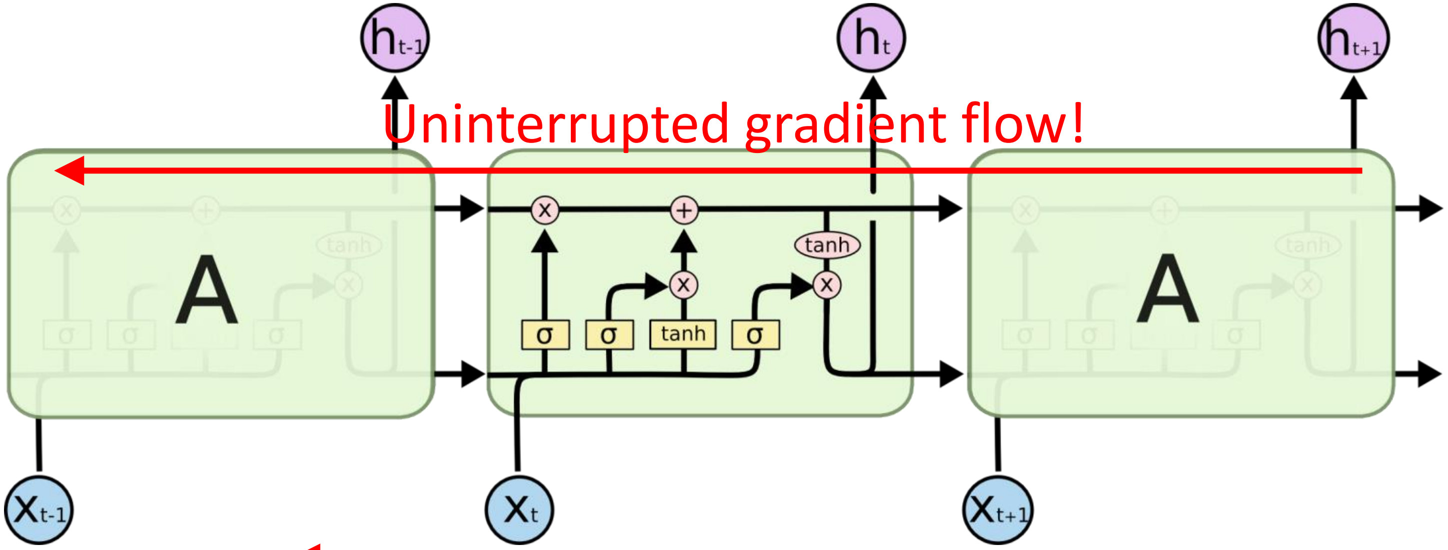
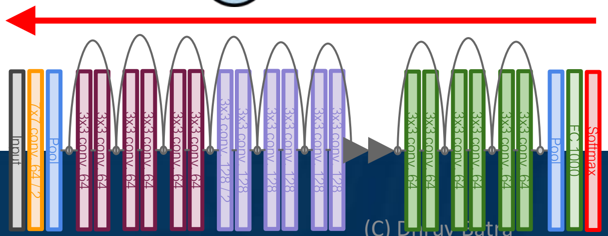
LSTMs Intuition: Additive Updates





Similar to ResNet!

Image Credit: Christopher Olah (http://colah.github.io/posts/2015-08-Understanding-LSTMs/)

LSTMs

- A pretty sophisticated cell

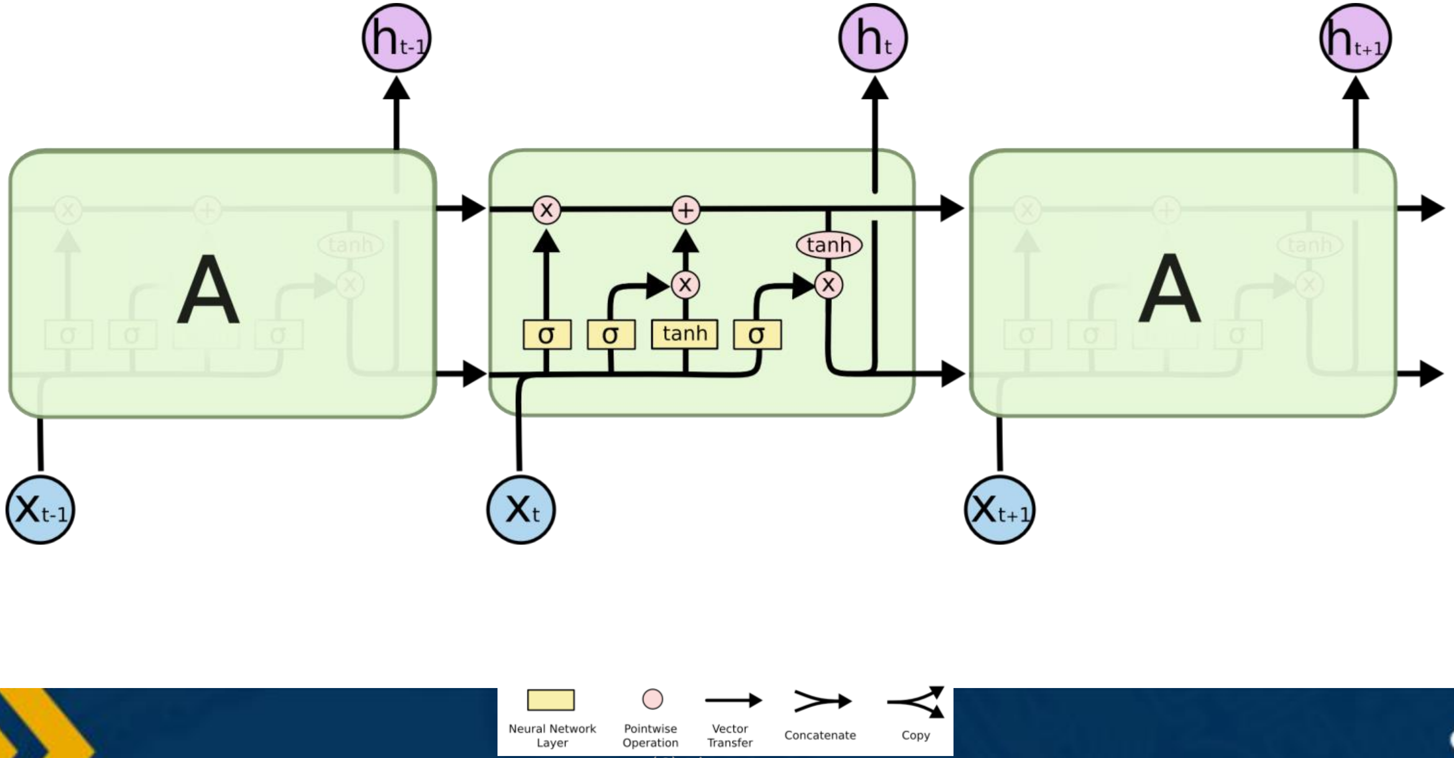


Image Credit: Christopher Olah (http://colah.github.io/posts/2015-08-Understanding-LSTMs/)

Other RNN Variants

[*An Empirical Exploration of Recurrent Network Architectures,* Jozefowicz et al., 2015]

MUT1:

$$
\begin{aligned}
z &= \mathrm{sigm}(W_{\mathrm{xz}} x_t + b_{\mathrm{z}}) \\
r &= \mathrm{sigm}(W_{\mathrm{xr}} x_t + W_{\mathrm{hr}} h_t + b_{\mathrm{r}}) \\
h_{t+1} &= \tanh(W_{\mathrm{hh}}(r \odot h_t) + \tanh(x_t) + b_{\mathrm{h}}) \odot z \\
&+ h_t \odot (1 - z)
\end{aligned}
$$

MUT2:

$$
\begin{aligned}
z &= \mathrm{sigm}(W_{\mathrm{xz}} x_t + W_{\mathrm{hz}} h_t + b_{\mathrm{z}}) \\
r &= \mathrm{sigm}(x_t + W_{\mathrm{hr}} h_t + b_{\mathrm{r}}) \\
h_{t+1} &= \tanh(W_{\mathrm{hh}}(r \odot h_t) + W_{xh} x_t + b_{\mathrm{h}}) \odot z \\
&+ h_t \odot (1 - z)
\end{aligned}
$$

MUT3:

$$
\begin{aligned}
z &= \mathrm{sigm}(W_{\mathrm{xz}} x_t + W_{\mathrm{hz}} \tanh(h_t) + b_{\mathrm{z}}) \\
r &= \mathrm{sigm}(W_{\mathrm{xr}} x_t + W_{\mathrm{hr}} h_t + b_{\mathrm{r}}) \\
h_{t+1} &= \tanh(W_{\mathrm{hh}}(r \odot h_t) + W_{xh} x_t + b_{\mathrm{h}}) \odot z \\
&+ h_t \odot (1 - z)
\end{aligned}
$$