

Topics:

- Attention and Transformers

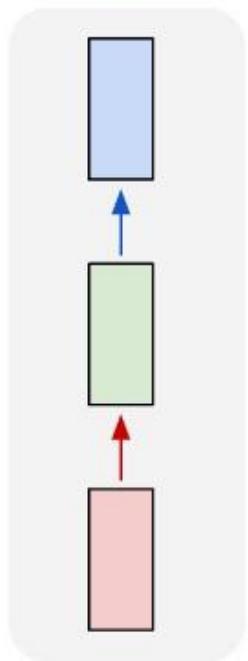
CS 4644-DL / 7643-A

ZSOLT KIRA

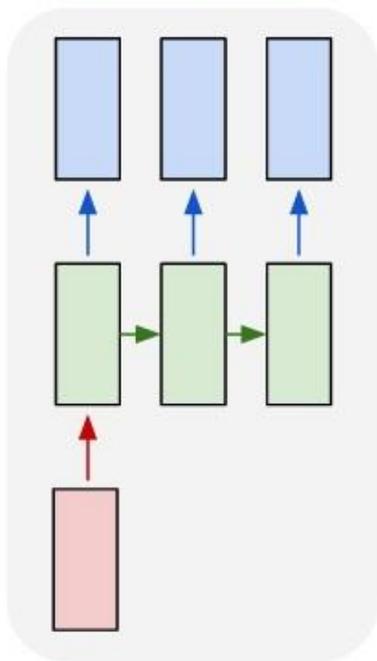
- **Assignment 3 out**
 - **Due March 14th 11:59pm EST**
- **Project milestone**
 - **Due March 14th 11:59pm EST**

Sequence Modeling with RNNs

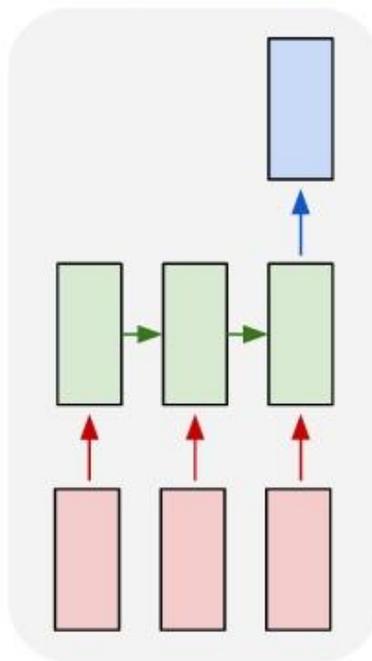
one to one



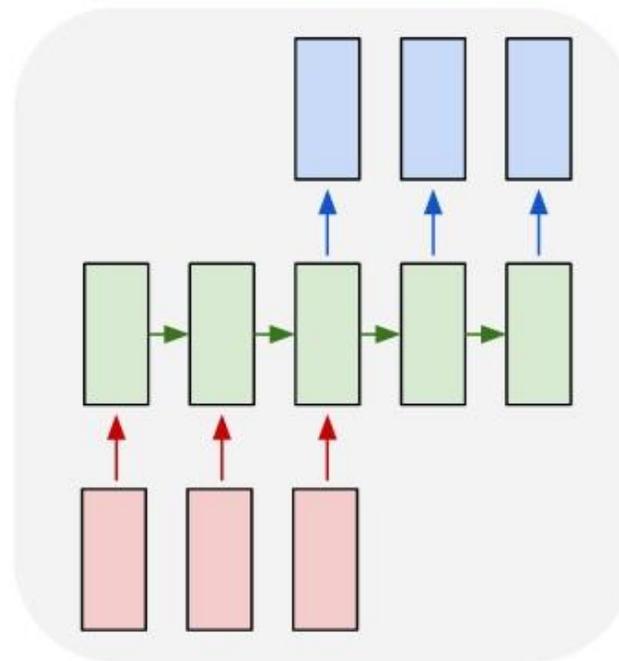
one to many



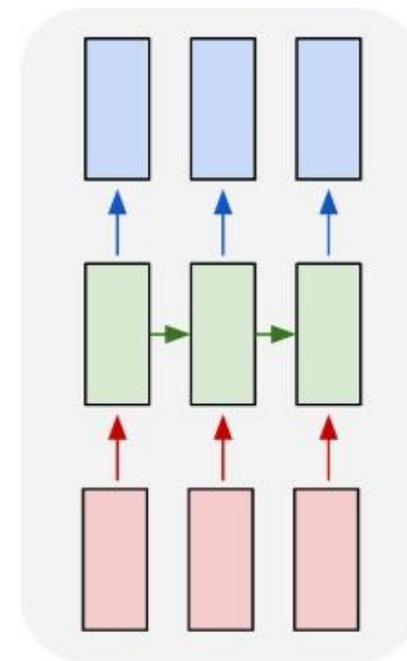
many to one



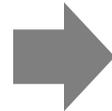
many to many



many to many



Machine Translation

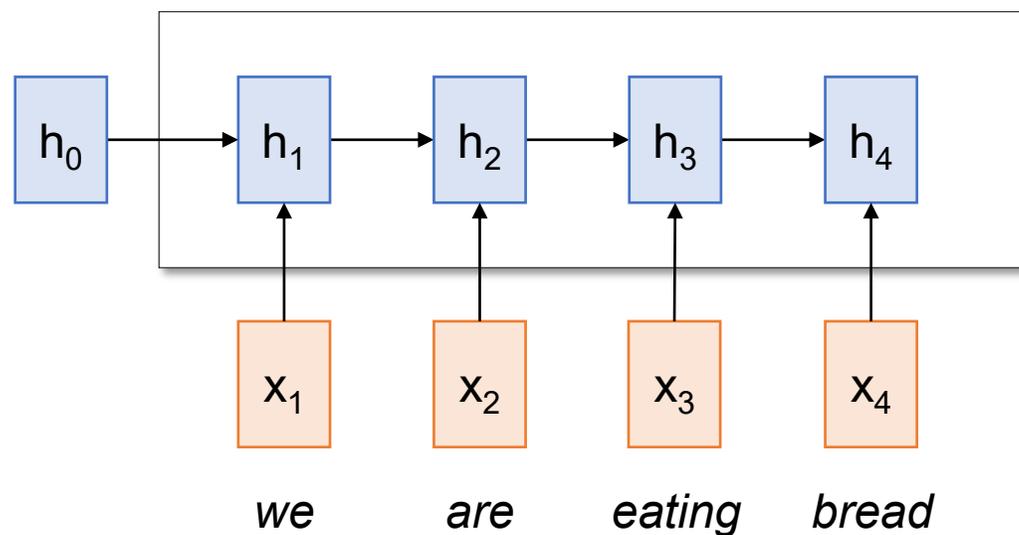


we are eating bread

estamos comiendo pan

Machine Translation with RNNs

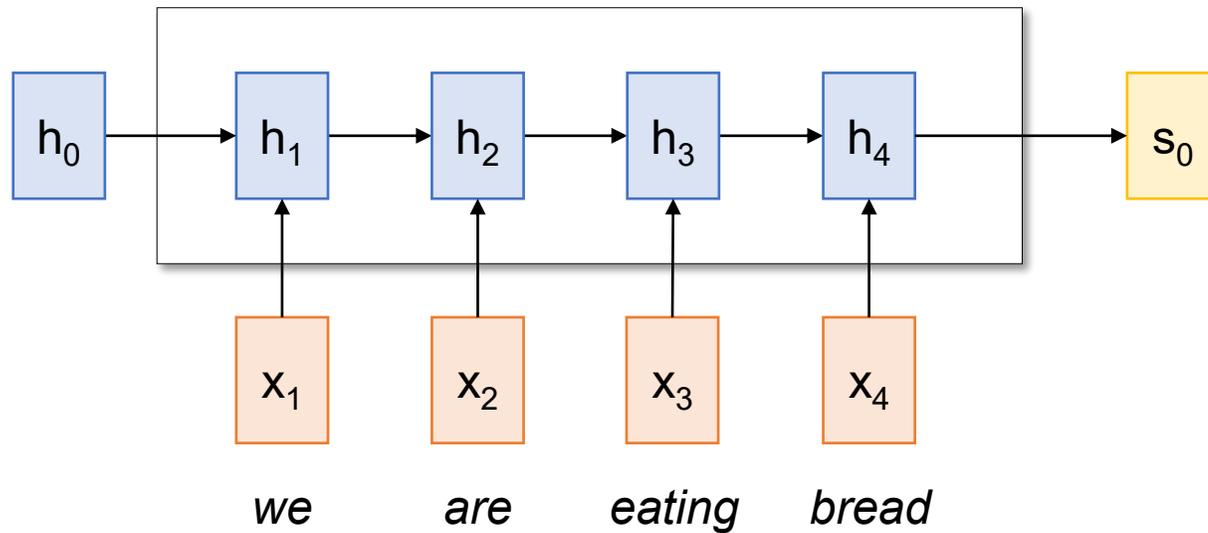
Encoder: $h_t = f_W(x_t, h_{t-1})$



Machine Translation with RNNs

Encoder: $h_t = f_W(x_t, h_{t-1})$

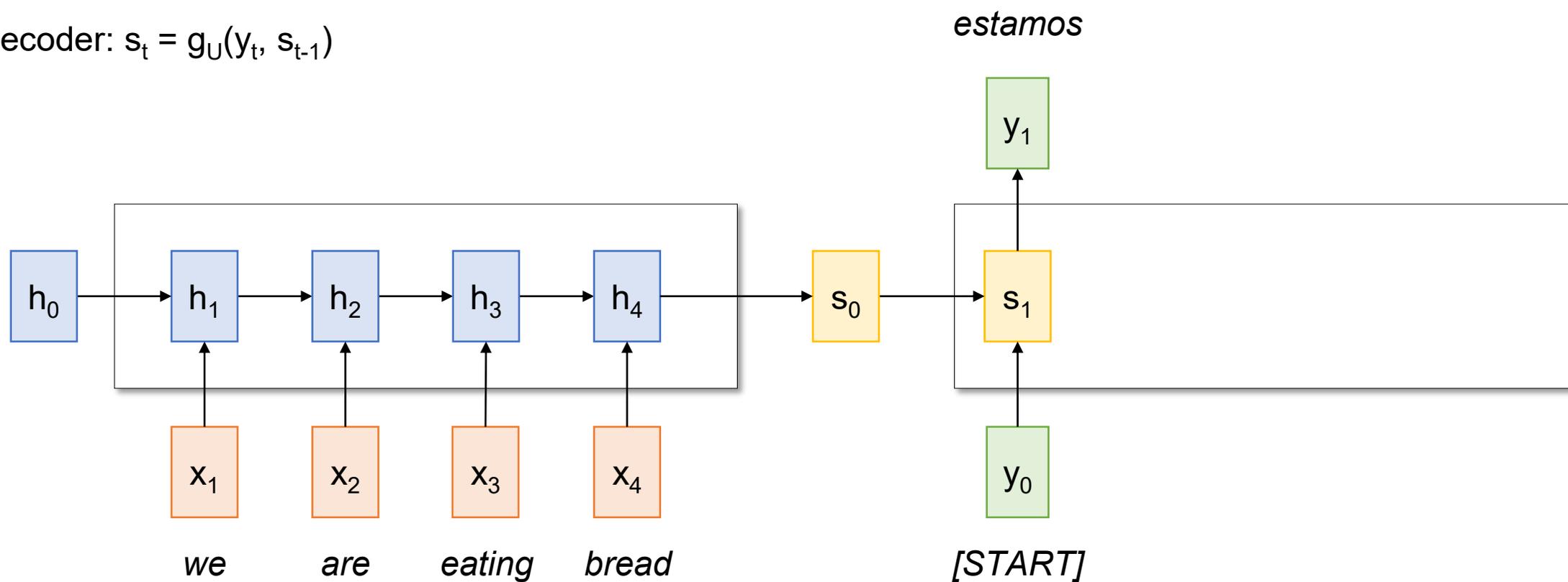
$s_0 = h_4$



Machine Translation with RNNs

Encoder: $h_t = f_W(x_t, h_{t-1})$

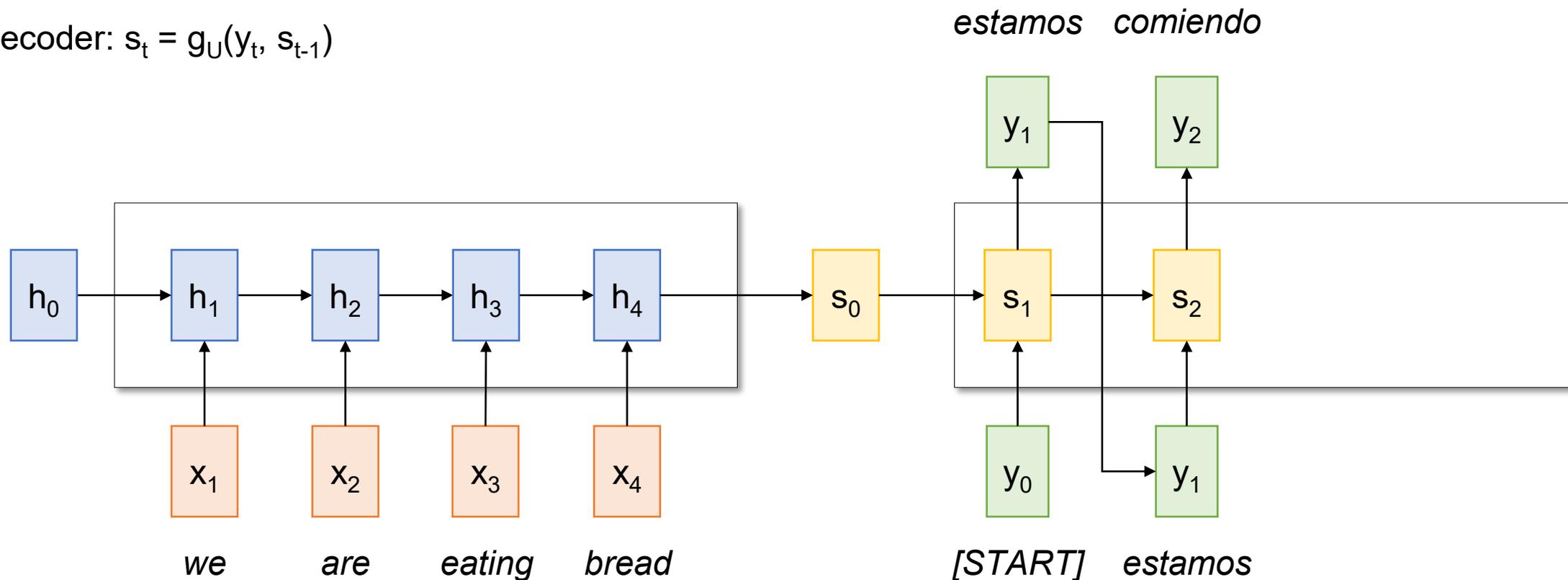
Decoder: $s_t = g_U(y_t, s_{t-1})$



Machine Translation with RNNs

Encoder: $h_t = f_W(x_t, h_{t-1})$

Decoder: $s_t = g_U(y_t, s_{t-1})$

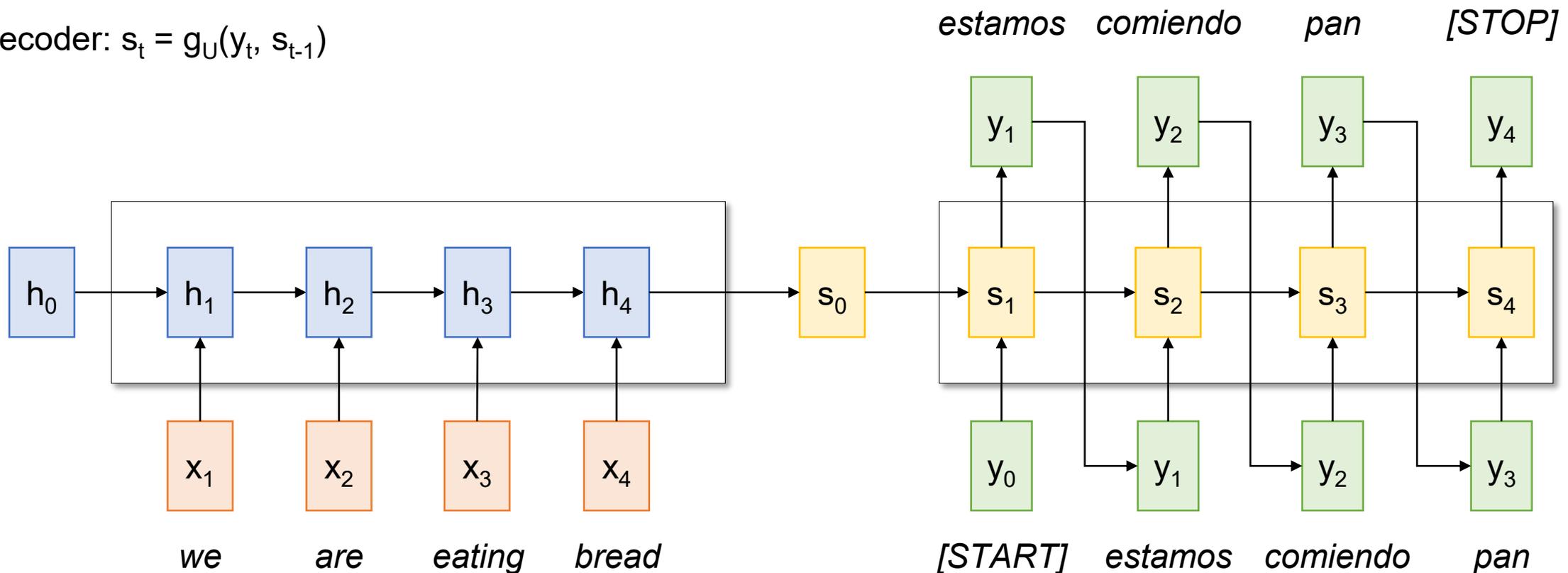


Machine Translation with RNNs

Note [START]/[STOP] words.
This can be treated as
representation for entire sentence

Encoder: $h_t = f_W(x_t, h_{t-1})$

Decoder: $s_t = g_U(y_t, s_{t-1})$

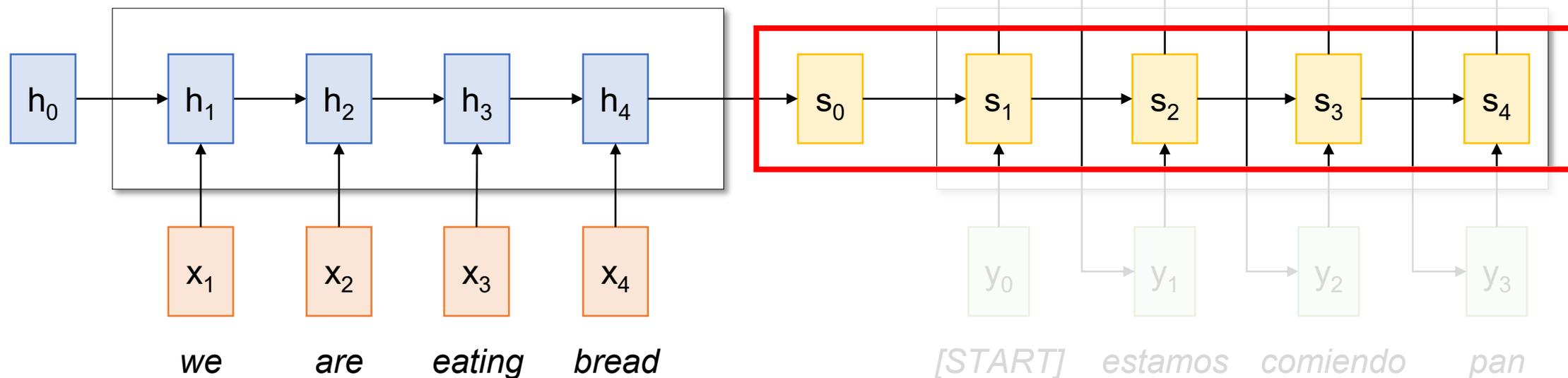


Machine Translation with RNNs

Encoder: $h_t = f_W(x_t, h_{t-1})$

Decoder: $s_t = g_U(y_t, s_{t-1})$

Problem: s_i is used to encode input and maintain decoder state



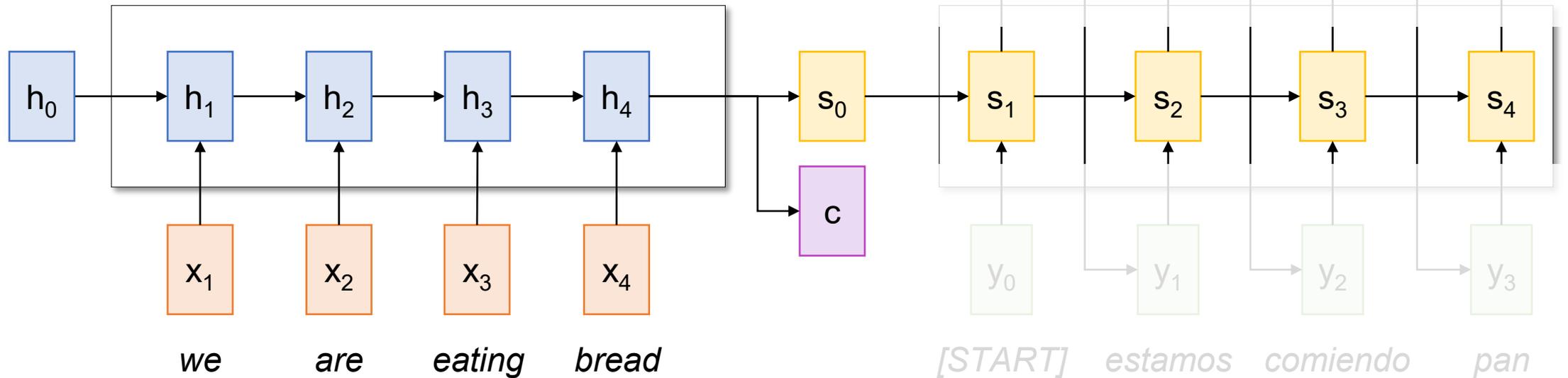
Machine Translation with RNNs

Encoder: $h_t = f_W(x_t, h_{t-1})$

Decoder: $s_t = g_U(y_t, s_{t-1},$

$c)$

Solution: add a context vector $c = h_4$ and predict s_0 from h_4



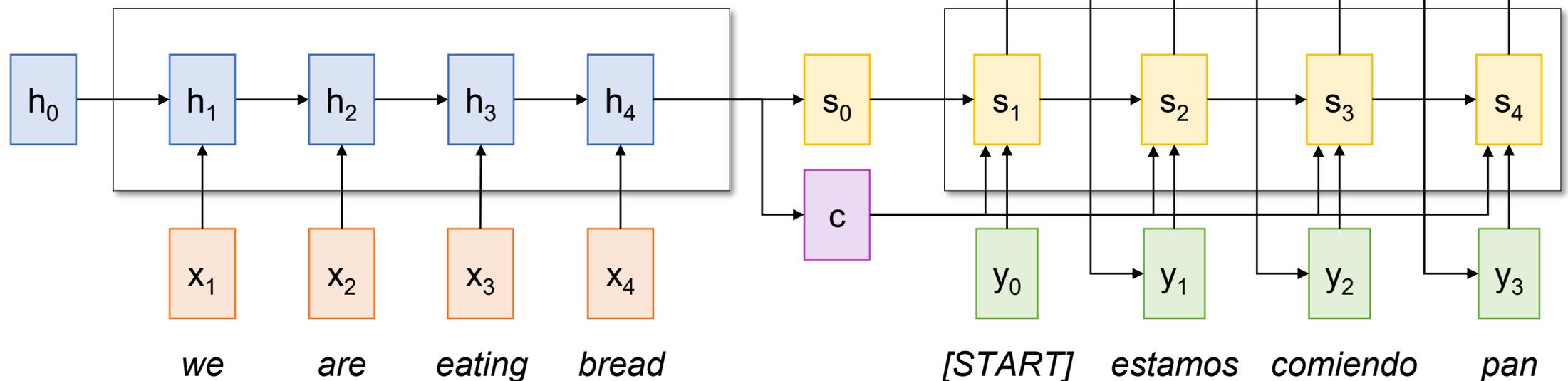
Machine Translation with RNNs

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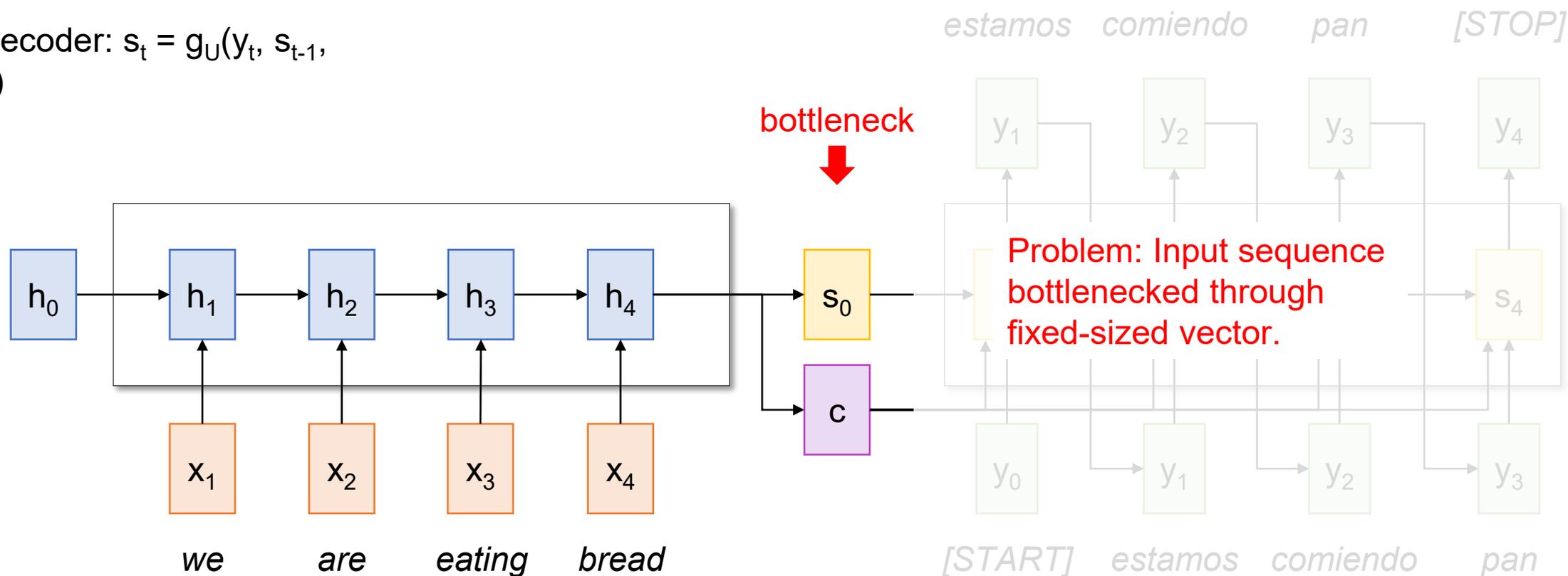


Machine Translation with RNNs

Encoder: $h_t = f_W(x_t, h_{t-1})$

Decoder: $s_t = g_U(y_t, s_{t-1},$

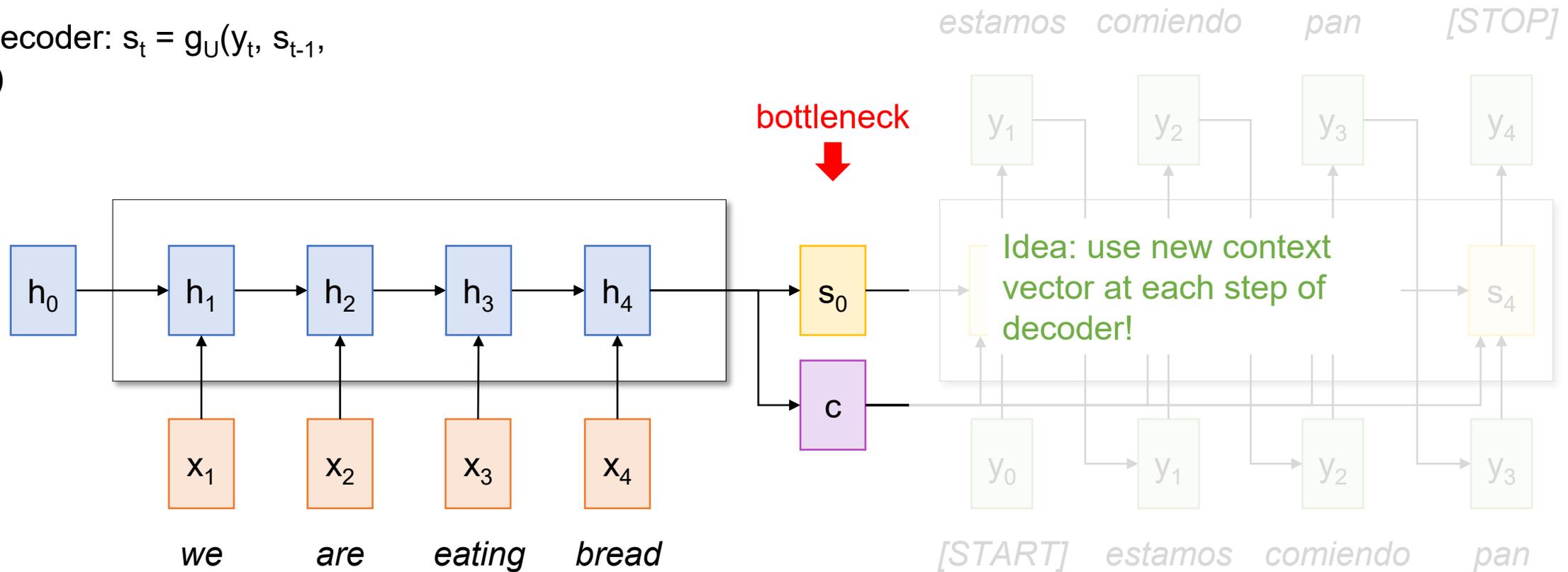
c)



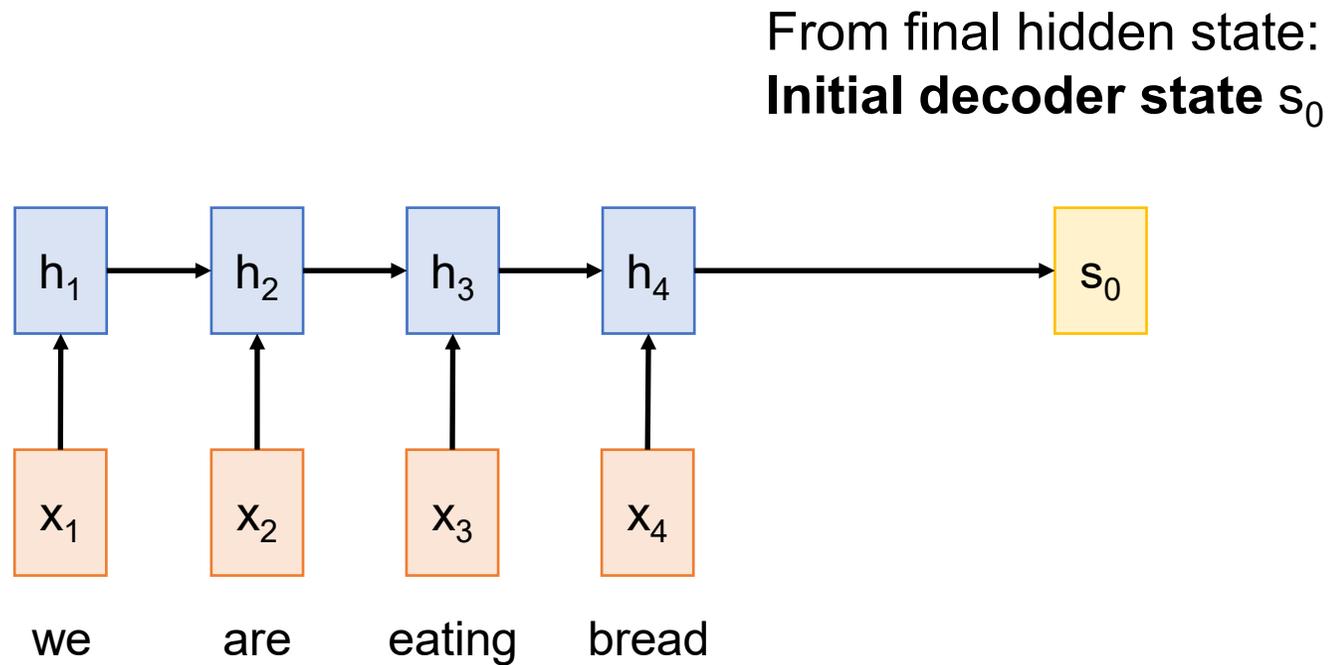
Machine Translation with RNNs

Encoder: $h_t = f_W(x_t, h_{t-1})$

Decoder: $s_t = g_U(y_t, s_{t-1}, c)$

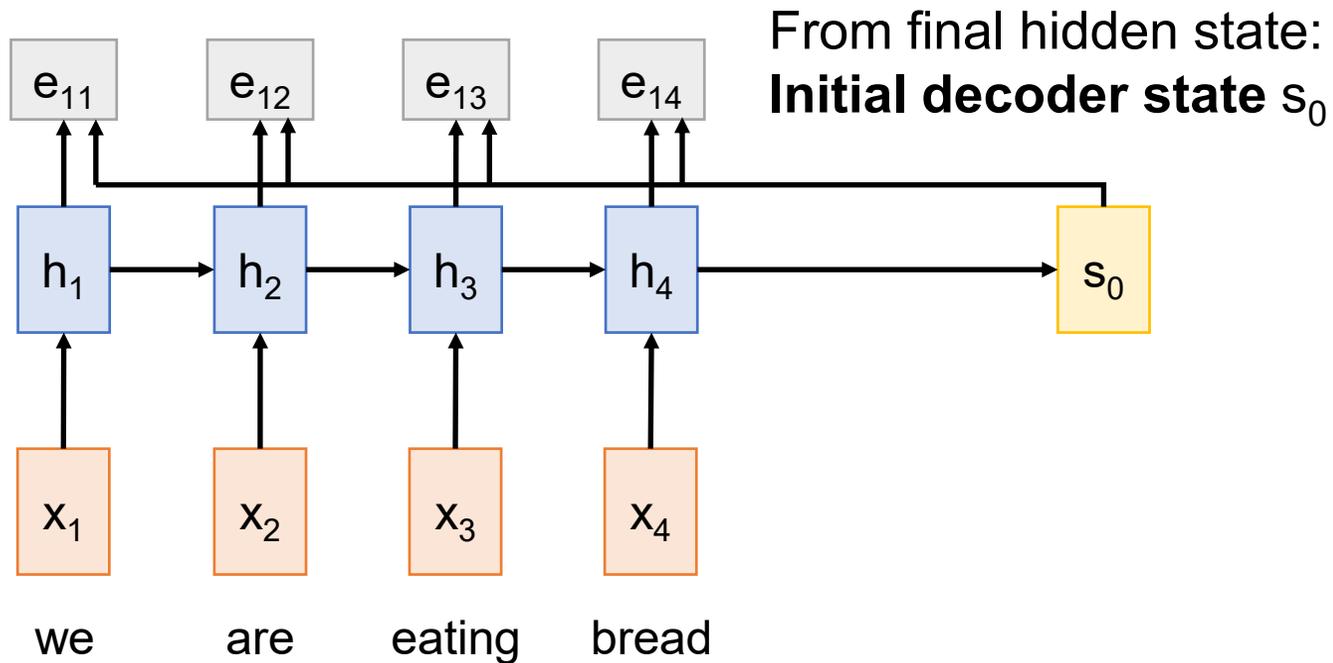


Machine Translation with RNNs and Attention

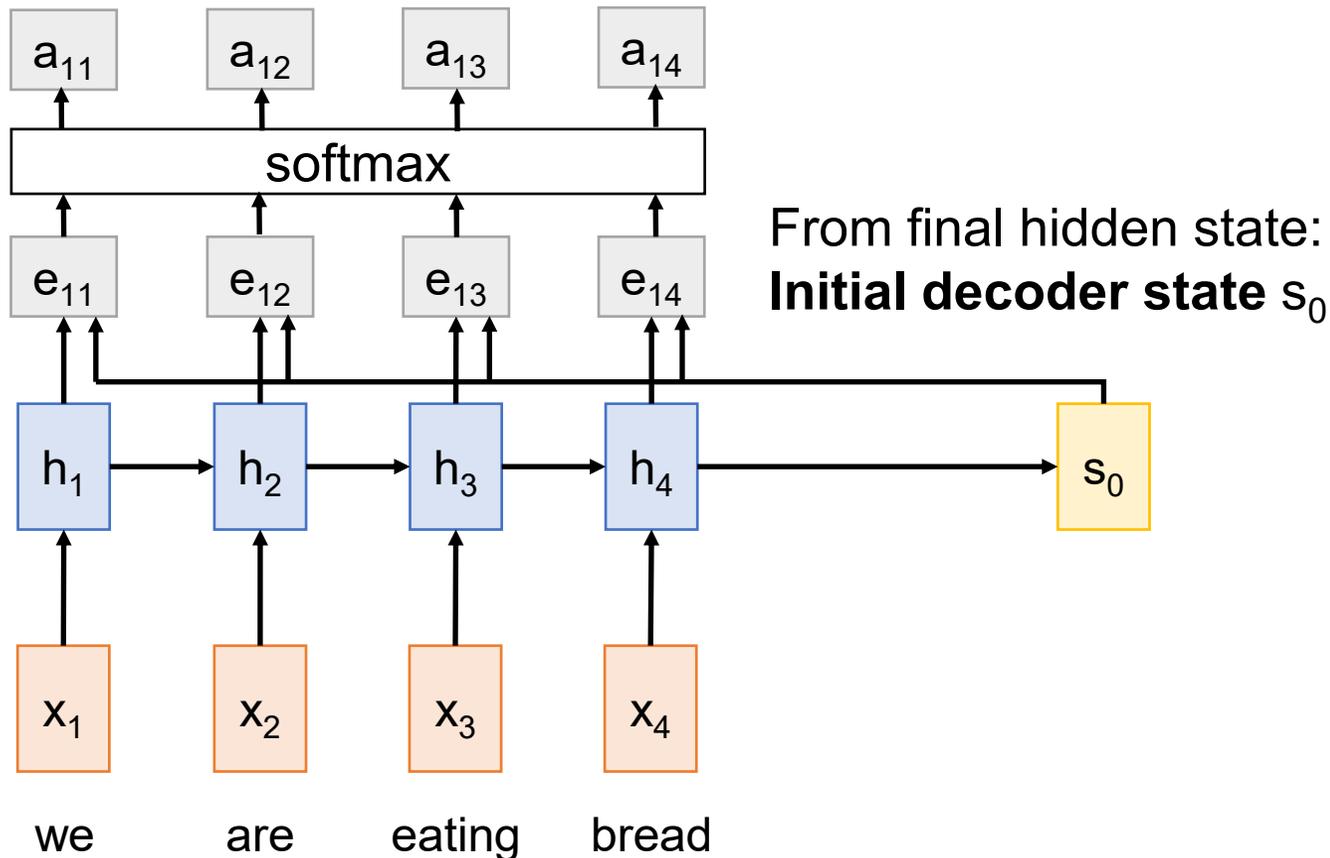


Machine Translation with RNNs and Attention

Compute **alignment scores**
 $e_{t,i} = f_{\text{att}}(s_{t-1}, h_i)$ (f_{att} is an
MLP)



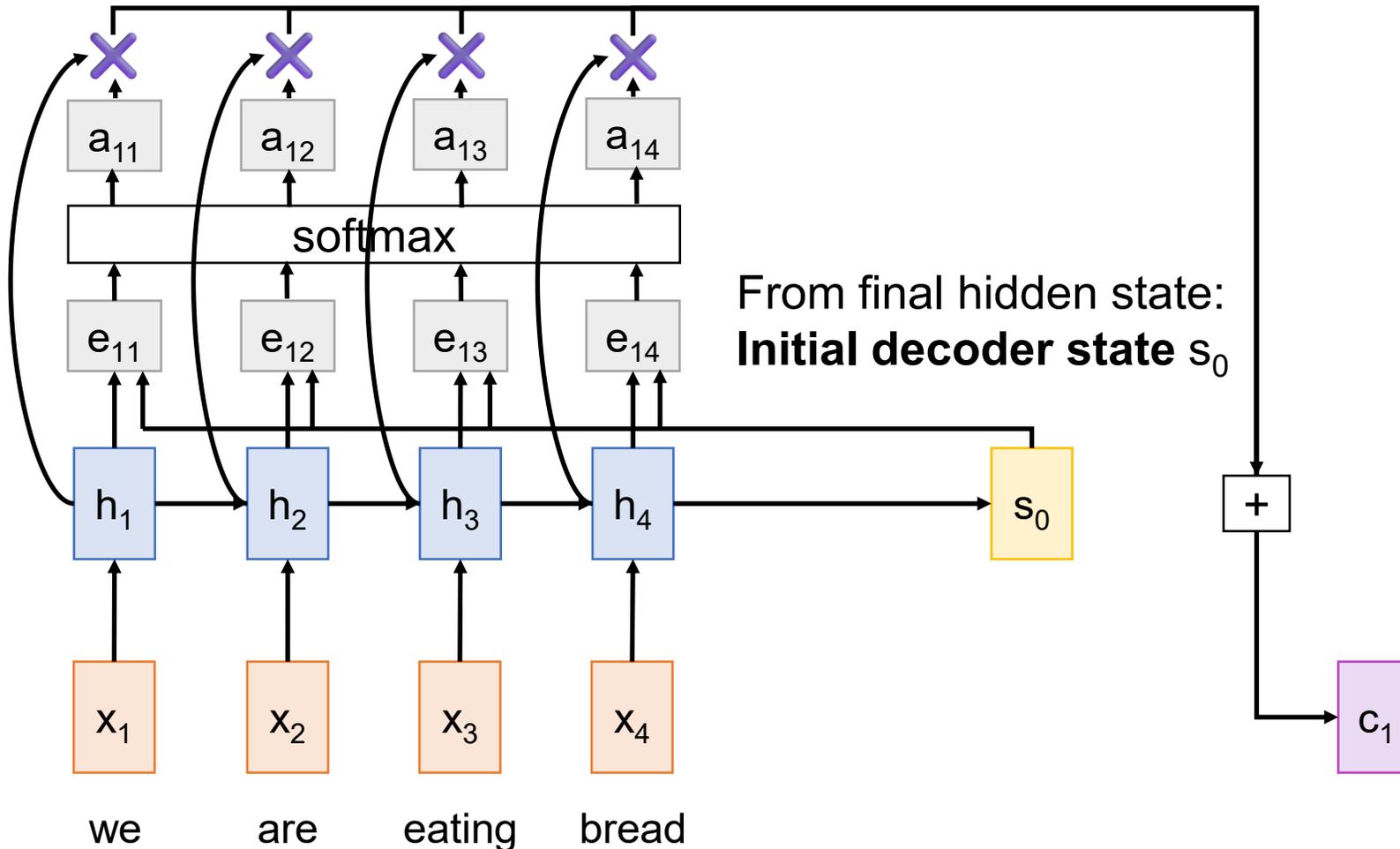
Machine Translation with RNNs and Attention



Compute **alignment scores**
 $e_{t,i} = f_{\text{att}}(s_{t-1}, h_i)$ (f_{att} is an **MLP**)

Normalize to get **attention weights**
 $0 < a_{t,i} < 1$ $\sum_i a_{t,i} = 1$

Machine Translation with RNNs and Attention

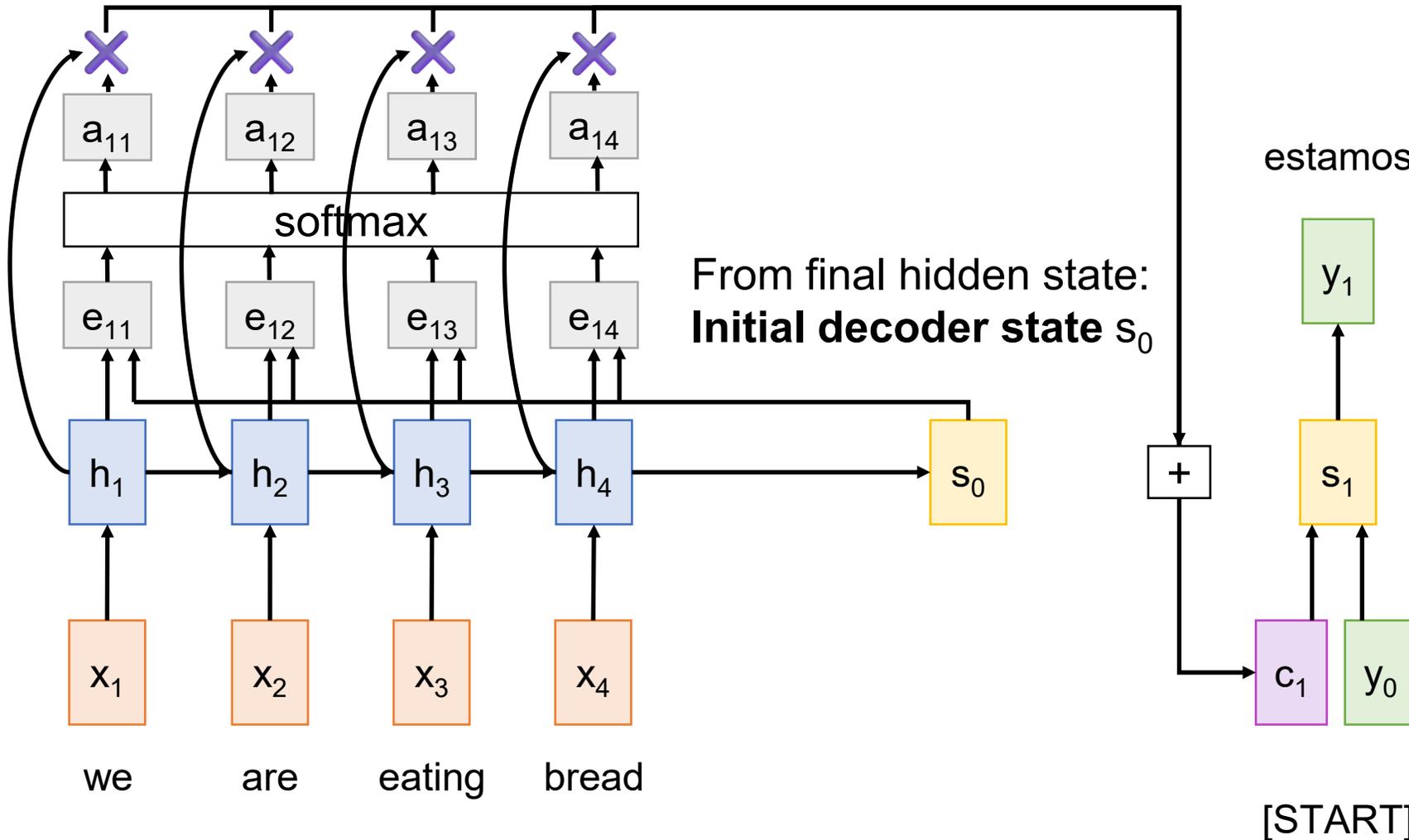


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Normalize to get
attention weights
 $0 < a_{t,i} < 1 \quad \sum_i a_{t,i} = 1$

Set context vector \mathbf{c} to a
linear combination of hidden
states
 $c_t = \sum_i a_{t,i} h_i$

Machine Translation with RNNs and Attention

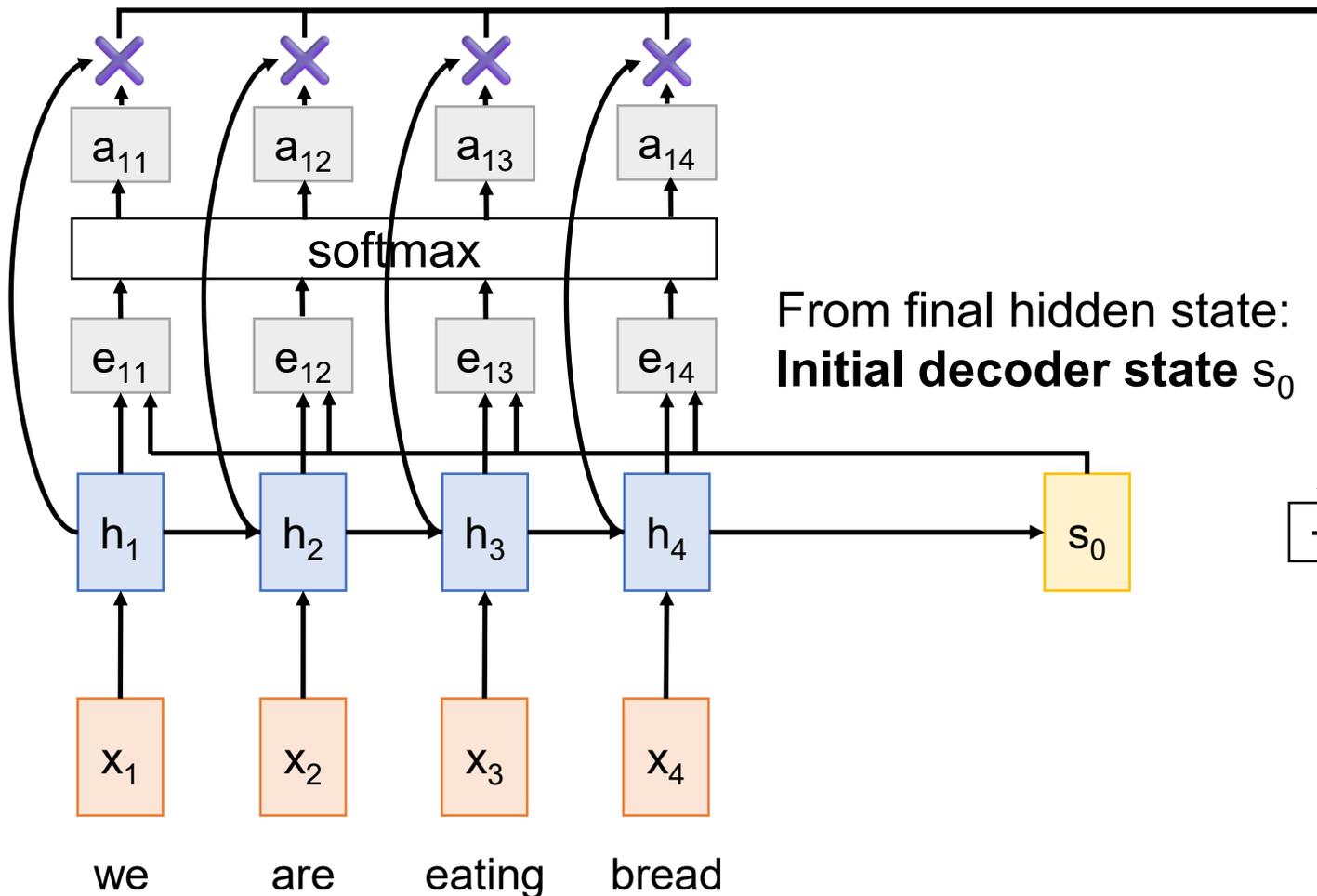


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Machine Translation with RNNs and Attention



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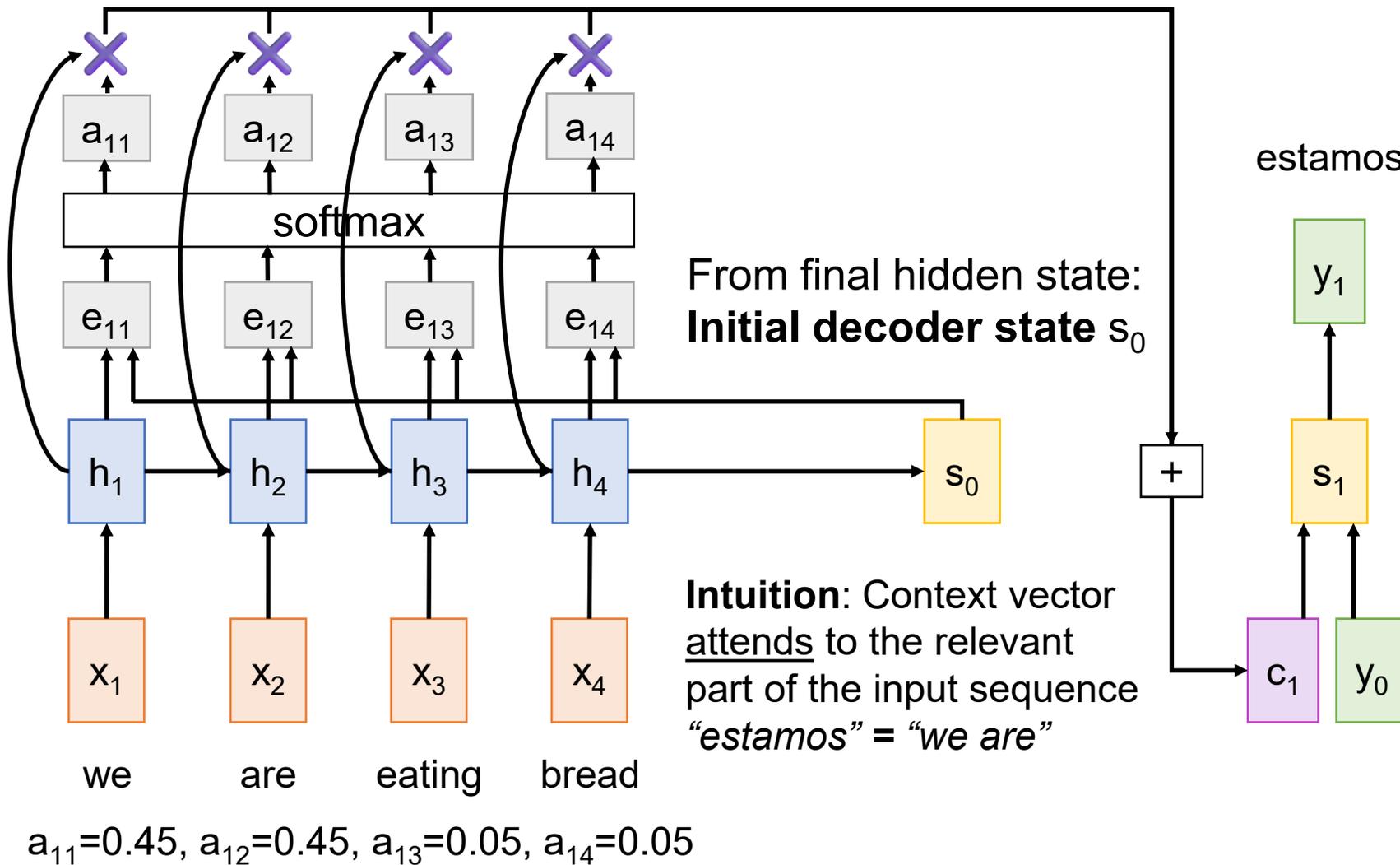
Set context vector \mathbf{c} to a linear combination of hidden states

$$c_t = \sum_i a_{t,i} h_i$$

This is all differentiable! Do not supervise attention weights – backprop through everything

Can be seen as a input-dependent weighting (rather than MLP)

Machine Translation with RNNs and Attention



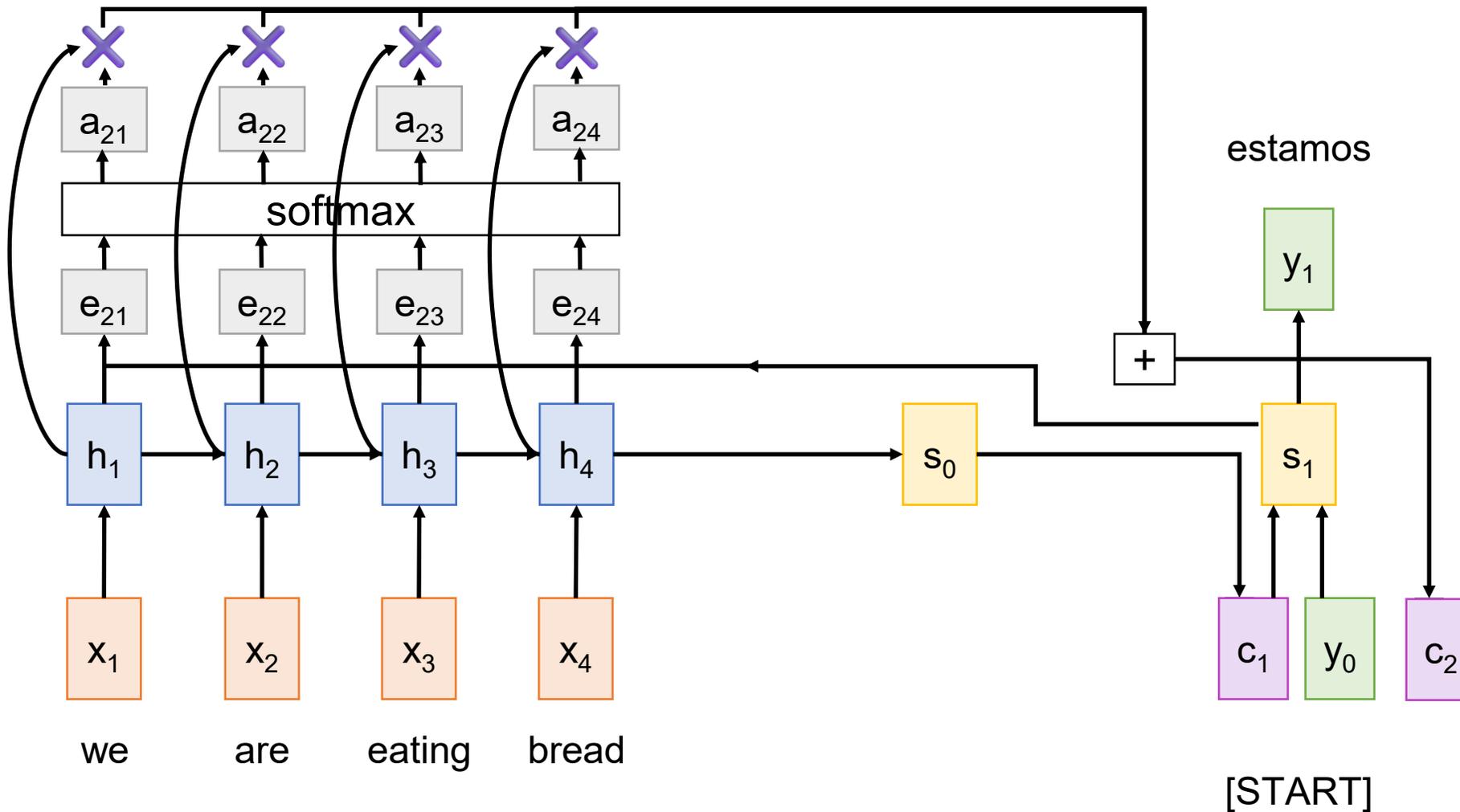
Compute **alignment scores**
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Normalize to get **attention weights**
 $0 < a_{t,i} < 1 \quad \sum_i a_{t,i} = 1$

Set context vector \mathbf{c} to a linear combination of hidden states
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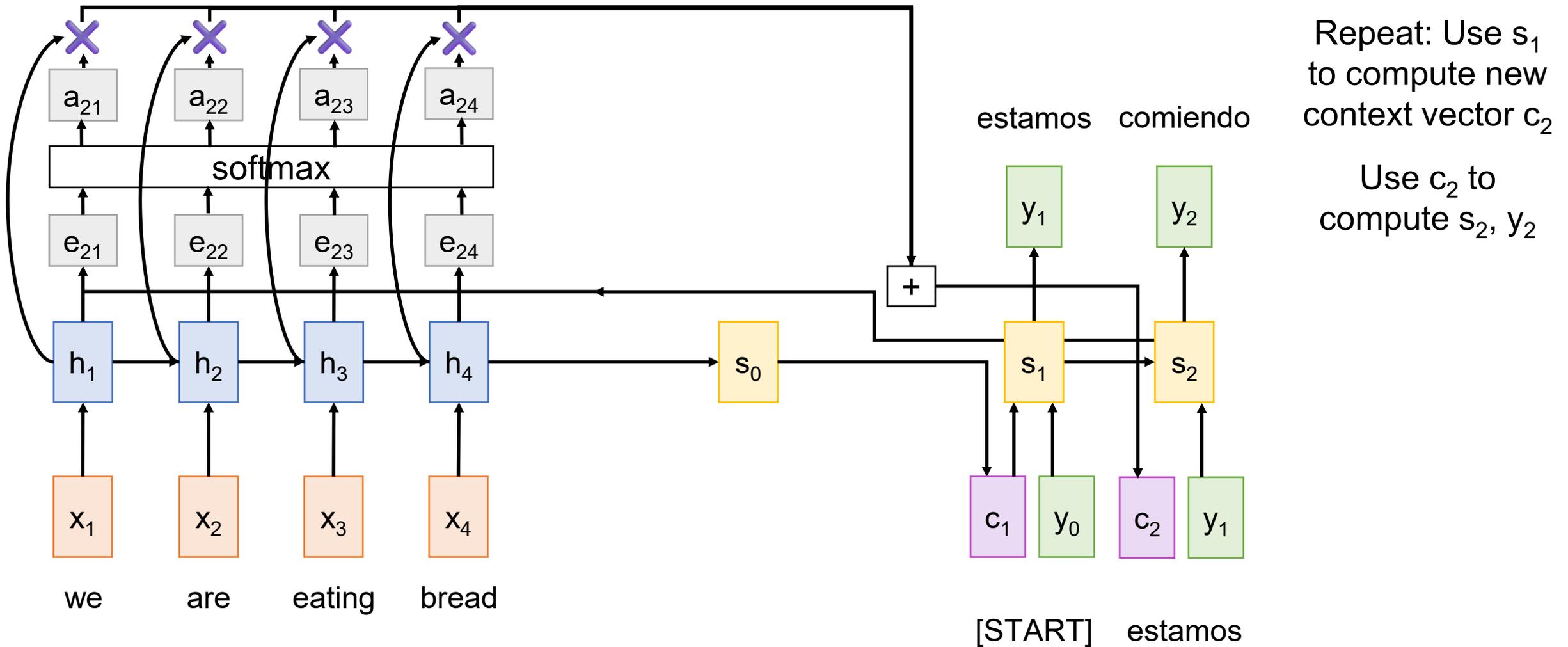
This is an inductive bias we think is reasonable for this task. Need to verify

Machine Translation with RNNs and Attention

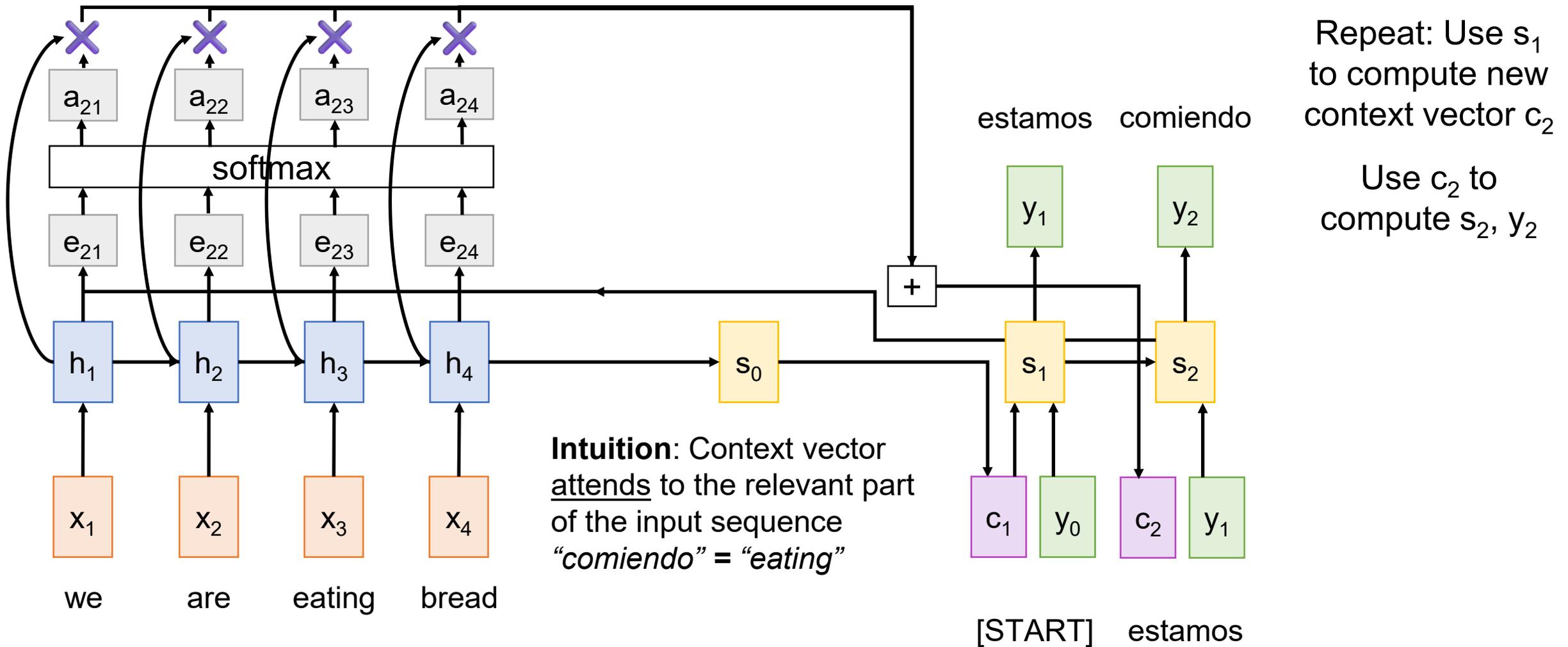


Repeat: Use s_1 to compute new context vector c_2

Machine Translation with RNNs and Attention

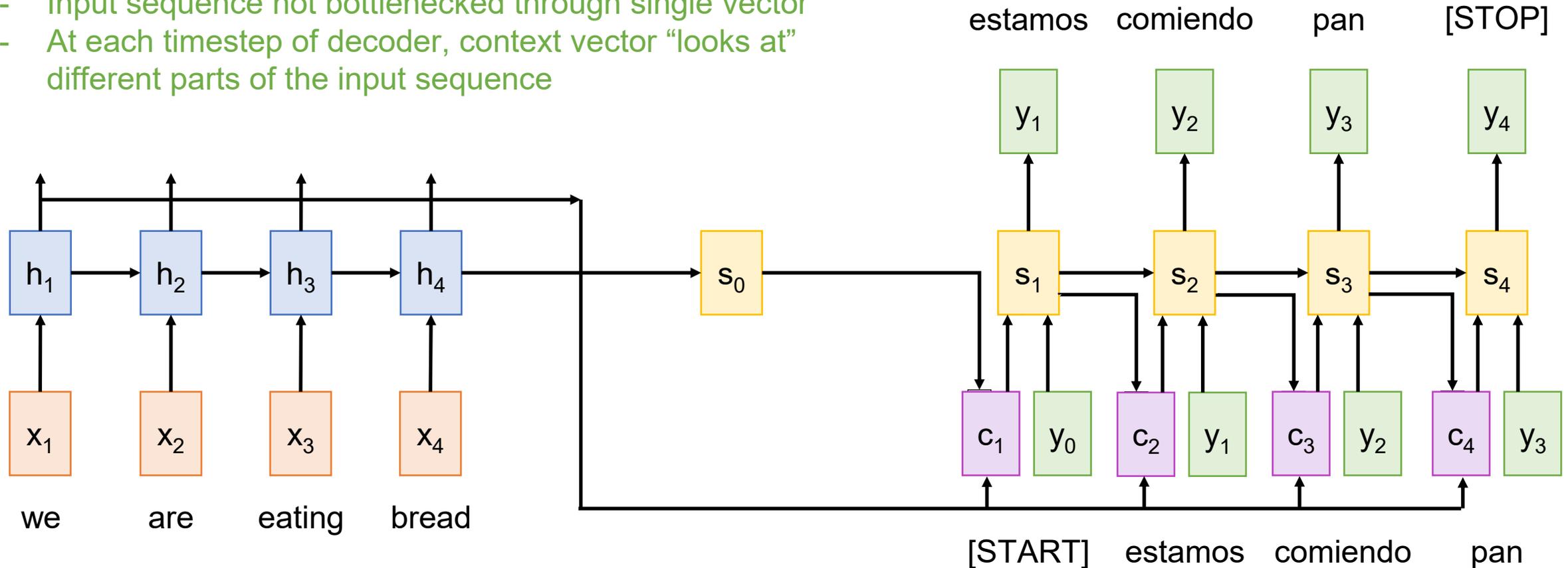


Machine Translation with RNNs and Attention



Machine Translation with RNNs and Attention

- Use a different context vector in each timestep of decoder
- Input sequence not bottlenecked through single vector
 - At each timestep of decoder, context vector “looks at” different parts of the input sequence



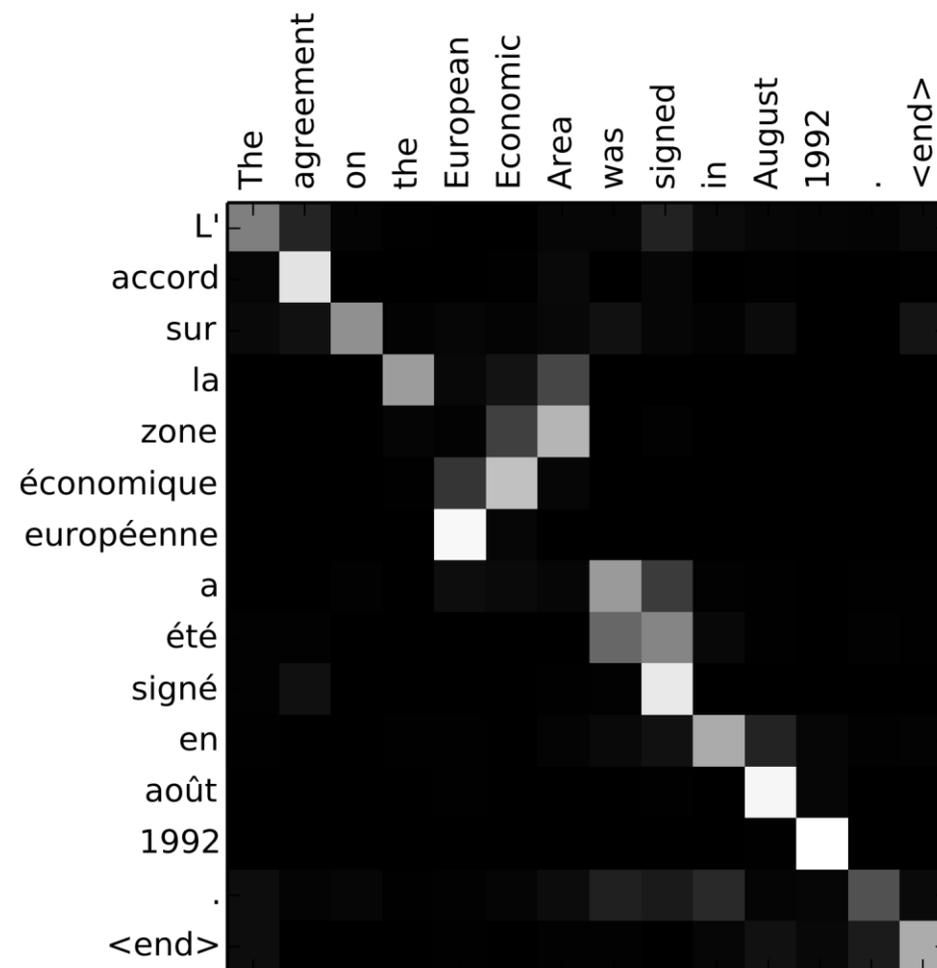
Machine Translation with RNNs and Attention

Example: English to French translation

Input: “The agreement on the European Economic Area was signed in August 1992.”

Output: “L’accord sur la zone économique européenne a été signé en août 1992.”

Visualize attention weights $a_{t,i}$



Machine Translation with RNNs and Attention

Example: English to French translation

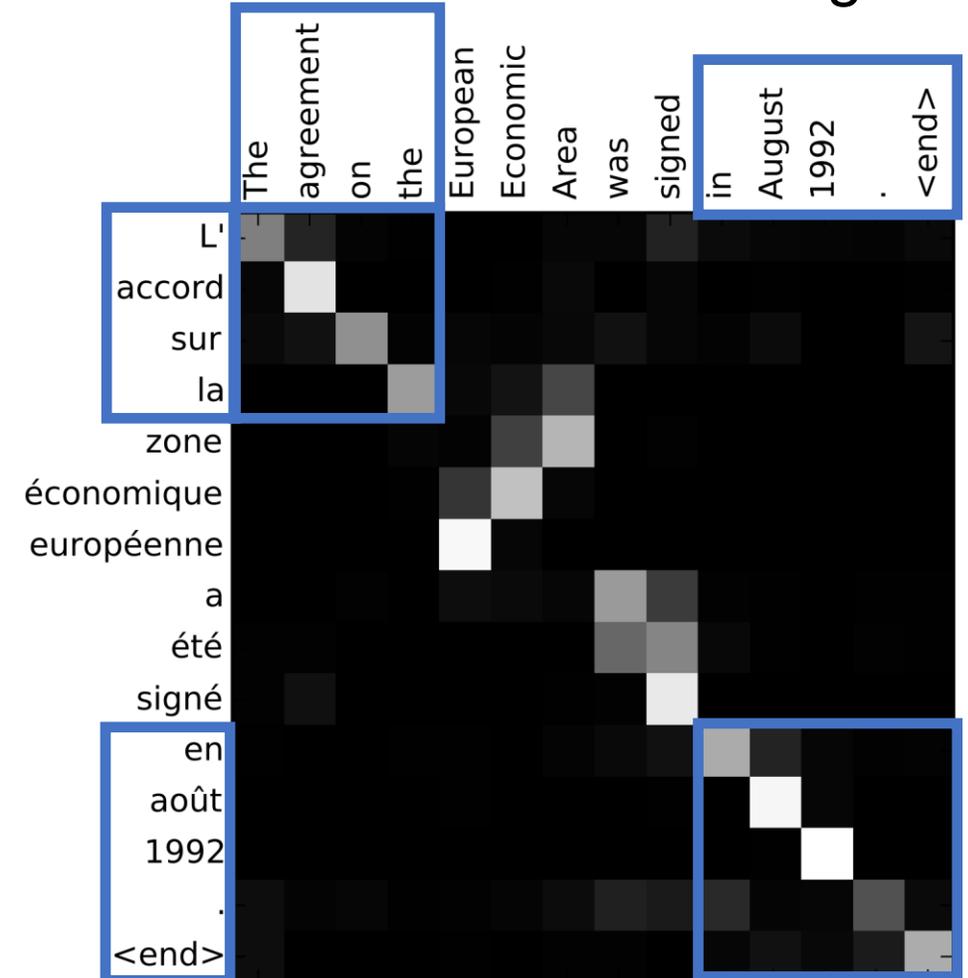
Input: “**The agreement on the** European Economic Area was signed **in August 1992.**”

Output: “**L'accord sur la** zone économique européenne a été signé **en août 1992.**”

Diagonal attention means words correspond in order

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Visualize attention weights $a_{t,i}$



Machine Translation with RNNs and Attention

Example: English to French translation

Input: “The agreement on the European Economic Area was signed in August 1992.”

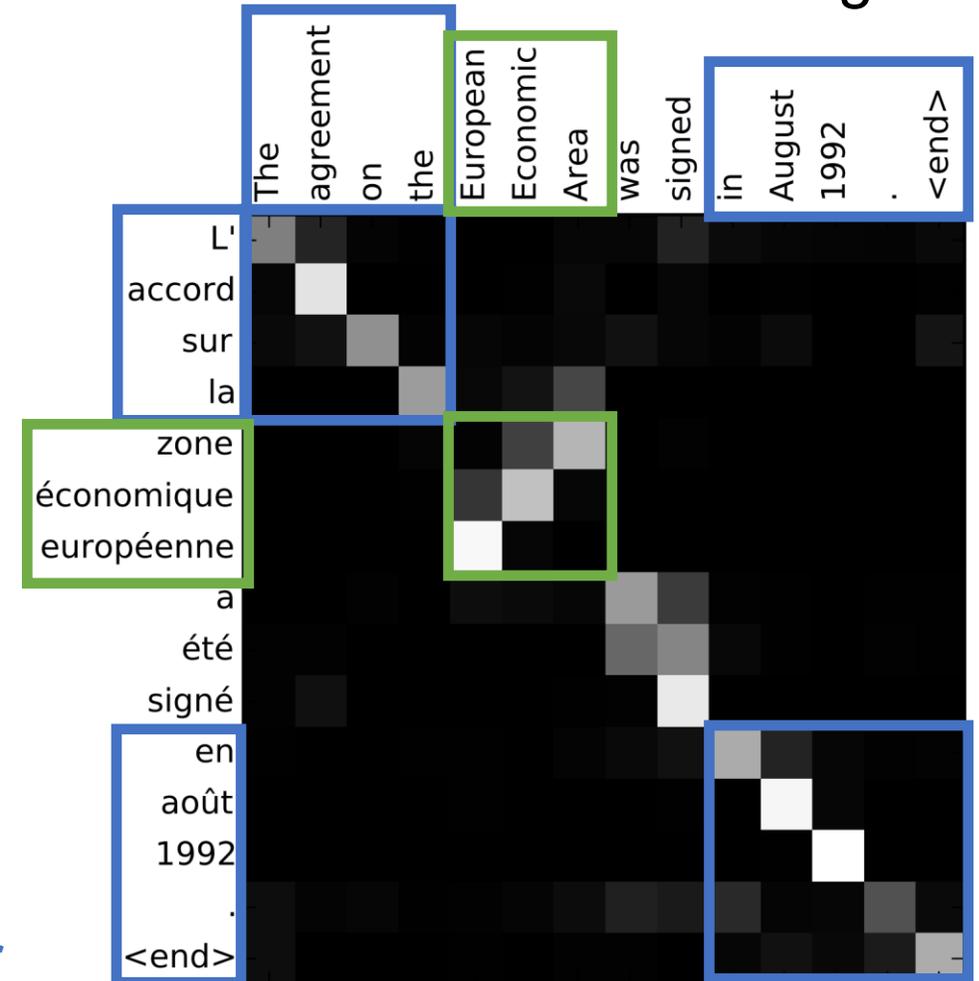
Output: “L'accord sur la zone économique européenne a été signé en août 1992.”

Diagonal attention means words correspond in order

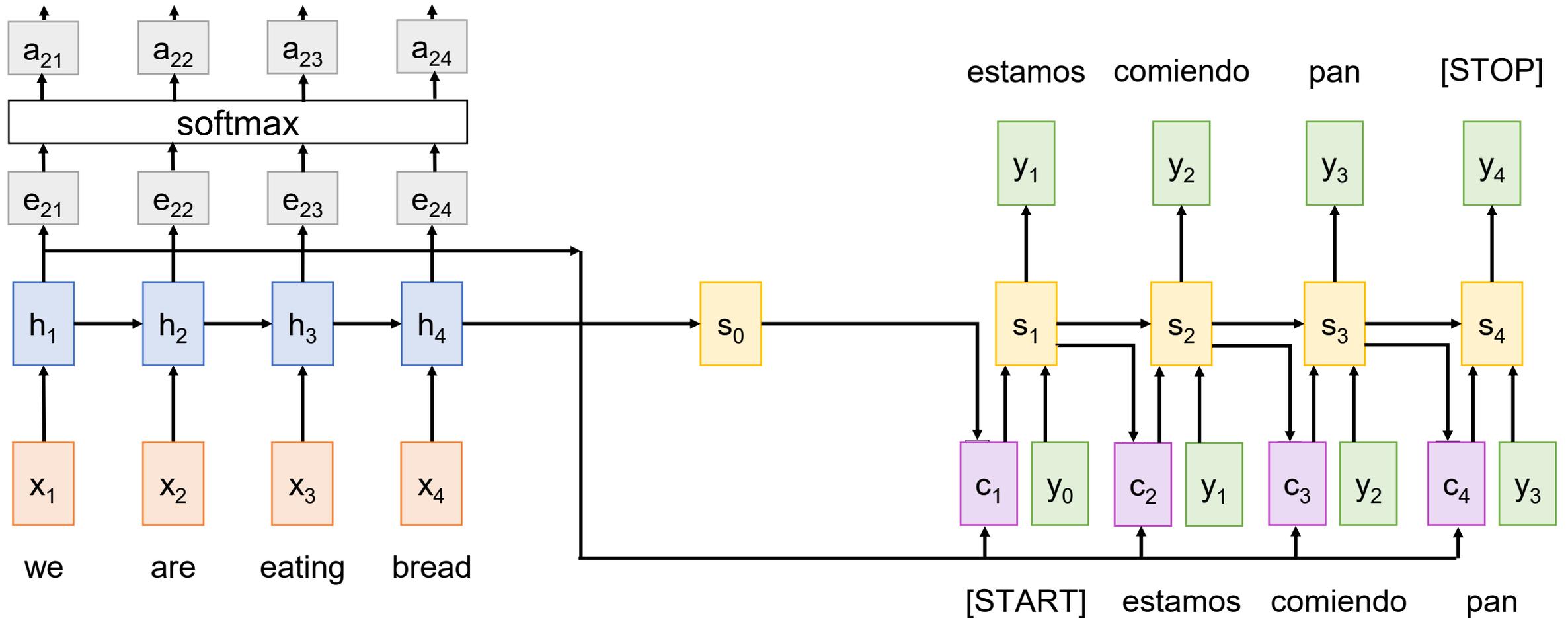
Attention figures out different word orders

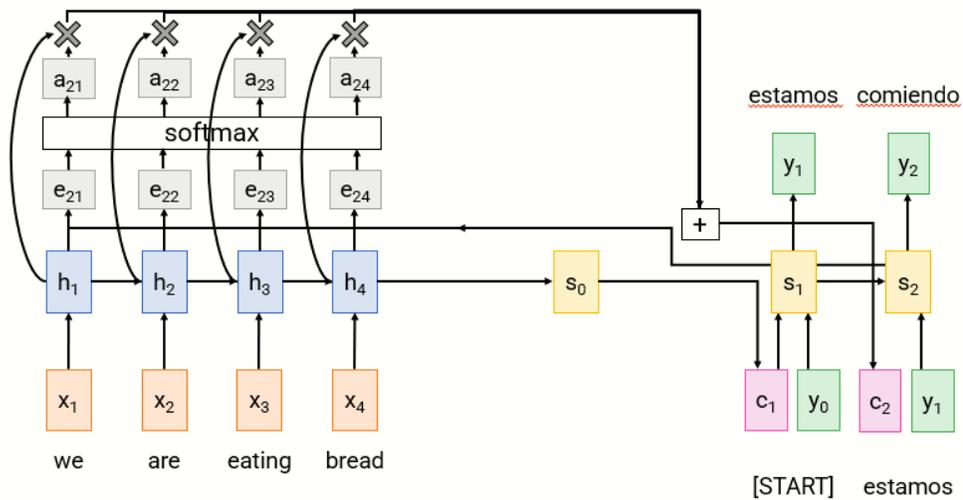
Diagonal attention means words correspond in order

Visualize attention weights $a_{t,i}$

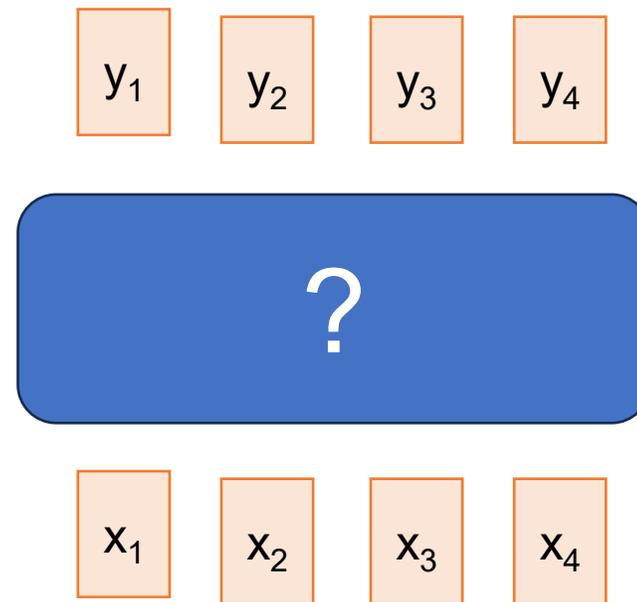


Machine Translation with RNNs and Attention





Idea: Can we use **attention** as a fundamental building block for a generic sequence (input) to sequence (output) layer?



Note: We just want a generic sequence-in, sequence-out model that will represent each input *contextualized* with rest of inputs, and encode meaning of entire sequence

We will progressively develop a generic mechanism using idea of attention.
Don't try to map to RNN translation example!

Attention Layer

Inputs:

State vector: \mathbf{s}_i (Shape: D_Q)

Hidden vectors: \mathbf{h}_i (Shape: $N_X \times D_H$)

Similarity function: f_{att}

Computation:

Similarities: e (Shape: N_X) $e_i = f_{\text{att}}(\mathbf{s}_{t-1}, \mathbf{h}_i)$

Attention weights: $a = \text{softmax}(e)$ (Shape: N_X)

Output vector: $y = \sum_i a_i \mathbf{h}_i$ (Shape: D_X)

Attention Layer

Inputs:

Query vector: \mathbf{q} (Shape: D_Q)

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Similarity function: f_{att}

Make the module generic:

Input (\mathbf{X}), Query (\mathbf{q})

Output (Weighted sum of inputs)

Computation:

Similarities: \mathbf{e} (Shape: N_X) $e_i = f_{\text{att}}(\mathbf{q}, \mathbf{X}_i)$

Attention weights: $\mathbf{a} = \text{softmax}(\mathbf{e})$ (Shape: N_X)

Output vector: $\mathbf{y} = \sum_i a_i \mathbf{X}_i$ (Shape: D_X)

Attention Layer

Inputs:

Query vector: \mathbf{q} (Shape: D_Q)

Input vectors: \mathbf{X} (Shape: $N_X \times D_Q$)

Similarity function: dot product

Computation:

Similarities: e (Shape: N_X) $e_i = \mathbf{q} \cdot \mathbf{X}_i$

Attention weights: $a = \text{softmax}(e)$ (Shape: N_X)

Output vector: $y = \sum_i a_i \mathbf{X}_i$ (Shape: D_X)

Changes:

- Use dot product for similarity

Attention Layer

Inputs:

Query vector: \mathbf{q} (Shape: D_Q)

Input vectors: \mathbf{X} (Shape: $N_X \times D_Q$)

Similarity function: scaled dot product

Computation:

Similarities: e (Shape: N_X) $e_i = \mathbf{q} \cdot \mathbf{X}_i / \sqrt{D_Q}$

Attention weights: $a = \text{softmax}(e)$ (Shape: N_X)

Output vector: $y = \sum_i a_i \mathbf{X}_i$ (Shape: D_X)

Changes:

- Use **scaled** dot product for similarity

Attention Layer

Inputs:

Query vectors: **Q** (Shape: $N_Q \times D_Q$)

Input vectors: **X** (Shape: $N_X \times D_Q$)

Make the module generic:

Sequence Input (**X**), Sequence Query (**Q**)

Output: Sequence (Weighted sum/mixture of inputs)

Computation:

Similarities: $E = \mathbf{QX}^T$ (Shape: $N_Q \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{X}_j / \text{sqrt}(D_Q)$

Attention weights: $A = \text{softmax}(E, \text{dim}=1)$ (Shape: $N_Q \times N_X$)

Output vectors: $Y = \mathbf{AX}$ (Shape: $N_Q \times D_X$) $Y_i = \sum_j A_{i,j} X_j$

Changes:

- Use dot product for similarity
- Multiple **query** vectors

Attention Layer

Inputs:

Query vectors: \mathbf{Q} (Shape: $N_Q \times D_Q$)

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Separate concerns:

- 1) *Matching* (similarity) -> Key,
- 2) Output given weighting -> Value

Computation:

Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

Value vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_Q \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_Q \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_Q \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$

Changes:

- Use dot product for similarity
- Multiple **query** vectors
- Separate **key** and **value**

Attention Layer

Inputs:

Query vectors: \mathbf{Q} (Shape: $N_Q \times D_Q$)

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Computation:

Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

Value vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_Q \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_Q \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_Q \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$

X_1

X_2

X_3

Q_1

Q_2

Q_3

Q_4

Attention Layer

Inputs:

Query vectors: \mathbf{Q} (Shape: $N_Q \times D_Q$)

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Computation:

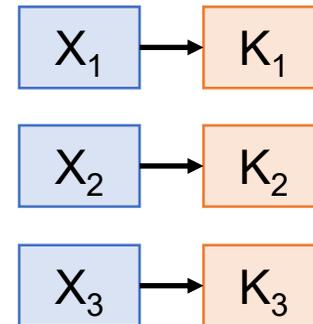
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Attention Layer

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Computation:

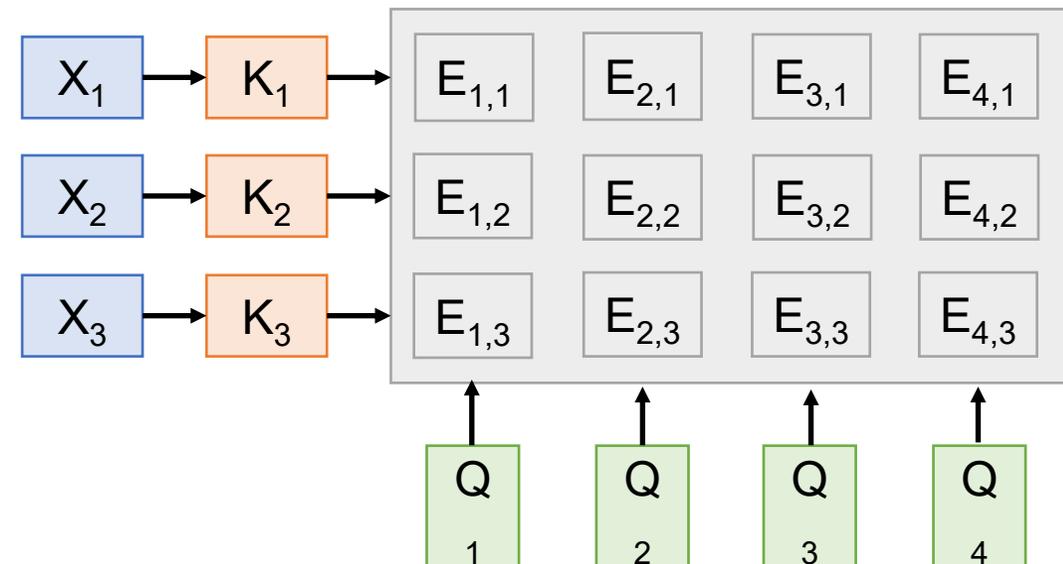
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Computation:

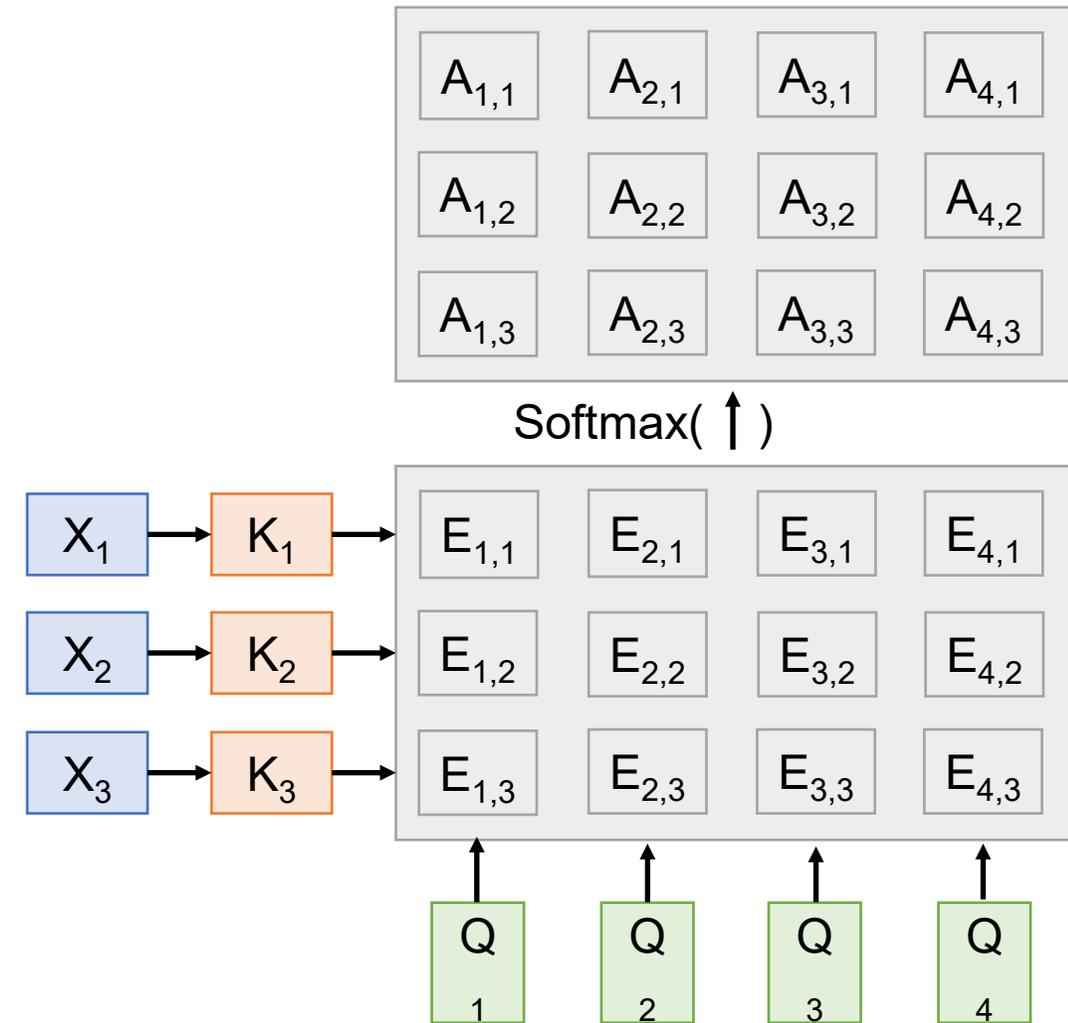
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Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Computation:

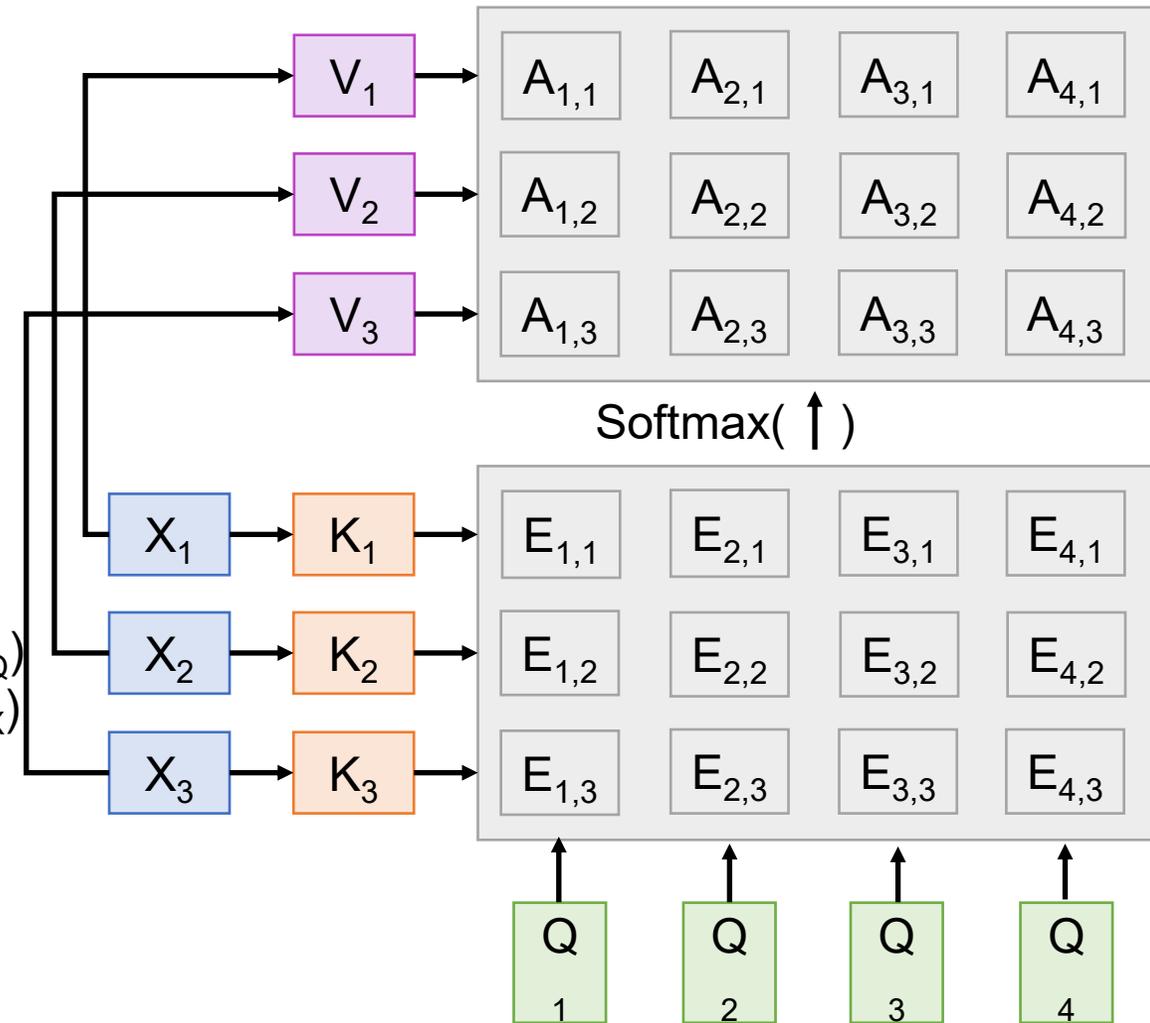
Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

Value vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_Q \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_Q \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_Q \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$



Attention Layer

Inputs:

Query vectors: \mathbf{Q} (Shape: $N_Q \times D_Q$)

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Computation:

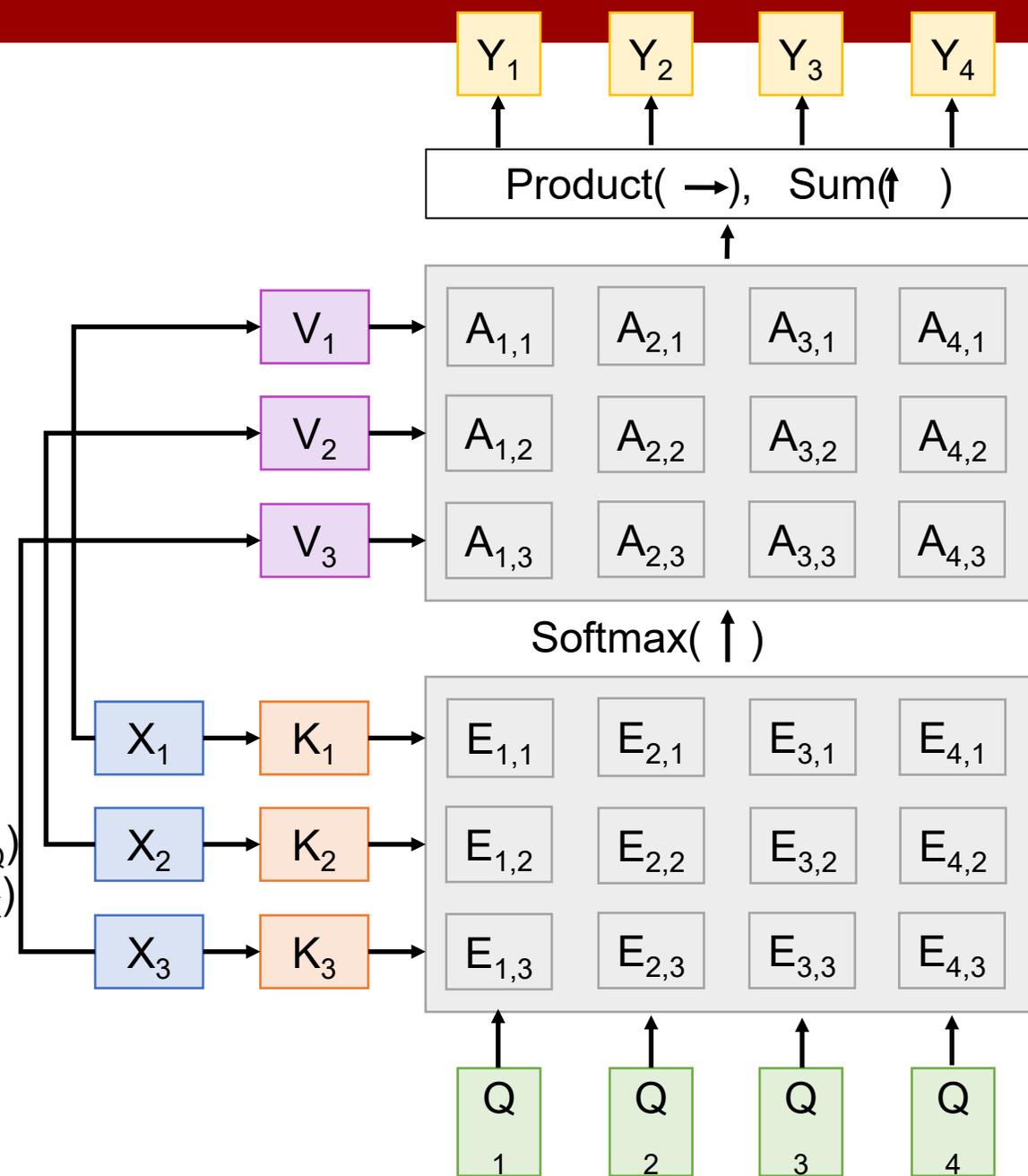
Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

Value vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_Q \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_Q \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_Q \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$



Self-Attention Layer

One **query** per **input vector**

Inputs:

Input vectors: X (Shape: $N_x \times D_x$)

Key matrix: W_K (Shape: $D_x \times D_Q$)

Value matrix: W_V (Shape: $D_x \times D_V$)

Query matrix: W_Q (Shape: $D_x \times D_Q$)

Make the module generic:

Input: Sequence (X)

Output: Sequence (Weighted sum/mixture of inputs)

Computation:

Query vectors: $Q = XW_Q$

Key vectors: $K = XW_K$ (Shape: $N_x \times D_Q$)

Value vectors: $V = XW_V$ (Shape: $N_x \times D_V$)

Similarities: $E = QK^T$ (Shape: $N_x \times N_x$) $E_{i,j} = Q_i \cdot K_j / \text{sqrt}(D_Q)$

Attention weights: $A = \text{softmax}(E, \text{dim}=1)$ (Shape: $N_x \times N_x$)

Output vectors: $Y = AV$ (Shape: $N_x \times D_V$) $Y_i = \sum_j A_{i,j} V_j$

X_1

X_2

X_3

Self-Attention Layer

One **query** per **input vector**

Inputs:

Input vectors: \mathbf{X} (Shape: $N_x \times D_x$)

Key matrix: \mathbf{W}_k (Shape: $D_x \times D_Q$)

Value matrix: \mathbf{W}_v (Shape: $D_x \times D_V$)

Query matrix: \mathbf{W}_Q (Shape: $D_x \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{XW}_Q$

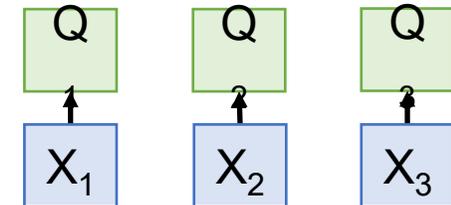
Key vectors: $\mathbf{K} = \mathbf{XW}_k$ (Shape: $N_x \times D_Q$)

Value vectors: $\mathbf{V} = \mathbf{XW}_v$ (Shape: $N_x \times D_V$)

Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_x \times N_x$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_x \times N_x$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_x \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$



Self-Attention Layer

One **query** per **input vector**

Inputs:

Input vectors: \mathbf{X} (Shape: $N_x \times D_x$)

Key matrix: \mathbf{W}_K (Shape: $D_x \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_x \times D_V$)

Query matrix: \mathbf{W}_Q (Shape: $D_x \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{XW}_Q$

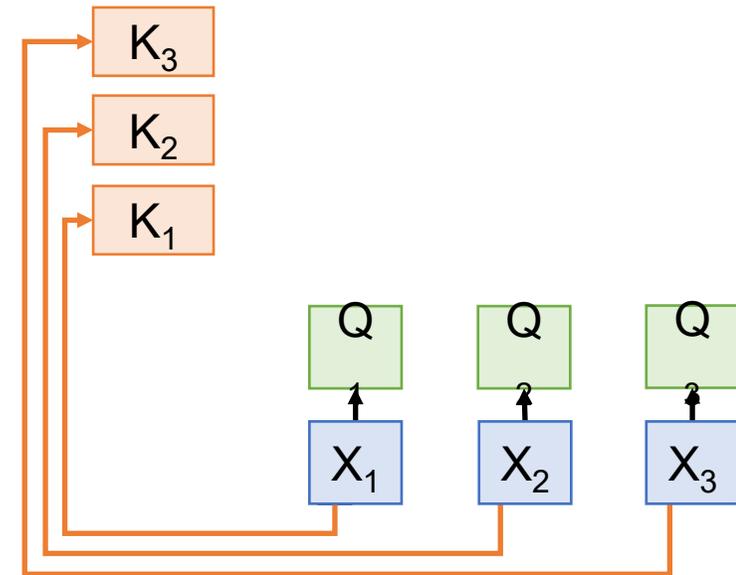
Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_x \times D_Q$)

Value vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_x \times D_V$)

Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_x \times N_x$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_x \times N_x$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_x \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$



Self-Attention Layer

One **query** per **input vector**

Inputs:

Input vectors: \mathbf{X} (Shape: $N_x \times D_x$)

Key matrix: \mathbf{W}_K (Shape: $D_x \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_x \times D_V$)

Query matrix: \mathbf{W}_Q (Shape: $D_x \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{XW}_Q$

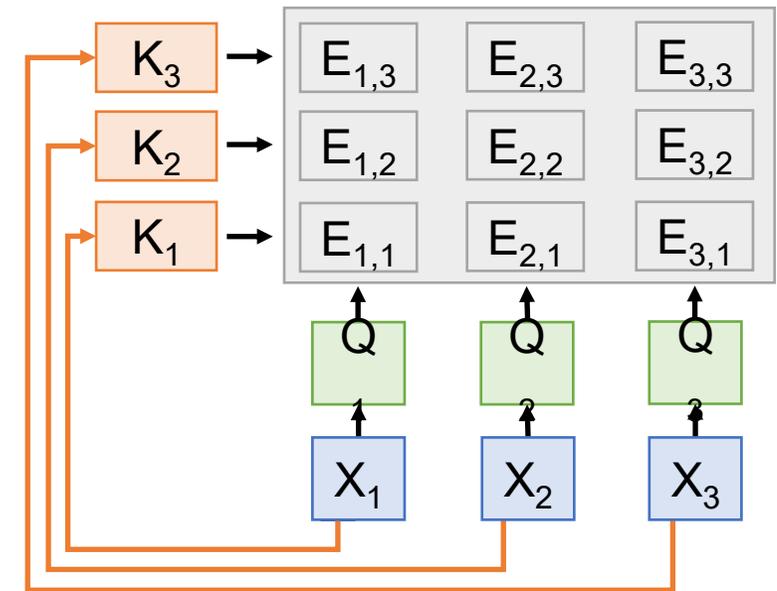
Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_x \times D_Q$)

Value vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_x \times D_V$)

Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_x \times N_x$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_x \times N_x$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_x \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$



Self-Attention Layer

One **query** per **input vector**

Inputs:

Input vectors: \mathbf{X} (Shape: $N_x \times D_x$)

Key matrix: \mathbf{W}_k (Shape: $D_x \times D_Q$)

Value matrix: \mathbf{W}_v (Shape: $D_x \times D_v$)

Query matrix: \mathbf{W}_Q (Shape: $D_x \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{XW}_Q$

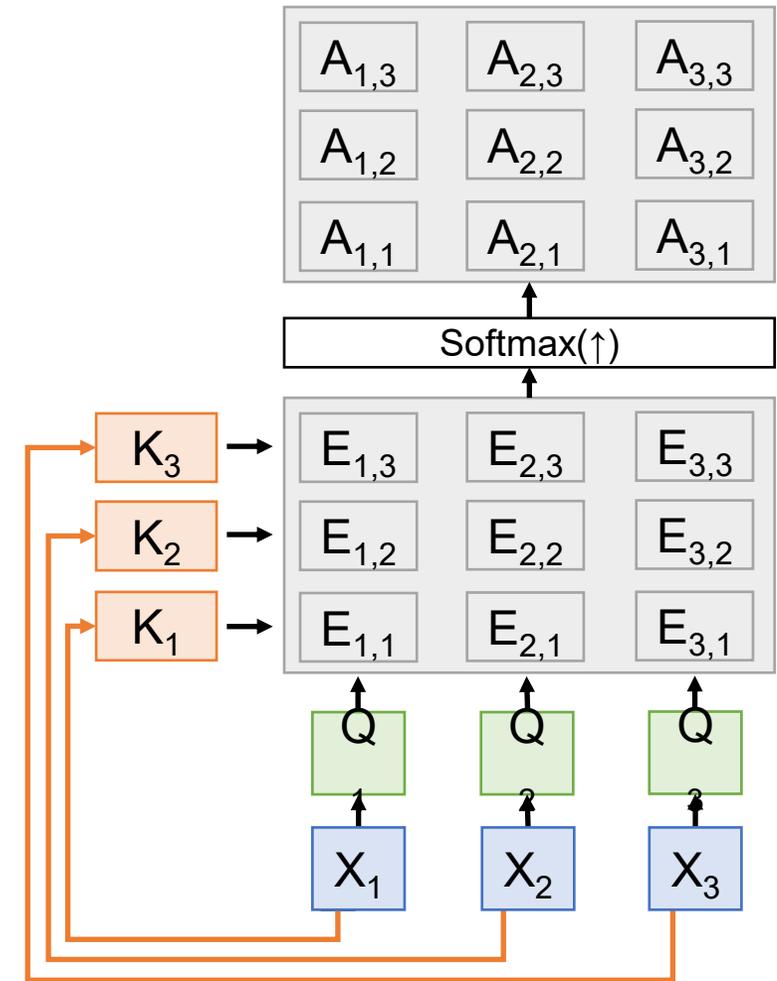
Key vectors: $\mathbf{K} = \mathbf{XW}_k$ (Shape: $N_x \times D_Q$)

Value vectors: $\mathbf{V} = \mathbf{XW}_v$ (Shape: $N_x \times D_v$)

Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_x \times N_x$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_x \times N_x$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_x \times D_v$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$



Self-Attention Layer

One **query** per **input vector**

Inputs:

Input vectors: X (Shape: $N_x \times D_x$)

Key matrix: W_K (Shape: $D_x \times D_Q$)

Value matrix: W_V (Shape: $D_x \times D_V$)

Query matrix: W_Q (Shape: $D_x \times D_Q$)

Computation:

Query vectors: $Q = XW_Q$

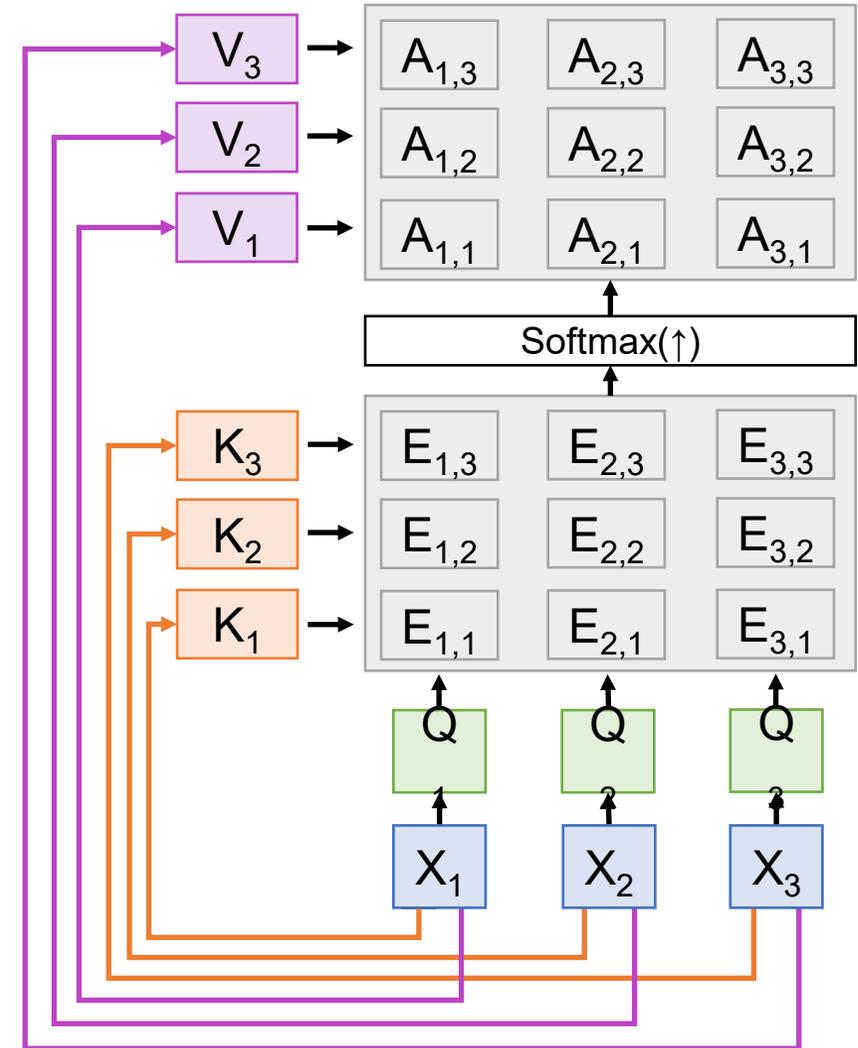
Key vectors: $K = XW_K$ (Shape: $N_x \times D_Q$)

Value vectors: $V = XW_V$ (Shape: $N_x \times D_V$)

Similarities: $E = QK^T$ (Shape: $N_x \times N_x$) $E_{i,j} = Q_i \cdot K_j / \text{sqrt}(D_Q)$

Attention weights: $A = \text{softmax}(E, \text{dim}=1)$ (Shape: $N_x \times N_x$)

Output vectors: $Y = AV$ (Shape: $N_x \times D_V$) $Y_i = \sum_j A_{i,j} V_j$



Self-Attention Layer

One **query** per **input vector**

Inputs:

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Query matrix: \mathbf{W}_Q (Shape: $D_X \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{XW}_Q$

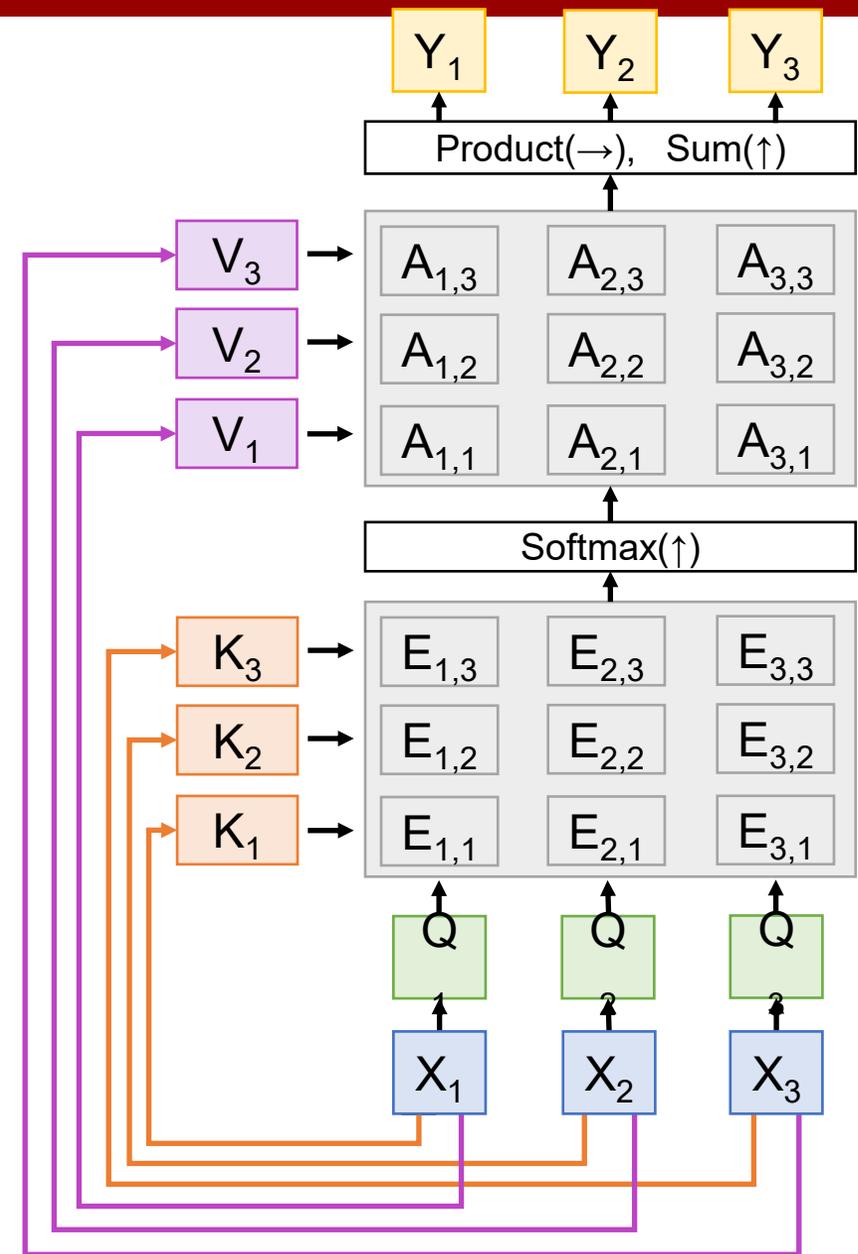
Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

Value vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_X \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_X \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_X \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$



Self-Attention Layer

Inputs:

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Query matrix: \mathbf{W}_Q (Shape: $D_X \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{XW}_Q$

Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

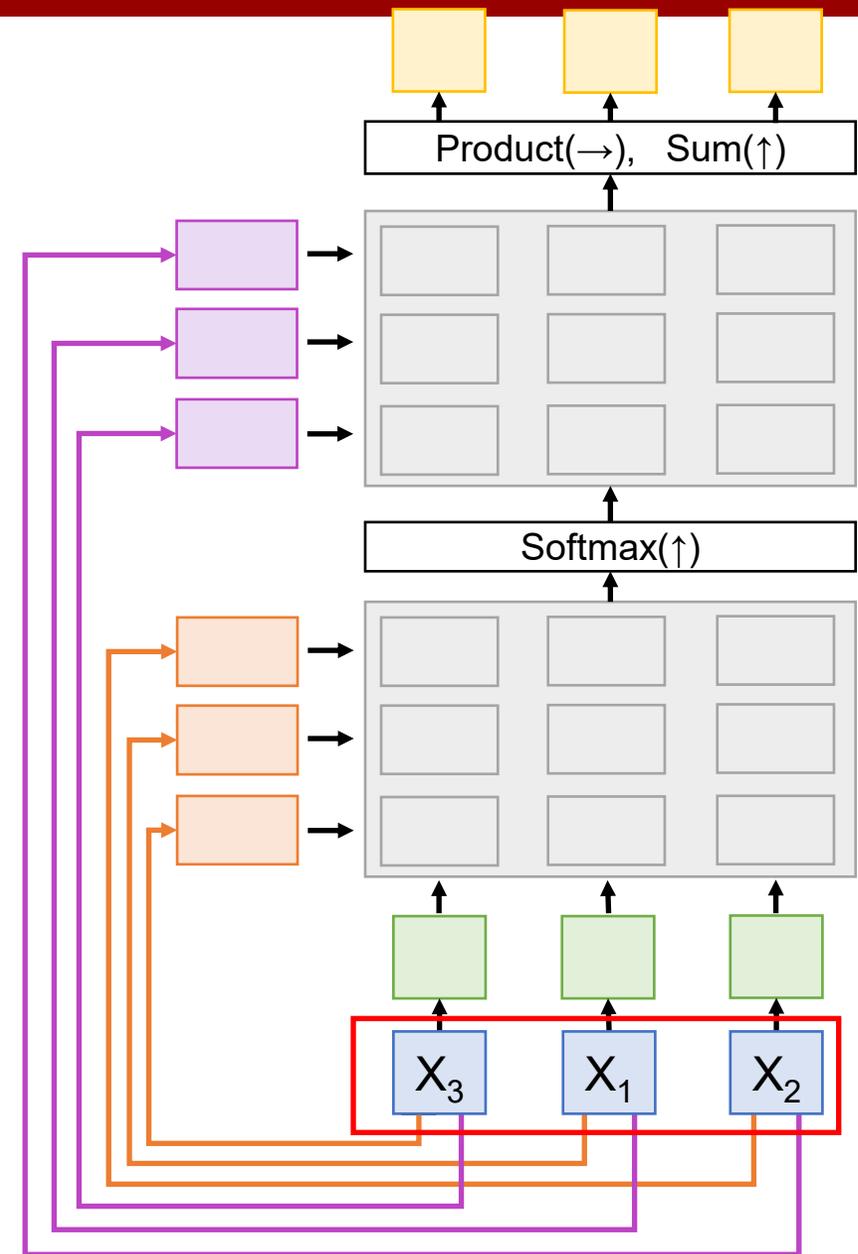
Value vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_X \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_X \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_X \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$

Consider **permuting**
the input vectors:



Self-Attention Layer

Inputs:

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Query matrix: \mathbf{W}_Q (Shape: $D_X \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{XW}_Q$

Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

Value Vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

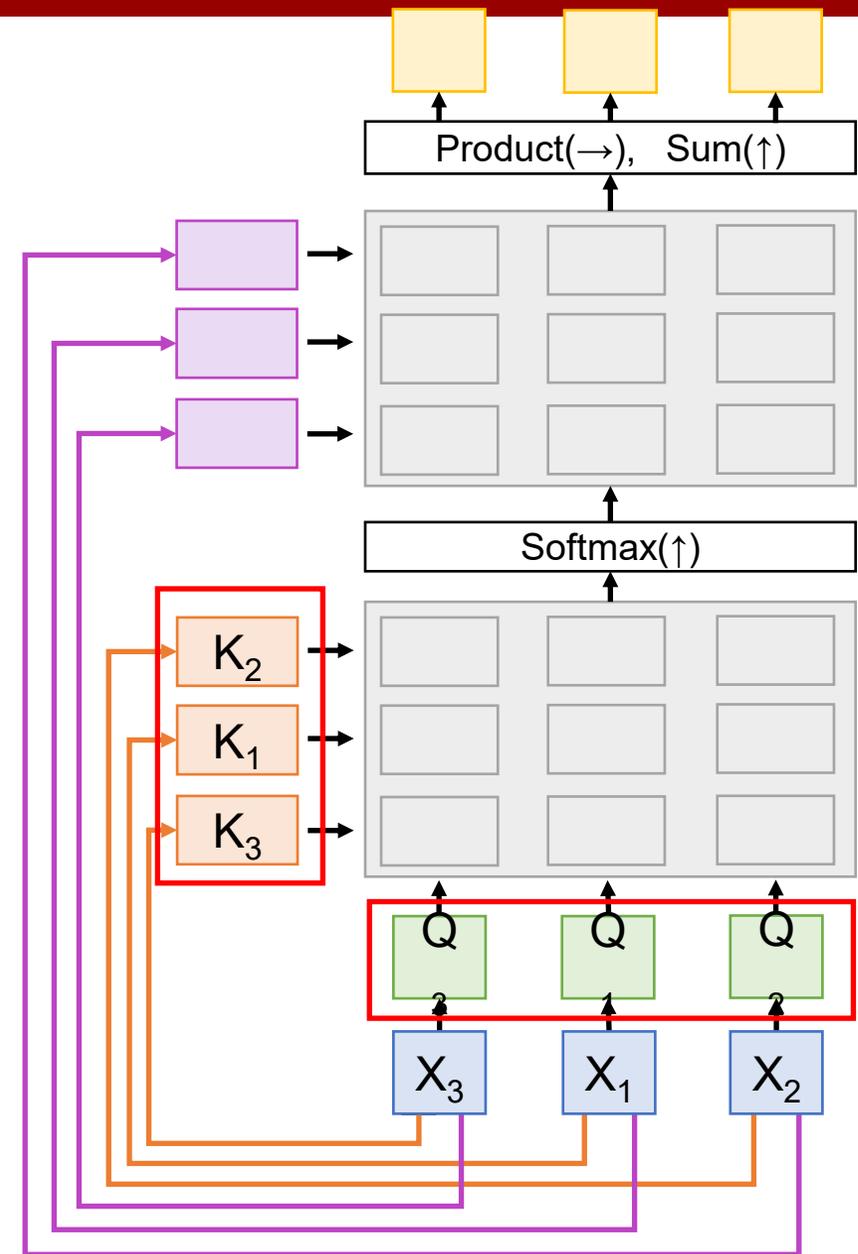
Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_X \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_X \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_X \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$

Consider **permuting**
the input vectors:

Queries and Keys will
be the same, but
permuted



Self-Attention Layer

Inputs:

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Query matrix: \mathbf{W}_Q (Shape: $D_X \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{XW}_Q$

Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

Value vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

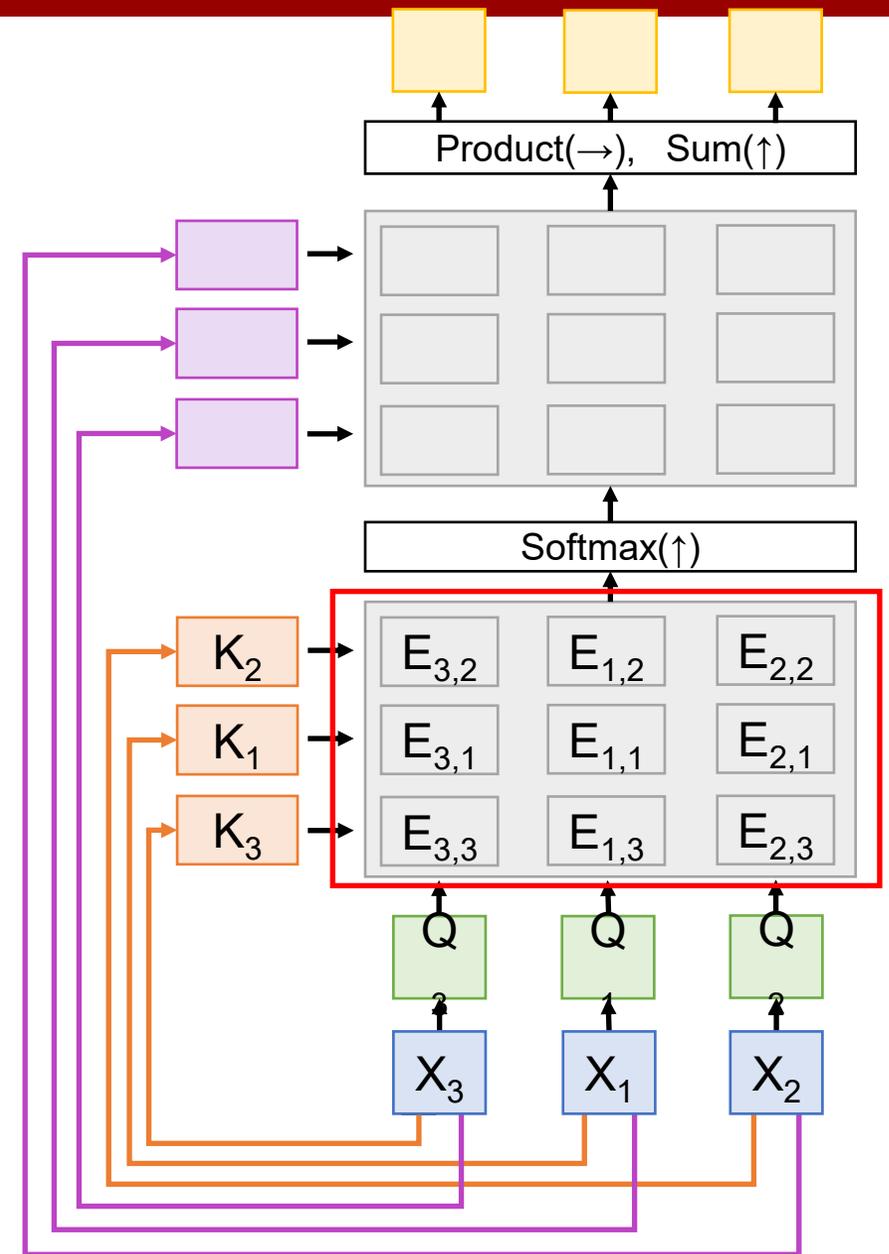
Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_X \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_X \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_X \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$

Consider **permuting**
the input vectors:

Similarities will be the
same, but permuted



Self-Attention Layer

Inputs:

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Query matrix: \mathbf{W}_Q (Shape: $D_X \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{XW}_Q$

Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

Value vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

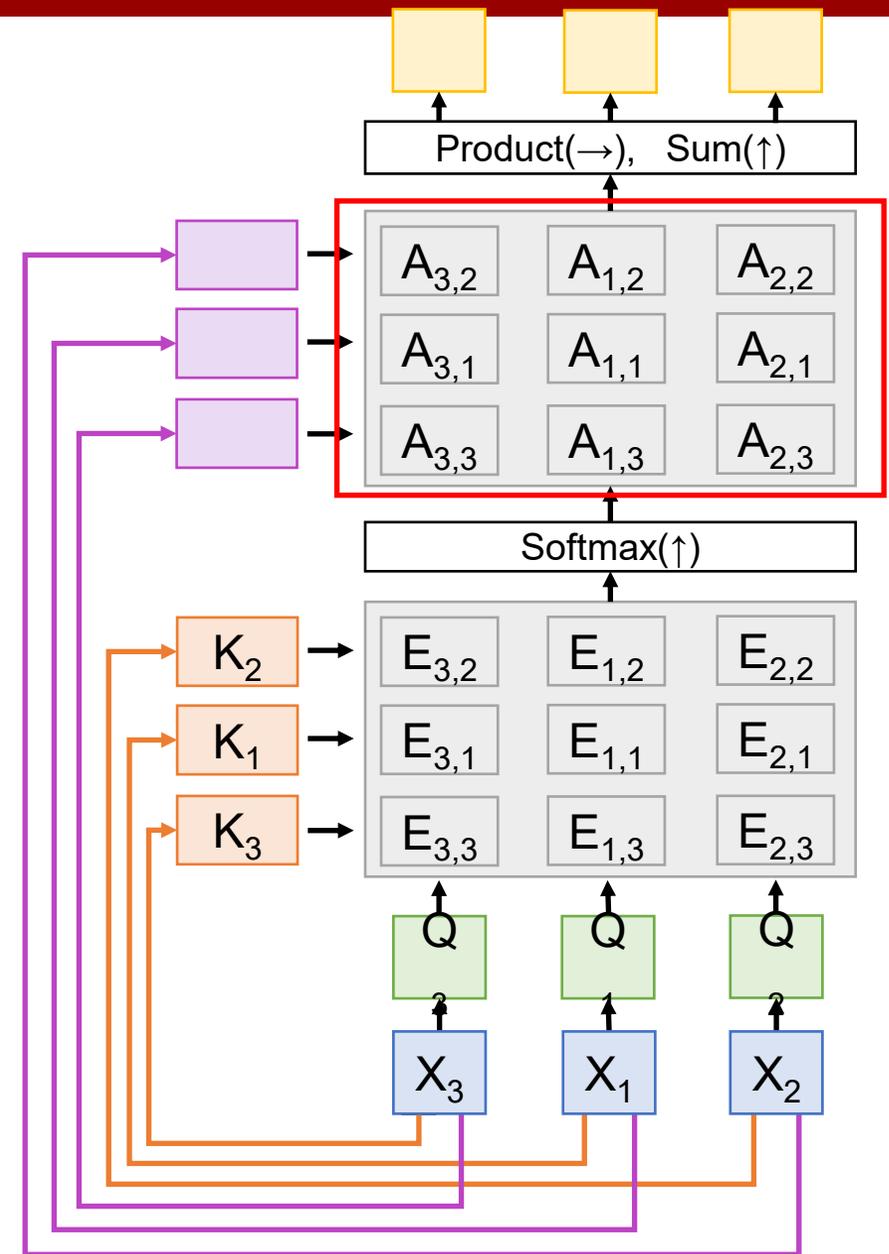
Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_X \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_X \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_X \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$

Consider **permuting**
the input vectors:

Attention weights will
be the same, but
permuted



Self-Attention Layer

Inputs:

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Query matrix: \mathbf{W}_Q (Shape: $D_X \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{XW}_Q$

Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

Value vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

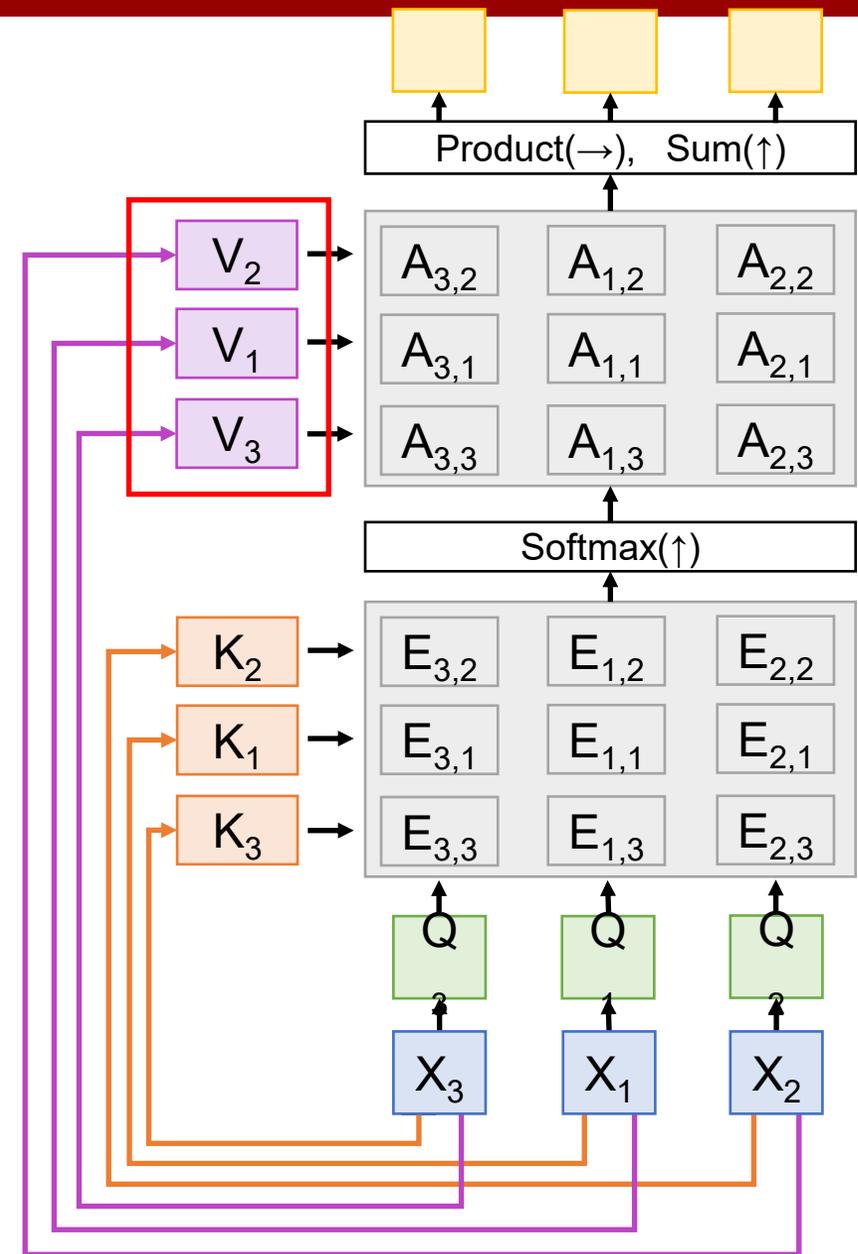
Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_X \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_X \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_X \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$

Consider **permuting**
the input vectors:

Values will be the
same, but permuted



Self-Attention Layer

Inputs:

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Query matrix: \mathbf{W}_Q (Shape: $D_X \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{XW}_Q$

Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

Value vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

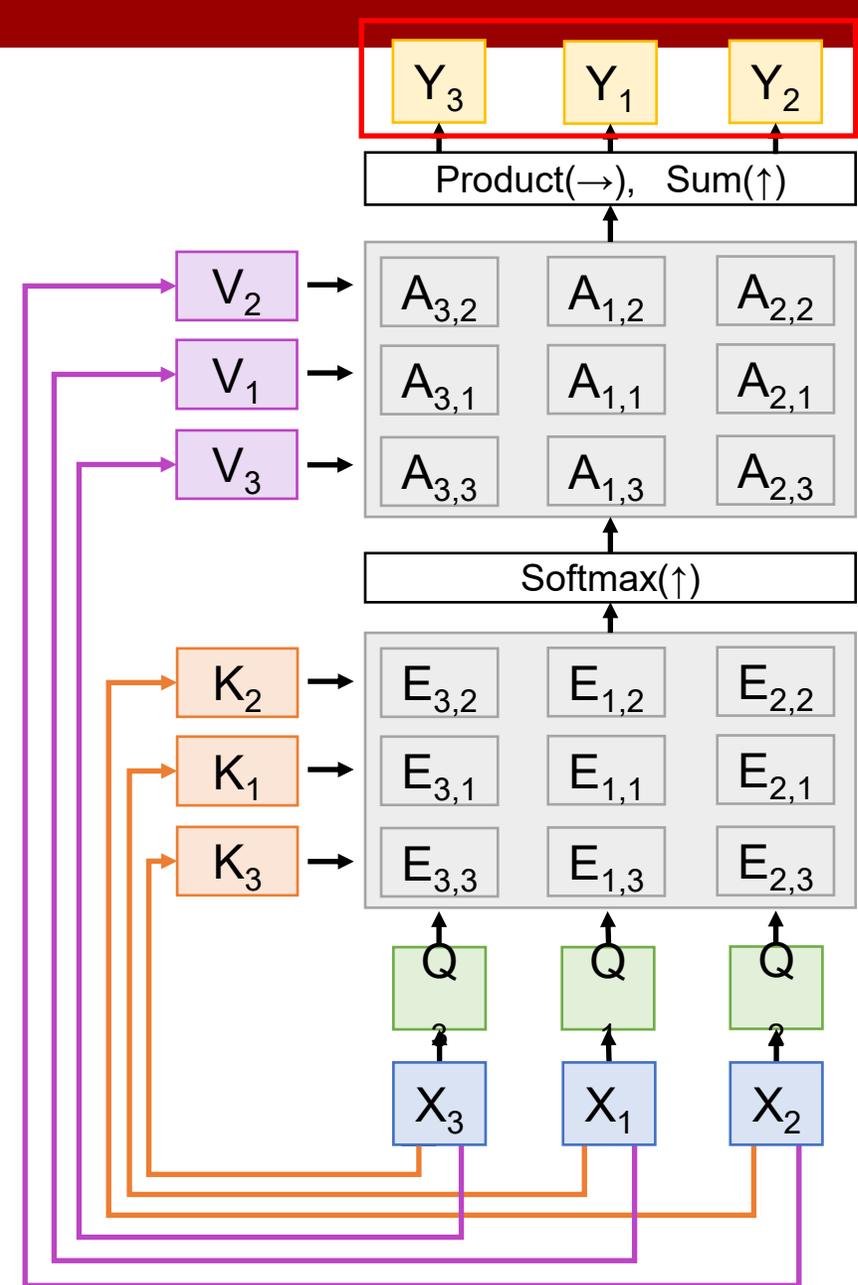
Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_X \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_X \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_X \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$

Consider **permuting**
the input vectors:

Outputs will be the
same, but permuted



Self-Attention Layer

Inputs:

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Query matrix: \mathbf{W}_Q (Shape: $D_X \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{XW}_Q$

Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

Value vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_X \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

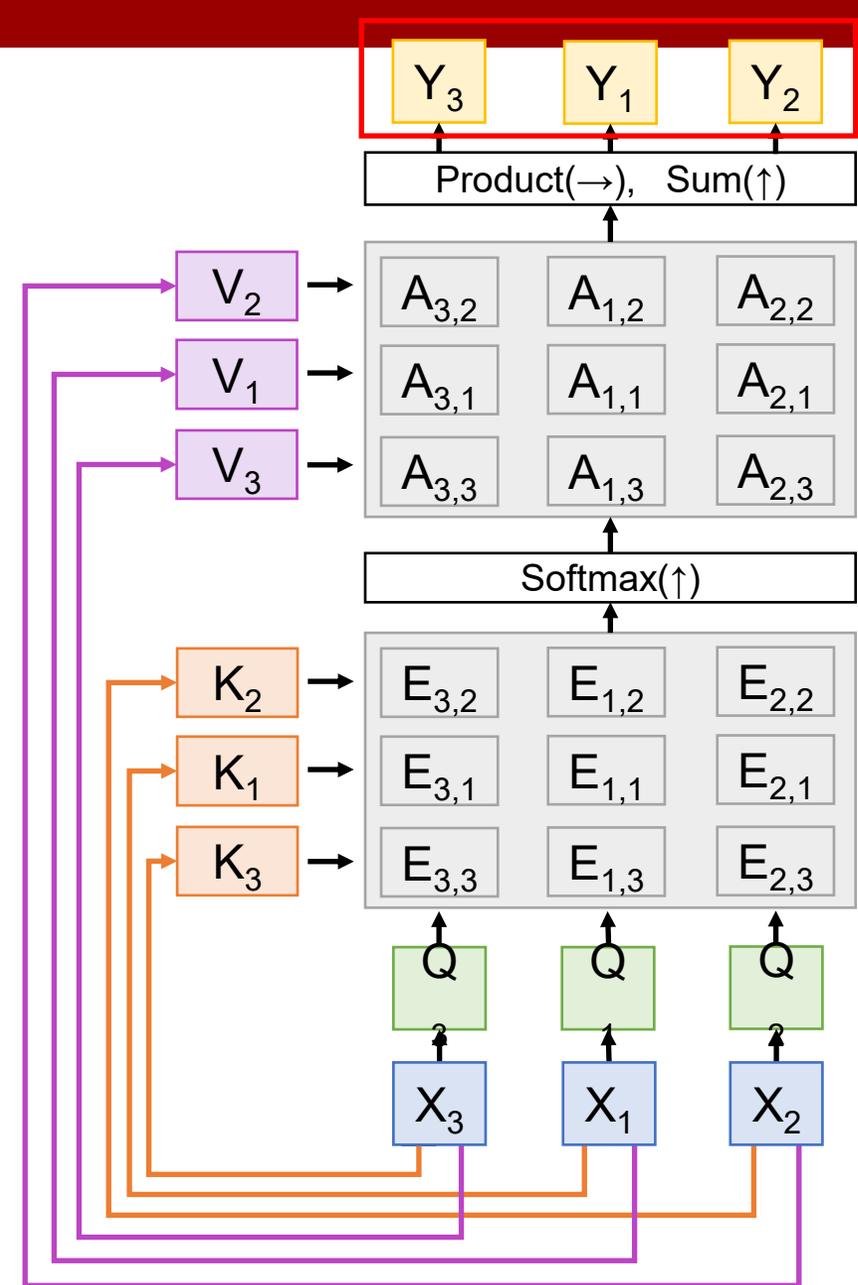
Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_X \times N_X$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_X \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$

Consider **permuting**
the input vectors:

Outputs will be the
same, but permuted

Self-attention layer is
**Permutation
Equivariant**
 $f(s(x)) = s(f(x))$



Self-Attention Layer

Inputs:

Input vectors: X (Shape: $N_X \times D_X$)

Key matrix: W_K (Shape: $D_X \times D_Q$)

Value matrix: W_V (Shape: $D_X \times D_V$)

Query matrix: W_Q (Shape: $D_X \times D_Q$)

Computation:

Query vectors: $Q = XW_Q$

Key vectors: $K = XW_K$ (Shape: $N_X \times D_Q$)

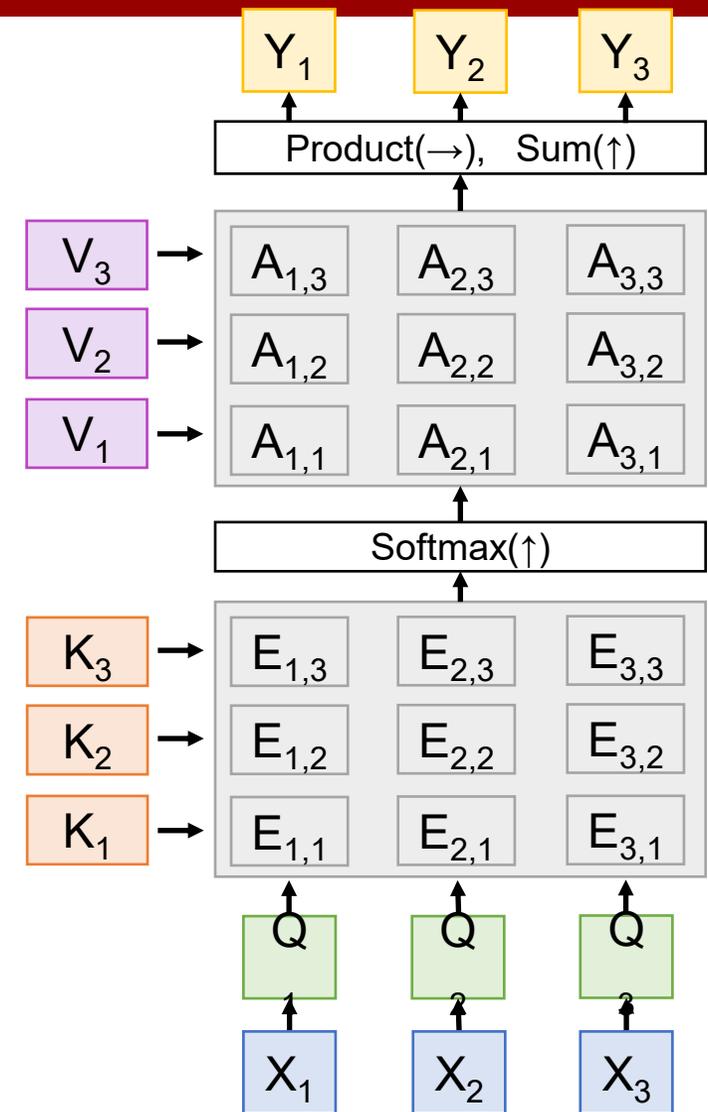
Value vectors: $V = XW_V$ (Shape: $N_X \times D_V$)

Similarities: $E = QK^T$ (Shape: $N_X \times N_X$) $E_{i,j} = Q_i \cdot K_j / \text{sqrt}(D_Q)$

Attention weights: $A = \text{softmax}(E, \text{dim}=1)$ (Shape: $N_X \times N_X$)

Output vectors: $Y = AV$ (Shape: $N_X \times D_V$) $Y_i = \sum_j A_{i,j} V_j$

Self attention doesn't "know" the order of the vectors it is processing!



Self-Attention Layer

Inputs:

Input vectors: X (Shape: $N_x \times D_x$)

Key matrix: W_K (Shape: $D_x \times D_Q$)

Value matrix: W_V (Shape: $D_x \times D_V$)

Query matrix: W_Q (Shape: $D_x \times D_Q$)

Computation:

Query vectors: $Q = XW_Q$

Key vectors: $K = XW_K$ (Shape: $N_x \times D_Q$)

Value vectors: $V = XW_V$ (Shape: $N_x \times D_V$)

Similarities: $E = QK^T$ (Shape: $N_x \times N_x$) $E_{i,j} = Q_i \cdot K_j / \text{sqrt}(D_Q)$

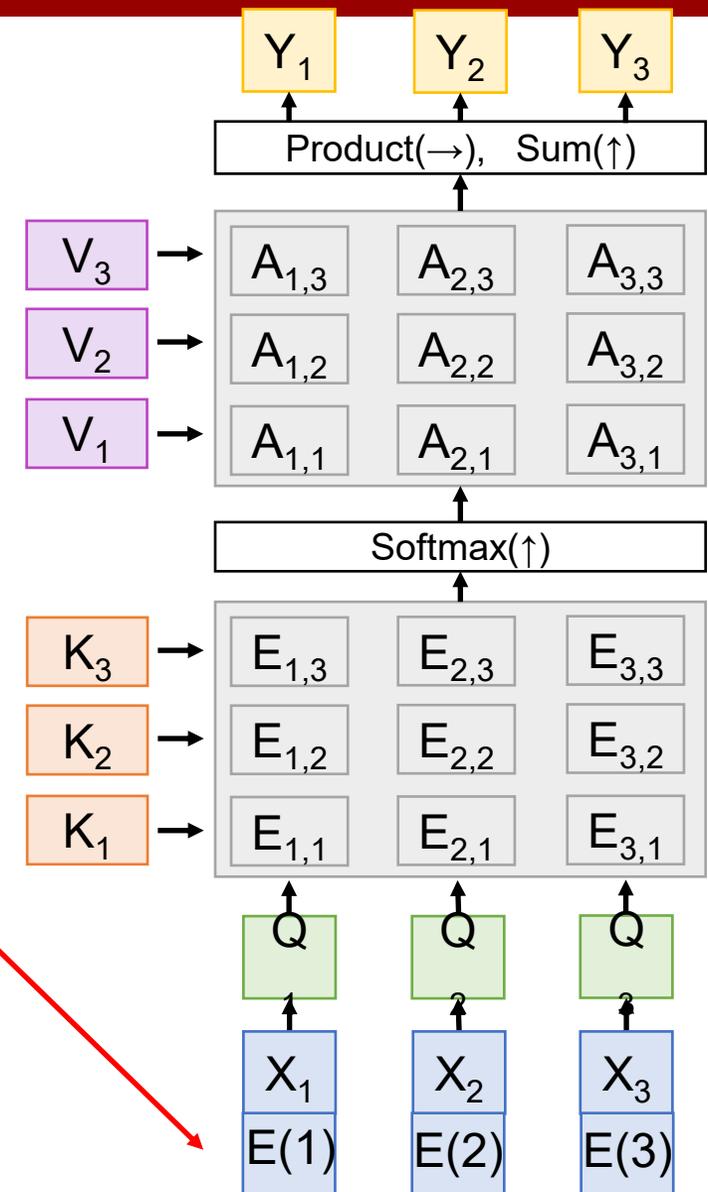
Attention weights: $A = \text{softmax}(E, \text{dim}=1)$ (Shape: $N_x \times N_x$)

Output vectors: $Y = AV$ (Shape: $N_x \times D_V$) $Y_i = \sum_j A_{i,j} V_j$

Self attention doesn't "know" the order of the vectors it is processing!

In order to make processing position-aware, concatenate input with **positional encoding**

E can be learned lookup table, or fixed function



Summary

- We have made a generic sequence-in to sequence-out layer
 - This is what we want for language processing!
 - Each output is a contextualized representation of the corresponding input word
 - Vector for stop word can be treated as representation of entire sentence (e.g. project its output to classifier and add loss)
- Unlike RNNs/LSTMs, it processes all inputs (e.g. entire sentence) **at once**
 - **Highly parallelizable**
 - **-> SCALE! -> Reduction of loss -> Magic**
- Next time: Entire transformer architecture that combines this new layer with other layers/concepts we know about (fully-connected, normalization, residual/skip connections)

Masked Self-Attention Layer

Inputs:

Input vectors: \mathbf{X} (Shape: $N_x \times D_x$)

Key matrix: \mathbf{W}_K (Shape: $D_x \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_x \times D_V$)

Query matrix: \mathbf{W}_Q (Shape: $D_x \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{XW}_Q$

Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_x \times D_Q$)

Value vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_x \times D_V$)

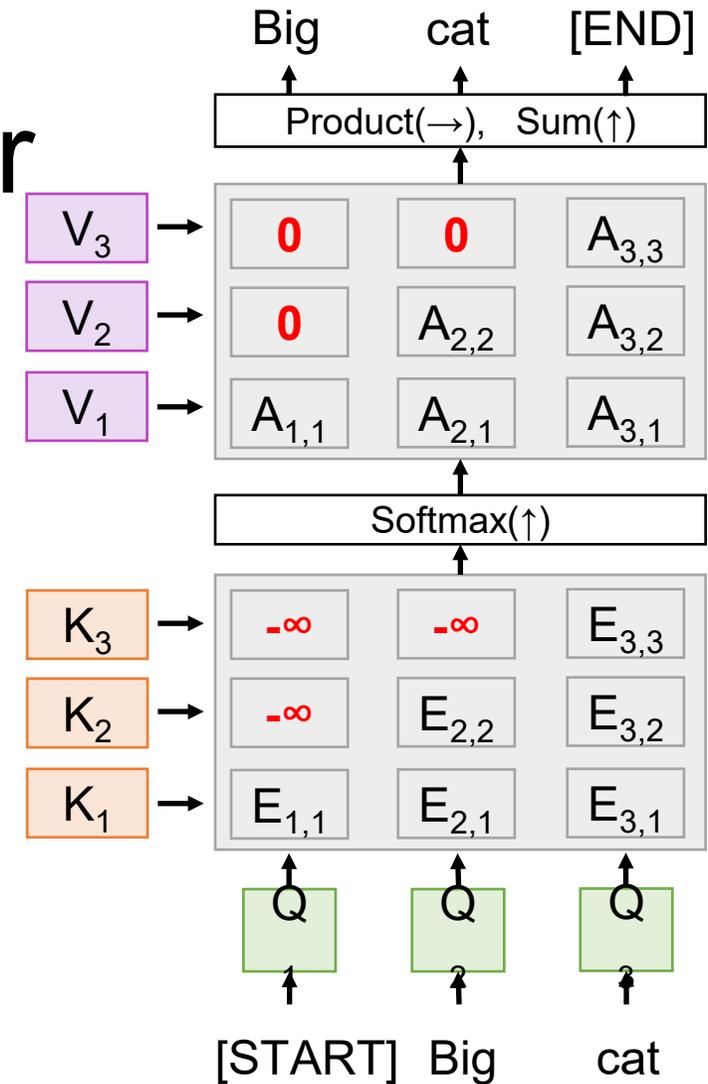
Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_x \times N_x$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

Attention weights: $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$ (Shape: $N_x \times N_x$)

Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_x \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$

Don't let vectors "look ahead" in the sequence

Used for language modeling (predict next word)



Multihead Self-Attention Layer

Inputs:

Input vectors: \mathbf{X} (Shape: $N_X \times D_X$)

Key matrix: \mathbf{W}_K (Shape: $D_X \times D_Q$)

Value matrix: \mathbf{W}_V (Shape: $D_X \times D_V$)

Query matrix: \mathbf{W}_Q (Shape: $D_X \times D_Q$)

Computation:

Query vectors: $\mathbf{Q} = \mathbf{XW}_Q$

Key vectors: $\mathbf{K} = \mathbf{XW}_K$ (Shape: $N_X \times D_Q$)

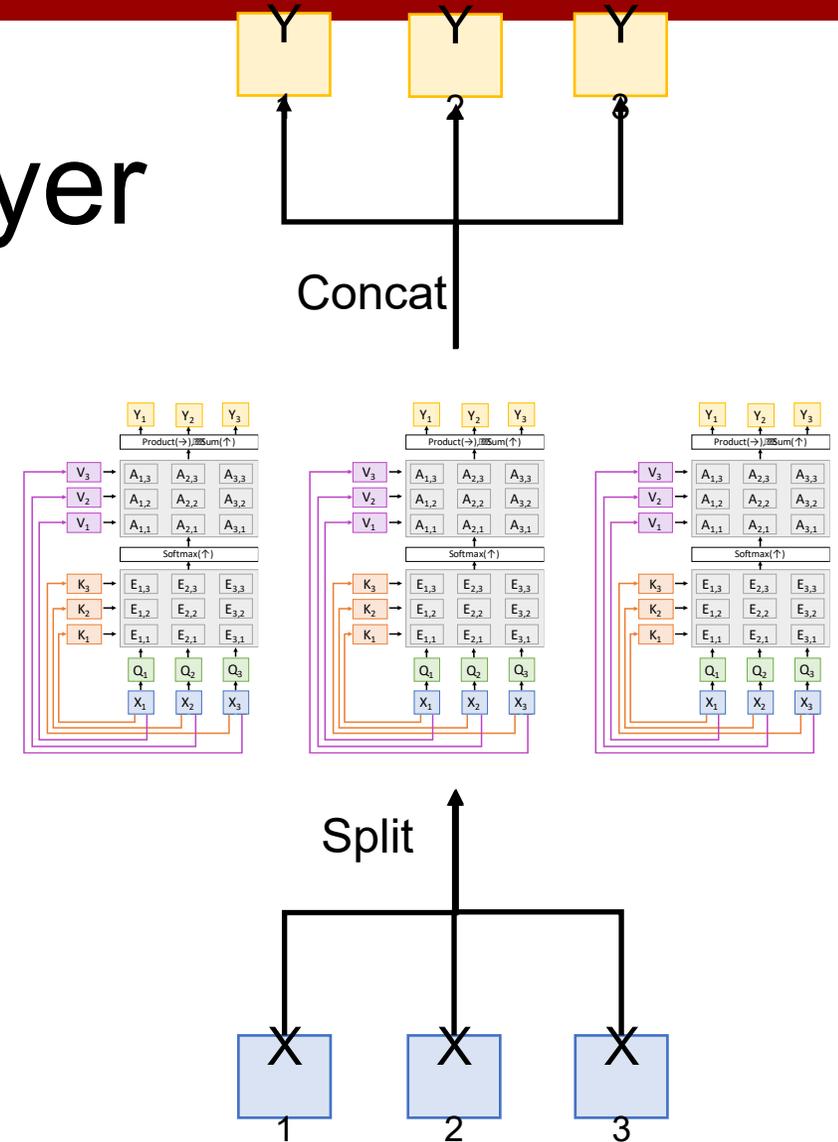
Value vectors: $\mathbf{V} = \mathbf{XW}_V$ (Shape: $N_X \times D_V$)

Similarities: $\mathbf{E} = \mathbf{QK}^T$ (Shape: $N_X \times N_X$) $E_{i,j} = \mathbf{Q}_i \cdot \mathbf{K}_j / \text{sqrt}(D_Q)$

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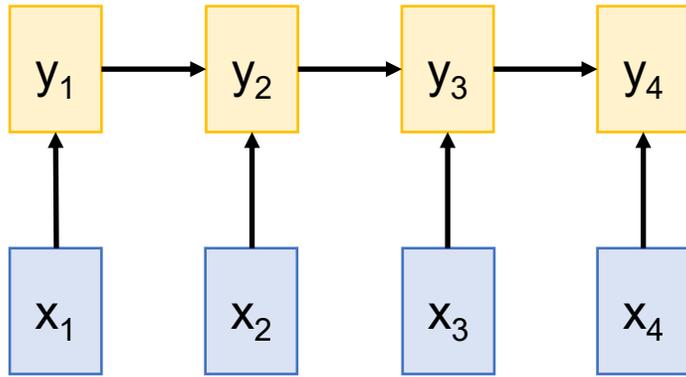
Output vectors: $\mathbf{Y} = \mathbf{AV}$ (Shape: $N_X \times D_V$) $Y_i = \sum_j A_{i,j} \mathbf{V}_j$

Use H independent
“Attention Heads” in
parallel



Three Ways of Processing Sequences

Recurrent Neural Network



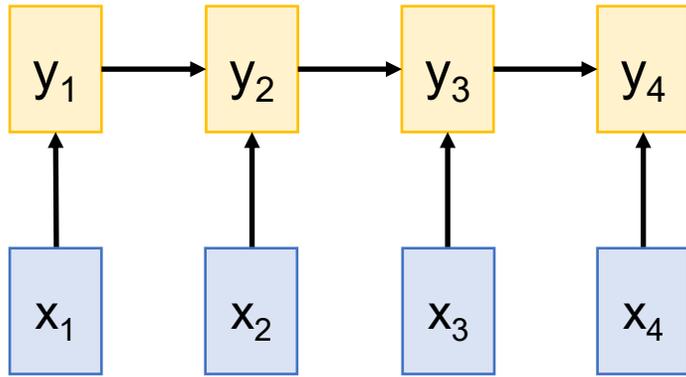
Works on **Ordered Sequences**

(+) **Good at long sequences:**
After one RNN layer, h_T "sees"
the whole sequence

(-) **Not parallelizable: need to
compute hidden states
sequentially**

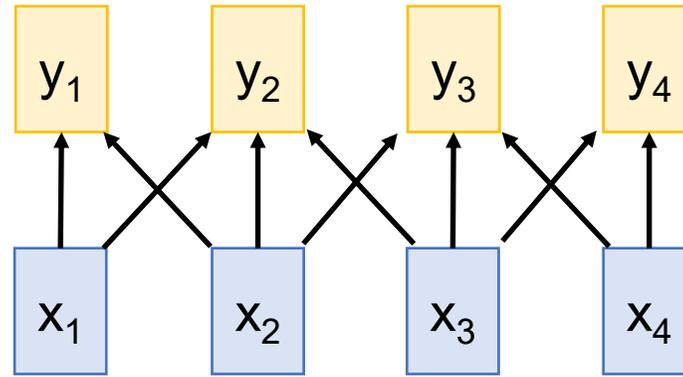
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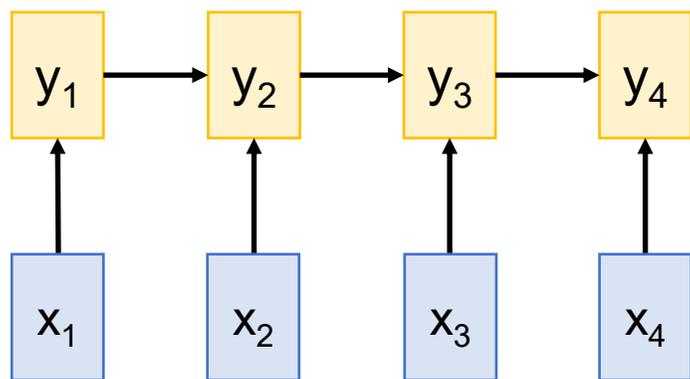
1D Convolution



Works on **Multidimensional
Grids**
(-) **Bad at long sequences:** Need
to stack many conv layers for
outputs to "see" the whole
sequence
(+) **Highly parallel:** Each output
can be computed in parallel

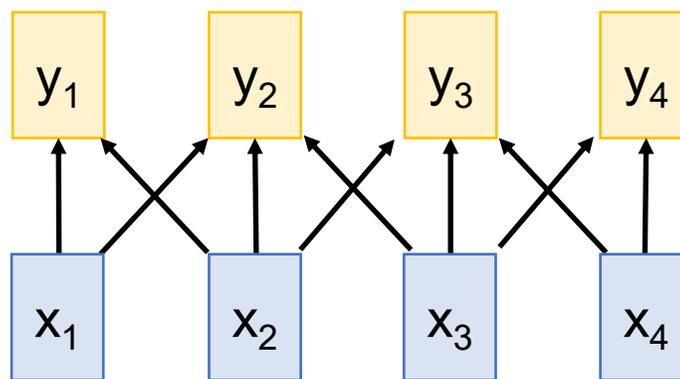
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Recurrent Neural Network



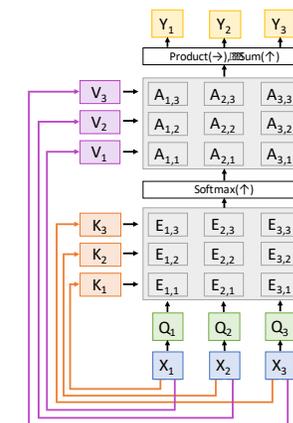
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1D Convolution



Works on **Multidimensional Grids**
(-) **Bad at long sequences:** Need to stack many conv layers for outputs to "see" the whole sequence
(+) **Highly parallel:** Each output can be computed in parallel

Self-Attention



Works on **Sets of Vectors**
(+) **Good at long sequences:** after one self-attention layer, each output "sees" all inputs!
(+) **Highly parallel:** Each output can be computed in parallel
(-) **Very memory intensive**

Three Ways of Processing Sequences

Recurrent Neural Network

1D Convolution

Self-Attention

Attention is all you need

Vaswani et al, NeurIPS 2017

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